On Neutrosophic Feebly Open Set In Neutrosophic Topological Spaces

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Abstract: The focus of this paper is to introduce the concept of Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic feebly open sets and Neutrosophic feebly closed sets in Neutrosophic Topological spaces. Also we analyse their characterizations and investigate their properties. This concept is the generalization of intuitionistic topological spaces and fuzzy topological spaces. Using this neutrosophic feebly open sets and neutrosophic feebly closed sets, we define a new class of functions namely neutrosophic feebly continuous functions. Further, relationships between this new class and the other classes of functions are established.

Keywords: Neutrosophic sets, Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic Topological spaces, Neutrosophic feebly open set, Neutrosophic feebly closed set and Neutrosophic continuous functions.

INTRODUCTION

Theory of fuzzy sets [18], theory of intuitionistic fuzzy sets [1-3], theory of neutrosophic sets [9] and the theory of interval neutrosophic sets [12] can be considered as tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in [12]. In 1965, Zadeh [18] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. The intuitionistic fuzzy set was introduced by Atanassov [2] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The neutrosophic set was introduced by Smarandache [9] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [16], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama, Smarandache and Valeri [17] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

In this paper, we introduce and study the concept of neutrosophic feebly open sets and neutrosophic feebly continuous functions in neutrosophic topological spaces. This paper consists of four sections. The Section I consists of the basic definitions and the operations of neutrosophic sets which are used in the later sections. The Section II deals with the concept of Neutrosophic point, Neutrosophic quasi coincident, Neutrosophic quasi neighbourhood, Neutrosophic feebly open sets in Neutrosophic topological space and study their properties. The Section III deals with the complement of neutrosophic feebly open set namely neutrosophic feebly closed set. The Section IV consists of neutrosophic feebly continuous functions in neutrosophic topological spaces and its relations with other functions.

I. PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

Definition 1.1 [16] Let X be a non-empty fixed set. A neutrosophic set (NF for short) A is an object having the form A = { (x, µA(x), σA(x), γA(x)) : x ∈ X } where µA(x), σA(x) and γA(x) which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element x ∈ X to the set A.

Remark 1.2 [16] A neutrosophic set A = { (x, µA(x), σA(x), γA(x)) : x ∈ X } can be identified to an ordered triple (µA, σA, γA) in [0,1]* on X.

Remark 1.3 [16] For the sake of simplicity, we shall use the symbol A = { (x, µA, σA, γA) } for the neutrosophic set A = { (x, µA(x), σA(x), γA(x)) : x ∈ X }.

Example 1.4 [16] Every intuitionistic fuzzy Set A is a non-empty set in X is obviously on neutrosophic set having the form A = { (x, µA(x), 1- (µA(x) + γA(x))), γA(x)) : x ∈ X }. Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the neutrosophic set 0_N and 1_N in X as follows:

0_N may be defined as:
(0_1) 0_N = { (x, 0, 0, 1) : x ∈ X }
(0_2) 0_N = { (x, 0, 1, 1) : x ∈ X }
We can easily generalize the operations of intersection and union in Definition 1.8 to arbitrary family of neutrosophic set as follows:

**Definition 1.9** [16] Let \{ A_j; j \in J \} be an arbitrary family of neutrosophic set in X. Then \( \bigcap_{j} A_j \) may be defined as:

(i) \( \cap A_j = \left\{ \left(x, \bigwedge_{j} \mu_{A_j}(x), \bigwedge_{j} \sigma_{A_j}(x), \bigwedge_{j} \gamma_{A_j}(x) \right) : x \in X \right\} \)

(ii) \( \cap A_j = \left\{ \left(x, \bigwedge_{j} \mu_{A_j}(x), \sigma_{A_j}(x), \gamma_{A_j}(x) \right) : x \in X \right\} \)

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(ii) \( \cap A_j = \left\{ \left(x, \bigwedge_{j} \mu_{A_j}(x), \bigwedge_{j} \sigma_{A_j}(x), \bigwedge_{j} \gamma_{A_j}(x) \right) : x \in X \right\} \)

**Proposition 1.10** [16] For all A and B are two neutrosophic sets then the following conditions are true:

1. \( C \cap (A \cup B) = (C \cup A) \cap (C \cup B) \)
2. \( C \cup (A \cap B) = (C \cap A) \cup (C \cap B) \)

**Definition 1.11** [16] A neutrosophic topology (NTS for short) is a non-empty set X is a family \( \tau \) of neutrosophic subsets in X satisfying the following axioms:

(i) \( \tau \) is a neutrosophic topology on \( X \) in the sense of Chang is obviously a NTS in the form \( \tau = \{ A : \mu_A \in T_1 \} \) whenever we identify a fuzzy set in X whose membership function is \( \mu_A \) with its counterpart.

**Remark 1.13** [16] Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allowing more general functions to be members of fuzzy topology.

**Example 1.14** [16] Let \( X = \left\{ x \right\} \) and \( A = \left\{ \left(x, 0.5, 0.4, 0.5 \right) : x \in X \right\} \)

\( B = \left\{ \left(x, 0.4, 0.6, 0.8 \right) : x \in X \right\} \)

\( D = \left\{ \left(x, 0.5, 0.6, 0.4 \right) : x \in X \right\} \)

\( C = \left\{ \left(x, 0.4, 0.5, 0.8 \right) : x \in X \right\} \)

Then the family \( \tau = \{ 0_N, A, B, C, D, I_N \} \) of neutrosophic sets in X is neutrosophic topology on X.
NCl (A) = ∩ \{K : K is a NCS in X and A \subseteq K \}
NInt (A) = ∪ \{G : G is a NOS in X and G \subseteq A \}.
It can be also shown that NCl (A) is NCS and NInt (A) is a NOS in X. That is,
a) A is NCS in X if and only if A = NCl (A).
b) A is NOS in X if and only if A = NInt (A).

Proposition 1.17 [17] For any neutrosophic set A in (X, τ) we have
(a) NCl (C (A)) = C (NInt (A)),
(b) NInt (C (A)) = C (NCl (A)).

Proposition 1.18 [17] Let (X, τ) be a NTS and A, B be two neutrosophic sets in X. Then the following properties holds :
(a) NInt (A) ⊆ A,
(b) A ⊆ NCl (A),
(c) A ⊆ B ⇒ NInt (A) ⊆ NInt (B),
(d) A ⊆ B ⇒ NCl (A) ⊆ NCl (B),
(e) NInt (A ∪ B) = NInt (A) ∪ NInt (B),
(f) NCl (A ∪ B) = NCl (A) ∩ NCl (B),
(g) NInt (1_A) = 1_X,
(h) NCl (0_A) = 0_X,
(i) A ⊆ B ⇒ C (B) ⊆ C (A),
(j) NInt (A ∩ B) ⊆ NCl (A) ∩ NCl (B),
(k) NInt (A ∪ B) ⊆ NInt (A) ∪ NInt (B).

Definition 1.19 [5] A Neutrosophic subset A is Neutrosophic semi open if A ⊆ NClNInt A.

Definition 1.20 [5] A Neutrosophic topological space (X, τ) is product related to another Neutrosophic topological space (Y, σ) if for any Neutrosophic subset v of X and \( \zeta \) of Y, whenever \( \lambda^v \geq v \) and \( \mu^{\zeta} \geq \zeta \) imply \( \lambda^v \times \zeta \geq \nu \times \zeta \), where \( \lambda \) and \( \mu \) exist and \( \lambda \in \mathbb{R}^m \), \( \mu \in \mathbb{R}^n \).

Definition 1.21 [5] Let X and Y be two nonempty neutrosophic sets and f : X → Y be a function.
(i) If B = \{(y, \mu(y), \sigma(y), \gamma(y)) : y \in Y \} is a Neutrosophic set in Y , then the pre image of B under f is denoted and defined by \( f^{-1}(B) = \{(x, f^{-1}(\mu(y)), f^{-1}(\sigma(y)), f^{-1}(\gamma(y)) : x \in X \} \).
(ii) If A = \{(x, \alpha(x), \delta(x), \lambda(x)) : x \in X \} is a NOS in X, then the image of A under f is denoted and defined by \( f(A) = \{(y, f(\alpha(x)), f(\delta(x)), f(\lambda(x))) : y \in Y \} \) where \( f(\alpha(x)) = C(f(C(A))) \).

Definition 1.22 [5] Let \( f_1 : X_1 \rightarrow Y_1 \) and \( f_2 : X_2 \rightarrow Y_2 \). The Neutrosophic product \( f_1 \times f_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) is defined by \( f_1 \times f_2(x_1, x_2) = (f_1(x_1), f_2(x_2)) \) for all \( (x_1, x_2) \in X_1 \times X_2 \).

Definition 1.23 [5] Let A, A_i (i ∈ J ) be NSs in X and B, B_j (j ∈ K ) be NSs in Y and f : X → Y be a function.
(i) f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j),
(ii) f^{-1}(\bigcap B) = \bigcap f^{-1}(B),
(iii) f^{-1}(1_B) = 1_{f^{-1}(B)} = 0_N,
(iv) f^{-1}(C(B)) = C(f^{-1}(B)),
(v) f(\bigcup A_i) = \bigcup f(A_i).

Definition 1.24 [5] Let f : X → Y be a function. The Neutrosophic graph g : X → X × Y of f is defined by g(x) = (x, f(x)) for all x ∈ X.

Lemma 1.25 [5] Let \( f_i : X_i → Y_i \) (i = 1, 2) be functions and A, B be Neutrosophic subsets of \( Y_1, Y_2 \) respectively. Then \( f_1 \times f_2 \) is defined on A × B.

Lemma 1.26 [5] Let g : X → X × Y be the graph of a function f : X → Y . If A is the NS of X and B is the NS of Y , then g^{-1}(A × B)(x) = (A \cap f^{-1}(B))(x).

II. NEUTROSOPHIC FEELY OPEN SET

In this section, the concept of Neutrosophic feebly open set is introduced.

Definition 2.1 Let \( \alpha, \beta, \gamma \in [0, 1] \) and \( \alpha + \beta + \gamma \leq 1 \). A Neutrosophic point with support x (a, b, c) ∈ X is a Neutrosophic set of X is defined by x ((a, b, c), y = x) = \{(x, \mu(X), \sigma(X), \gamma(X)) : x \in X \} is denoted by two ways
(i) x (a, b, c) ∈ A if \( \mu(a(X)) \leq \beta \leq \mu(c(X)) \) and \( \gamma \geq \gamma(X) \).
(ii) x (a, b, c) ∈ A if \( \alpha \leq \mu(a(X)) \), \( \beta \leq \sigma(X) \) and \( \gamma \geq \gamma(X) \).

Clearly a Neutrosophic point can be represented by an ordered triple of Neutrosophic set as follows : x ((a, b, c), y = x (a, b, c) ∈ X). A class of all neutrosophic points in X is denoted as NP(X).

Definition 2.2 For any two Neutrosophic subsets A and B, we shall write AqB to mean that A is quasi-coincident (q- coincident, for short) with B if there exists x ∈ X such that A(x) + B(x) > 1. That is \{x, x(a(X)) + x(a(X)) + x(a(X)) + x(a(X)) : x ∈ X \} > 1.

Definition 2.3 Let \( \lambda \) and \( \mu \) be any two Neutrosophic subsets of a Neutrosophic topological space. Then A is q- neighbourhood with B (q- ndb, for short) if there exists a Neutrosophic open set O with AqO ≤ B.
Proposition 2.4 Let \((X, \tau)\) be a Neutrosophic topological space. Then for a Neutrosophic set \(A\) of a Neutrosophic topological space \(X\), \(N\text{SCIA}\) is the union of all Neutrosophic points \(x \triangleq (\alpha, \beta, \gamma)\) such that every Neutrosophic semi open set \(O\) with \(x \triangleq (\alpha, \beta, \gamma)\) \(qO\) is Neutrosophic \(q\)-coincident with \(A\).

**Proof:** Let \(x_0 \in \text{NSCIA}\). Suppose there is a Neutrosophic semi open set \(O\) such that \(x \triangleq (\alpha, \beta, \gamma) \notin qO\) and \(q \notin A\). That implies that \(O^C \supseteq A\), where \(O^C\) is Neutrosophic semi closed. \(O^C \supseteq \text{NSCIA}\). By using Definition 2.6, \(x \triangleq (\alpha, \beta, \gamma) \in \text{NSCIA}\) implies that \(x \triangleq (\alpha, \beta, \gamma) \notin \text{NSCIA}\). This is a contradiction to our assumption. Therefore for every semi open \(O\) with \(x \triangleq (\alpha, \beta, \gamma) \notin qO\) is \(q\)-coincident with \(A\).

Conversely, for every semi open \(O\) with \(x \triangleq (\alpha, \beta, \gamma) \notin qO\) is \(q\)-coincident with \(A\). Suppose \(x_0 \in \text{NSCIA}\). Then there is a neutrosophic semi closed set \(G \supseteq A\) with \(x \triangleq (\alpha, \beta, \gamma) \notin G\). \(G^C\) is neutrosophic semi open set with \(x \triangleq (\alpha, \beta, \gamma) \notin G^C\). That is \(A(x) > (G^C)^C \subseteq G\). This is a contradiction to the assumption. Therefore \(x \triangleq (\alpha, \beta, \gamma) \notin \text{NSCIA}\).

Proposition 2.5 Let \((X, \tau)\) be a Neutrosophic topological space. Let \(A\) and \(B\) be Neutrosophic subsets of a Neutrosophic topological space \(X\). Then if \(A \wedge B = \emptyset\) then \(A \vee B\)

\[
A \wedge B \Rightarrow x \in (\alpha, \beta, \gamma) \vee \forall qA, A \vee B \Rightarrow (x) \vee A \\
\]

**Proof:** Let \((A \wedge B)(x) = 0\). Then min \(\{A(x), B(x)\} = 0\). This implies that \(A(x) = 0\) and \(B(x) = 0\). For every \(x \in (\alpha, \beta, \gamma) \vee \forall qA\).

Let \(A \wedge B \Rightarrow (\alpha, \beta, \gamma) \vee \forall qA\). This implies that \(\alpha \vee \beta \vee \gamma\). Now \(x \notin qA\) implies that \(x \notin qB\). Therefore \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). That implies \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). This is a contradiction. Therefore \(A(x) \supseteq B(x)\). This proves (ii).

By using Definition 2.2, \(A \vee B\) if and only if for each \(x \in X\), \(A(x) \supseteq B(x)^C\). That is \(A \supseteq B^C\). This proves (iii).

Now \(x_0 \in (\alpha, \beta, \gamma) \vee \forall qA\) if and only if \((x_0 \in (\alpha, \beta, \gamma) \vee \forall qA)\). That is \((\alpha, \beta, \gamma)^C < (\alpha, \beta, \gamma)^C\). This implies that \(x_0 \in (\alpha, \beta, \gamma) \vee \forall qA\). This shows that \(\{\alpha, \beta, \gamma\} \in \text{NSCIA}\). That shows \(\text{NSCIA} \subseteq \text{NCl}\).

Proposition 2.6 Let \((X, \tau)\) be a Neutrosophic topological space. Let \(A\) be a Neutrosophic subset of a Neutrosophic topological space \(X\). Then \(\text{NIntNClNIntNInt} \cap \text{NCIA} = \text{NIntNCIA}\) and \(\text{NIntNClNInt} \cap \text{NCIA} = \text{NIntNClA}\).

**Proof:** We know that \(\text{NIntNCIA} \subseteq \text{NCIA}\). By using Definition 2.6, \(\text{NIntNClNInt} \subseteq \text{NCl}\). This implies that \(\text{NIntNClNInt} = \text{NIntNCIA}\). Since \(\text{NIntNCIA} = \text{NCIA}\), \(\text{NIntNCIA} = \text{NIntNCIA} = \text{NIntNCIA}\). From the above \(\text{NIntNClNIntNCIA} = \text{NIntNCIA}\). This proves (i).

(ii) follows from Proposition 1.17 [2].

Proposition 2.7 Let \((X, \tau)\) be a Neutrosophic topological space.

(a) Let \(x_0\) and \(A\) be a Neutrosophic point, a Neutrosophic subset, resp., of a Neutrosophic topological space \(X\). Then \(x_0 \cap (\alpha, \beta, \gamma) \subseteq A\).

(b) Let \(A\) and \(B\) be any two Neutrosophic open subsets of a Neutrosophic topological space \(X\) with \(A \subseteq B\). Then \(A \subseteq \text{NCIA} \cap \text{NCIA} \subseteq B\).

**Proof:** Let \(x_0 \in (\alpha, \beta, \gamma) \subseteq A\).

Suppose \(x_0 \in (\alpha, \beta, \gamma) \subseteq A\) if and only if \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). This implies that \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). Taking complement on both sides implies \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). This shows that \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). Therefore \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). That shows \(A(x) \supseteq (\alpha, \beta, \gamma)^C\).

By using Definition 2.1, \(x_0 \in A \subseteq B\). This proves (a).

Suppose \(A \subseteq B\). This implies that \(A(x) \subseteq B(x)^C\). This proves (b).

Proposition 2.8 Let \((X, \tau)\) be a Neutrosophic topological space. Let \(A\) be a Neutrosophic subset of a Neutrosophic topological space \(X, \tau\).

**Proof:** Let \(x_0 \in (\alpha, \beta, \gamma) \subseteq A\).

Suppose \(x_0 \in (\alpha, \beta, \gamma) \subseteq A\) if and only if \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). This implies that \(A(x) \supseteq (\alpha, \beta, \gamma)^C\). This shows that \(\{\alpha, \beta, \gamma\} \in \text{NSCIA}\). That shows \(\text{NSCIA} \subseteq \text{NCl}\).

Therefore \(\text{NSCIA} \subseteq \text{NCl}\).

Theorem 2.9 Let \((X, \tau)\) be a Neutrosophic topological space. If a Neutrosophic subset \(A\) is Neutrosophic open, then \(\text{NIntNCIA} = \text{NSCIA}\).

**Proof:** By using Proposition 2.8, it suffices to show that \(\text{NSCIA} \subseteq \text{NIntNCIA}\). Let \(x_0 \in (\alpha, \beta, \gamma) \subseteq \text{NSCIA}\).
Then \( x_{\alpha, \beta, \gamma} \) \( q \) \( (\text{NIntNCI} A)^C \). By using Proposition 2.4, \( x_{\alpha, \beta, \gamma} \) \( q \) \( (\text{NCICI} A^T) \). By using Proposition 2.5, \( \text{NCI NInt} A^T = \text{NCI Nnt} \) \( C \). This can be written as \( \text{NCI Nnt} A^C \subseteq \text{NCI Nnt} (\text{NCI Nnt} A^T) \). By using Definition 1. 19, \( \text{NCI Nnt} A^C \) is Neutrosophic semi open. By using Proposition 2.6, \( A \) \( q \) \( (\text{NCICI} A^T) \), that implies \( x_{\alpha, \beta, \gamma} \) \( \not\in \) \( \text{NCSCI} \). That shows \( \text{NSCIA} \subseteq \text{NCI Nnt} \). Therefore \( \text{NIntNCl} = \text{NCSCI} \).

**Definition 2.10** A Neutrosophic subset \( A \) of a Neutrosophic topological Space \( (X, \tau) \) is Neutrosophic feebly open if there is a Neutrosophic open set \( U \) in \( X \) such that \( U \subseteq A \subseteq \text{NntNCI} \).

**Proposition 2.11** A Neutrosophic subset \( A \) is Neutrosophic feebly open if \( \text{Int} \) \( \text{A} \subseteq \text{NntNCI} \).

**Proof:** If \( A \) is Neutrosophic feebly open, then by Definition 2.10, there is a Neutrosophic open set \( U \) such that \( U \subseteq A \subseteq \text{NntNCI} \). Now \( U \subseteq A \subseteq \text{NntNCI} \). Since \( U \) is Neutrosophic open, \( U = \text{Nnt} U \subseteq \text{Nnt} A \), it follows that \( \text{NCI} U \subseteq \text{NCI} A \). This implies that \( \text{NntNCI} U \subseteq \text{NntNCI} \). Hence \( U \subseteq A \subseteq \text{NntNCI} \). Therefore \( A \) is Neutrosophic feebly open.

**Example 2.12** The following example is one of the Neutrosophic feebly-open set.

Let \( X = \{x\} \) and \( \tau = (\{0, 0.1\}, \{1, 1.0\}, \{0.1, 0.2, 0.6\}, \{0.7, 0.4, 0.8\}, \{0.7, 0.4, 0.6\}, \{0.5, 0.2, 0.8\}) \). Then \( (X, \tau) \) is a Neutrosophic topological space.

Let \( A = (0.7, 0.6, 0.8) \). Then \( \text{NInt} A = (0.7, 0.4, 0.8) \). The corresponding Neutrosophic closed sets \( \tau^T = (1.0, 0.0) \), \( \{0.1, 1.1\}, \{0.6, 0.2, 0.5\} \), \( \{0.8, 0.4, 0.7\} \), \( \{0.6, 0.2, 0.5\} \). Now \( \text{NCICI} A = (0.7, 0.4, 0.8) \). \( A \subseteq \text{NntNCI} \). Hence \( A = (0.7, 0.6, 0.8) \) is Neutrosophic feebly open set.

**Proposition 2.13** Every Neutrosophic open set is Neutrosophic feebly-open set.

**Proof:** Let \( A \) be a Neutrosophic open set in \( X \). Then \( A = \text{Nnt} A \). Since \( A \subseteq \text{NCI} A \), \( A \subseteq \text{NCICI} \). Since \( \text{Nnt} A \subseteq \text{NntNCI} \), \( A \subseteq \text{NntNCI} \). Hence \( A \) is Neutrosophic feebly open set.

**Example 2.14** The following example shows that the reverse implication is not true. That is, \( A \) is Neutrosophic feebly open set but \( A \) is not a Neutrosophic open set. Let \( X = \{x\} \) and \( \tau = (\{0.0, 0.1\}, \{1.1, 1.0\}, \{0.5, 0.2, 0.6\}, \{0.4, 0.6, 0.8\}, \{0.5, 0.6, 0.4\}, \{0.4, 0.5, 0.8\}) \). Then \( (X, \tau) \) is a Neutrosophic topological space. Let \( A = (0.7, 0.6, 0.8) \) is not a Neutrosophic open set. Then \( \text{NInt} A = (0.7, 0.4, 0.8) \). The corresponding Neutrosophic closed sets \( \tau^T = (1.0, 0.0) \), \( (0.1, 1.1) \), \( (0.6, 0.2, 0.5) \), \( (0.8, 0.4, 0.7) \), \( (0.6, 0.2, 0.5) \). Now \( \text{NCICI} A = (0.8, 0.5, 0.4) \). \( A \subseteq \text{NntNCI} \). Hence \( A = (0.5, 0.5, 0.4) \). This implies that \( A \subseteq \text{NntNCI} \). Hence \( A \) is Neutrosophic feebly open set.

**Proposition 2.15** If \( A \) and \( B \) be two Neutrosophic feebly open set then \( A \cup B \) is Neutrosophic feebly open set.

**Proof:** If \( A \) and \( B \) be two Neutrosophic feebly open set. Then by Proposition 2.11, \( A \subseteq \text{NntNCI} \) and \( B \subseteq \text{NntNCI} \). Now \( \text{AUB} \subseteq \text{NntNCI} \cup \text{NntNCI} \). Since \( \text{NCI} U \subseteq \text{NCI} A \), \( \text{NCI} U \subseteq \text{NCI} B \), \( \text{NCI} U \subseteq \text{NCI} \). Hence \( A \subseteq \text{NntNCI} \). Thus \( A \subseteq \text{NntNCI} \). Therefore \( A \) is Neutrosophic feebly open set.

**Proposition 2.16** Arbitrary union of Neutrosophic feebly open sets is a Neutrosophic feebly open set.

**Proof:** Let \( \{A_\alpha\} \) be a collection of Neutrosophic feebly open sets of a Neutrosophic topological space \( X \). Then by Definition 2.10, there exists a Neutrosophic open set \( V_\alpha \) such that \( V_\alpha \subseteq A_\alpha \subseteq \text{NSCI} \) for each \( \alpha \). Now, \( U \subseteq \cup A_\alpha \subseteq \text{UNSCI} \). By Proposition 6.5 in [6], \( U \cup \alpha \subseteq \cup A_\alpha \subseteq \text{NSCI} \cup V_\alpha \). Hence \( U \) is a Neutrosophic feebly open set.

**Example 2.17** Intersection of any two Neutrosophic feebly open sets need not be a Neutrosophic feebly open set as shown by the following example.

Let \( X = \{x\} \) and \( \tau = \{0.0, 0.1\}, \{1.1, 1.0\}, \{0.5, 0.5, 0.4\}, \{0.4, 0.6, 0.8\}, \{0.5, 0.6, 0.4\}, \{0.4, 0.5, 0.8\}\). Then \( (X, \tau) \) is a Neutrosophic topological space. Let \( A = (0.5, 0.5, 0.4) \). Then \( \text{NInt} A = (0.5, 0.5, 0.4) \). The corresponding Neutrosophic closed sets \( \tau^T = \{0.1, 1.0\}, \{0.1, 1.1\}, \{0.4, 0.6, 0.4\}\). Now \( \text{NCICI} A = (0.8, 0.5, 0.4) \). \( A \subseteq \text{NntNCI} \). Hence \( A = (0.5, 0.5, 0.4) \) is Neutrosophic feebly open set.

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\(\{x, 0.4, 0.5, 0.8\}\) \(\cap\) \(\NClNInt(A \cap B) = \{x, 0.4, 0.5, 0.8\}\). \(A \cap B \subseteq \NClNInt(A \cap B)\). Hence \(A \cap B\) \(= \{x, 0.5, 0.5, 0.6\}\) is not a Neutrosophic feebly open set.

**Example 2.18** The following example shows that Intersection of a Neutrosophic feebly open set with a Neutrosophic open set may fail to be a Neutrosophic feebly open set.

Let \(X = \{(x, 0.4, 0.6, 0.8)\}\), \(\tau = \{(x, 0.4, 0.6, 0.8)\}\). Then \(\NIntNCl A = (x, 0.4, 0.6, 0.8)\) \(\subseteq \{x, 0.5, 0.5, 0.6\}\). Hence \((X, \tau)\) is a Neutrosophic topological space. Let \(A = \{(x, 0.8, 0.6, 0.5)\}\). Then \(\NClNInt A = (x, 0.7, 0.5, 0.6)\). This implies that \(A \subseteq \NClNInt A\). Hence \(A = \{(x, 0.8, 0.6, 0.5)\}\) is a Neutrosophic feebly open subset of \(A\) and \(A \subseteq \NClNInt A\). Hence \(A\) is Neutrosophic feebly open.

**Proposition 2.19** The Neutrosophic closure of a Neutrosophic open set is a Neutrosophic feebly open set.

**Proof:** Let \(A\) be a Neutrosophic open set in \(X\). Then \(A = \NIntNCl A\) \(\subseteq \NClNInt A\). Take complement on both sides. Then \(\NIntNCl A \cup \NIntNCl A \subseteq \NClNInt A\) \(\cup \NClNInt A\). Hence \(A\) is Neutrosophic feebly open.

**Definition 3.1** A Neutrosophic subset \(A\) of a Neutrosophic topological space \((X, \tau)\) is Neutrosophic feebly closed if there is a Neutrosophic closed set \(U\) in \(X\) such that \(\NInt NCl U \subseteq A \subseteq U\).

**Proposition 3.2** A Neutrosophic subset \(A\) is Neutrosophic feebly closed if \(\NClNInt NCl A \subseteq A\).

**Proof:** Necessity: If \(A\) is Neutrosophic feebly closed, then by Definition 3.1, there is a Neutrosophic closed set \(U\) such that \(\NInt NCl U \subseteq A \subseteq U\). Now \(\NClNInt NCl A \subseteq U\). Hence \(\NClNInt NCl A \subseteq \NClNInt NCl U \subseteq A\). Therefore \(A\) is a Neutrosophic feebly closed set.

**Proposition 3.3** Let \(A\) be a Neutrosophic feebly closed set if \(A^C\) is Neutrosophic feebly open set.

**Example 3.4** The following example is one of the Neutrosophic feebly closed set.

Let \(X = \{(x, 0.4, 0.5, 0.8)\}\) and \(\tau = \{(x, 0.4, 0.5, 0.8)\}\). Then \((X, \tau)\) is a Neutrosophic topological space. Let \(A = \{(x, 0.8, 0.4, 0.5)\}\). Then \(\NInt NCl A = \{(x, 0.8, 0.4, 0.5)\}\). The corresponding Neutrosophic closed set \(U = \{(x, 1, 0.0)\}\). Hence \(A \subseteq U\). Therefore \(A\) is a Neutrosophic feebly closed set.

**Theorem 2.21** Let \((X, \tau)\) and \((Y, \sigma)\) be any two Neutrosophic topological spaces such that \(X\) is product related to \(Y\). Then the product \(A_1 \times A_2\) of a Neutrosophic feebly open set \(A_1\) of \(X\) and a Neutrosophic feebly open set \(A_2\) of \(Y\) is a Neutrosophic feebly open set of the Neutrosophic product space \(X \times Y\).

**Proof:** Let \(A_1\) be a Neutrosophic feebly open subset of \(X\) and \(A_2\) be a Neutrosophic feebly open subset of \(Y\). Then by using Proposition 2.11, we have \(A_1 \subseteq \NInt NCl NInt A_1\) and \(A_2 \subseteq \NInt NCl NInt A_2\). By using Theorem 2.17 in [6], implies that \(A_1 \times A_2 \subseteq \NInt NCl NInt (A_1 \times A_2)\). By using Proposition 2.11, \(A_1 \times A_2\) is a Neutrosophic feebly open set of the Neutrosophic product space \(X \times Y\).

**III. NEUTROSOPHIC FEEBLY CLOSED SET**

In this section, the concept of Neutrosophic feebly closed set is introduced.

**Proposition 3.5** Every Neutrosophic closed set is a Neutrosophic feebly closed set.

**Proof:** Let \(A\) be a Neutrosophic closed set in \(X\). Then \(A = \NCl A\). Since \(\NInt A = A\), \(\NInt NCl A \subseteq A\). That implies \(\NCl NInt NCl A \subseteq A\). Thus \(A\) is a Neutrosophic feebly closed set.
Example 3.6 The following example shows that the reverse implication is not true. That is, A is Neutrosophic feebly closed set, but A is not a Neutrosophic closed set. Let \( X = \{x\} \) and \( \tau = \{(x,0.0,1), (x,1.1,0), (x,0.2,0.5,0.7), (x,0.7,0.5,0.4), (x,0.0,0.5,0.6)\} \). Then \( (X, \tau) \) is a Neutrosophic topological space. The corresponding Neutrosophic closed sets \( \tau = \{(x,1.1,0), (x,0.2,0.5,0.7), (x,0.5,0.5,0.7)\} \). Then \( NCl(X) = \{(x,0.0,0.5,0.6)\} \). Hence A is Neutrosophic feebly closed set.

Proposition 3.7 If A and B be two Neutrosophic feebly closed set then \( A \cap B \) is Neutrosophic feebly closed set.

Proof: If A and B be two Neutrosophic feebly closed set. Then by Proposition 3.2, NIntNClNClA \( \leq A \) and NClNIntNClB \( \leq B \). (NIntNClNClA) \( \cap (NClNIntNClB) \leq A \cap B \). By Proposition 1.18, NCl(NIntNClA \( \cap NClB \)) \( \leq A \cap B \). Again by Proposition 1.18, NIntNClNClA \( \cap NClB \) \( \leq A \cap B \). Hence \( A \cap B \) is Neutrosophic feebly closed set.

Proposition 3.8 Finite intersection of a Neutrosophic feebly closed sets is a Neutrosophic feebly closed set.

Proof: Let \( \{A_i\} \) be a collection of Neutrosophic feebly closed sets of a Neutrosophic topological space \( X \). Then by Definition 3.1, there exists a Neutrosophic closed set \( V_i \) such that \( NInt V_i \leq A_i \leq V_i \) for each \( i \). Now, \( \cap NInt V_i \leq \cap A_i \leq \cap V_i \). By Theorem 3.3 in [6], \( NInt(\cap V_i) \leq \cap A_i \leq \cap V_i \). Hence \( \cap A_i \) is a Neutrosophic feebly closed set.

Example 3.9 Union of any two Neutrosophic feebly closed sets need not be a Neutrosophic feebly closed set as shown by the following example.

Let \( X = \{x\} \) and \( \tau = \{(x,0.0,1), (x,1.1,0), (x,0.2,0.5,0.7), (x,0.8,0.4,0.5), (x,0.2,0.4,0.7), (x,0.8,0.5,0.5)\} \). Then \( (X, \tau) \) is a Neutrosophic topological space. The corresponding Neutrosophic closed sets \( \tau = \{(x,1.1,0), (x,0.2,0.5,0.7), (x,0.5,0.5,0.7)\} \). Then \( NCl(A) = \{(x,0.0,0.5,0.6)\} \). Now \( NIntNClA \leq A \). Hence A is Neutrosophic feebly closed set. Let \( B = \{(x,0.4,0.5,0.6)\} \). Then \( NIntNClB \leq B \). Now \( NIntNClB \leq B \). Hence B is Neutrosophic feebly closed set. AUB = \{(x,0.5,0.5,0.6)\}. Then \( NInt(AUB) \leq \{(x,0.5,0.5,0.6)\} \). This implies that \( NIntNCl(A \cup B) \leq A \cup B \). Hence AUB = \{(x,0.5,0.5,0.6)\} is not a Neutrosophic feebly closed set.

Example 3.10 Union of a Neutrosophic feebly closed set with a Neutrosophic open set may fail to be a Neutrosophic feebly closed set as shown by the following example.

Let \( X = \{x\} \) and \( \tau = \{(x,0.0,1), (x,1.1,0), (x,0.2,0.4,0.3), (x,0.7,0.5,0.6), (x,0.7,0.5,0.3), (x,0.2,0.4,0.6)\} \). Then \( (X, \tau) \) is a Neutrosophic topological space. Let \( A = \{(x,0.8,0.6,0.5)\} \). Then \( NIntA \leq \{(x,0.7,0.5,0.6)\} \). The corresponding Neutrosophic closed sets \( \tau = \{(x,1.0,0), (x,0.1), (x,0.3,0.4,0.2), (x,0.6,0.5,0.7), (x,0.3,0.5,0.7), (x,0.6,0.4,0.2)\} \). Then \( NIntNClA = \{(x,1.0,0)\} \). \( A \subset NIntA \). \( A \cap NIntA \leq A \). Hence \( A \) is a Neutrosophic feebly closed set.

Proposition 3.11 The neutrosophic interior of a neutrosophic closed set is a neutrosophic feebly closed set.

Proof: Let A be a Neutrosophic closed set in X. Then \( A \) is a Neutrosophic feebly closed set. By Proposition 2.15, \( NIntA \leq A \), NInt NCl A \( \leq A \), NClNIntNClA \( \leq A \). Hence A is Neutrosophic feebly closed set.

IV. NEUTROSOPHIC FEEBLY CONTINUOUS FUNCTIONS IN NEUTROSOPHIC TOPOLOGICAL SPACES

We shall now consider some possible definitions for neutrosophic feebly continuous functions.

Definition 4.1 [15] Let \( (X, \tau) \) and \( (Y, \sigma) \) be two NTSs. Then a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called neutrosophic continuous ( in short N-continuous ) function if the inverse image of every neutrosophic open set in \( (Y, \sigma) \) is neutrosophic open set in \( (X, \tau) \).

Definition 4.2 Let \( (X, \tau) \) and \( (Y, \sigma) \) be two neutrosophic topological space. Then a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called neutrosophic feebly continuous ( in short NF-continuous ) function if the inverse image of every neutrosophic open set in \( (Y, \sigma) \) is neutrosophic feebly open set in \( (X, \tau) \).

Theorem 4.3 Every N-continuous function is NF-continuous function.
Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be N-continuous function. Let \( V \) be a neutrosophic open set in \((Y, \sigma)\). Then \( f^{-1}(V) \) is neutrosophic open set in \((X, \tau)\). Since every neutrosophic open set is neutrosophic feebly open set, \( f^{-1}(V) \) is neutrosophic feebly open set in \((X, \tau)\). Hence \( f \) is neutrosophic feebly-continuous function.

Remark 4.4 The converse of the above theorem is need not be true as shown by following example.

Example 4.5 Let \( X = Y = \{ a, b, c \} \). Define the neutrosophic sets as follows:

\[
A = \{ (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) \}
\]

\[
B = \{ (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) \}
\]

\[
C = \{ (0.5, 0.4, 0.2), (0.2, 0.3, 0.1), (0.6, 0.9, 0.8) \}
\]

and

\[
D = \{ (0.4, 0.2, 0.5), (0.1, 0.1, 0.2), (0.5, 0.6, 0.8) \}.
\]

Now \( T = \{ 0_x, A, B, 1_y \} \) and \( S = \{ 0_x, C, D, 1_y \} \) are neutrosophic topologies on \( X \). Thus \((X, \tau)\) and \((Y, \sigma)\) are NTSs. Also we define \( f : (X, \tau) \rightarrow (Y, \sigma) \) as follows:

\[
f(a) = b, f(b) = a, f(c) = c.
\]

Clearly \( f \) is NF-continuous function. But \( f \) is not N-continuous function. Since \( E = \{ (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) \} \) is a neutrosophic open in \((Y, \sigma)\), \( f^{-1}(E) \) is not neutrosophic open set in \((X, \tau)\).

Definition 4.6 Let \((X, \tau)\) be NTS and \( A = \{ x, B_\alpha(x), \sigma_\alpha(x), \gamma_\alpha(x) \} \) be a NF in \( X \). Then the neutrosophic feebly-closure and neutrosophic feebly-interior of \( A \) are defined by

\[
\text{NFCl}(A) = \cap \{ K : K \text{ is a NFC set in } X \text{ and } A \subseteq K \}
\]

\[
\text{NFInt}(A) = \cup \{ G : G \text{ is a NFO set in } X \text{ and } G \subseteq A \}.
\]