Self-Centered Single Valued Neutrosophic Graphs

V.Krishnaraj
Research Scholar, Research & Development Centre,
Bharathiar University, Coimbatore - 641 046, India.
Orcid Id:0000-0001-8092-7524

R.Vikramaprasad
Assistant Professor, Department of Mathematics,
Government Arts College, Salem - 636 007, Tamil Nadu, India.

R.Dhavaseelan
Assistant Professor, Department of Mathematics,
Sona College of Technology, Salem - 636 005, Tamil Nadu, India.
Orcid Id:0000-0001-7035-4427

Abstract
In this paper, we introduce the concepts of length, distance, eccentricity, radius, diameter, status, total status, median and central vertex of a single valued neutrosophic graph. We present the concept of self-centered single valued neutrosophic graph. We investigated some properties of self-centered single valued neutrosophic graphs.

Keywords: Length; distance; eccentricity; radius; diameter; central vertex; status; median; self-centered single valued neutrosophic graph.

INTRODUCTION
Fuzzy set [19] theory plays a vital role in complex phenomena which is not effortlessly described by classical set theory. Atanassov introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs(IFGs). Parvathi and Karunambigai [13] introduced the concept of IFG elaborately and analyzed its components. Authors of [9] introduced the concept of self-centered IFG. Smarandache [6]-[7] introduced the idea of neutrosophic sets by combining the non-standard analysis. Neutrosophic set is a mathematical tool for dealing real life problems having imprecise, indeterminacy and inconsistent data. Neutrosophic set theory, as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory, is applied in a variety of fields, including control theory, decision making problems, topology, medicines and in many more real life problems. Wang et al. [16] presented the notion of single-valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. A single-valued neutrosophic set has three components: truth membership degree, indeterminacy membership degree and falsity membership degree. These three components of a single-valued neutrosophic set are not dependent and their values are contained in the standard unit interval [0, 1]. Single-valued neutrosophic sets are the generalization of intuitionistic fuzzy sets. Single-valued neutrosophic sets have been a new hot research topic and many researchers have addressed this issue. Akram et al.[1-4] has discussed several concepts related to single-valued neutrosophic graphs. Majumdar and Samanta [10] studied similarity and entropy of single-valued neutrosophic sets. Ye[18] proposed correlation coefficients of single-valued neutrosophic sets, and applied it to single-valued neutrosophic decision making problems.

In this paper, we introduce the concepts of length, distance, radius, eccentricity, diameter, status, total status, median and central vertex of a single valued neutrosophic graph. We present the concept of self-centered single valued neutrosophic graph. We also discuss some interesting properties besides giving some examples.

Definition 1.1 [17] Let X be a space of points. A neutrosophic set A in X is characterized by a truth-membership function \( T_A(x) \), an indeterminacy membership function \( I_A(x) \) and a falsity membership function \( F_A(x) \). The functions
\[
T_A(x), I_A(x) \text{ and } F_A(x)
\]
are real standard or non standard subsets of \([0^-, 1^+]\). That is, \( T_A(x): X \to [0^-, 1^+] \), \( I_A(x): X \to [0^-, 1^+] \) and \( F_A(x): X \to [0^-, 1^+] \). and \( 0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+ \).

From philosophical point view, the neutrosophic set takes the value from real standard or non standard subsets of \([0^-, 1^+]\). In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non standard subset of \([0^-, 1^+]\).

Definition 1.2 [3, 1] A single valued neutrosophic graph is a pair \( G = (A, B) \), where \( A: V \to [0,1] \) is single valued neutrosophic set in V and \( B: V \times V \to [0,1] \) is single valued
neutrosophic relation on $V$ such that $T_A(x, y) \leq \min(T_A(x), T_A(y))$ , $I_A(x, y) \leq \min(I_A(x), I_A(y))$ , and $F_A(x, y) \leq \max(F_A(x), F_A(y))$ for all $x, y \in V$. A is called single valued neutrosophic vertex set of $G$ and $B$ is called single valued neutrosophic edge set of $G$, respectively. We note that $B$ is symmetric single valued neutrosophic relation on $A$. If $B$ is not symmetric single valued neutrosophic relation on $A$, then $G = (A, B)$ is called a single valued neutrosophic directed graph.

**Definition 1.3** A single valued neutrosophic graph $G = (A, B)$ is said to be complete if $T_B(v_i, v_j) = \min(T_A(v_i), T_A(v_j))$, $I_B(v_i, v_j) = \min(I_A(v_i), I_A(v_j))$, and $F_B(v_i, v_j) = \max(F_A(v_i), F_A(v_j))$, for all $v_i, v_j \in V$.

**SELF-CENTERED SINGLE VALUED NEUTROSOPHIC GRAPHS**

**Definition 2.1** Let $G = (A, B)$ be a single valued neutrosophic graph. Then the order of $G$ is defined to be $O(G) = (O_T(G), O_I(G), O_F(G))$ where $O_T(G) = \sum_{u \in V} T_A(u)$,$O_I(G) = \sum_{u \in V} I_A(u),O_F(G) = \sum_{u \in V} F_A(u)$.

**Definition 2.2** The size of $G$ is defined to be $S(G) = (S_T(G), S_I(G), S_F(G))$ where $S_T(G) = \sum_{u \in V} T_B(u,v), S_I(G) = \sum_{u \in V} I_B(u,v), S_F(G) = \sum_{u \in V} F_B(u,v)$. The neighbourhood of any vertex $v$ is defined as $N(v) = (N_T(v), N_I(v), N_F(v))$ where $N_T(v) = \{u \in V : T_B(v,u) = \min(T_A(u), T_A(v))\}, N_I(v) = \{u \in V : I_B(v,u) = \min(I_A(u), I_A(v))\}, N_F(v) = \{u \in V : F_B(v,u) = \max(F_A(u), F_A(v))\}$ and $N[v] = N(v) \cup \{v\}$ is called closed neighbourhood of $v$.

**Definition 2.4** A path $P$ in a single valued neutrosophic graph $G = (A, B)$ is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$ such that either one of the following condition is satisfied (i) $T_B(v_i, v_j) > 0, I_B(v_i, v_j) > 0$ and $F_B(v_i, v_j) = 0$ for some $i$ and $j$. (ii) $T_B(v_i, v_j) = 0, I_B(v_i, v_j) = 0$ and $F_B(v_i, v_j) > 0$ for some $i$ and $j$.

**Definition 2.5** Let $G$ be a single valued neutrosophic graph. (i) [13]The length of a path $P: v_1, v_2, \ldots, v_n+1$ ($n > 0$) in $G$ is $n$. (ii) [13]The path $P: v_1, v_2, \ldots, v_{n+1}$ in $G$ is called a cycle if $v_1 = v_{n+1}$ and $n \geq 3$. (iii) An single valued neutrosophic graph $G$ is connected if any two vertices are joined by path.

**Definition 2.6** The strength of a path $P: v_1, v_2, \ldots, v_n$, is defined as $S(P) = (S_T(P), S_I(P), S_F(P))$ where, $S_T(P) = \min(T_B(v_i,v_j)), S_I(P) = \min(I_B(v_i,v_j))$ and $S_F(P) = \max(F_B(v_i,v_j))$ for all $i$ and $j$.

**Note 2.1** In other words, the strength of a path is defined to be the weight of the weakest edge of the path, i.e. the strength of a path $S(P)$.

**Definition 2.7** A single valued neutrosophic graph $G = (A, B)$ is said to be a single valued neutrosophic bipartite if the vertex set $V$ can be partitioned into two non-empty sets $V_1$ and $V_2$ such that (i) $T_B(v_i,v_j) = 0, I_B(v_i,v_j) = 0$ and $F_B(v_i,v_j) = 0$ , if $v_i \in V_1$ or $v_j \in V_2$, (ii) $T_B(v_i,v_j) > 0, I_B(v_i,v_j) > 0$ and $F_B(v_i,v_j) > 0$, if $v_i \in V_1$ or $v_j \in V_2$ for some $i$ and $j$ or $T_B(v_i,v_j) = 0, I_B(v_i,v_j) = 0$ and $F_B(v_i,v_j) > 0$ , if $v_i \in V_1$ or $v_j \in V_2$ for some $i$ and $j$.

**Definition 2.8** A single valued neutrosophic bipartite graph $G = (A, B)$ is said to be complete if $T_B(v_i,v_j) = \min(T_A(v_i), T_A(v_j)), I_B(v_i,v_j) = \min(I_A(v_i), I_A(v_j))$ and $F_B(v_i,v_j) = \max(F_A(v_i), F_A(v_j))$ for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{V_1 \cup V_1}$.

**Definition 2.9** Let single valued neutrosophic graph $H = (A', B')$ is said to be a single valued neutrosophic subgraph of a connected single valued neutrosophic graph $G = (A, B)$. If $T_{A'}(v_i) = T_A(v_i), I_{A'}(v_i) = I_A(v_i), F_{A'}(v_i) = F_A(v_i)$ for all $v_i \in V'$ and $T_{B'}(v_i,v_j) = T_B(v_i,v_j), I_{B'}(v_i,v_j) = I_B(v_i,v_j)$, $F_{B'}(v_i,v_j) = F_B(v_i,v_j)$ for all $v_i, v_j \in V'$.

**Definition 2.10** Let $G = (A, B)$ be a connected single valued neutrosophic graph. (i) The $T$-length of a path $P: v_1, v_2, \ldots, v_n$ in $G$, $l_T(P)$ is defined as $l_T(P) = \sum_{i=1}^{n-1} \frac{1}{T_B(v_i,v_{i+1})}$. (ii) The $I$-length of a path $P: v_1, v_2, \ldots, v_n$ in $G$, $l_I(P)$ is defined as $l_I(P) = \sum_{i=1}^{n-1} \frac{1}{I_B(v_i,v_{i+1})}$. (iii) The $F$-length of a path $P: v_1, v_2, \ldots, v_n$ in $G$, $l_F(P)$ is defined as $l_F(P) = \sum_{i=1}^{n-1} \frac{1}{F_B(v_i,v_{i+1})}$. The $T,I,F$-length of a path $P: v_1, v_2, \ldots, v_n$ in $G$, $l_{T,I,F}(P)$ is defined as $l_{T,I,F}(P) = (l_T(P), l_I(P), l_F(P))$.

**Definition 2.11** Let $G = (A, B)$ be a connected single valued neutrosophic graph. (i) The $T$-distance $\delta_T(v_i, v_j)$ is the minimum of the $T$-length of all the paths joining $v_i$ and $v_j$ in $G$. where $v_i, v_j \in V$. i.e $\delta_T(v_i, v_j) = \min(l_T(P))$, P is a path between $v_i$ and $v_j$.
(ii) The I-distance $\delta_i(v_i, v_j)$ is the minimum of the I-length of all the paths joining $v_i$ and $v_j$ in $G$, where $v_i, v_j \in V$. i.e.$\delta_i(v_i, v_j) = \min\{l_i(P): P$ is a path between $v_i$ and $v_j\}$.

(iii) The F-distance $\delta_F(v_i, v_j)$ is the minimum of the F-length of all the paths joining $v_i$ and $v_j$ in $G$, where $v_i, v_j \in V$. i.e.$\delta_F(v_i, v_j) = \min\{l_F(P): P$ is a path between $v_i$ and $v_j\}.

The distance $\delta_{(T,F)}(v_i, v_j)$ is defined as $\delta_{(T,F)}(v_i, v_j) = (\delta_T, \delta_F)$.

**Definition 2.12** Let $G = (A, B)$ be a connected single valued neutrosophic graph.

(i) For each $v_i \in V$, the T-eccentricity of $v_i$, denoted by $e_T(v_i)$ and is defined as $e_T(v_i) = \max\{\delta_T(v_i, v_j): v_i \in V, v_i \neq v_j\}$.

(ii) For each $v_i \in V$, the I-eccentricity of $v_i$, denoted by $e_I(v_i)$ and is defined as $e_I(v_i) = \max\{\delta_I(v_i, v_j): v_i \in V, v_i \neq v_j\}$.

(iii) For each $v_i \in V$, the F-eccentricity of $v_i$, denoted by $e_F(v_i)$ and is defined as $e_F(v_i) = \min\{\delta_F(v_i, v_j): v_i \in V, v_i \neq v_j\}$.

For each $v_i \in V$, the eccentricity of $v_i$ denoted by $e(v_i)$ and is defined as $e(v_i) = (e_T(v_i), e_I(v_i), e_F(v_i))$.

**Definition 2.13** Let $G = (A, B)$ be a connected single valued neutrosophic graph.

(i) The T-radius of $G$ is denoted by $r_T(G)$ and is defined as $r_T(G) = \min\{e_T(v_i): v_i \in V\}$.

(ii) The I-radius of $G$ is denoted by $r_I(G)$ and is defined as $r_I(G) = \min\{e_I(v_i): v_i \in V\}$.

(iii) The F-radius of $G$ is denoted by $r_F(G)$ and is defined as $r_F(G) = \min\{e_F(v_i): v_i \in V\}$.

The radius of $G$ is denoted by $r(G)$ and is defined as $r(G) = (r_T(G), r_I(G), r_F(G))$.

**Definition 2.14** Let $G = (A, B)$ be a connected single valued neutrosophic graph.

(i) The T-diameter of $G$ is denoted by $d_{\beta}(G)$ and is defined as $\text{diam}_T(G) = \max\{e_T(v_i): v_i \in V\}$.

(ii) The I-diameter of $G$ is denoted by $d_{\iota}(G)$ and is defined as $\text{diam}_I(G) = \max\{e_I(v_i): v_i \in V\}$.

(iii) The F-diameter of $G$ is denoted by $d_{\phi}(G)$ and is defined as $\text{diam}_F(G) = \max\{e_F(v_i): v_i \in V\}$.

The diameter of $G$ is denoted by $\text{diam}(G)$ and is defined as $\text{diam}(G) = (\text{diam}_T(G), \text{diam}_I(G), \text{diam}_F(G))$.

**Example 2.1** Consider a single valued neutrosophic graph, $G = (A, B)$ such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_1), (v_3, v_4), (v_4, v_5), (v_5, v_2)\}$.

Then the eccentricity of $v_i$ are $e(v_1) = (13, 18, 4)$, $e(v_2) = (13, 14, 5)$, $e(v_3) = (13, 18, 5)$, $e(v_4) = (13, 13, 4)$, $e(v_5) = (13, 18, 5)$. Radius of $G$ is $d(G) = (13, 13, 4)$ and Diameter of $G$ is $d(G) = (13, 18, 5)$.

**Definition 2.15** A vertex $v_i \in V$ is called a

(i) T-central vertex of a connected single valued neutrosophic graph $G$ if $r_{\beta}(G) = e_{\beta}(v_i)$.

(ii) I-central vertex of a connected single valued neutrosophic graph $G$ if $r_{\iota}(G) = e_{\iota}(v_i)$.

(iii) F-central vertex of a connected single valued neutrosophic graph $G$ if $r_{\phi}(G) = e_{\phi}(v_i)$.

(iv) Central vertex of a connected single valued neutrosophic graph $G$ if $r_{\beta}(G) = e_{\beta}(v_i)$, $r_{\iota}(G) = e_{\iota}(v_i)$ and $r_{\phi}(G) = e_{\phi}(v_i)$ and the set of all central vertices of a single valued neutrosophic graph is denoted by $C(G)$.

**Definition 2.16** $<C(G) >= H: (A', B')$ is a single valued neutrosophic subgraph of $G = (A, B)$ induced by the central vertices of $G$ is called the center of $G$.

**Definition 2.17** A connected single valued neutrosophic graph $G$ is

(i) T- self-centered single valued neutrosophic graph, if every vertex of $G$ is a T- central vertex. (i.e) $r_{\beta}(G) = e_{\beta}(v_i), \forall v_i \in V$. 

15538
(ii) I- self-centered single valued neutrosophic graph, if every vertex of \( G \) is a I- central vertex. (i.e) \( r_i(G) = e_i(v), \forall v \in V \).

(iii) F- self-centered single valued neutrosophic graph, if every vertex of \( G \) is a F- central vertex. (i.e) \( r_f(G) = e_f(v), \forall v \in V \).

(iv) Single valued neutrosophic self-centered graph, if every vertex of \( G \) is a central vertex. (i.e) \( r_i(G) = e_i(v) \) and \( r_f(G) = e_f(v), \forall v \in V \).

Example 2.2 Consider a single valued neutrosophic graph, \( G = (A, B) \) such that \( V = \{v_1, v_2, v_3, v_4\} \),

\[ E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3)\}. \]

![Graph Image]

Then the eccentricity of \( v_i \) are \( e(v_1) = (11,11,4) \), \( e(v_2) = (11,11,4) \), \( e(v_3) = (11,11,4) \), \( e(v_4) = (11,11,4) \). Radius of \( G \) is \( r(G) = (11,11,4) \) and Diameter of \( G \) is \( d(G) = (11,11,4) \). Here \( r(G) = e(v_1), \forall v_i \in V \). Hence \( G \) is a self-centered single valued neutrosophic graph.

Definition 2.18 Let \( G = (A, B) \) be a connected single valued neutrosophic graph.

(i) The T-status of a node \( u \) of \( G \) is denoted by \( s_{\tau}(u) \) and is defined as \( s_{\tau}(u) = \sum_{v \in V} \delta_{\tau}(u, v) \).

(ii) The I-status of a node \( u \) of \( G \) is denoted by \( s_{\iota}(u) \) and is defined as \( s_{\iota}(u) = \sum_{v \in V} \delta_{\iota}(u, v) \).

(iii) The F-status of a node \( u \) of \( G \) is denoted by \( s_{\varphi}(u) \) and is defined as \( s_{\varphi}(u) = \sum_{v \in V} \delta_{\varphi}(u, v) \).

(iv) The status of a node \( u \) of \( G \) is defined as \( s(u) = (s_{\tau}(u), s_{\iota}(u), s_{\varphi}(u)). \)

Definition 2.19 Let \( G = (A, B) \) be a connected single valued neutrosophic graph.

(i) The minimum T-status of \( G \) is defined as \( m[s_{\tau}(u)]: u \in V \).

(ii) The minimum I-status of \( G \) is defined as \( m[s_{\iota}(u)]: u \in V \).

(iii) The minimum F-status of \( G \) is defined as \( m[s_{\varphi}(u)]: u \in V \).

(iv) The minimum status of \( G \) is denoted by \( m[s(G)] \) and is defined as \( m[s(G)] = (m[s_{\tau}(G)], m[s_{\iota}(G)], m[s_{\varphi}(G)]). \)

Definition 2.20 Let \( G = (A, B) \) be a connected single valued neutrosophic graph.

(i) The maximum T-status of \( G \) is defined as \( M[s_{\tau}(u)]: u \in V \).

(ii) The maximum I-status of \( G \) is defined as \( M[s_{\iota}(u)]: u \in V \).

(iii) The maximum F-status of \( G \) is defined as \( M[s_{\varphi}(u)]: u \in V \).

(iv) The maximum status of \( G \) is denoted by \( M[s(G)] \) and is defined as \( M[s(G)] = (M[s_{\tau}(G)], M[s_{\iota}(G)], M[s_{\varphi}(G)]). \)

Definition 2.21 Let \( G = (A, B) \) be a connected single valued neutrosophic graph.

The total T-status of a node \( u \) of \( G \) is denoted by \( t[s_{\tau}(u)] \) and is defined as \( t[s_{\tau}(u)] = \sum_{v \in V} s_{\tau}(u) \).

Then the eccentricity of \( v_i \) are \( e(v_i) = (11,11,4) \), \( e(v_2) = (11,11,4) \), \( e(v_3) = (11,11,4) \), \( e(v_4) = (11,11,4) \). Radius of \( G \) is \( r(G) = (11,11,4) \) and Diameter of \( G \) is \( d(G) = (11,11,4) \). Here \( r(G) = e(v_1), \forall v_i \in V \). Hence \( G \) is a self-centered single valued neutrosophic graph.

Example 2.3 Consider a single valued neutrosophic -graph, \( G = (A, B) \) such that \( V = \{v_1, v_2, v_3, v_4\} \), \( E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3)\} \).

Here, status of the nodes are \( s(v_1) = (22,19,17) \), \( s(v_2) = (27,28,23) \), \( s(v_3) = (26,16,14) \), \( s(v_4) = (29,27,20) \). The minimum status of \( G \) is \( m[s(G)] = (22,16,14) \). The maximum status of \( G \) is \( M[s(G)] = (29,28,23) \). The total status of \( G \) is \( t[s(G)] = (104,90,74) \). The median is \( M(G) = (104,90,74) \).
Definition 2.23 A connected single valued neutrosophic graph \( G = (A, B) \) is a self-median if all the nodes have the same status. In other words, a connected single valued neutrosophic graph \( G = (A, B) \) is self-median if and only if \( m[s(G)] = M[s(G)] \).

Example 2.4 Consider a single valued neutrosophic graph, \( G = (A, B) \) such that \( V = \{v_1, v_2, v_3, v_4\} \), \( E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\} \). Here, status of the nodes are \( (v_1) = (20,23,15), s(v_2) = (20,23,15), s(v_3) = (20,23,15), s(v_4) = (20,23,15) \). The minimum status of \( G \) is \( m[s(G)] = (20,23,15) \). The maximum status of \( G \) is \( M[s(G)] = (20,23,15) \). The total status of \( G \) is \( t[s(G)] = (80,92,60) \).

The median is \( M(G) = \{\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_4\}\} \). Hence \( G = (A, B) \) is called the self-median graph.

Theorem 2.1 If \( G = (A, B) \) is a bipartite single valued neutrosophic cycle of odd length.

Proof. Let \( G \) be a bipartite single valued neutrosophic graph with bipartition \( V_1 \) and \( V_2 \). Suppose that it contains a strong cycle of odd length, say \( v_1, v_2, \ldots, v_n, v_1 \) for some odd \( n(\text{vertices}) \). Without loss of generality, let \( v_1 \in V_1 \). Since \((v_i, v_{i+1}) \) is strong single valued neutrosophic for \( i = 1, 2, \ldots, n - 1 \) and the nodes are alternatively in \( V_1 \) and \( V_2 \), we have \( v_n \) and \( v_1 \in V_1 \). But this implies that \((v_n, v_1)\) is an edge in \( V_2 \) which contradicts the assumption that \( G \) is a bipartite single valued neutrosophic graph. Hence bipartite single valued neutrosophic graph has no single valued neutrosophic strong cycle of odd length.

Theorem 2.2 Every complete single valued neutrosophic graph \( G \) is a self-centered single valued neutrosophic graph and \( r(G) = \frac{1}{\tau_{A}(x)} \) where \( T_{A}(x) \) and \( l_{A}(x) \) are the least value and \( F_{A}(x) \) is greatest value.

Proof. Let \( G \) be complete single valued neutrosophic graph \( G \). To prove that \( G \) is a self-centered single valued neutrosophic graph. That is we have to show that every vertex is a central vertex. First we claim that \( G \) is a self-centered single valued neutrosophic graph and \( r(G) = \frac{1}{\tau_{A}(v)} \), where \( T_{A}(v) \) is the least. Now fix a vertex \( v_i \in V \) such that \( T_{A}(v_i) \) is least vertex membership value of \( G \).

Case (i) : If \( n = 1 \), then \( T_{B}(v_1, v_1) = \min(T_{A}(v_1, v_1)) = T_{A}(v_i) \). Therefore, the T-length of \( P = l_{A}(P) = \frac{1}{\tau_{A}(v_i)} \).

Case (ii) : If \( n > 1 \), then one of the edges of \( P \) possesses the T-strength \( T_{A}(v_i) \) and hence, T-length of a \( v_i - v_j \) path will exceed \( \frac{1}{\tau_{A}(v_i)} \). That is T-length of \( P = l_{A}(P) > \frac{1}{\tau_{A}(v_i)} \).

Hence \( \delta_{A}(v_i, v_j) = \min(l_{A}(P)) = \frac{1}{\tau_{A}(v_i)}, \forall v_j \in V \). (1)

(2) Let \( v_k \neq v_i \in V \). Consider all \( v_k - v_j \) paths \( Q \) of length \( n \) in \( G \), \( \forall v_j \in V \).

Case (i) : If \( n = 1 \), then \( T_{B}(v_k, v_j) = \min(T_{A}(v_k, v_j)) = T_{A}(v_i) \), since \( T_{A}(v_i) \) is the least. Hence T-length of \( Q = l_{A}(Q) = \frac{1}{\tau_{B}(v_k, v_j)} \leq \frac{1}{\tau_{A}(v_i)} \).

Case (ii) : If \( n = 2 \), then \( l_{A}(Q) = \frac{1}{\tau_{B}(v_k, v_{k-1})} + \frac{1}{\tau_{B}(v_{k-1}, v_j)} \leq \frac{2}{\tau_{A}(v_i)} \), since \( T_{A}(v_i) \) is the least.

Case (iii) : If \( n > 2 \), then \( l_{A}(Q) \leq \frac{n}{\tau_{A}(v_i)} \), since \( T_{A}(v_i) \) is the least.

Hence \( \delta_{A}(v_k, v_j) = \min(l_{A}(Q)) \leq \frac{1}{\tau_{A}(v_i)}, \forall v_k, v_j \in V \). (2)

From Equations (1) and (2), we have \( e_{A}(v_i) = \max(\delta_{A}(v_i, v_j)) = \frac{1}{\tau_{A}(v_i)}, \forall v_j \in V \). (3)
Hence G is a T-self-centered single valued neutrosophic graph.

Now, \( r(\mathcal{T}(G)) = \min(e(\mathcal{T}(v_i))) = \frac{1}{I_{\mathcal{T}(v_i)})}, \) since by equation (3)

\[
r(\mathcal{T}(G)) = \frac{1}{I_{\mathcal{T}(v_i)})}, \text{ where } T_{\mathcal{T}(v_i)} \text{ is least.}
\]

Next, we claim that G is a I- self-centered single valued neutrosophic graph and \( r(\mathcal{T}(G)) = \frac{1}{I_{\mathcal{T}(v_i)})}, \) where \( I_{\mathcal{T}(v_i)} \) is the least. Now fix a vertex \( v_i \in V \) such that \( I_{\mathcal{T}(v_i)} \) is least vertex membership value of G.

(1) Consider all the \( v_i - v_j \) paths P of length n in G, \( \forall v_j \in V \).

Case (i) : If \( n = 1 \), then \( I_{\mathcal{T}(v_i, v_j)} = \min(I_{\mathcal{T}(v_i)}) = I_{\mathcal{T}(v_i)}) \). Therefore, the I-length of P = l \( (P) = \frac{1}{I_{\mathcal{T}(v_i)})} \).

Case (ii) : If \( n > 1 \), then one of the edges of P possesses the I-strength \( I_{\mathcal{T}(v_i)} \), and hence, I-length of a \( v_i - v_j \) path will exceed \( \frac{1}{I_{\mathcal{T}(v_i)})} \). That is I-length of P = l \( (P) > \frac{1}{I_{\mathcal{T}(v_i)})} \).

Hence \( \delta_{\mathcal{T}}(v_i, v_j) = \min(l \( (P) \)) = \frac{1}{I_{\mathcal{T}(v_i)})}, \forall v_j \in V \). (4)

(2) Let \( v_k \not= v_j \) in V. Consider all \( v_k - v_j \) paths Q of length n in G, \( \forall v_j \in V \).

Case (i) : If \( n = 1 \), then \( I_{\mathcal{T}(v_k, v_j)} = \min(I_{\mathcal{T}(v_k)}) \geq I_{\mathcal{T}(v_i)}) \), since \( I_{\mathcal{T}(v_i)} \) is the least. Hence I-length of Q = l \( (Q) = \frac{1}{I_{\mathcal{T}(v_k, v_j)})} \leq \frac{1}{I_{\mathcal{T}(v_j)})} \).

Case (ii) : If \( n = 2 \), then \( l \( (Q) = \frac{1}{I_{\mathcal{T}(v_k, v_j)})} + \frac{1}{I_{\mathcal{T}(v_j)})} \leq \frac{2}{I_{\mathcal{T}(v_j)})} \) since \( I_{\mathcal{T}(v_j)}) \) is the least.

Case (iii) : If \( n > 2 \), then \( l \( (Q) \leq \frac{n}{I_{\mathcal{T}(v_j)})} \), since \( I_{\mathcal{T}(v_i)} \) is the least. Hence \( \delta_{\mathcal{T}}(v_k, v_j) = \min(l \( (Q) \)) = \frac{1}{I_{\mathcal{T}(v_k, v_j)})}, \forall v_k, v_j \in V \). (5)

From Equations (4) and (5), we have \( e_{\mathcal{T}}(v_i) = \max(\delta_{\mathcal{T}}(v_i, v_j)) = \frac{1}{I_{\mathcal{T}(v_j)})}, \forall v_i \in V \). (6)

Hence G is a I-self-centered single valued neutrosophic graph.

Now, \( r(\mathcal{T}(G)) = \min(e(\mathcal{T}(v_i))) = \frac{1}{I_{\mathcal{T}(v_i)})}, \) since by equation (6)

\[
r(\mathcal{T}(G)) = \frac{1}{I_{\mathcal{T}(v_i)})}, \text{ where } r_{\mathcal{T}(v_i)} \text{ is least.}
\]

Next, we claim that G is a F - self-centered single valued neutrosophic graph and \( r(\mathcal{F}(G)) = \frac{1}{F_{\mathcal{F}(v_i)})} \), where \( F_{\mathcal{F}(v_i)} \) is the greatest. Now fix a vertex \( v_i \in V \) such that \( F_{\mathcal{F}(v_i)} \) is greatest vertex membership value of G.

(1) Consider all the \( v_i - v_j \) paths P of length n in G, \( \forall v_j \in V \).

Case (i) : If \( n = 1 \), then \( F_{\mathcal{F}(v_i, v_j)} = \max(F_{\mathcal{F}(v_i)}) = F_{\mathcal{F}(v_i)}) \). Therefore, the F - length of P = \( l \( (P) = \frac{1}{F_{\mathcal{F}(v_i)})} \).

Case (ii) : If \( n > 1 \), then one of the edges of P possesses the F-strength \( F_{\mathcal{F}(v_i)} \), and hence, F-length of a \( v_i - v_j \) path will exceed \( \frac{1}{F_{\mathcal{F}(v_i)})} \). That is F-length of P = \( l \( (P) > \frac{1}{F_{\mathcal{F}(v_i)})} \).

Hence \( \delta_{\mathcal{F}}(v_i, v_j) = \min(l \( (P) \)) = \frac{1}{F_{\mathcal{F}(v_i)})}, \forall v_j \in V \). (7)

(2) Let \( v_k \not= v_j \) in V. Consider all \( v_k - v_j \) paths Q of length n in G, \( \forall v_j \in V \).

Case (i) : If \( n = 1 \), then \( F_{\mathcal{F}(v_k, v_j)} = \max(F_{\mathcal{F}(v_k)}) \leq F_{\mathcal{F}(v_i)}) \), since \( F_{\mathcal{F}(v_i)} \) is the greatest. Hence F-length of Q = \( l \( (Q) = \frac{1}{F_{\mathcal{F}(v_k, v_j)})} \geq \frac{1}{F_{\mathcal{F}(v_i)})} \).

Case (ii) : If \( n = 2 \), then \( l \( (Q) = \frac{1}{F_{\mathcal{F}(v_k, v_j)})} + \frac{1}{F_{\mathcal{F}(v_j)})} \geq \frac{2}{F_{\mathcal{F}(v_i)})} \), since \( F_{\mathcal{F}(v_i)} \) is the greatest.

Case (iii) : If \( n > 2 \), then \( l \( (Q) \geq \frac{n}{F_{\mathcal{F}(v_i)})} \), since \( F_{\mathcal{F}(v_i)} \) is the greatest. Hence \( \delta_{\mathcal{F}}(v_k, v_j) = \min(l \( (Q) \)) = \frac{1}{F_{\mathcal{F}(v_k, v_j)})}, \forall v_k, v_j \in V \). (8)

From Equations (7) and (8), we have \( e_{\mathcal{F}}(v_i) = \max(\delta_{\mathcal{F}}(v_i, v_j)) = \frac{1}{F_{\mathcal{F}(v_j)})}, \forall v_i \in V \). (9)

Hence G is a F-self-centered single valued neutrosophic graph.

Now, \( r(\mathcal{F}(G)) = \min(e(\mathcal{F}(v_i))) = \frac{1}{F_{\mathcal{F}(v_i)})}, \) since by equation (9)

\[
r(\mathcal{F}(G)) = \frac{1}{F_{\mathcal{F}(v_i)})}, \text{ where } r_{\mathcal{F}(v_i)} \text{ is greatest.}
\]

From equations (3),(6), and (9), every vertex of G is a central vertex. Hence G is a self-centered single valued neutrosophic graph.

**Theorem 2.3** A single valued neutrosophic graph \( G = (A, B) \) is a self-centered single valued neutrosophic graph iff \( \delta_{\mathcal{F}}(v_i, v_j) \leq r_{\mathcal{F}(v_i)} \text{ and } \delta_{\mathcal{T}}(v_i, v_j) \leq r_{\mathcal{T}(v_i)} \) and \( \delta_{\mathcal{T}}(v_i, v_j) \geq r_{\mathcal{T}(v_i)} \forall v_i, v_j \in V \).

**Proof** We assume that G is self-centered single valued neutrosophic graph G. That is \( e_{\mathcal{F}}(v_i) = e_{\mathcal{F}(v_i)}, e_{\mathcal{T}}(v_i) = e_{\mathcal{T}(v_i)}, e_{\mathcal{F}}(v_j) = e_{\mathcal{F}(v_j)}, \forall v_i, v_j \in V, r_{\mathcal{T}(G)} = r_{\mathcal{T}(G)} \forall v_i \in V \).

Now we wish to show that \( \delta_{\mathcal{F}}(v_i, v_j) \leq r_{\mathcal{F}(v_i)} \text{ and } \delta_{\mathcal{T}}(v_i, v_j) \leq r_{\mathcal{T}(v_i)} \).
\[ r(T(G), \delta(I(v, v)) \leq r(T(G) \text{ and } \delta_f(v, v) \geq r(T(G), \forall v, v \in V). \]

By the definition of eccentricity, we obtain, \[ \delta(T(G), \delta(I(v, v)) \leq e(T), (v, v) \leq e(T) \text{ and } \delta_f(v, v) \geq e(T), \forall v, v \in V. \]

When \[ e(T(v)) = e(T) \cdot e(T(v)) = e(T), \forall v, v \in V. \] Since G is self-centered single valued neutrosophic graph, the above inequality states \[ \delta(T(v, v)) \leq r(T(G)), \delta(I(v, v)) \leq r(T(G)) \text{ and } \delta_f(v, v) \geq r(T(G)). \]

\[ \iff \]

Asymptote, \[ \delta(T(v, v)) \leq r(T(G)), \delta(I(v, v)) \leq r(T(G)) \text{ and } \delta_f(v, v) \geq r(T(G)). \]

\[ \Rightarrow \]

From equations (10), (14), the complement of G has two components and each is a complete single valued neutrosophic graph. Therefore, \[ G \text{ is self-centered single valued neutrosophic graph.} \]

\[ \text{Suppose that } G \text{ is not self-centered single valued neutrosophic graph. Then} r(T(G) = e(I(v)), r(I) = e(I(v)) \text{ and } r(T(G) = e(I(v)), \text{ for some } v, v \in V. \]

Let us assume that \[ e(I(v)), e(I(v)) \text{ and } e(I(v)) \text{ is the least value among all other eccentricities. That is} \]

\[ r(T(G) = e(I(v)), r(I) = e(I(v)) \text{ and } r(T(G) = e(I(v)), \text{ for some } v, v \in V. \]

\[ \text{Hence from equations (10) and (11), we have } \delta(T(v, v)) > r(T(G)), \delta(I(v, v)) > r(T(G)) \text{ and } \delta_f(v, v) > r(T(G). \]
\[\min\left(\frac{1}{f_g(v_1, v_1)}\right) = \min\left(\frac{1}{f_g(v_2, v_2)}\right) = \min\left(\frac{1}{f_g(v_n, v_n)}\right) = \ldots = \min\left(\frac{1}{f_g(v, v)}\right).\]

\[\max\{\delta_T(v_1, v_1)\} = \max\{\delta_T(v_2, v_2)\} = \max\{\delta_T(v_3, v_3)\} = \ldots = \max\{\delta_T(v_n, v_n)\}.\]

\[\max\{\delta_I(v_1, v_1)\} = \max\{\delta_I(v_2, v_2)\} = \max\{\delta_I(v_3, v_3)\} = \ldots = \max\{\delta_I(v_n, v_n)\}.\]

\[\min\{\delta_F(v_1, v_1)\} = \min\{\delta_F(v_2, v_2)\} = \min\{\delta_F(v_3, v_3)\} = \ldots = \min\{\delta_F(v_n, v_n)\}.\]

\[e(v_1) = e(v_2) = e(v_3) = \ldots = e(v_n).\]

Therefore G is self-centered.

CONCLUSION

In this paper, the concepts of length, distance, eccentricity, radius, diameter, status, total status, median and central vertex of a single valued neutrosophic graph have been investigated. We have presented the concept of self-centered single valued neutrosophic graph. Also some interesting properties of self-centered single valued neutrosophic graphs followed by some examples.

REFERENCES


