On Neutrosophic Supra Pre-Continuous Functions in Neutrosophic Topological Spaces

M. Parimala¹, M. Karthika², R. Dhavaseelan³, S. Jafari⁴

¹Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam-638401, Tamil Nadu, India.
Email: rishwanthpari@gmail.com.

²Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam-638401, Tamil Nadu, India.
Email: karthikamuthusamy1991@gmail.com.

³Department of Mathematics, Sona College of Technology, Salem-636005, Tamil Nadu, India.
Email: dhavaseelan.r@gmail.com.

⁴College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark.
Email: jafaripersia@gmail.com.

ABSTRACT
In this paper, we introduce and investigate a new class of sets and functions between topological space called neutrosophic supra pre-continuous functions. Furthermore, the concepts of neutrosophic supra pre-open maps and neutrosophic supra pre-closed maps in terms of neutrosophic supra pre-open sets and neutrosophic supra pre-closed sets, respectively, are introduced and several properties of them are investigated.

KEYWORDS: Neutrosophic supra topological spaces, neutrosophic supra pre-open sets and neutrosophic supra pre-continuous maps.

1 INTRODUCTION AND PRELIMINARIES

Intuitionistic fuzzy set is defined by Atanassov (1986) as a generalization of the concept of fuzzy set given by Zadeh (1965). Using the notation of intuitionistic fuzzy sets, Çoker

In this paper, we introduce and investigate a new class of sets and functions between topological space called neutrosophic supra semi-open set and neutrosophic supra semi-open continuous functions respectively.

**Definition 1.** Let $T, I, F$ be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$

\[
sup_I = i_{sup}, inf_I = i_{inf}
\]

\[
sup_F = f_{sup}, inf_F = f_{inf}
\]

\[
n - sup = t_{sup} + i_{sup} + f_{sup}
\]

\[
n - inf = t_{inf} + i_{inf} + f_{inf}.
\]

$T, I, F$ are neutrosophic components.

**Definition 2.** Let $X$ be a nonempty fixed set. A neutrosophic set [NS for short] $A$ is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set $A$.

**Remark 1.**

1. A neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A \rangle$ in $]0^-, 1^+[$ on $X$.

2. For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ for the neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$.

**Definition 3.** Let $X$ be a nonempty set and the neutrosophic sets $A$ and $B$ in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$, $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(c) \( \bar{A} = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \} \); [Complement of A]

(d) \( A \cap B = \{ \langle x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \land \sigma_B(x), \gamma_A(x) \lor \gamma_B(x) \rangle : x \in X \} \);

(e) \( A \cup B = \{ \langle x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \gamma_A(x) \land \gamma_B(x) \rangle : x \in X \} \);

(f) \( \{ \} A = \{ \langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \} \);

(g) \( \emptyset A = \{ \langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \} \).

**Definition 4.** Let \( \{ A_i : i \in J \} \) be an arbitrary family of neutrosophic sets in X. Then

(a) \( \bigcap A_i = \{ \langle x, \land \mu_{A_i}(x), \land \sigma_{A_i}(x), \lor \gamma_{A_i}(x) \rangle : x \in X \} \); 

(b) \( \bigcup A_i = \{ \langle x, \lor \mu_{A_i}(x), \lor \sigma_{A_i}(x), \land \gamma_{A_i}(x) \rangle : x \in X \} \).

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets \( 0_N \) and \( 1_N \) in \( X \) as follows:

**Definition 5.** (Dhavaseelan & Jafari, in press) \( 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \} \) and \( 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \} \).

**Definition 6.** (Dhavaseelan & Jafari, in press) A neutrosophic topology (NT) on a nonempty set \( X \) is a family \( T \) of neutrosophic sets in \( X \) satisfying the following axioms:

(i) \( 0_N, 1_N \in T \),

(ii) \( G_1 \cap G_2 \in T \) for any \( G_1, G_2 \in T \),

(iii) \( \cup G_i \in T \) for arbitrary family \( \{ G_i : i \in \Lambda \} \subseteq T \).

In this case the ordered pair \((X, T)\) or simply \( X \) is called a neutrosophic topological space (NTS) and each neutrosophic set in \( T \) is called a neutrosophic open set (NOS). The complement \( \overline{A} \) of a NOS \( A \) in \( X \) is called a neutrosophic closed set (NCS) in \( X \).

**Definition 7.** (Dhavaseelan & Jafari, in press) Let \( A \) be a neutrosophic set in a neutrosophic topological space \( X \). Then

\[ \text{Nint}(A) = \bigcup \{ G \mid G \text{ is a neutrosophic open set in } X \text{ and } G \subseteq A \} \]

is called the neutrosophic interior of \( A \);

\[ \text{Ncl}(A) = \bigcap \{ G \mid G \text{ is a neutrosophic closed set in } X \text{ and } G \supseteq A \} \]

is called the neutrosophic closure of \( A \).

**Definition 8.** Let \( X \) be a nonempty set. If \( r, t, s \) be real standard or non standard subsets of \( [0^-, 1^+] \) then the neutrosophic set \( x_{r,t,s} \) is called a neutrosophic point (in short NP) in \( X \) given by

\[ x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases} \]
for $x_p \in X$ is called the support of $x_{r,t,s}$, where $r$ denotes the degree of membership value, $t$ denotes the degree of indeterminacy, and $s$ is the degree of non-membership value of $x_{r,t,s}$.

Now we shall define the image and preimage of neutrosophic sets. Let $X$ and $Y$ be two nonempty sets and $f : X \rightarrow Y$ be a function.

**Definition 9.** (Dhavaseelan & Jafari, in press)

(a) If $B = \{ (y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y \}$ is a neutrosophic set in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is the neutrosophic set in $X$ defined by $f^{-1}(B) = \{ (x, f^{-1}(\mu_B(x)), f^{-1}(\sigma_B(x)), f^{-1}(\gamma_B(x)) : x \in X \}$.

(b) If $A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \}$ is a neutrosophic set in $X$, then the image of $A$ under $f$, denoted by $f(A)$, is the neutrosophic set in $Y$ defined by $f(A) = \{ (y, f(\mu_A(y)), f(\sigma_A(y)), (1 - f(1 - \gamma_A)(y)) : y \in Y \}$, where

$$
\begin{align*}
    f(\mu_A)(y) &= \begin{cases} 
        \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\
        0, & \text{otherwise},
    \end{cases} \\
    f(\sigma_A)(y) &= \begin{cases} 
        \sup_{x \in f^{-1}(y)} \sigma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\
        0, & \text{otherwise},
    \end{cases} \\
    (1 - f(1 - \gamma_A))(y) &= \begin{cases} 
        \inf_{x \in f^{-1}(y)} \gamma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\
        1, & \text{otherwise},
    \end{cases}
\end{align*}
$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $1 - f(1 - \gamma_A)$.

**Corollary 1.** (Dhavaseelan & Jafari, in press) Let $A_i (i \in J)$ be neutrosophic sets in $X$, $B_i (i \in K)$ be neutrosophic sets in $Y$ and $f : X \rightarrow Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,
(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
(c) $A \subseteq f^{-1}(f(A)) \{ \text{If } f \text{ is injective, then } A = f^{-1}(f(A)) \}$,
(d) $f(f^{-1}(B)) \subseteq B \{ \text{If } f \text{ is surjective, then } f(f^{-1}(B)) = B \}$,
(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,
(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,
(g) $f(\bigcup A_i) = \bigcup f(A_i)$,
(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i) \{ \text{If } f \text{ is injective, then } f(\bigcap A_i) = \bigcap f(A_i) \}$. 

374
(i) \( f^{-1}(1_N) = 1_N \),
(j) \( f^{-1}(0_N) = 0_N \),
(k) \( f(1_N) = 1_N \), if \( f \) is surjective
(l) \( f(0_N) = 0_N \),
\( \overline{f(A)} \subseteq f(A) \), if \( f \) is surjective,
\( f^{-1}(B) = \overline{f^{-1}(B)} \).

2 NEUTROSOPHIC SUPRA PRE-OPEN SET.

In this section, we introduce a new class of open sets called neutrosophic supra pre-open sets and study some of their basic properties.

**Definition 2.1.** Let \((X, \tau)\) be an neutrosophic supra topological space. A set \(A\) is called an neutrosophic supra pre-open set (briefly NSPOS) if \(A \subseteq s-Nint(s-Ncl(A))\). The complement of an neutrosophic supra pre-open set is called an neutrosophic supra pre-closed set (briefly NSPCS).

**Theorem 2.2.** Every neutrosophic supra-open set is neutrosophic supra pre-open.

**Proof.** Let \(A\) be an neutrosophic supra-open set in \((X, \tau)\). Then \(A \subseteq s-Nint(A)\), we get \(A \subseteq s-Nint(s-Ncl(A))\) then \(s-Nint(A) \subseteq s-Nint(s-Ncl(A))\). Hence \(A\) is neutrosophic supra pre-open in \((X, \tau)\).

The converse of the above theorem need not be true as shown by the following example.

**Example 2.3.**

Let
\[ X = \{a, b\}, A = \{x, (0.5, 0.2), (0.5, 0.2), (0.3, 0.4)\}, B = \{x, (0.3, 0.4), (0.3, 0.4), (0.6, 0.5)\} \]
and \( C = \{x, (0.3, 0.4), (0.3, 0.4), (0.2, 0.5)\}, \tau = \{0, 1, A, B, A \cup B\} \). Then \(C\) is called neutrosophic supra pre-open set but it is not neutrosophic supra-open set.

**Theorem 2.4.** Every neutrosophic supra \(\alpha\)-open set is neutrosophic supra pre-open

**Proof.** Let \(A\) be an neutrosophic supra \(\alpha\)-open set in \((X, \tau)\). Then \(A \subseteq s-Nint(s-Ncl(s-Nint(A)))\), it is obvious that \(s-Nint(s-Ncl(s-Nint(A))) \subseteq s-Nint(s-Ncl(A))\) and \(A \subseteq s-Nint(s-Ncl(A))\). Hence \(A\) is neutrosophic supra pre-open in \((X, \tau)\).

The converse of the above theorem need not be true as shown by the following example.

**Example 2.5.**

Let \(X = \{a, b\}, A = \{x, (0.3, 0.5), (0.3, 0.5), (0.4, 0.5)\}\), \(B = \{x, (0.4, 0.3), (0.4, 0.3), (0.5, 0.4)\}\) and \(C = \{x, (0.4, 0.5), (0.4, 0.5), (0.5, 0.4)\}\),
\( \tau = \{0, 1, A, B, A \cup B\} \). Then \( C \) is called neutrosophic supra \( \text{pre-open} \) set but it is not neutrosophic supra \( \alpha \)-open set.

**Theorem 2.6.** Every neutrosophic supra \( \text{pre-open} \) set is neutrosophic supra \( \beta \)-open

**Proof.** Let \( A \) be an neutrosophic supra \( \text{pre-open} \) set in \((X, \tau)\). It is obvious that \( s-Nint(s-Ncl(A)) \subseteq s-Ncl(s-Nint(s-Ncl(A))) \). Then \( A \subseteq s-Nint(s-Ncl(A)) \). Hence \( A \subseteq s-Ncl(s-Nint(s-Ncl(A))) \).

The converse of the above theorem need not be true as shown by the following example.

**Example 2.7.**

Let \( X = \{a, b\}, A = \{x, (0.2, 0.3), (0.2, 0.3), (0.5, 0.3)\}, B = \{x, (0.1, 0.2), (0.1, 0.2), (0.6, 0.5)\} \) and \( C = \{x, (0.2, 0.3), (0.2, 0.3), (0.2, 0.3)\} \), \( \tau = \{0, 1, A, B, A \cup B\} \). Then \( C \) is called neutrosophic supra \( \text{pre-open} \) set but it is not neutrosophic supra \( \text{pre-open} \) set.

**Theorem 2.8.** Every neutrosophic supra \( \text{pre-open} \) set is neutrosophic supra \( \beta \)-open

**Proof.** Let \( A \) be an neutrosophic supra \( \text{pre-open} \) set in \((X, \tau)\). It is obvious that \( s-Nint(s-Ncl(A)) \subseteq s-Ncl(s-Nint(s-Ncl(A))) \). Then \( A \subseteq s-Nint(s-Ncl(A)) \). Hence \( A \subseteq s-Ncl(s-Nint(s-Ncl(A))) \).

The converse of the above theorem need not be true as shown by the following example.

**Example 2.9.**

Let \( X = \{a, b\}, A = \{x, (0.5, 0.2), (0.5, 0.4), (0.3, 0.4)\}, B = \{x, (0.3, 0.4), (0.3, 0.4), (0.6, 0.5)\} \) and \( C = \{x, (0.3, 0.4), (0.3, 0.4), (0.4, 0.4)\} \), \( \tau = \{0, 1, A, B, A \cup B\} \). Then \( C \) is called neutrosophic supra \( \text{pre-open} \) set but it is not neutrosophic supra \( \text{pre-open} \) set.

**Theorem 2.10.**

(i) Arbitrary union of neutrosophic supra \( \text{pre-open} \) sets is always neutrosophic supra \( \text{pre-open} \).

(ii) Finite intersection of neutrosophic supra \( \text{pre-open} \) sets may fail to be neutrosophic supra \( \text{pre-open} \).

**Proof.**

(i) Let \( A \) and \( B \) to be neutrosophic supra \( \text{pre-open} \) sets. Then \( A \subseteq s-Nint(s-Ncl(A)) \) and \( B \subseteq s-Nint(s-Ncl(B)) \). Then \( A \cup B \subseteq s-Nint(s-Ncl(A)) \). Therefore, \( A \cup B \) is neutrosophic supra \( \text{pre-open} \) sets.

(ii) Let \( X = \{a, b\}, A = \{x, (0.3, 0.4), (0.3, 0.4), (0.2, 0.5)\}, B = \{x, (0.3, 0.4), (0.3, 0.4), (0.4, 0.4)\} \)
and $\tau = \{0, 1, A, B, A \cup B\}$.

Hence $A$ and $B$ are neutrosophic supra pre-open but $A \cap B$ is not neutrosophic supra pre-open set.

**Theorem 2.11.**

(i) Arbitrary intersection of neutrosophic supra pre-closed sets is always neutrosophic supra pre-closed.

(ii) Finite union of neutrosophic supra pre-closed sets may fail to be neutrosophic supra pre-closed.

**Proof.**

(i) This proof immediately from Theorem 2.10

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.3 \rangle, \langle 0.2, 0.4 \rangle\}$, $B = \{x, \langle 0.5, 0.4 \rangle, \langle 0.5, 0.4 \rangle, \langle 0.4, 0.5 \rangle\}$ and $\tau = \{0, 1, A, B, A \cup B\}$. Hence $A$ and $B$ are neutrosophic supra pre-closed but $A \cup B$ is not neutrosophic supra pre-closed set.

**Definition 2.12.** The neutrosophic supra pre-closure of a set $A$, denoted by $s$-pre-Ncl$(A)$, is the intersection of neutrosophic supra pre-closed sets including $A$. The neutrosophic supra pre-interior of a set $A$, denoted by $s$-pre-Nint$(A)$, is the union of neutrosophic supra pre-open sets included in $A$.

**Remark 2.** It is clear that $s$-pre-Nint$(A)$ is an neutrosophic supra pre-open set and $s$-pre-Ncl$(A)$ is an neutrosophic supra pre-closed set.

**Theorem 2.14.**

(i) $A \subseteq s$-pre-Ncl$(A)$; and $A$ = $s$-pre-Ncl$(A)$ iff $A$ is an neutrosophic supra pre-closed set;

(ii) $s$-pre-Nint$(A) \subseteq A$; and $s$-pre-Nint$(A) = A$ iff $A$ is an neutrosophic supra pre-open set;

(iii) $X - s$-pre-Nint$(A) = s$-pre-Ncl$(X - A)$;

(iv) $X - s$-pre-Ncl$(A) = s$-pre-Nint$(X - A)$.

**Proof.** It is obvious.

**Theorem 2.15.**
(i) $s$-pre-$Nint(A) \cup s$-pre-$Nint(B) \subseteq s$-pre-$Nint(A \cup B)$;

(ii) $s$-pre-$Ncl(A \cap B) \subseteq s$-pre-$Ncl(A) \cap s$-pre-$Ncl(B)$.

Proof It is obvious.

The inclusions in (i) and (ii) in Theorem 2.15 can not replaced by equalities by let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.4 \rangle, \langle 0.3, 0.4 \rangle, \langle 0.2, 0.5 \rangle\}$, $B = \{x, \langle 0.3, 0.4 \rangle, \langle 0.3, 0.4 \rangle, \langle 0.4, 0.4 \rangle\}$ and $\tau = \{0, 1, 2, 3\}$, $A, B, A \cup B$, where $s$-pre-$Nint(A) = \{x, \langle 0.2, 0.5 \rangle, \langle 0.2, 0.5 \rangle, \langle 0.3, 0.4 \rangle\}$, $s$-pre-$Nint(B) = \{x, \langle 0.5, 0.4 \rangle, \langle 0.5, 0.4 \rangle, \langle 0.4, 0.5 \rangle\}$ and $s$-pre-$Nint(A \cup B) = \{x, \langle 0.5, 0.5 \rangle, \langle 0.5, 0.5 \rangle, \langle 0.3, 0.4 \rangle\}$.

Then $s$-pre-$Ncl(A) \cap s$-pre-$Ncl(B) = \{x, \langle 0.3, 0.4 \rangle, \langle 0.3, 0.4 \rangle, \langle 0.2, 0.5 \rangle\}$ and $s$-pre-$Ncl(A)$ = $s$-pre-$Ncl(B)$ = $1$, $\mu$.

Proposition 2.16.

(i) The intersection of an neutrosophic supra open set and an neutrosophic supra $pre$-open set is an neutrosophic supra $pre$-open set.

(ii) The intersection of an neutrosophic supra $\alpha$-open set and an neutrosophic supra $pre$-open set is an neutrosophic supra $pre$-open set.

3 NEUTROSOHIC SUPRA PRE-CONTINUOUS MAPPINGS.

In this section, we introduce a new type of continuous mapings called a neutrosophic supra $pre$-continuous mappings and obtain some of their properties and characterizations.

Definition 3.1. Let $(X, \tau)$ and $(Y, \sigma)$ be the two topological sets and $\mu$ be an associated neutrosophic supra topology with $\tau$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an neutrosophic supra $pre$-continuous mapping if the inverse image of each open set in $Y$ is an neutrosophic supra $pre$-open set in $X$.

Theorem 3.2. Every neutrosophic supra continuous map is an neutrosophic supra $pre$-continuous map.

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic continuous map and $A$ is an open set in $Y$. Then $f^{-1}(A)$ is an open set in $X$. Since $\mu$ is associated with $\tau$, then $\tau \subseteq \mu$. Therefore, $f^{-1}(A)$ is an neutrosophic supra open set in $X$ which is an neutrosophic supra $pre$-open set in $X$. Hence $f$ is an neutrosophic supra $pre$-continuous map.

The converse of the above theorem is not true as shown in the following example.

Example 3.3. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = \{\langle 0.5, 0.2 \rangle, \langle 0.5, 0.2 \rangle, \langle 0.3, 0.4 \rangle\}$, $B = \{\langle 0.3, 0.4 \rangle, \langle 0.3, 0.4 \rangle, \langle 0.6, 0.5 \rangle\}$,
Then the neutrosophic supra topology \( \sigma = \{0 \sim, 1 \sim, C, D, C \cup D\} \). Define a mapping \( f(X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The inverse image of the open set in \( Y \) is not an neutrosophic supra open in \( X \) but it is an neutrosophic supra \( pre \)-open. Then \( f \) is an neutrosophic supra \( pre \)-continuous map but may not be an neutrosophic supra continuous map.

The following example shows that neutrosophic supra \( pre \)-continuous map but may not be an neutrosophic supra \( \alpha \)-continuous map.

**Example 3.4.** Let \( X = \{a, b\} \) and \( Y = \{u, v\} \),
\[
\tau = \{0_{\sim}, 1_{\sim}, \{0.5, 0.2\}, \{05, 0.2\}, \{0.3, 0.4\}\},
\{0.3, 0.4\}, \{0.3, 0.4\}, \{06, 0.5\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\} \text{be a neutrosophic supra topology on } X.
\]
Then the neutrosophic supra topology \( \sigma \) on \( Y \) is defined as follows:
\[
\sigma = \{0_{\sim}, 1_{\sim}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.6, 0.5\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\}.
\]
Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The inverse image of the open set in \( Y \) is not an neutrosophic supra \( \alpha \)-open in \( X \) but it is an neutrosophic supra \( pre \)-open. Then \( f \) is an neutrosophic supra \( pre \)-continuous map but may not be an neutrosophic supra \( \alpha \)-continuous map.

The following example shows that neutrosophic supra \( b \)-continuous map but may not be an neutrosophic supra \( pre \)-continuous map.

**Example 3.5.** Let \( X = \{a, b\} \) and \( Y = \{u, v\} \),
\[
\tau = \{0_{\sim}, 1_{\sim}, \{0.5, 0.2\}, \{0.5, 0.2\}, \{0.3, 0.4\}\}, \{0.3, 0.4\}, \{0.3, 0.4\}, \{06, 0.5\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\} \text{be a neutrosophic supra topology on } X.
\]
Then the neutrosophic supra topology \( \sigma \) on \( Y \) is defined as follows:
\[
\sigma = \{0_{\sim}, 1_{\sim}, \{0.5, 0.2\}, \{0.5, 0.2\}, \{0.3, 0.4\}\}, \{0.3, 0.4\}, \{0.3, 0.4\}, \{06, 0.5\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\}.
\]
Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The inverse image of the open set in \( Y \) is not an neutrosophic supra \( pre \)-open in \( X \) but it is an neutrosophic supra \( b \)-open. Then \( f \) is an neutrosophic supra \( b \)-continuous map but may not be an neutrosophic supra \( pre \)-continuous map.

The following example shows that neutrosophic supra \( \beta \)-continuous map but may not be an neutrosophic supra \( pre \)-continuous map.

**Example 3.6.** Let \( X = \{a, b\} \) and \( Y = \{u, v\} \),
\[
\tau = \{0_{\sim}, 1_{\sim}, \{0.5, 0.2\}, \{0.5, 0.2\}, \{0.3, 0.4\}\}, \{0.3, 0.4\}, \{0.3, 0.4\}, \{06, 0.5\}\}, \{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}\} \text{be a neutrosophic supra topology on } X.
\]
Then the neutro-
Sophistic supra topology $\sigma$ on $Y$ is defined as follows:

$$
\sigma = \{0, \infty, 1, \infty, \{0.5, 0.2\}, \{0.5, 0.2\}, \{0.3, 0.4\}, \{0.3, 0.4\}, \{0.6, 0.5\}\},
\{0.5, 0.4\}, \{0.5, 0.4\}, \{0.3, 0.4\}.
$$

Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The inverse image of the open set in $Y$ is not an neutrosophic supra $\beta$-open but it is an neutrosophic supra $\beta$-open. Then $f$ is an neutrosophic supra $\beta$-continuous map but may not be an neutrosophic supra $\beta$-continuous map.

From the above discussion we have the following diagram in which the converses of the implications need not be true (cts. is the abbreviation of continuity).

**Theorem 3.7.** Let $(X, \tau)$ and $(Y, \sigma)$ be the two topological spaces and $\mu$ be an associated neutrosophic supra topology with $\tau$. Let $f$ be a map from $X$ into $Y$. Then the following are equivalent:

(i) $f$ is an neutrosophic supra $\beta$-continuous map.

(ii) The inverse image of a closed sets in $Y$ is an neutrosophic supra $\beta$-closed set in $X$;

(iii) $s$-$\text{pre} - \text{Ncl}(f^{-1}(A)) \subseteq f^{-1}(\text{Ncl}(A))$ for every set $A$ in $Y$;

(iv) $f(s$-$\text{pre} - \text{Ncl}(A)) \subseteq \text{Ncl}(f(A))$ for every set $A$ in $X$;

(v) $f^{-1}(\text{Nint}(B)) \subseteq s$-$\text{pre} - \text{Nint}(f^{-1}(B))$ for every set $B$ in $Y$.

**Proof.** (i)⇒(ii): Let $A$ be a closed set in $Y$, then $Y - A$ is open set in $Y$. Then $f^{-1}(Y - A) = X - f^{-1}(A)$ is $s$-$\text{pre}$-open set in $X$. It follows that $f^{-1}(A)$ is a supra $\beta$-closed subset of $X$.

(ii)⇒(iii): Let $A$ be any subset of $Y$. Since $\text{Ncl}(A)$ is closed in $Y$, then it follows that $f^{-1}(\text{Ncl}(A))$ is supra $\beta$-closed set in $X$. Therefore $s$-$\text{pre} - \text{Ncl}(f^{-1}(A)) \subseteq f^{-1}(\text{Ncl}(A))$.

(iii)⇒(iv): Let $A$ be any subset of $X$. By (iii) we have $f^{-1}(\text{Ncl}(f(A))) \supseteq s$-$\text{pre} - \text{Ncl}(f^{-1}(f(A))) \supseteq s$-$\text{pre} - \text{Ncl}(A)$ and hence $f(s$-$\text{pre} - \text{Ncl}(A)) \subseteq \text{Ncl}(f(A))$.

(iv)⇒(v): Let $B$ be any subset of $Y$. By (iv) we have $f^{-1}(s$-$\text{pre} - \text{Ncl}(X - f^{-1}(B))) \subseteq \text{Ncl}(f(X - f^{-1}(B)))$ and $f(X - s$-$\text{pre} - \text{Nint}(f^{-1}(B))) \subseteq \text{Ncl}(Y - B) = Y - \text{Nint}(B))$. Therefore we have $X - s$-$\text{pre} - \text{Nint}(f^{-1}(B)) \subseteq f^{-1}(Y - \text{Nint}(B))$ and hence $f^{-1}(\text{Nint}(B)) \subseteq s$-$\text{pre} - \text{Nint}(f^{-1}(B))$.

(v)⇒(i): Let $B$ be an open set in $Y$ and $f^{-1}(\text{Nint}(B)) \subseteq s$-$\text{pre} - \text{Nint}(f^{-1}(B))$, hence $f^{-1}(B) \subseteq s$-$\text{pre} - \text{Nint}(f^{-1}(B))$. Then $f^{-1}(B) = s$-$\text{pre} - \text{Nint}(f^{-1}(B))$. But, $s$-$\text{pre} - \text{Nint}(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = s$-$\text{pre} - \text{Nint}(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an neutrosophic supra $\beta$-open set in $Y$.

**Theorem 3.8.** If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $s$-$\text{pre}$-continuous and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is continuous, then $(g \circ f)$ is $s$-$\text{pre}$-continuous.

**Proof.** It is Obvious.

**Theorem 3.9.** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an neutrosophic $s$-$\text{pre}$-continuous map if one of the following holds:
(i) \( f^{-1}(s\text{-pre-Nint}(B)) \subseteq Nint(f^{-1}(B)) \) for every set \( B \) in \( Y \),

(ii) \( Ncl(f^{-1}(A)) \subseteq f^{-1}(s\text{-pre-Ncl}(B)) \) for every set \( B \) in \( Y \),

(iii) \( f(Ncl(A)) \subseteq s\text{-pre-Ncl}(f(B)) \) for every \( A \) in \( X \).

**Proof.** Let \( B \) be any open set of \( Y \), if the condition (i) is satisfied, then \( f^{-1}(s\text{-pre-Nint}(B)) \subseteq Nint(f^{-1}(B)) \). We get, \( f^{-1}(B) \subseteq Nint(f^{-1}(B)) \). Therefore \( f^{-1}(B) \) is a neutrosophic open set. Every neutrosophic open set is neutrosophic supra \( pre \)-open set. Hence \( f \) is an neutrosophic \( s\text{-pre-continuous} \).

If condition (ii) is satisfied, then we can easily prove that \( f \) is an neutrosophic supra \( pre \)-continuous.

Let condition (iii) is satisfied and \( B \) be any open set in \( Y \). Then \( f^{-1}(B) \) is a set in \( X \) and then we can easily prove that \( f \) is an neutrosophic \( s\text{-pre-continuous} \) function. If condition (iii) is satisfied, and \( B \) is any open set of \( Y \). Then \( f^{-1}(B) \) is a set in \( X \) and \( f(Ncl(f^{-1}(B))) \subseteq s\text{-pre-Ncl}(f(f^{-1}(B))) \). This implies \( f(Ncl(f^{-1}(B))) \subseteq s\text{-pre-Ncl}(B) \). This is nothing but condition (ii). Hence \( f \) is an neutrosophic \( s\text{-pre-continuous} \).

4 NEUTROSOPHIC SUPRA \( PRE \)-OPEN MAPS AND NEUTROSOPHIC SUPRA \( PRE \)-CLOSED MAPS.

**Definition 4.1.**

A map \( f : X \rightarrow Y \) is called neutrosophic supra \( pre \)-open (resp.neutrosophic supra \( pre \)-closed) if the image of each open (resp.closed) set in \( X \), is neutrosophic supra \( pre \)-open(resp.neutrosophic supra \( pre \)-closed)in \( Y \).

**Theorem 4.2.**

A map \( f : X \rightarrow Y \) is called an neutrosophic supra \( pre \)-open if and only if \( f(Nint(A)) \subseteq s\text{-pre-Nint}(A) \) for every set \( A \) in \( X \).

**Proof.** Suppose that \( f \) is an neutrosophic supra \( pre \)-open map. Since \( Nint(A) \subseteq f(A) \).

By hypothesis \( f(Nint(A)) \) is a neutrosophic supra \( pre \)-open set and \( s\text{-pre-Nint}(f(A)) \) is the largest neutrosophic supra \( pre \)-open set contained in \( f(A) \), then \( f(Nint(A)) \subseteq s\text{-pre-Nint}(f(A)) \)

Conversely, let \( A \) be a open set in \( X \). Then \( f(Nint(A)) \subseteq s\text{-pre-Nint}(f(A)) \). Since \( Nint(A) = A \), then \( f(A) \subseteq s\text{-pre-Nint}(f(A)) \). Therefore \( f(A) \) is an neutrosophic supra \( pre \)-open set in \( Y \) and \( f \) is an neutrosophic supra \( pre \)-open.

**Theorem 4.3.** A map \( f : X \rightarrow Y \) is called a neutrosophic supra \( pre \)-closed if and only if \( f(Ncl(A)) \subseteq s\text{-pre-Ncl}(A) \) for every set \( A \) in \( X \).

**Proof.** Suppose that \( f \) is an neutrosophic supra \( pre \)-closed map. Since for each set \( A \) in \( X \), \( Ncl(A) \) is closed set in \( X \), then \( f(Ncl(A)) \) is an neutrosophic supra \( pre \)-closed set in \( Y \).
Also, since \( f(A) \subseteq f(Ncl(A)) \), then \( s\text{-}pre\text{-}Ncl(f(A)) \subseteq f(Ncl(A)) \).

Conversely, let \( A \) be a closed set in \( X \). Since \( s\text{-}pre\text{-}Ncl(f(A)) \) is the smallest neutrosophic supra \( pre \)-closed set containing \( f(A) \), then \( f(A) \subseteq s\text{-}pre\text{-}Ncl(f(A)) \subseteq f(Ncl(A)) = f(A) \). Thus \( f(A) = s\text{-}pre\text{-}Ncl(f(A)) \). Hence \( f(A) \) is an neutrosophic supra \( pre \)-closed set in \( Y \).

Therefore \( f \) is a neutrosophic supra \( pre \)-closed map.

**Theorem 4.4.** Let \( f : X \rightarrow Y \) and \( g : y \rightarrow Z \) be two maps.

(i) If \( g \circ f \) is a neutrosophic supra \( pre \)-open and \( f \) is continuous surjective, then \( g \) is an neutrosophic semi-supra \( pre \)-open.

(ii) If \( g \circ f \) is open and \( g \) is a neutrosophic supra \( pre \)-continuous injective, then \( f \) is neutrosophic supra \( pre \)-open.

**Theorem 4.5** Let \( f : X \rightarrow Y \) be a map. Then the following are equivalent;

(i) \( f \) is a neutrosophic supra \( pre \)-open map;

(ii) \( f \) is a neutrosophic supra \( pre \)-closed map;

(iii) \( f \) is a neutrosophic supra \( pre \)-continuous map.

**Proof.** (i)\(\Rightarrow\) (ii). Suppose \( B \) is a closed set in \( X \). Then \( X - B \) is an open set in an open set in \( X \). By (1), \( f(X - B) \) is a neutrosophic supra \( pre \)-open set in \( X \). Since \( f \) is bijective, then \( f(X - B) = Y - f(B) \). Hence \( f(B) \) is a neutrosophic supra \( pre \)-closed set in \( Y \). Therefore \( f \) is an neutrosophic supra \( pre \)-closed map.

(ii)\(\Rightarrow\) (iii). Let \( f \) is a neutrosophic supra \( pre \)-closed map and \( B \) be closed set \( X \). Since \( f \) is bijective, then \( (f^{-1})^{-1}(B) = f(B) \) is a neutrosophic supra \( pre \)-closed set in \( Y \). By Theorem 3.7 \( f \) is a neutrosophic supra \( pre \)-continuous map.

(iii)\(\Rightarrow\) (i). Let \( A \) be an open set in \( X \). Since \( f^{-1} \) is a neutrosophic supra \( pre \)-continuous map, then \( (f^{-1})^{-1}(A) = f(A) \) is an neutrosophic supra \( pre \)-open set in \( Y \). Hence \( f \) is an neutrosophic supra \( pre \)-open.

**REFERENCES**


