Supervised pattern recognition using similarity measure between two interval valued neutrosophic soft sets

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ABSTRACT. F. Smarandache introduced the concept of neutrosophic set in 1995 and P. K. Maji introduced the notion of neutrosophic soft set in 2013, which is a hybridization of neutrosophic set and soft set. Irfan Deli introduced the concept of interval valued neutrosophic soft sets. Interval valued neutrosophic soft sets are most efficient tools to deals with problems that contain uncertainty such as problem in social, economic system, medical diagnosis, pattern recognition, game theory, coding theory and so on. In this article we introduce similarity measure between two interval valued neutrosophic soft sets and study some basic properties of similarity measure. An algorithm is developed in interval valued neutrosophic soft set setting using similarity measure. Using this algorithm a model is constructed for supervised pattern recognition problem using similarity measure.

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1. INTRODUCTION

the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The words “neutrosophy” and “neutrosophic” were introduced by F. Smarandache in his 1998 book. Etymologically, “neutro-sophy” (noun) [French neutre < Latin neuter, neutral and Greek sophia, skill/wisdom] means knowledge of neutral thought. While “neutrosophic” (adjective), means having the nature of, or having the characteristic of Neutrosophy. Maji[7] combined neutrosophic set and soft set and established some operations on these sets. Wang et al. [15] introduced interval neutrosophic sets. Deli[5] introduced the concept of interval-valued neutrosophic soft sets.

Similarity measure is a very effective method to measure degree of similarity between two fuzzy sets and between two soft sets. We can measure degree of similarity between two objects or patterns whenever they can be modeled as fuzzy sets or soft sets or neutrosophic sets etc. Said Broumi and Florentin Smarandache[4] introduced the concept of several similarity measures of neutrosophic sets and Jun Ye [16] introduced the concept of similarity measures between interval neutrosophic sets. Recently A. Mukherjee and S. Sarkar[9, 10] introduced several methods of similarity measure for neutrosophic soft sets and interval valued neutrosophic soft sets.

Pattern recognition problem is a very important and highly attractive topic of research for nearly last five decades and growing day by day due to its emerging applications in various fields. Basically pattern recognition problem consists of two kinds of problems: supervised pattern recognition and unsupervised pattern recognition. In supervised pattern recognition input pattern or unknown pattern is identified as a member of a predefined class. In unsupervised pattern recognition problem the pattern is assigned to a hitherto unknown class. Thus pattern recognition problem is a problem of classification or categorization of patterns, where the classes are defined by the system designer in case of supervised pattern recognition problem and in case of unsupervised pattern recognition pattern problems patterns are learned based on the similarity of patterns.

In this article we introduce similarity measure between two interval valued neutrosophic soft sets and study some basic properties of similarity measure. An algorithm is developed in interval valued neutrosophic soft set setting using similarity measure. Using this algorithm a model is constructed for supervised pattern recognition problem using similarity measure.

2. Preliminaries

In this section we briefly review some basic definitions which will be used in the rest of the paper.

**Definition 2.1 (17).** Let $X$ be a non empty collection of objects denoted by $x$. Then a fuzzy set $\alpha$ in $X$ is a set of ordered pairs having the form $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$, where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also called degree of compatibility or degree of truth) of $x$ in $\alpha$. The interval $M = [0,1]$ is called membership space.
Definition 2.2 ([18]). Let \(D[0,1]\) be the set of closed sub-intervals of the interval \([0,1]\). An interval-valued fuzzy set in \(X\), \(X \neq \Phi\) and \(\text{Card}(X)=n\), is an expression \(A\) given by \(A = \{(x,M_A(x)) : x \in X\}\), where \(M_A : X \to D[0,1]\).

Definition 2.3 ([3]). Let \(X\) be a non empty set. Then an intuitionistic fuzzy set (IFS for short) \(A\) is a set having the form \(A = \{(x,\mu_A(x),\gamma_A(x)) : x \in X\}\) where the functions \(\mu_A : X \to [0,1]\) and \(\gamma_A : X \to [0,1]\) represents the degree of membership and the degree of non-membership respectively of each element \(x \in X\) and \(0 \leq \mu_A(x) \leq \gamma_A(x) \leq 1\) for each \(x \in X\).

Definition 2.4 ([6, 8]). Let \(U\) be an initial universe and \(E\) be a set of parameters. Let \(P(U)\) denotes the power set of \(U\) and \(A \subseteq U\). Then the pair \((F,A)\) is called a soft set over \(U\), where \(F\) is a mapping given by \(F : A \to P(U)\).

Definition 2.5 ([12, 13, 14]). A neutrosophic set \(A\) on the universe of discourse \(X\) is defined as \(A = \{(x,T_A(x),I_A(x),F_A(x)) : x \in X\}\) where \(T,I,F : X \to [-0,1]^+\) and \(-0 \leq T_A(x) \leq I_A(x) \leq F_A(x) \leq 3^+\)

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of \([-0,1]^+[\). But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of \([-0,1]^+[\). Hence we consider the neutrosophic set which takes the value from the subset of \([0,1]\) that is

\[0 \leq T_A(x) \leq I_A(x) \leq F_A(x) \leq 3\]

Definition 2.6 ([7]). Let \(U\) be the universal set and \(E\) be the set of parameters. Also let \(A \subseteq E\) and \(P(U)\) be the set of all neutrosophic sets of \(U\). Then the collection \((F,A)\) is called neutrosophic soft set over \(U\), where \(F\) is a mapping given by \(F : A \to P(U)\).

Definition 2.7 ([7]). Let \(E = \{e_1, e_2, e_3, \ldots, e_m\}\) be the set of parameters, then the set denoted by \(|E|\) and defined by \(|E| = \{\beta e_1, \beta e_2, \ldots, \beta e_m\}\), where \(\beta e_i = \text{note} e_i\) is called NOT set of the set of parameters \(E\). Where \(|\) and \(\gamma\) are different operators.

Definition 2.8 ([15]). Let \(U\) be a space of points (objects), with a generic element in \(U\). An interval valued neutrosophic set (IVN-set) \(A\) in \(U\) is characterized by truth membership function \(T_A\), a indeterminacy-membership function \(I_A\) and a falsity-membership function \(F_A\). For each point \(u \in U\); \(T_A, I_A\) and \(F_A \subseteq [0,1]\). Thus a IVN-set \(A\) over \(U\) is represented as

\[A = \{(T_A(u), I_A(u), F_A(u)) : u \in U\}\]

Where \(0 \leq \sup(T_A(u)) + \sup(I_A(u)) + \sup(F_A(u)) \leq 3\) and \((T_A(u), I_A(u), F_A(u))\) is called interval value neutrosophic number for all \(u \in U\).

Definition 2.9 ([5]). Let \(U\) be an initial universe set, \(E\) be a set of parameters and \(A \subseteq E\). Let \(IVNS(U)\) denotes the set of all interval valued neutrosophic subsets of \(U\). The collection \((F,A)\) is called the interval valued neutrosophic soft set over \(U\), where \(F\) is a mapping given by \(F : A \to IVNS(U)\).

The interval valued neutrosophic soft set defined over a universe is denoted by IVNSS.
Definition 2.10 ([5]). Let $U$ be the universal set, $E$ be the set of parameters and $(N_1, E), (N_2, E)$ be two interval valued neutrosophic soft sets over $U(E)$. Then $(N_1, E)$ is called an interval valued neutrosophic soft sub set of $(N_2, E)$ if

\begin{align*}
\inf T_{N_1}(x_i)(e_j) &\leq \inf T_{N_2}(x_i)(e_j) \\
\sup T_{N_1}(x_i)(e_j) &\geq \sup T_{N_2}(x_i)(e_j) \\
\inf I_{N_1}(x_i)(e_j) &\geq \inf I_{N_2}(x_i)(e_j) \\
\sup I_{N_1}(x_i)(e_j) &\geq \sup I_{N_2}(x_i)(e_j) \\
\inf F_{N_1}(x_i)(e_j) &\geq \inf F_{N_2}(x_i)(e_j) \\
\sup F_{N_1}(x_i)(e_j) &\geq \sup F_{N_2}(x_i)(e_j)
\end{align*}

3. Similarity measure for interval valued neutrosophic soft sets (IVNSS)

In this section we introduce a new method for measuring similarity measure and weighted similarity measure for IVNSS and some basic properties are also studied.

Definition 3.1. Let $U = \{x_1, x_2, x_3, \ldots, x_n\}$ be the universe of discourse and $E = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of parameters and $(N_1, E), (N_2, E)$ be two interval valued neutrosophic soft sets over $U(E)$. Then the similarity measure between two IVNSSs $(N_1, E)$ and $(N_2, E)$ is denoted by $S(N_1, N_2)$ and is defined as follows:

\[ S(N_1, N_2) = \frac{1}{3mn} \sum_{i=1}^{n} \sum_{j=1}^{m} \left( 3 - |T_{N_1}(x_i)(e_j) - T_{N_2}(x_i)(e_j)| - |I_{N_1}(x_i)(e_j) - I_{N_2}(x_i)(e_j)| - F_{N_1}(x_i)(e_j) - F_{N_2}(x_i)(e_j) \right) \]

where

\begin{align*}
T_{N_1}(x_i)(e_j) &= \sqrt{\inf T_{N_1}(x_i)(e_j) \sup T_{N_1}(x_i)(e_j)} \\
I_{N_1}(x_i)(e_j) &= \sqrt{\inf I_{N_1}(x_i)(e_j) \sup I_{N_1}(x_i)(e_j)} \\
F_{N_1}(x_i)(e_j) &= \sqrt{\inf F_{N_1}(x_i)(e_j) \sup F_{N_1}(x_i)(e_j)} \\
T_{N_2}(x_i)(e_j) &= \sqrt{\inf T_{N_2}(x_i)(e_j) \sup T_{N_2}(x_i)(e_j)} \\
I_{N_2}(x_i)(e_j) &= \sqrt{\inf I_{N_2}(x_i)(e_j) \sup I_{N_2}(x_i)(e_j)} \\
F_{N_2}(x_i)(e_j) &= \sqrt{\inf F_{N_2}(x_i)(e_j) \sup F_{N_2}(x_i)(e_j)}
\end{align*}

Theorem 3.2. If $S(N_1, N_2)$ be the similarity measure between two IVNSSs $(N_1, E)$ and $(N_2, E)$ then

(i) $0 \leq S(N_1, N_2) \leq 1$
(ii) $S(N_1, N_2) = S(N_2, N_1)$
(iii) $S(N_1, N_1) = 1$
(iv) if $(N_1, E) \subseteq (N_2, E) \subseteq (N_3, E)$ then $S(N_1, N_3) \leq S(N_2, N_3)$

Proof. (i) Obvious from definition 3.1.
(ii) Obvious from definition 3.1.
(iii) Obvious from definition 3.1.
(iv) Let $U = \{x_1, x_2, x_3, \ldots, x_n\}$ be the universe of discourse and $E = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of parameters and $(N_1, E), (N_2, E), (N_3, E)$ be three interval valued neutrosophic soft sets over $U(E)$, such that $(N_1, E) \subseteq (N_2, E) \subseteq (N_3, E)$. Now by definition of interval valued neutrosophicsoft sub sets [5] we have,
\[ \inf T_{N_1}(x_i)(e_j) \leq \inf T_{N_2}(x_i)(e_j) \leq \inf T_{N_3}(x_i)(e_j) \]
\[ \sup T_{N_1}(x_i)(e_j) \leq \sup T_{N_2}(x_i)(e_j) \leq \sup T_{N_3}(x_i)(e_j) \]
\[ \inf I_{N_1}(x_i)(e_j) \geq \inf I_{N_2}(x_i)(e_j) \geq \inf I_{N_3}(x_i)(e_j) \]
\[ \sup I_{N_1}(x_i)(e_j) \geq \sup I_{N_2}(x_i)(e_j) \geq \sup I_{N_3}(x_i)(e_j) \]
\[ \inf F_{N_1}(x_i)(e_j) \geq \inf F_{N_2}(x_i)(e_j) \geq \inf F_{N_3}(x_i)(e_j) \]
\[ \sup F_{N_1}(x_i)(e_j) \geq \sup F_{N_2}(x_i)(e_j) \geq \sup F_{N_3}(x_i)(e_j) \]

\[ \Rightarrow [T_{N_1}(x_i)(e_j) - T_{N_3}(x_i)(e_j)] \geq [T_{N_2}(x_i)(e_j) - T_{N_3}(x_i)(e_j)] \]
\[ |I_{N_1}(x_i)(e_j) - I_{N_3}(x_i)(e_j)| \geq |I_{N_2}(x_i)(e_j) - I_{N_3}(x_i)(e_j)| \]
\[ |F_{N_1}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \geq |F_{N_2}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \]

\[ \Rightarrow (3 - [T_{N_1}(x_i)(e_j) - T_{N_3}(x_i)(e_j)]) - [I_{N_1}(x_i)(e_j) - I_{N_3}(x_i)(e_j)] - 
\quad |F_{N_1}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \leq (3 - [T_{N_2}(x_i)(e_j) - T_{N_3}(x_i)(e_j)]) - 
\quad |I_{N_2}(x_i)(e_j) - I_{N_3}(x_i)(e_j)| - |F_{N_2}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \]

\[ \Rightarrow \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left( 3 - [T_{N_1}(x_i)(e_j) - T_{N_3}(x_i)(e_j)] - [I_{N_1}(x_i)(e_j) - 
\quad |F_{N_1}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \right) \leq \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left( 3 - [T_{N_2}(x_i)(e_j) - T_{N_3}(x_i)(e_j)] - 
\quad |I_{N_2}(x_i)(e_j) - I_{N_3}(x_i)(e_j)| - |F_{N_2}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \right) \]

\[ \Rightarrow S(N_1, N_3) \leq S(N_2, N_3) \]  [By equation (1)]

**Definition 3.3.** Let \( U = \{x_1, x_2, x_3, \ldots, x_n\} \) be the universe of discourse and \( E = \{e_1, e_2, e_3, \ldots, e_m\} \) be the set of parameters and \((N_1, E), (N_2, E)\) be two interval valued neutrosophic soft sets over \( U(E) \). Now if we consider weights \( w_i \) of \( x_i (i = 1, 2, 3, \ldots, n) \) then the weighted similarity measure between IVNSSs \((N_1, E)\) and \((N_2, E)\) denoted by \( WS(N_1, N_2) \) is defined as follows:

\[
WS(N_1, N_2) = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} w_i \left( 3 - [T_{N_1}(x_i)(e_j) - T_{N_3}(x_i)(e_j)] - [I_{N_1}(x_i)(e_j) - |F_{N_1}(x_i)(e_j) - F_{N_3}(x_i)(e_j)| \right) 
\]

Where \( w_i \in [0, 1], i = 1, 2, 3, \ldots, n \) and \( \sum_{i=1}^{n} w_i = 1 \). In particular if we take \( w_i = \frac{1}{n} \) then \( WS(N_1, N_2) = S(N_1, N_2) \)

**Theorem 3.4.** If \( WS(N_1, N_2) \) be the weighted similarity measure between two IVNSSs \((N_1, E)\) and \((N_2, E)\) then

(i) \( 0 \leq WS(N_1, N_2) \leq 1 \)
(ii) \( WS(N_1, N_2) = WS(N_2, N_1) \)
(iii) \( WS(N_1, N_1) = 1 \)
(iv) if \((N_1, E) \subseteq (N_2, E) \subseteq (N_3, E)\) then \( WS(N_1, N_3) \leq WS(N_2, N_3) \)
Proof. (i) Obvious from definition 3.3.
(ii) Obvious from definition 3.3.
(iii) Obvious from definition 3.3.
(iv) Similar to proof of (iv) of theorem 3.2. □

Definition 3.5. Let \((N_1, E)\) and \((N_2, E)\) be two IVNSSs over the universe \(U\). Then \((N_1, E)\) and \((N_2, E)\) are said be \(\alpha\)-similar , denoted by \((N_1, E) \alpha \sim (N_2, E)\) if and only if \(S(N_1, N_2) > \alpha\) for \(\alpha \in (0, 1)\). We call the two IVNSSs significantly similar if \(S(N_1, N_2) > \frac{1}{2}\).

Definition 3.6. Let \((N_1, E)\) and \((N_2, E)\) be two IVNSSs over the universe \(U\). Then \((N_1, E)\) and \((N_2, E)\) are said be substantially-similar if \(S(N_1, N_2) > 0.95\) and is denoted by \((N_1, E) \equiv (N_2, E)\).

4. Application of similarity measure of IVNSSs in pattern recognition problem

An algorithm for supervised pattern recognition problems using similarity measure between two interval valued neutrosophic soft sets is developed in interval valued neutrosophic soft set setting.

Steps of the proposed algorithm are as follows:

Step 1: construct IVNSS(s) \(\hat{\mu}_i\)\((i = 1,2,3,\ldots, n)\) as ideal pattern(s).

Step 2: construct IVNSS(s) \(\hat{\nu}_j\)\((j = 1,2,3,\ldots, m)\) for sample pattern(s) which is/are to be recognized.

Step 3: calculate similarity measure between IVNSS(s) for ideal pattern(s) and sample pattern(s).

Step 4: recognize sample pattern(s) under certain predefined conditions.

Example 4.1. To demonstrate the proposed algorithm for supervised pattern recognition we consider a problem of medical diagnosis. A fictitious numerical example is given to determine whether a patient with some visible symptoms is suffering from cancer or not. For this we assume that if the similarity measure between the ideal pattern for the disease and sample pattern for the patient lies in the interval \([0.7, 0.9]\)(which can be decided with help of medical expert person) then patient is possibly suffering from the disease, if the similarity measure is greater than 0.90 then the patient is surely suffering from the disease and if similarity measure is less than 0.7 then the patient may not suffering from the disease.

Let \(U = \{u_1, u_2, u_3\}\) be the universe of discourse , where \(u_1 =\) intense , \(u_2 =\) duration and \(u_3 =\) metamorphosis and \(E = \{e_1, e_2, e_2\}\) be the set of parameters(certain visible symptoms), where \(e_1 =\) weakness, \(e_2 =\) fatigue and \(e_3 =\) headache.

**Step 1:** construct an IVNSS (ideal pattern) for the disease cancer, which can be constructed with the help of a medical expert person:

<table>
<thead>
<tr>
<th>((\mu, E))</th>
<th>(e_1)</th>
<th>(e_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>((0.4,0.6), (0.2,0.3), (0.3,0.4))</td>
<td>((0.7,0.8), (0.2,0.4), (0.3,0.4))</td>
</tr>
<tr>
<td>(u_2)</td>
<td>((0.6,0.7), (0.1,0.2), (0.2,0.3))</td>
<td>((0.8,0.9), (0.1,0.2), (0.4,0.5))</td>
</tr>
<tr>
<td>(u_3)</td>
<td>((0.5,0.6), (0.05,0.1), (0.3,0.4))</td>
<td>((0.4,0.6), (0.2,0.3), (0.2,0.3))</td>
</tr>
</tbody>
</table>
Step 2: construct an IVNSS (sample pattern) for the patient $P$:

<table>
<thead>
<tr>
<th>$(\nu_1, E)$</th>
<th>$e_1$</th>
<th>$e_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>(0.4,0.55), (0.2,0.3),(0.25,0.35)</td>
<td>(0.7,0.9), (0.2,0.3),(0.25,0.35)</td>
</tr>
<tr>
<td>$u_2$</td>
<td>(0.5,0.7), (0.1,0.15),(0.2,0.35)</td>
<td>(0.75,0.9), (0.1,0.15),(0.4,0.5)</td>
</tr>
<tr>
<td>$u_3$</td>
<td>(0.4,0.6), (0.1,0.15),(0.2,0.4)</td>
<td>(0.5,0.6), (0.15,0.25),(0.2,0.4)</td>
</tr>
</tbody>
</table>

Construct an IVNSS (sample pattern) for the patient $Q$:

<table>
<thead>
<tr>
<th>$(\nu_2, E)$</th>
<th>$e_1$</th>
<th>$e_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>(0.1,0.3), (0.1,0.2),(0.4,0.5)</td>
<td>(0.1,0.2), (0.05,0.1),(0.8,0.9)</td>
</tr>
<tr>
<td>$u_2$</td>
<td>(0.5,0.7), (0.05,0.2),(0.35,0.45)</td>
<td>(0.2,0.3), (0.4,0.5),(0.8,0.9)</td>
</tr>
<tr>
<td>$u_3$</td>
<td>(0.8,0.9), (0.1,0.2),(0.2,0.3)</td>
<td>(0.2,0.3), (0.6,0.7),(0.7,0.8)</td>
</tr>
</tbody>
</table>

Step 3: calculate similarity measure between $\hat{\mu}$ and $\hat{\nu}_1$ and between $\hat{\mu}$ and $\hat{\nu}_2$ :

Now by definition $3.1$ similarity measure between $\hat{\mu}$ and $\hat{\nu}_1$ is given by $S(\hat{\mu}, \hat{\nu}_1)=0.96$ and similarity measure between $\hat{\mu}$ and $\hat{\nu}_2$ is given by $S(\hat{\mu}, \hat{\nu}_2)=0.56$.

Step 4: since similarity measure $S(\hat{\mu}, \hat{\nu}_1)=0.96 > 0.9$, therefore patient $P$ is surely suffering from the disease cancer. But similarity measure $S(\hat{\mu}, \hat{\nu}_2)=0.56 < 0.7$, therefore patient $Q$ possibly not suffering from the disease cancer.

5. Conclusions

In this paper we proposed similarity measure and weighted similarity measure for interval valued neutrosophic soft sets. We also study some definitions and basic properties of similarity measure. An algorithm is developed in interval valued neutrosophic soft set setting for supervised pattern recognition problem using similarity measure. A fictitious numerical example is given to demonstrate the possible application of proposed model in a medical diagnosis problem.

References


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