

$$I_{\tilde{j}}(J) = \begin{cases} \frac{J-b_1}{b_2-b_1}, & b_1 \leq J \leq b_2, \\ \frac{b_2-J}{b_3-b_2}, & b_2 \leq J \leq b_3, \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

and its falsity-membership function is defined as

$$F_{\tilde{j}}(J) = \begin{cases} \frac{J-c_1}{c_2-c_1}, & c_1 \leq J \leq c_2, \\ \frac{c_2-J}{c_3-c_2}, & c_2 \leq J \leq c_3, \\ 1, & \text{otherwise.} \end{cases} \quad (24)$$

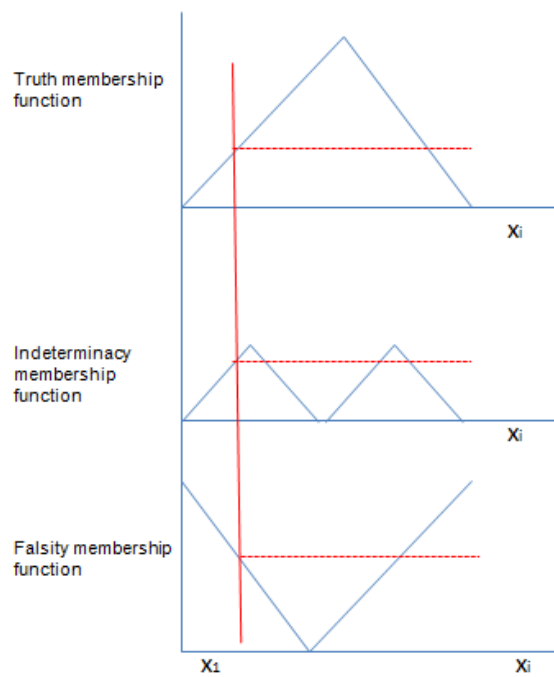


Fig. 4: Neutrosophication process [11]

3 Formation of The Problem

The multi-objective linear programming problem and the multi- objective neutrosophic linear programming problem are described in this section.

A. Multi-objective Programming Problem (MOPP)

In this chapter, the general mathematical model of the MOPP is as follows [6]:

$$\min/\max \left[z_1(x_1, \dots, x_n), z_2(x_1, \dots, x_n), \dots, z_p(x_1, \dots, x_n) \right] \quad (8)$$

subject to $x \in S, x \geq 0$

$$S = \left\{ x \in \mathbb{R}^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, X \geq 0. \right\} \quad (25)$$

B. Neutrosophic Multi-objective Programming Problem (NMOPP)

If an imprecise aspiration level is introduced to each of the objectives of MOPP, then these neutrosophic objectives are termed as neutrosophic goals.

Let $z_i \in [z_i^L, z_i^U]$ denote the imprecise lower and upper bounds respectively for the i^{th} neutrosophic objective function.

For maximizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i) = \begin{cases} 1, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \leq z_i^L \end{cases} \quad (26)$$

$$\sigma_i^I(z_i) = \begin{cases} 1, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \leq z_i^L \end{cases} \quad (27)$$

$$\nu_i^I(z_i) = \begin{cases} 0, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \leq z_i^L \end{cases} \quad (28)$$

For minimizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^U - z_i}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \geq z_i^U \end{cases} \quad (29)$$

$$\sigma_i^I(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^U - z_i}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \geq z_i^U \end{cases} \quad (30)$$

$$\nu_i^I(z_i) = \begin{cases} 0, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^U - z_i}{z_i^U - z_i^L}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \geq z_i^U \end{cases} \quad (31)$$

4 Algorithm for Neutrosophic Multi-Objective Programming Problem

The computational procedure and proposed algorithm of presented model is given as follows:

Step 1. Determine $x_i^* = (x_{i1}^*, x_{i2}^*, \dots, x_{in}^*)$ that is used to maximize or minimize the i^{th} truth membership function $\mu_i^I(X)$, the indeterminacy membership $\sigma_i^I(X)$, and the falsity membership functions $\nu_i^I(X)$. $i=1,2,\dots,p$ and n is the number of variables.

Step 2. Transform the truth membership, indeterminacy membership, falsity membership functions by using first-order Taylor polynomial series

$$\mu_i^I(x) \cong \mu_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial \mu_i^I(x_i^*)}{\partial x_j} \quad (32)$$

$$\sigma_i^I(x) \cong \sigma_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial \sigma_i^I(x_i^*)}{\partial x_j}$$

(33)

$$v_i^I(x) \equiv v_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial v_i^I(x_i^*)}{\partial x_j} \quad (34)$$

Step 3. Find satisfactory $x_i^* = (x_{i1}^*, x_{i2}^*, \dots, x_{in}^*)$ by solving the reduced problem to a single objective for the truth membership, indeterminacy membership, falsity membership functions respectively.

$$\begin{aligned} p(x) &= \sum_{i=1}^p \left[\mu_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial \mu_i^I(x_i^*)}{\partial x_j} \right] \\ q(x) &= \sum_{i=1}^p \left[\sigma_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial \sigma_i^I(x_i^*)}{\partial x_j} \right] \\ h(x) &= \sum_{i=1}^p \left[v_i^I(x_i^*) + \sum_{j=1}^n (x_j - x_{ij}^*) \frac{\partial v_i^I(x_i^*)}{\partial x_j} \right] \end{aligned} \quad (35)$$

Thus neutrosophic multiobjective linear programming problem is converted into a new mathematical model and is given below:

Maximize or Minimize $p(x)$

Maximize or Minimize $q(x)$

Maximize or Minimize $h(x)$,

where $\mu_i^I(X)$, $\sigma_i^I(X)$ and $v_i^I(X)$ calculate using equations (10), (11), and (12) or equations (13), (14), and (15) according to type functions maximum or minimum respectively.

4.1 Illustrative Example

A multi-criteria supplier selection is selected from [2]. For supplying a new product to a market assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers' performance on these criteria are not known precisely. The neutrosophic values of their cost, quality and service level are presented in Table 1.

The multi-objective linear formulation of numerical example is presented as $\min z_1, \max z_2, z_3$:

$$\begin{aligned}
 \min z_1 &= 5x_1 + 7x_2 + 4x_3, \\
 \max z_2 &= 0.80x_1 + 0.90x_2 + 0.85x_3, \\
 \max z_3 &= 0.90x_1 + 0.80x_2 + 0.85x_3, \\
 s.t.: \\
 x_1 + x_2 + x_3 &= 800, \\
 x_1 &\leq 400, \\
 x_2 &\leq 450, \\
 x_3 &\leq 450, \\
 x_i &\geq 0, \quad i = 1, 2, 3.
 \end{aligned}$$

Table 1: Suppliers quantitative information

	Z1 Cost	Z2Quality (%)	Z3 Service (%)	Capacity
Supplier 1	5	0.80	0.90	400
Supplier 2	7	0.90	0.80	450
Supplier 3	4	0.85	0.85	450

The truth membership, indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular. When they depend on three scalar parameters (a1, m, a2). z_1 depends on neutrosophic aspiration levels (3550,4225,4900), when z_2 depends on neutrosophic aspiration levels (660,681.5,702.5), and z_3 depends on neutrosophic aspiration levels (657.5,678.75,700).

The truth membership functions of the goals are obtained as follows:

$$\mu_1^I(z_1) = \begin{cases} 0, & \text{if } z_1 \leq 3550, \\ \frac{4225 - z_1}{4225 - 3550}, & \text{if } 3550 \leq z_1 \leq 4225, \\ \frac{4900 - z_1}{4900 - 4225}, & \text{if } 4225 \leq z_1 \leq 4900, \\ 0, & \text{if } z_1 \geq 4900 \end{cases}$$

$$\mu_2^I(z_2) = \begin{cases} 0, & \text{if } z_2 \geq 702.5, \\ \frac{z_2 - 681.5}{702.5 - 681.5}, & \text{if } 681.5 \leq z_2 \leq 702.5, \\ \frac{z_2 - 660}{681.5 - 660}, & \text{if } 660 \leq z_2 \leq 681.5, \\ 0, & \text{if } z_2 \leq 660. \end{cases}$$

$$\mu_3^I(z_3) = \begin{cases} 0, & \text{if } z_3 \geq 700, \\ \frac{z_3 - 678.75}{700 - 678.75}, & \text{if } 678.75 \leq z_3 \leq 700, \\ \frac{z_3 - 657.5}{678.75 - 657.5}, & \text{if } 657.5 \leq z_3 \leq 678.75, \\ 0, & \text{if } z_3 \leq 657.5. \end{cases}$$

$$\text{If } \mu_1^I(z_1) = \max\left(\min\left(\frac{4225 - (5x_1 + 7x_2 + 4x_3)}{675}, \frac{4900 - (5x_1 + 7x_2 + 4x_3)}{675}, 0\right)\right)$$

$$\mu_2^I(z_2) = \min\left(\max\left(\frac{(0.8x_1 + 0.9x_2 + 0.85x_3) - 681.5}{21}, \frac{(0.8x_1 + 0.9x_2 + 0.85x_3) - 660}{21}, 1\right)\right)$$

$$\mu_3^I(z_3) = \min\left(\max\left(\frac{(0.9x_1 + 0.8x_2 + 0.85x_3) - 678.75}{21.25}, \frac{(0.9x_1 + 0.8x_2 + 0.85x_3) - 657.5}{21.25}, 1\right)\right)$$

$$\text{Then } \mu_1^{I*}(350, 0, 450), \mu_2^{I*}(0, 450, 350), \mu_3^{I*}(400, 0, 400)$$

The truth membership functions are transformed by using first-order Taylor polynomial series

$$\begin{aligned} \hat{\mu}_1^I(x) = & \mu_1^I(350, 0, 450) + \left[(x_1 - 350) \frac{\partial \mu_1^I(350, 0, 450)}{\partial x_1} \right] \\ & + \left[(x_2 - 0) \frac{\partial \mu_1^I(350, 0, 450)}{\partial x_2} \right] + \left[(x_3 - 450) \frac{\partial \mu_1^I(350, 0, 450)}{\partial x_3} \right] \end{aligned}$$

$$\hat{\mu}_1^I(x) = -0.00741x_1 - 0.0104x_2 - 0.00593x_3 + 5.2611$$

In the similar way, we get

$$\hat{\mu}_2^I(x) = 0.0381x_1 + 0.0429x_2 + 0.0405x_3 - 33.405$$

$$\hat{\mu}_3^I(x) = 0.042x_1 + 0.037x_2 + 0.0395x_3 - 32.512$$

The $p(x)$ is

$$p(x) = \hat{\mu}_1^I(x) + \hat{\mu}_2^I(x) + \hat{\mu}_3^I(x)$$

$$P(x) = 0.07259x_1 + 0.0695x_2 + 0.0741x_3 - 60.6559$$

s.t.:

$$x_1 + x_2 + x_3 = 800,$$

$$x_1 \leq 400,$$

$$x_2 \leq 450,$$

$$x_3 \leq 450,$$

$$x_i \geq 0, i = 1, 2, 3.$$

The linear programming software LINGO 15.0 is used to solve this problem. The problem is solved and the optimal solution for the truth membership model is obtained is as follows: $(x_1, x_2, x_3) = (350, 0, 450)$ $z_1=3550$, $z_2=662.5$, $z_3=697.5$.

The truth membership values are $\mu_1 = 1$, $\mu_2 = 0.1163$, $\mu_3 = 0.894$. The truth membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 100%, 11.63% and 89.4% respectively for the obtained solution which is $x_1=350$; $x_2=0$, $x_3=450$.

In the similar way, we get $\sigma_i^I(X)$, $q(x)$. Consequently, we get the optimal solution for the indeterminacy membership model is obtained is as follows: $(x_1, x_2, x_3) = (350, 0, 450)$ $z_1=3550$, $z_2=662.5$, $z_3=697.5$ and the indeterminacy membership values are $\mu_1 = 1$, $\mu_2 = 0.1163$, $\mu_3 = 0.894$. The indeterminacy membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 100%, 11.63% and 89.4% respectively for the obtained solution which is $x_1=350$; $x_2=0$, $x_3=450$.

In the similar way, we get $v_i^I(X)$ and $h(x)$ Consequently, we get the optimal solution for the falsity membership model is obtained is as follows: $(x_1, x_2, x_3) = (350, 0, 450)$ $z_1=3550$, $z_2=662.5$, $z_3=697.5$ and the falsity membership values are $\mu_1 = 0$, $\mu_2 = 0.8837$, $\mu_3 = 0.106$. The falsity membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 0%, 88.37% and 10.6% respectively for the obtained solution which is $x_1=350$; $x_2=0$, $x_3=450$.

5 Conclusions and Future Work

In this chapter, we have proposed a solution to Neutrosophic Multiobjective Programming Problem (NMOPP). The truth membership, indeterminacy membership, falsity membership functions associated with each objective of the problem are transformed by using the first order Taylor polynomial series. The neutrosophic multi-objective programming problem is

reduced to an equivalent multiobjective programming problem by the proposed method. The solution obtained from this method is very near to the solution of MOPP. Hence this method gives a more accurate solution as compared with other methods. Therefore, the complexity in solving NMOPP has reduced to easy computation. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

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References

- [1] El-Hefenawy, N., Metwally, M. A., Ahmed, Z. M., & El-Henawy, I. M. A Review on the Applications of Neutrosophic Sets. *Journal of Computational and Theoretical Nanoscience*, 13(1), (2016), pp. 936-944.
- [2] X. Xu, Y. Lei, and W. Dai. Intuitionistic fuzzy integer programming based on improved particle swarm optimization. In: *Journal of Computer Applications*, vol. 9, 2008, p. 062.
- [3] A. Yücel, and Ali Fuat Güneri. A weighted additive fuzzy programming approach for multi-criteria supplier selection. In: *Expert Systems with Applications*, 38, no. 5 (2011): 6281-6286.
- [4] Mohamed, Mai, et al. "Neutrosophic Integer Programming Problem." *Neutrosophic Sets & Systems* 15 (2017).
- [5] L. Yi, L. Wei-min, and X. Xiao-lai. Intuitionistic Fuzzy Bilevel Programming by Particle Swarm Optimization. In: *Computational Intelligence and Industrial Application*, 2008. PACIIA'08. Pacific-Asia Workshop on, IEEE, 2008, pp. 95–99.
- [6] Abdel-Baset, Mohamed, and Ibrahim M. Hezam. An Improved Flower Pollination Algorithm for Ratios Optimization Problems. In: *Applied Mathematics & Information Sciences Letters An International Journal* , 3, no. 2 (2015): 83-91.
- [7] R. Irene Hepzibah, R and Vidhya. Intuitionistic Fuzzy Multi-Objective Linear Programming Problem(IFMOLPP) using Taylor Series. In: *International Journal of Scientific and Engineering Research (IJSER)*, Vol. 3, Issue 6, June 2014.
- [8] A. Amid, S. H. Ghodsypour, and Ch O'Brien. Fuzzy multiobjective linear model for supplier selection in a supply chain. In: *International Journal of Production Economics* 104, no. 2 (2006): 394-407.
- [9] D. Pintu, Tapan K. R. Multi-objective non-linear programming problem based on Neutrosophic Optimization Technique and its application in Riser Design Problem. In: *Neutrosophic Sets and Systems*, Vol. 9, 2015: 88-95.
- [10] Abdel-Basset, M., Mohamed, M. & Sangaiah, A.K. *J Ambient Intell Human Comput* (2017). DOI: <https://doi.org/10.1007/s12652-017-0548-7>
- [11] S. Aggarwal, Ranjit B., and A. Q. Ansari. Neutrosophic modeling and control. In: *Computer and Communication Technology (ICCCT)*, 2010 International Conference on, pp. 718-723. IEEE, 2010.
- [12] Abdel-Baset, M., Hezam, I. M., & Smarandache, F. Neutrosophic Goal Programming. In: *Neutrosophic Sets & Systems*, vol. 11 (2016).

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[13] Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. Infinite Study, 2005.