Multi-objective Geometric Programming Problem Based on Neutrosophic Geometric Programming Technique

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Abstract

The chapter aims to give computational algorithm to solve a multi-objective non-linear programming problem using Neutrosophic geometric programming technique. As the Neutrosophic optimization technique utilizes degrees of truth-membership, falsity-membership and indeterminacy-membership functions, we made a study of correspondence among those membership functions to see its impact on optimization. Also, we made a comparative study of optimal solution between intuitionistic fuzzy geometric programming and Neutrosophic geometric programming technique. The developed algorithm has been illustrated by a numerical example. Finally, an application of proposed Neutrosophic geometric programming technique on gravel box design problem is presented.

Keywords

Neutrosophic set, Single valued Neutrosophic set, Multi-objective non-linear programming, Neutrosophic geometric programming.

1 Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. Since the fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy sets use one real value $\mu_A(x) \in [0, 1]$ to represents the truth membership function of a fuzzy set a defined on universe X. In some applications, we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov [3], [5] introduced the intuitionistic fuzzy sets which is a generalisation of fuzzy sets. The intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information. In IFS, sum of membership-degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems. Hence further generalization of fuzzy set as well as intuitionistic fuzzy sets are required. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophy was introduced by Florentin Smarandache in 1995 [4] which is actually generalization of different types of FS and IFS. The term "neutrosophy" means knowledge of neutral thought. This neutral concept makes the different between NS and other sets like FS, IFS. Modeling of most of real life problems involving optimization process turns out to be a multi-objective programming problem in a natural way. In this field, a paper named Multi-objective geometric programming problem with weighted-sum method by A.K. Ojha, A.K. Das has been published in the journal of computing 2010 [12]. In 1971 L.D. Paschal and A. Ben. Israel [16] developed a vector valued criteria in geometric programming. In 1978 a paper Fuzzy linear programming with several objective functions has been published by H.J Zimmermann [15]. In 1992 M.P. Bishal [13] and in 1990 R.k. Verma [14] has studied fuzzy programming technique to solve multi-objective geometric programming problems. In 2007 B. Jana and T.K. Roy [9] has studied multi-objective intuitionistic fuzzy linear programming problem and its application in Transportation model and in 2009 G.S. Mahapatra and T.K. Roy [10] developed multi-objective intuitionistic fuzzy mathematical programming problem and its application in Reliability optimization model. In this present study, a new approach of Neutrosophic Optimization (NO) is proposed. A multi-objective non-linear programming problem is solved by geometric programming technique.

2 Some Preliminaries

2.1 Definition -1 (Fuzzy set) [1]

Let X is a fixed set. A fuzzy set A of X is an object having the form $\tilde{A} = \{(x, \mu_A(x)), x \in X\}$ where the function $\mu_A(x)$: X → [0, 1] defines the truth membership of the element x ∈ X to the set A.

2.2 Definition-2 (Intuitionistic fuzzy set) [3]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, \mu_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1] and $\nu_A(x)$: X→ [0, 1] define the Truth-membership and Falsity-membership respectively , for every element of x∈ X , $0 \le \mu_A(x) + \nu_A(x) \le 1$.

2.3 Definition-3 (Neutrosophic set) [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a Truth-membership function $\mu_A(x)$, an indeterminacy-membership function $\sigma_A(x)$ and a falsity-membership function $\nu_A(x)$ and having the form $\tilde{A}^n = \{< X, \mu_A(x),\ \sigma_A(x),\ \nu_A(x) > / x \in X\}$. $\mu_A(x),\ \sigma_A(x)\ and\ \nu_A(x)$ are real standard or non-standard subsets of
$]0^-, 1^+[$. that is

$$\mu_A(x) : X \to]0^-, 1^+[$$
$$\sigma_A(x) : X \to]0^-, 1^+[$$
$$\nu_A(x) : X \to]0^-, 1^+[$$

There is no restriction on the sum of $\mu_A(x),\ \sigma_A(x)\ and\ \nu_A(x)$, so
$0^- \le \sup \mu_A(x) + \sup \sigma_A(x) + \sup \nu_A(x) \le 3^+$

2.4 Definition-3 (Single valued Neutrosophic sets) [6]

Let X be a universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< X, \mu_A(x),\ \sigma_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1], $\sigma_A(x)$: X→[0, 1] and $\nu_A(x)$: X→ [0, 1] with $0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3$ for all x ∈ X.

Example1: Assume that X = [x_1, x_2, x_3]. X_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, $x_2$and x_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of “good service”, a degree of indeterminacy and a degree of “poor service”. A is a single valued neutrosophic set of X defined by

A = $\langle 0.3,0.4,0.5\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.7,0.2,0.2\rangle/x_3$

2.5 Definition- 4(Complement): [6]

The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by

$$\mu_{c(A)}(x) = \nu_A(x)$$
$$\sigma_{c(A)}(x) = 1 - \sigma_A(x)$$
$$\nu_{c(A)}(x) = \mu_A(x) \qquad \text{for all x in X.}$$

Example 2: let A be a single valued neutrosophic set defined in example 1. Then, c (A) = ⟨0.5,0.6,0.3⟩/x_1 + ⟨0.3,0.8,0.5⟩/x_2 +⟨0.2,0.8,0.7⟩/x_3.

2.6 Definition 5(Union):[6]

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_c(x)$ = max $(\mu_A(x), \mu_B(x))$
$\sigma_c(x)$ =max $(\sigma_A(x), \sigma_B(x))$
$\nu_c(x) = \min(\nu_A(x), \nu_B(x))$ for all x in X

Example 3: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∪ B = ⟨0.6,0.4,0.2⟩/x_1 + ⟨0.5,0.2,0.3⟩/x_2 + ⟨0.7,0.2,0.2⟩/x_3.

2.7 Definition 6(Intersection):[6]

The Intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∩ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_c(x)$ = min $(\mu_A(x), \mu_B(x))$
$\sigma_c(x)$ = min $(\sigma_A(x), \sigma_B(x))$
$\nu_c(x)$ = max $(\nu_A(x), \nu_B(x))$ for all x in X

Example 4: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∩ B = ⟨0.3,0.1,0.5⟩/x_1 + ⟨0.3,0.2,0.6⟩/x_2 + ⟨0.4,0.1,0.5⟩/x_3.

Here, we notice that by the definition of complement, union and intersection of single valued neutrosophic sets, single valued neutrosophic sets satisfy the most properties of classic set, fuzzy set and intuitionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude [17-21].

3 Multi-objective Geometric Programming Problem

A multi-objective geometric programming problem can be defined as

Find $x=(x_1,x_2,\ldots\ldots\ldots\ldots x_n)^T$,so as to (1)

Min $f_{k0}(x) = \sum_{t=1}^{T_{k0}} C_{k0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}$ k=1,2,…………,p

such that $f_i(x) = \sum_{t=1}^{T_i} c_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ i=1 ,2, ,m

$x_j > 0$ j= 1,2, n

where $c_{k0t} > 0$ for all k and t . a_{itj} ,a_{k0tj} are all real ,for all i, k, t, j.

4 Computational Algorithm

Step 1: Solve the MONLP problem (1) as a single objective non-linear problem p times for each problem by taking one of the objectives at a time and ignoring the others. These solutions are known as ideal solutions. Let x^k be the respective optimal solution for the kth different objective and evaluate each objective value for all these kth optimal solution.

Step 2: From the result of step-1, determine the corresponding values for every objective for each derived solution. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows.

$$\begin{bmatrix} f_1{}^*(x^1) & f_2(x^1) \ldots\ldots\ldots f_p(x^1) \\ f_1(x^2) & f_2{}^*(x^2) \ldots\ldots\ldots f_p(x^2) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. & \\ f_1(x^p) & f_2(x^p) \ldots\ldots\ldots f_p{}^*(x^p) \end{bmatrix}$$

Step 3. For each objective $f_k(x)$, find lower bound $L_k{}^\mu$ and the upper bound $U_k{}^\mu$.

$U_k{}^\mu$ = max $\{f_k(x^{r^*})\}$ and $L_k{}^\mu$ = min $\{f_k(x^{r^*})\}$ where $1 \le r \le k$

For truth membership of objectives.

Step 4. We represent upper and lower bounds for indeterminacy and falsity membership of objectives as follows:

$U_k{}^\nu = U_k{}^\mu$ and $L_k{}^\nu = L_k{}^\mu + t\,(U_k{}^\mu - L_k{}^\mu)$

$L_k{}^\sigma = L_k{}^\mu$ and $U_k{}^\sigma = L_k{}^\mu + s\,(U_k{}^\mu - L_k{}^\mu)$

Here t and s are to predetermined real number in (0, 1).

Step 5. Define Truth-membership, Indeterminacy-membership, Falsity-membership functions as follows:

$$\mu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \le L_k{}^\mu \\ \frac{U_k{}^\mu - f_k(x)}{U_k{}^\mu - L_k{}^\mu} & if\ L_k{}^\mu \le f_k(x) \le U_k{}^\mu \\ 0 & if\ f_k(x) \ge U_k{}^\mu \end{cases},$$

$\nu_k(f_{k0}(x)) = 1 - \frac{1}{1-t}\mu_k(f_{k0}(x))$ and $\sigma_k(f_{k0}(x)) = \frac{1}{s}\mu_k(f_{k0}(x)) - \frac{1-s}{s}$

for k= 1, 2, ………,p.

It is obvious that

$$\sigma_k(f_k(x)) = \begin{cases} 1 & if \ f_k(x) \le L_k{}^{\sigma} \\ \frac{U_k{}^{\sigma} - f_k(x)}{U_k{}^{\sigma} - L_k{}^{\sigma}} & if \ L_k{}^{\sigma} \le f_k(x) \le U_k{}^{\sigma} \\ 0 & if \ f_k(x) \ge U_k{}^{\sigma} \end{cases},$$

$$\nu_k(f_k(x)) = \begin{cases} 0 & if \ f_k(x) \le L_k{}^{\nu} \\ \frac{f_k(x) - L_k{}^{\nu}}{U_k{}^{\nu} - L_k{}^{\nu}} & if \ L_k{}^{\nu} \le f_k(x) \le U_k{}^{\nu} \\ 1 & if \ f_k(x) \ge U_k{}^{\nu} \end{cases}$$

and $0 \le \mu_k(f_{k0}(x)) + \nu_k(f_{k0}(x)) + \sigma_k(f_k(x)) \le 3$
for k= 1, 2, …………,p.

Step 7. Now a Neutrosophic geometric programming technique for multi-objective non-linear programming problem with the linear Truth-membership, Falsity-membership and Indeterminacy functions can be written as

Maximize $(\mu_1(f_{10}(x)), \mu_2(f_{20}(x)), \ldots\ldots, \mu_p(f_{p0}(x)))$ (2)

Minimize $(\nu_1(f_{10}(x))\ \nu_2(f_{20}(x)) \ldots\ldots, \nu_p(f_{p0}(x)))$

Maximize $(\sigma_1(f_{10}(x)), \sigma_2(f_{20}(x)), \ldots\ldots, \sigma_p(f_{p0}(x)))$

Subject to $f_i(x) = \sum_{t=1}^{T_i} c_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ for i=1, 2,…………m

$x_j > 0$, j= 1,2,…………n.

Using weighted sum method, the multi-objective non-linear programming problem (2) reduces to

$$\text{Min } V_{MA}(x) = \sum_{k=1}^{p} w_k(\nu_k(f_{k0}(x)) - \mu_k(f_{k0}(x)) - \sigma_k(f_{k0}(x))) \quad (3)$$

$$\text{Min } V_{MA}(x) = \left(1 + \frac{1}{1-t} + \frac{1}{s}\right) \sum_{k=1}^{p} w_k \frac{\sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j{}^{a_{k0tj}}}{U_k{}^{\mu} - L_k{}^{\mu}} - \left\{\left(\left(1 + \frac{1}{1-t} + \frac{1}{s}\right) \sum_{k=1}^{p} w_k \frac{U_k{}^{\mu}}{U_k{}^{\mu} - L_k{}^{\mu}} \right) - \frac{1}{s}\right\}$$

Subject to $f_i(x) = \sum_{t=1}^{T_i} c_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ i=1, 2,…………m

$x_j > 0$, j= 1,2,…………n.

Excluding the constant term, the above (3) reduces to the following geometric programming problem

$$\text{Min } V_{MA1}(x) = \left(1 + \frac{1}{1-t} + \frac{1}{s}\right) \sum_{k=1}^{p} w_k \frac{\sum_{t=1}^{T_{k0}} c_{k0t} \prod_{j=1}^{n} x_j{}^{a_{k0tj}}}{U_k{}^{\mu} - L_k{}^{\mu}} \quad (4)$$

Such that $f_i(x) = \sum_{t=1}^{T_i} c_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \le 1$ i=1, 2,……...m;

$x_j > 0$, j=1,2,………n.

Here t, s ϵ (0, 1) are pre-determined real numbers.

where $V_{MA}(f_{k0}(x))$= V_{MA1} $(f_{k0}(x))$-$\left\{ (1+\frac{1}{1-t}+\frac{1}{s}) \left(\sum_{k=1}^{p} w_k \frac{{U_k}^{\mu}}{{U_k}^{\mu}-{L_k}^{\mu}} \right) - \frac{1}{s} \right\}$.

Here (4) is a posynomial geometric programming problem with

DD = $\sum_{k=1}^{p} T_{k0} + \sum_{i=1}^{m} T_i - n - 1$.

It can be solved by usual geometric programming technique.

Definition: Neutrosophic Pareto (or NS Pareto) optimal solution

A decision variable x^* Є X is said to be a NS Pareto optimal solution to the Neutrosophic GPP (2) if there does not exit another x Є X such that $\mu_k(f_{k0}(x)) \leq \mu_k(f_{k0}(x^*))$, $\nu_k(f_{k0}(x)) \geq \nu_k(f_{k0}(x^*))$ and $\sigma_k(f_{k0}(x)) \leq \sigma_k(f_{k0}(x^*))$ for all k=1,2,……..p. and $\mu_j(f_{j0}(x)) \neq \mu_j\left(f_{j0}(x^*)\right)$, $\nu_j\left(f_{j0}(x)\right) \neq \nu_j\left(f_{j0}(x^*)\right)$ and $\sigma_j(f_{j0}(x)) \neq \sigma_j\left(f_{j0}(x^*)\right)$ for at least one j, j = 1,2,……..p.

Some basic theorems on M-N Pareto optimal solutions are introduced below.

Theorem 1 The solution of (2) based on weighted sum method Neutrosophic GP problem (3) is weakly NS Pareto optimal.

Proof. Let x^* ϵ X be a solution of the Neutrosophic GP problem. Let us suppose that it is not weakly M-N Pareto optimal. In this case there exit another x ϵ X such that $\mu_k(f_{k0}(x)) < \mu_k(f_{k0}(x^*)), \nu_k(f_{k0}(x)) > \nu_k(f_{k0}(x^*))$ and $\sigma_k(f_{k0}(x)) < \sigma_k(f_{k0}(x^*))$. for all k=1,2,……..p. Observing that $\mu_k(f_{k0}(x))$ is strictly monotone decreasing function with respect to $f_{k0}(x)$, this implies $\mu_k(f_{k0}(x)) > \mu_k(f_{k0}(x^*))$ and $\nu_k(f_{k0}(x))$ is strictly monotone increasing function with respect to $f_{k0}(x)$, this implies $\nu_k(f_{k0}(x)) < \nu_k(f_{k0}(x^*))$ and also $\sigma_k(f_{k0}(x)) > \sigma_k(f_{k0}(x^*))$. Thus we have $\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x)) > \sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x^*))$, $\sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x)) < \sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x^*))$ and $\sum_{k=1}^{p} w_k\, \sigma_k(f_{k0}(x)) > \sum_{k=1}^{p} w_k\, \sigma_k(f_{k0}(x^*))$. This is a contradiction to the assumption that x^* is a solution of the Neutrosophic GP problem (2). Thus x^* is weakly NS Pareto optimal.

Theorem 2 The unique solution of Neutrosophic GP problem (3) based on weighted sum method is weakly NS Pareto optimal.

Proof. Let x^* Є X be a unique solution of the Neutrosophic GP problem. Let us suppose that it is not weakly NS Pareto optimal. In this case there exit another x Є X such that $\mu_k(f_{k0}(x)) \leq \mu_k(f_{k0}(x^*))$, $\nu_k(f_{k0}(x)) \geq \nu_k(f_{k0}(x^*))$ for all k=1,2,……..p and $\mu_l(f_{l0}(x)) < \mu_l(f_{l0}(x^*))$, $\nu_l(f_{l0}(x)) > \nu_l(f_{l0}(x^*))$ for at least one l. Observing that $\mu_k(f_{k0}(x))$ is strictly monotone decreasing function with respect to $f_{k0}(x)$, this implies $\mu_k(f_{k0}(x)) > \mu_k\big(f_{k0}(x^*)\big)$ and $\nu_k(f_{k0}(x))$ is strictly monotone increasing function with respect to $f_{k0}(x)$, this implies $\nu_k(f_{k0}(x)) < \nu_k\big(f_{k0}(x^*)\big)$ and also $\sigma_k(f_{k0}(x))$ is strictly monotone decreasing function with respect to $f_{k0}(x)$, this implies $\sigma_k(f_{k0}(x)) > \sigma_k\big(f_{k0}(x^*)\big)$. Thus we have $\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x)) \geq \sum_{k=1}^{p} w_k\, \mu_k\big(f_{k0}(x^*)\big)$ and $\sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x)) \leq \sum_{k=1}^{p} w_k\, \nu_k\big(f_{k0}(x^*)\big)$ and $\sum_{k=1}^{p} w_k\, \sigma_k(f_{k0}(x)) \geq \sum_{k=1}^{p} w_k\, \sigma_k\big(f_{k0}(x^*)\big)$.

On the other hand, the uniqueness of x^* means that:

$\sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x^*)) < \sum_{k=1}^{p} w_k\, \mu_k(f_{k0}(x))$, $\sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x^*)) > \sum_{k=1}^{p} w_k\, \nu_k(f_{k0}(x))$ and $\sum_{k=1}^{p} w_k\, \sigma_k(f_{k0}(x^*)) < \sum_{k=1}^{p} w_k\, \sigma_k(f_{k0}(x))$.

The two sets inequalities above are contradictory and thus x^* is weakly NS Pareto optimal.

5 Illustrated Example

Min $f_1(x_1, x_2) = {x_1}^{-1}{x_2}^{-2}$

Min $f_2(x_1, x_2) = 2\, {x_1}^{-2}{x_2}^{-3}$

Such that $x_1 + x_2 \leq 1$

Here pay-off matrix is $\begin{bmatrix} 6.75 & 60.78 \\ 6.94 & 57.87 \end{bmatrix}$

Define truth-membership, falsity-membership and indeterminacy-membership functions are as follows:

$$\mu_1(f_1(x)) = \begin{cases} 1 & if\ \ {x_1}^{-1}{x_2}^{-2} \leq 6.75 \\ \frac{6.94 - {x_1}^{-1}{x_2}^{-2}}{0.19} & if\ \ 6.75 \leq {x_1}^{-1}{x_2}^{-2} \leq 6.94 \\ 0 & if\ {x_1}^{-1}{x_2}^{-2} \geq 6.94 \end{cases}$$

$$\mu_2(f_2(x)) = \begin{cases} 1 & if\ \ 2\, {x_1}^{-2}{x_2}^{-3} \leq 57.87 \\ \frac{60.78 - 2{x_1}^{-2}{x_2}^{-3}}{2.91} & if\ \ 57.87 \leq 2\, {x_1}^{-2}{x_2}^{-3} \leq 60.78 \\ 0 & if\ 2\, {x_1}^{-2}{x_2}^{-3} \geq 60.78 \end{cases}$$

$\nu_1(f_1(x)) = 1 - \frac{1}{1-t}\, \mu_1(f_1(x))$, and $\nu_2(f_2(x)) = 1 - \frac{1}{1-t}\, \mu_2(f_2(x))$

$\sigma_1(f_1(x)) = \frac{1}{s}\, \mu_1(f_1(x)) - \frac{1-s}{s}$, $\sigma_2(f_2(x)) = \frac{1}{s}\, \mu_2(f_2(x)) - \frac{1-s}{s}$

Table 1. Optimal values of primal, dual variables and objective functions from neutrosophic geometric programming problem for different weights.

Weights W_1, W_2	optimal dual variables $w_{01}^*, w_{02}^*, w_{11}^*, w_{12}^*$	optimal primal variables x_1^*	x_2^*	optimal objectives $f_1^*(x_1^*, x_2^*)$	$f_2^*(x_1^*, x_2^*)$	Sum of optimal objectives $f_1^*(x_1^*, x_2^*) + f_2^*(x_1^*, x_2^*)$
0.5, 0.5	0.6491609, 0.3508391, 1.3508391, 2.3508391	0.3649261	0.6491609	6.794329	58.53371	65.32803
0.9, 0.1	0.9415706, 0.0584294, 1.0584294, 2.0584294	0.3395821	0.6604179	6.751768	60.21212	66.96388
0.1, 0.9	0.1745920, 0.8254080, 1.8254080, 2.8254080	0.3924920	0.6075080	6.903434	57.90451	64.80794

Table 2. Comparison of optimal solutions by IFGP and NSGP technique.

optimization techniques	optimal decision variables $x_1^*,\ x_2^*$	optimal objective functions $f_1^*(x_1^*, x_2^*)$, $f_2^*(x_1^*, x_2^*)$	sum of optimal objective functions $f_1^*(x_1^*, x_2^*) + f_2^*(x_1^*, x_2^*)$
Intuitionistic Fuzzy Geometric Programming (IFGP)	0.36611, 0.63389	6.797678 58.58212	65.37980
proposed Neutrosophic Geometric Programming(NSGP)	0.3649261, 0.6491609	6.794329 58.53371	65.32803

In Table.2, it is seen that NSGP technique gives better optimal result than IFGP technique.

6 Application of Neutrosophic Optimization in Gravel box Design Problem

Gravel box problem: A total of 800 cubic-meters of gravel is to be ferried across a river on a barrage. A box (with an open top) is to be built for this purpose. After the entire gravel has been ferried, the box is to be discarded. The transport cost per round trip of barrage of box is Rs 1 and the cost of materials of the ends of the box are Rs20/m^2 and the cost of materials of other two sides and bottom are Rs 10/m^2 and Rs 80/m^2. Find the dimension of the box that is to be built for this purpose and the total optimal cost. Let length = x_1 m, width = x_2 m, height = x_3 m. The area of the end of the gravel box =x_2x_3 m^2. Area of the sides =x_1x_3 m^2. Area of the bottom =x_1x_2 m^2 .The volume of the gravel box=$x_1x_2x_3$ m^3. Transport cost: Rs $\frac{80}{x_1x_2x_3}$. Material cost: $40x_2x_3$. So, the multi-objective geometric programming problem is

$$\text{Min } g_{01} = \frac{80}{x_1x_2x_3} + 40x_2x_3$$

$$\text{Min } g_{02} = \frac{80}{x_1x_2x_3}$$

$$\text{Such that } x_1x_2 + 2x_1x_3 \leq 4.$$

Here pay-off matrix is $\begin{bmatrix} 95.24 & 63.78 \\ 120 & 40 \end{bmatrix}$

Table. 3: Comparison of optimal solutions by IFGP and NSGP technique.

Optimization techniques	Optimal Decision Variables x_1^*, x_2^*, x_3^*	Optimal Objective Functions g_{01}^*, g_{02}^*	Sum of optimal objective values
Intuitionistic fuzzy geometric programming (IFGP)	1.2513842, 1.5982302, 0.7991151	101.1421624 50.0553670	151.1975294
Proposed neutrosophic geometric programming(NSGP)	1.2513843, 1.5982300, 0.7991150	101.1421582 50.0553655	151.1975237

7 Conclusion

In view of comparing the Neutrosophic geometric programming technique with Intuitionistic fuzzy geometric programming technique, we also obtained the solution of the undertaken numerical problem by Intuitionistic fuzzy optimization method and took the best result obtained for comparison with present study.

The objectives of the present study are to give the effective algorithm for Neutrosophic geometric programming method for getting optimal solutions to a multi-objective non-linear programming problem. Further the comparisons of results obtained for the undertaken problem clearly show the superiority of Neutrosophic geometric programming technique over Intuitionistic fuzzy geometric programming technique.

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