On Neutrosophic Semi-Supra Open Set and Neutrosophic Semi-Supra Continuous Functions

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Abstract: In this paper, we introduce and investigate a new class of sets and functions between topological space called neutrosophic semi-supra open set and neutrosophic semi-supra open continuous functions respectively.

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1 Introduction and Preliminaries


The purpose of this paper is to introduce and investigate a new class of sets and functions between topological space called neutrosophic semi-supra open set and neutrosophic semi-supra open continuous functions, respectively.

Definition 1.1. Let \( T, I, F \) be real standard or non standard sub-
sets of \( [0^-,1^+] \), with \( sup_T = \sup_t, inf_T = \inf_t \)
\( sup_I = i_{sup}, inf_I = i_{inf} \)
\( sup_F = f_{sup}, inf_F = f_{inf} \)

\( n - sup = t_{sup} + i_{sup} + f_{sup} \)
\( n - inf = t_{inf} + i_{inf} + f_{inf} \).
\( T, I, F \) are neutrosophic components.

Definition 1.2. Let \( X \) be a nonempty fixed set. A neutrosophic set \( \{ x, \mu_A(x), \sigma_A(x), \gamma_A(x) : x \in X \} \) where \( \mu_A(x), \sigma_A(x) \) and \( \gamma_A(x) \) represent the degree of membership function (namely \( \mu_A(x) \)), the degree of indeterminacy (namely \( \sigma_A(x) \)) and the degree of nonmembership (namely \( \gamma_A(x) \)) respectively of each element \( x \in X \) to the set \( A \).

Remark 1.1. (1) A neutrosophic set \( A \) is an object having the form \( A = \{ x, \mu_A(x), \sigma_A(x), \gamma_A(x) : x \in X \} \) can be identified to an ordered triple \( \langle \mu_A, \sigma_A, \gamma_A \rangle \) in \([0^-,1^+] \) on \( X \).

(2) For the sake of simplicity, we shall use the symbol \( A = \langle \mu_A, \sigma_A, \gamma_A \rangle \) for the neutrosophic set \( A = \{ x, \mu_A(x), \sigma_A(x), \gamma_A(x) : x \in X \} \).

Definition 1.3. Let \( X \) be a nonempty set and the neutrosophic sets \( A \) and \( B \) in the form
\( A = \{ x, \mu_A(x), \sigma_A(x), \gamma_A(x) : x \in X \}, B = \{ x, \mu_B(x), \sigma_B(x), \gamma_B(x) : x \in X \} \). Then

(a) \( A \subseteq B \) iff \( \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \) and \( \gamma_A(x) \geq \gamma_B(x) \) for all \( x \in X \);
(b) \( A = B \) iff \( A \subseteq B \) and \( B \subseteq A \);
(c) \( \overline{A} = \{ x, \gamma_A(x), \sigma_A(x), \mu_A(x) : x \in X \}; \) [Complement of \( A \)]
(d) \( A \cap B = \{ x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \land \sigma_B(x), \gamma_A(x) \lor \gamma_B(x) : x \in X \}; \)
In this case the ordered pair $s$ for Definition 1.5.

Definition 1.4. Let $\{ A_i : i \in J \}$ be an arbitrary family of neutrosophic sets in $X$. Then

(a) $\bigcap A_i = \{ (x, \wedge \mu_{A_i}(x), \wedge \sigma_{A_i}(x), \wedge \nu_{A_i}(x)) : x \in X \};$

(b) $\bigcup A_i = \{ (x, \vee \mu_{A_i}(x), \vee \sigma_{A_i}(x), \vee \nu_{A_i}(x)) : x \in X \}.

Definition 1.6. [5] A neutrosophic topology (NT) on a nonempty set $X$ is a family $T$ of neutrosophic sets in $X$ satisfying the following axioms:

(i) $0_N, 1_N \in T,$

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T,$

(iii) $\bigcup G_i \in T$ for arbitrary family $\{ G_i \ | \ i \in \Lambda \} \subseteq T.$

In this case the ordered pair $(X, T)$ or simply $X$ is called a neutrosophic topological space (NTS) and each neutrosophic set in $T$ is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS $A$ in $X$ is called a neutrosophic closed set (NCS) in $X$.

Definition 1.7. [5] Let $A$ be a neutrosophic set in a neutrosophic topological space $X$. Then

$\text{Int}(A) = \bigcup \{ G \ | \ G$ is a neutrosophic open set in $X \text{ and } G \subseteq A \}$ is called the neutrosophic interior of $A$;

$\text{Cl}(A) = \bigcap \{ G \ | \ G$ is a neutrosophic closed set in $X \text{ and } G \supseteq A \}$ is called the neutrosophic closure of $A$.

Definition 1.8. Let $X$ be a nonempty set. If $r, t, s$ be real standard or non standard subsets of $\{ 0^-, 1^+ \}$, then the neutrosophic set $x_{r, t, s}$ is called a neutrosophic point (in short NP) in $X$ given by

$$x_{r, t, s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r, t, s}$, where $r$ denotes the degree of membership value, $t$ denotes the degree of indeterminacy and $s$ is the degree of non-membership value of $x_{r, t, s}$.

Now we shall define the image and preimage of neutrosophic sets. Let $X$ and $Y$ be two nonempty sets and $f : X \rightarrow Y$ be a function.

Definition 1.9. [5]

(a) If $B = \{ (y, \mu_y(y), \sigma_y(y), \nu_y(y)) : y \in Y \}$ is a neutrosophic set in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is the neutrosophic set in $X$ defined by

$$f^{-1}(B) = \{ x, f^{-1}(\mu_y(x)), f^{-1}(\sigma_y(x)), f^{-1}(\nu_y(x)) : x \in X \}.$$

(b) If $A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \}$ is a neutrosophic set in $X$, then the image of $A$ under $f$, denoted by $f(A)$, is the neutrosophic set in $Y$ defined by

$$f(A) = \{ (y, f(\mu_A(y)), f(\sigma_A(y)), f(\nu_A(y)), (1 - f(1 - \nu_A(y)) : y \in Y \}.$$

For the sake of simplicity, let us use the symbol $f_{-}((\gamma_A))$ for $1 - f(1 - \gamma_A)$.

Corollary 1.1. [5] Let $A, A_i (i \in J)$ be neutrosophic sets in $X, Y, B_i (i \in K)$ be neutrosophic sets in $Y$ and $f : X \rightarrow Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(c) $A \subseteq f^{-1}(f(A))$ \{ If $f$ is injective, then $A = f^{-1}(f(A))$ \},

(d) $f(f^{-1}(B)) \subseteq B$ \{ If $f$ is surjective, $f(f^{-1}(B)) = B$ \},

(e) $f^{-1}(\bigcup B_i) = \bigcup f^{-1}(B_i)$,

(f) $f^{-1}(\bigcap B_i) = \bigcap f^{-1}(B_i)$,

(g) $f(\bigcup A_i) = \bigcup f(A_i)$,

(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ \{ If $f$ is injective, then $f(\bigcap A_i) = \bigcap f(A_i)$ \},

(i) $f^{-1}(1_N) = 1_N$,

(j) $f^{-1}(0_N) = 0_N$,

(k) $f(1_N) = 1_N$ if $f$ is surjective

(l) $f(0_N) = 0_N$,

(m) $\overline{f(A)} \subseteq f(\overline{A})$, if $f$ is surjective,

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$. 
2 Main Results

Definition 2.1. A neutrosophic set A in a neutrosophic topological space \((X, T)\) is called

1) a neutrosophic semiopen set (NSOS) if \(A \subseteq Ncl(Nint(A))\).

2) a neutrosophic \(\alpha\) open set (NoOS) if \(A \subseteq Nint(Ncl(Nint(A)))\).

3) a neutrosophic preopen set (NPOS) if \(A \subseteq Ncl(Ncl(A))\).

4) a neutrosophic regular open set (NROS) if \(A = Nint(Ncl(A))\).

5) a neutrosophic semipreopen or \(\beta\) open set (N\(\beta\)OS) if \(A \subseteq Ncl(Nint(Ncl(A)))\).

A neutrosophic set \(A\) is called a neutrosophic semiclosed set, neutrosophic \(\alpha\) closed set, neutrosophic preclosed set, neutrosophic regular closed set and neutrosophic \(\beta\) closed set, respectively (NSCS, NoCS, NPCS, NRCS and \(N\beta\)CS, resp), if the complement of \(A\) is a neutrosophic semiclosed set, neutrosophic \(\alpha\)-open set, neutrosophic preopen set, neutrosophic regular open set, and neutrosophic \(\beta\)-open set, respectively.

Definition 2.2. Let \((X, T)\) ba a neutrosophic topological space. A neutrosophic set \(A\) is called a neutrosophic semi-supra open set (brieﬂy NSSOS) if \(A \subseteq s-Ncl(s-Nint(A))\). The complement of a neutrosophic semi-supra open set is called a neutrosophic semi-supra closed set.

Proposition 2.1. Every neutrosophic supra open set is neutrosophic semi-supra open set.

\(\square\)

Proof. Let \(A\) be a neutrosophic supra open set in \((X, T)\). Since \(A \subseteq s-Ncl(A)\), we get \(A \subseteq s-Ncl(s-Nint(A))\). Then \(s-Nint(A) \subseteq s-Ncl(s-Nint(A))\). Hence \(A \subseteq s-Ncl(s-Nint(A))\).

The converse of Proposition 2.1., need not be true as shown in Example 2.1.

Example 2.1. Let \(X = \{a, b\}\). Define the neutrosophic sets \(A, B\), and \(C\) in \(X\) as follows:
\[
A = \langle x, (a_1, b_1), (a_2, b_2), (a_3, b_3)\rangle, \quad B = \langle x, (a_4, b_4), (a_5, b_5)\rangle, \quad C = \langle x, (a_6, b_6), (a_7, b_7)\rangle.
\]
Then the families \(T = \{0_N, 1_N, A, B, A \cup B\}\) is neutrosophic topology on \(X\). Thus, \((X, T)\) is a neutrosophic topological space. Then \(C\) is called neutrosophic semi-supra open but not neutrosophic supra open set.

Proposition 2.2. Every neutrosophic \(\alpha\)-supra open is neutrosophic semi-supra open.

\(\square\)

Proof. Let \(A\) be a neutrosophic \(\alpha\)-supra open in \((X, T)\), then \(A \subseteq s-Nint(s-Ncl(s-Nint(A)))\). It is obvious that \(s-Nint(s-Ncl(s-Nint(A))) \subseteq s-Ncl(s-Nint(A))\). Hence \(A \subseteq s-Ncl(s-Nint(A))\).

The converse of Proposition 2.2., need not be true as shown in Example 2.2.

Example 2.2. Let \(X = \{a, b\}\). Define the neutrosophic sets \(A, B\) and \(C\) in \(X\) as follows:
\[
A = \langle x, (a_1, b_1), (a_2, b_2), (a_3, b_3)\rangle, \quad B = \langle x, (a_4, b_4), (a_5, b_5), (a_6, b_6)\rangle,
\]
and \(C = \langle x, (a_7, b_7), (a_8, b_8), (a_9, b_9)\rangle\). Then the families \(T = \{0_N, 1_N, A, B, A \cup B\}\) is neutrosophic topology on \(X\). Thus, \((X, T)\) is a neutrosophic topological space. Then \(C\) is called neutrosophic semi-supra open but not neutrosophic \(\alpha\)-supra open set.

Proposition 2.3. Every neutrosophic regular supra open set is neutrosophic semi-supra open set.

\(\square\)

Proof. Let \(A\) be a neutrosophic regular supra open set in \((X, T)\). Then \(A \subseteq s-Ncl(A)\). Hence \(A \subseteq s-Ncl(s-Nint(A))\).

The converse of Proposition 2.3., need not be true as shown in Example 2.3.

Example 2.3. Let \(X = \{a, b\}\). Define the neutrosophic sets \(A, B\) and \(C\) in \(X\) as follows:
\[
A = \langle x, (a_1, b_1), (a_2, b_2), (a_3, b_3)\rangle, \quad B = \langle x, (a_4, b_4), (a_5, b_5), (a_6, b_6)\rangle,
\]
and \(C = \langle x, (a_7, b_7), (a_8, b_8), (a_9, b_9)\rangle\). Then the families \(T = \{0_N, 1_N, A, B, A \cup B\}\) is neutrosophic topology on \(X\). Thus, \((X, T)\) is a neutrosophic topological space. Then \(C\) is called neutrosophic semi-supra open but not neutrosophic regular-supra open set.

Definition 2.3. The neutrosophic semi-supra closure of a set \(A\) is denoted by \(semi-s-Ncl(A) = \bigcup\{G : G\) is an neutrosophic semi-supra open set in \(X\) and \(G \subseteq A\}\) and the neutrosophic semi-supra interior of a set \(A\) is denoted by \(semi-s-Nint(A) = \bigcap\{G : G\) is a neutrosophic semi-supra closed set in \(X\) and \(G \subseteq A\}\).

Remark 2.1. It is clear that \(semi-s-Nint(A)\) is a neutrosophic semi-supra open set and \(semi-s-Ncl(A)\) is a neutrosophic semi-supra closed set.

Proposition 2.4. i) \(semi-s-Nint(A) = semi-s-Ncl(\overline{A})\)

\(\square\)

ii) \(semi-s-Ncl(A) = semi-s-Ncl(\overline{A})\)

\(\square\)

iii) if \(A \subseteq B\) then \(semi-s-Ncl(A) \subseteq semi-s-Ncl(B)\) and \(semi-s-Nint(A) \subseteq semi-s-Nint(B)\)

\(\square\)

Proposition 2.5. (i) The intersection of a neutrosophic supra open set and a neutrosophic supra open set is a neutrosophic semi-supra open set.
(ii) The intersection of a neutrosophic semi-supra open set and aneutrosophic pre-supra open set is a neutrosophic pre-supra open set.

**Proof.** It is obvious. □

**Definition 2.4.** Let \((X,T)\) and \((Y,S)\) be two neutrosophic semi-supra open sets and \(R\) be a associated supra topology with \(T\). A map \(f : (X,T) \rightarrow (Y,S)\) is called neutrosophic semi-supra continuous map if the inverse image of each neutrosophic open set in \(Y\) is a neutrosophic semi-supra open set in \(X\).

**Proposition 2.6.** Every neutrosophic supra continuous map is neutrosophic semi-supra continuous map.

**Proof.** Let \(f : (X,T) \rightarrow (Y,S)\) be a neutrosophic supra continuous map and \(A\) is a neutrosophic open set in \(Y\). Then \(f^{-1}(A)\) is a neutrosophic open set in \(X\). Since \(R\) is associated with \(T\). Then \(T \subseteq R\). Therefore \(f^{-1}(A)\) is a neutrosophic supra open set in \(X\) which is a neutrosophic supra open set in \(Y\). Hence \(f\) is aneutrosophic semi-supra continuous map. □

**Remark 2.2.** Every neutrosophic semi-supra continuous map need not be neutrosophic supra continuous map.

**Proposition 2.7.** Let (\(X,T\)) and \((Y,S)\) be two neutrosophic topological spaces and \(R\) be a associated neutrosophic supra topology with \(T\). Let \(f\) be a map from \(X\) into \(Y\). Then the following are equivalent.

i) \(f\) is a neutrosophic semi-supra continuous map.

ii) The inverse image of a neutrosophic closed sets in \(Y\) is a neutrosophic semi-closed set in \(X\).

iii) Semi-Ncl\((f^{-1}(A)) \subseteq f^{-1}(Ncl(A))\) for every neutrosophic set \(A\) in \(X\).

iv) \(f(\text{semi-s-Ncl}(A)) \subseteq \text{Ncl}(f(A))\) for every neutrosophic set \(A\) in \(X\).

v) \(f^{-1}(Nint(B)) \subseteq \text{semi-s-Nint}(f^{-1}(B))\) for every neutrosophic set \(B\) in \(Y\).

**Proof.** (i) ⇒ (ii) : Let \(A\) be a neutrosophic closed set in \(Y\). Then \(A\) is neutrosophic open in \(Y\). Thus \(f^{-1}(A) = \overline{f^{-1}(A)}\) is neutrosophic semi-open in \(X\). It follows that \(f^{-1}(A)\) is a neutrosophic semi-closed set of \(X\).

(ii) ⇒ (iii) : Let \(A\) be any subset of \(X\). Since \(Ncl(A)\) is neutrosophic closed in \(X\) then it follows that \(f^{-1}(Ncl(A))\) is neutrosophic semi-closed in \(X\). Therefore, \(f^{-1}(Ncl(A)) = \text{semi-s-Ncl}(f^{-1}(Ncl(A)) \supseteq \text{semi-s-Ncl}(f^{-1}(A))\)

(iii) ⇒ (iv) : Let \(A\) be any subset of \(X\). By (iii) we obtain \(f^{-1}(Ncl(f(A))) \supseteq \text{semi-s-Ncl}(f^{-1}(f(A))) \supseteq \text{semi-s-Ncl}(A)\) and hence \(f(\text{semi-s-Ncl}(A)) \subseteq Ncl(f(A))\).

(iv) ⇒ (v) : Let \(f(\text{semi-s-Ncl}(A)) \subseteq \text{Ncl}(f(A))\) for every neutrosophic set \(A\) in \(X\). Then \(\text{semi-s-Ncl}(A) \subseteq f^{-1}(Ncl(f(A))) \supseteq \text{semi-s-Ncl}(A) \supseteq f^{-1}(Ncl(f(A)))\)

and \(\text{semi-s-Nint}(A) \supseteq f^{-1}(Nint(f(A)))\). Then \(\text{semi-s-Nint}(f^{-1}(B)) \supseteq f^{-1}(Nint(B))\). Therefore \(f^{-1}(Nint(B)) \subseteq s-Nint(f^{-1}(B))\) for every \(B\) in \(Y\).

\((v) \Rightarrow (i)\) : Let \(A\) be a neutrosophic open set in \(Y\). Therefore \(f^{-1}(Nint(A)) \subseteq \text{semi-s-Nint}(f^{-1}(A))\), hence \(f^{-1}(A) \subseteq \text{semi-s-Nint}(f^{-1}(A))\). But we know that \(\text{semi-s-Nint}(f^{-1}(A)) \subseteq f^{-1}(A)\), then \(f^{-1}(A) = \text{semi-s-Nint}(f^{-1}(A))\). Therefore \(f^{-1}(A)\) is a neutrosophic semi-open set. □

**Proposition 2.8.** If a map \(f : (X,T) \rightarrow (Y,S)\) is a neutrosophic semi-s-continuous and \(g : (Y,S) \rightarrow (Z,R)\) is neutrosophic continuous, Then \(g \circ f\) is neutrosophic semi-s-continuous.

**Proof.** Obvious. □

**Proposition 2.9.** Let a map \(f : (X,T) \rightarrow (Y,S)\) be a neutrosophic semi-supra continuous map, then one of the following holds

i) \(f^{-1}(\text{semi-s-Nint}(A)) \subseteq \text{Nint}(f^{-1}(A))\) for every neutrosophic set \(A\) in \(Y\).

ii) \(Ncl(f^{-1}(A)) \subseteq f^{-1}(\text{semi-s-Ncl}(A))\) for every neutrosophic set \(A\) in \(Y\).

iii) \(f(Ncl(B)) \subseteq \text{semi-s-Ncl}(f(B))\) for every neutrosophic set \(B\) in \(X\).

**Proof.** Let \(A\) be any neutrosophic open set of \(Y\), then condition (i) is satisfied, then \(f^{-1}(\text{semi-s-Nint}(A)) \subseteq \text{Nint}(f^{-1}(A))\). We get, \(f^{-1}(A) \subseteq \text{Nint}(f^{-1}(A))\). Therefore \(f^{-1}(A)\) is a neutrosophic supra open set. Every neutrosophic supra open set is a neutrosophic semi supra open set. Hence \(f\) is a neutrosophic semi-s-continuous function. If condition (ii) is satisfied, then we can easily prove that \(f\) is a neutrosophic semi-s-continuous function if condition (iii) is satisfied, and \(A\) is any neutrosophic open set of \(Y\), then \(f^{-1}(A)\) is a set in \(X\) and \(f(Ncl(f^{-1}(A)) \subseteq \text{semi-s-Ncl}(f(f^{-1}(A)))\). This implies \(f(Ncl(f^{-1}(A))) \subseteq \text{semi-s-Ncl}(A)\). This is nothing but condition (ii). Hence \(f\) is a neutrosophic semi-s-continuous function. □

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