Neutrosophic vague soft expert set theory

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Abstract. In this paper, we first introduce the concept of neutrosophic vague soft expert sets (NVSESs for short) which combines neutrosophic vague sets and soft expert sets to be more effective and useful. We also define its basic operations, namely complement, union, intersection, AND and OR along with illustrative examples, and study some related properties with supporting proofs. Lastly, this concept is applied to a decision making problem and its effectiveness is demonstrated using a hypothetical example.

Keywords: Neutrosophic soft expert set, neutrosophic vague set, neutrosophic vague soft set, soft expert set

1. Introduction

In reality, the limitation of precise research is increasingly being recognized in many fields, such as economics, social science, and management science. In recent years, uncertain theories such as probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], vague set theory [3], rough set theory [4] and interval mathematics have been widely applied in uncertain and ambiguous environment. However, these theories do not handle the indeterminate and inconsistent information. Thus neutrosophic set (NS in short) is defined [5], as a new mathematical tool for dealing with problems involving incomplete, indeterminacy and inconsistent knowledge. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. Many research and applications based on neutrosophic set were undertaken such as aggregation operators of interval neutrosophic linguistic numbers [6], similarity measures between interval neutrosophic sets in multicriteria decision-making [7], aggregation operators for simplified neutrosophic sets [8] and improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making [9]. Molodtsov [10] firstly proposed soft set theory as a general mathematical tool to cope with uncertainty and vagueness. Since then, soft set has been developed rapidly to possibility fuzzy soft set [11], soft multisets theory [12], multiparameterized soft set [13], soft intuitionistic fuzzy sets [14], Q-fuzzy soft sets [15–17], and multi Q-fuzzy sets [18–21], thereby opening avenues to many applications [22, 23]. Cagman et al. [24, 25] studied fuzzy soft set theory and fuzzy parameterized fuzzy soft set theory with its applications. Deli and Cagman [26] introduced the concept of intuitionistic fuzzy parameterized soft set and gave its application in decision making. Deli and Karatas [27] also introduced interval valued intuitionistic fuzzy parameterized soft set theory and its decision making. Vague soft set theory was provided by Xu [28], while Alhazaymeh and Hassan [29] introduced the concept of generalized vague soft set followed by possibility vague soft set [30], and interval-valued vague soft sets [31]. They also introduced the concept of possibility interval-valued vague soft set [32]. Maji [33] introduced neutrosophic soft set, which was extended to relations on interval valued neutrosophic soft sets [34], distance and similarity measures of interval neutrosophic soft sets [35], neutrosophic soft relations and some
properties [36], neutrosophic soft matrices and NSM-decision making [37], interval-valued neutrosophic soft sets and its decision making [38] and interval valued neutrosophic parameterized soft set theory [39]. Alkhazaleh and Salleh then proceeded to introduce the notion of fuzzy soft expert sets [40], while Hassan and Alhazaymeh introduced the theory of vague soft expert sets [41], mapping on generalized vague soft expert set [42] and vague soft set relations [43].

In this paper we first introduce the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set [41], mapping on generalized vague soft expert sets [42] and vague soft set relations [43].

In this section, we recall some basic notions in neutrosophic vague set, neutrosophic vague set, soft expert set and neutrosophic soft expert set.

Definition 2.1. (see [44]) A neutrosophic vague set $A_{NV}$ (NVS) on the universe $X$ written as $A_{NV} = \{ \leq x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x) \}$ whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as

\[
\hat{T}_{A_{NV}}(x) = [T^{+}, T^{-}], \quad \hat{I}_{A_{NV}}(x) = [I^{-}, I^{+}], \quad \hat{F}_{A_{NV}}(x) = [F^{-}, F^{+}]
\]

where (1) $0 \leq T^{-} + I^{-} + F^{-} \leq 2$, (2) $I^{+} = 1 - F^{-}$, and (3) $T^{+} = 1 - T^{-}$.

Definition 2.2. (see [44]) If $\Psi_{NV}$ is a NVS of the universe $U$, where $\forall u_i \in U$, $\hat{T}_{\Psi_{NV}}(x) = [1, 1]$, $\hat{I}_{\Psi_{NV}}(x) = [0, 0]$, $\hat{F}_{\Psi_{NV}}(x) = [0, 0]$, then $\Psi_{NV}$ is called a unit NVS, where $1 \leq i \leq n$. If $\Phi_{NV}$ is a NVS of the universe $U$, where $\forall u_i \in U$, $\hat{T}_{\Phi_{NV}}(x) = [0, 0]$, $\hat{I}_{\Phi_{NV}}(x) = [1, 1]$, $\hat{F}_{\Phi_{NV}}(x) = [1, 1]$, then $\Phi_{NV}$ is called a zero NVS, where $1 \leq i \leq n$.

Definition 2.3. (see [44]) Let $A_{NV}$ and $B_{NV}$ be two NVSs of the universe $U$. If $\forall u_i \in U$, (1) $\hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i)$, (2) $\hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i)$ and (3) $\hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$, then the NVS $A_{NV}$ is equal to $B_{NV}$, denoted by $A_{NV} = B_{NV}$, where $1 \leq i \leq n$.

Definition 2.4. (see [44]) Let $A_{NV}$ and $B_{NV}$ be two NVSs of the universe $U$. If $\forall u_i \in U$, (1) $\hat{T}_{A_{NV}}(u_i) \leq \hat{T}_{B_{NV}}(u_i)$, (2) $\hat{I}_{A_{NV}}(u_i) \geq \hat{I}_{B_{NV}}(u_i)$ and (3) $\hat{F}_{A_{NV}}(u_i) \geq \hat{F}_{B_{NV}}(u_i)$, then the NVS $A_{NV}$ is included by $B_{NV}$, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 2.5. (see [44]) The complement of a NVS $A_{NV}$ is denoted by $A^{c}$ and is defined by

\[
\hat{T}_{A_{NV}}^{c}(x) = [1 - T^{+}, 1 - T^{-}], \quad \hat{I}_{A_{NV}}^{c}(x) = [1 - I^{-}, 1 - I^{+}] \quad \text{and} \quad \hat{F}_{A_{NV}}^{c}(x) = [1 - F^{+}, 1 - F^{-}].
\]

Definition 2.6. (see [44]) The union of two NVSs $A_{NV}$ and $B_{NV}$ is a NVS $C_{NV}$, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $A_{NV}$ and $B_{NV}$ given by

\[
\hat{T}_{C_{NV}}(x) = \max \left( \hat{T}_{A_{NV}}(x), \hat{T}_{B_{NV}}(x) \right), \quad \hat{I}_{C_{NV}}(x) = \min \left( \hat{I}_{A_{NV}}(x), \hat{I}_{B_{NV}}(x) \right) \quad \text{and} \quad \hat{F}_{C_{NV}}(x) = \min \left( \hat{F}_{A_{NV}}(x), \hat{F}_{B_{NV}}(x) \right).
\]

Definition 2.7. (see [44]) The intersection of two NVSs $A_{NV}$ and $B_{NV}$ is a NVS $C_{NV}$, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $A_{NV}$ and $B_{NV}$ given by

\[
\hat{T}_{C_{NV}}(x) = \min \left( \hat{T}_{A_{NV}}(x), \hat{T}_{B_{NV}}(x) \right), \quad \hat{I}_{C_{NV}}(x) = \max \left( \hat{I}_{A_{NV}}(x), \hat{I}_{B_{NV}}(x) \right) \quad \text{and} \quad \hat{F}_{C_{NV}}(x) = \max \left( \hat{F}_{A_{NV}}(x), \hat{F}_{B_{NV}}(x) \right).
\]
Definition 2.8. (see [44]) Let $U$ be an initial universal set and let $E$ be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of $U$ and let $A \subseteq E$. A collection of pairs $(\bar{F}, E)$ is called a neutrosophic vague soft set (NVSS) over $U$, where $\bar{F}$ is a mapping given by $\bar{F} : A \rightarrow NV(U)$, where $\bar{F}$ is a mapping given by $\bar{F} : A \rightarrow NV(U)$.

Let $U$ be a universe, $E$ a set of parameters, $X$ a set of experts (agents), and $O$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.9. (see [45]) A pair $(F, A)$ is called a soft expert set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$, where $P(U)$ denotes the power set of $U$.

Let $U$ be a universe, $E$ a set of parameters, $X$ a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.10. (see [46]) A pair $(F, A)$ is called a neutrosophic soft expert set (NSES in short) over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$, where $P(U)$ denotes the power neutrosophic set of $U$.

Definition 2.11. (see [46]) Let $(F, A)$ and $(G, B)$ be two NSESs over the common universe $U$. $(F, A)$ is said to be neutrosophic soft expert subset of $(G, B)$, if $A \subseteq B$ and $T_{F(e)}(X) \leq T_{G(e)}(X)$, $I_{F(e)}(X) \leq I_{G(e)}(X)$, $F_{F(e)}(X) \geq F_{G(e)}(X)$ for all $e \in A$, $X \in U$. We denote it by $(F, A) \subseteq (G, B)$.

$(F, A)$ is said to be neutrosophic soft expert subset of $(G, B)$ if $(G, B)$ is a neutrosophic soft expert subset of $(F, A)$. We denote it by $(F, A) \supseteq (G, B)$.

Definition 2.12. (see [46]) Two (NSESs) $(F, A)$ and $(G, B)$ over the common universe $U$ are said to be equal if $(F, A)$ is neutrosophic soft expert subset of $(G, B)$ and $(G, B)$ is neutrosophic soft expert subset of $(F, A)$. We denote it by $(F, A) = (G, B)$.

Definition 2.13. (see [46]) Let $E = \{-e_1, e_2, ..., e_n\}$ be a set of parameters. The NOT set of $E$ is denoted by $\neg E = \{-e_1, -e_2, ..., -e_n\}$, where $\neg e_i = \text{not} e_i$, $\forall i$.

Definition 2.14. (see [46]) The complement of a NSES $(F, A)$ denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, -A)$, where $F^c : -A \rightarrow P(U)$ is given by $F^c(x) = \text{neutrosophic soft expert complement with } T_{F^c(X)} = F_{F(X)}$, $I_{F^c(X)} = F_{I(X)}$, $F_{F^c(X)} = T_{F(X)}$.

Definition 2.15. (see [46]) An agree-NSES $(F, A)_1$ over $U$ is a neutrosophic soft expert subset of $(F, A)$ defined as $$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}.$$
and falsity-membership of \((K, A \times B)\) are as follows:

\[
T_{K}(\alpha, \beta)(m) = \min \left( T_{H}(\alpha)(m), T_{G}(\beta)(m) \right), \\
I_{K}(\alpha, \beta)(m) = \frac{I_{H}(\alpha)(m) + I_{G}(\beta)(m)}{2}, \\
F_{K}(\alpha, \beta)(m) = \max \left( F_{H}(\alpha)(m), F_{G}(\beta)(m) \right),
\]

\(\forall \alpha \in A, \forall \beta \in B.\)

**Definition 2.20.** (see [46]) Let \((H, A)\) and \((G, B)\) be two NSESs over the common universe \(U\). The “OR” operation on them is denoted by \((H, A) \lor (G, B)\) and is defined by \((H, A) \lor (G, B) = (O, A \times B)\), where the truth-membership, indeterminacy-membership and falsity-membership of \((O, A \times B)\) are as follows:

\[
T_{O}(\alpha, \beta)(m) = \max \left( T_{H}(\alpha)(m), T_{G}(\beta)(m) \right), \\
I_{O}(\alpha, \beta)(m) = \frac{I_{H}(\alpha)(m) + I_{G}(\beta)(m)}{2}, \\
F_{O}(\alpha, \beta)(m) = \min \left( F_{H}(\alpha)(m), F_{G}(\beta)(m) \right),
\]

\(\forall \alpha \in A, \forall \beta \in B.\)

### 3. Neutrosophic vague soft expert set

In this section, we introduce the definition of a neutrosophic vague soft expert set and give basic properties of this concept.

Let \(U\) be a universe, \(E\) a set of parameters, \(X\) a set of experts (agents), and \(O = \{1 = \text{agree}, 0 = \text{disagree}\}\) a set of opinions. Let \(Z = E \times X \times O \subseteq Z.\)

**Definition 3.1.** A pair \((F, A)\) is called a neutrosophic vague soft expert set over \(U\), where \(F\) is a mapping given by \(F : A \rightarrow NVU\), where \(NVU\) denotes the power neutrosophic vague set of \(U\).

Suppose \(F : A \rightarrow NVU\) is a function defined as \(F(a) = F(a)(u)\), \(\forall u \in U\). For each \(a_i \in A\), \(F(a_i) = F(a_i)(u)\), where \(F(a_i)\) represents the degree of belongingness, degree of indeterminacy and non-belongingness of the elements of \(U\) in \(F(a_i)\). Hence \(F(a_i)\) can be written as:

\[
F(a_i) = \left\{ \frac{\text{\(u_i\)}}{F(a_i)(u)} \right\}, \quad \text{for} \quad i = 1, 2, 3, \ldots
\]

where \(F(a_i)(u) = (F_{K}(a_i)(u_1), F_{K}(a_i)(u_2), F_{K}(a_i)(u_3))\) and \(\{F_{K}(a_i)(u_1), F_{K}(a_i)(u_2), F_{K}(a_i)(u_3)\}\) representing the truth-membership function, indeterminacy-membership function and falsity-membership function of each of the elements \(u_i \in U\), respectively.

**Example 3.2.** Suppose that a company produced new types of its products and wishes to take the opinion of some experts concerning these products. Let \(U = \{u_1, u_2, u_3, u_4\}\) be a set of products, \(E = \{e_1, e_2\}\) a set of decision parameters where \(e_1(i = 1, 2)\) denotes the decision “easy to use,” and “quality,” respectively, and let \(X = \{p, q\}\) be a set of experts. Suppose that the company has distributed a questionnaire to the two experts to make decisions on the company’s products, and we get the following:

\[
F(e_1, p, 1) = \left\{ \begin{array}{l}
\{[0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8]\} \\
\{[0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8]\}
\end{array} \right\}
\]

\[
F(e_1, q, 1) = \left\{ \begin{array}{l}
\{[0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8]\} \\
\{[0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8]\}
\end{array} \right\}
\]

\[
F(e_1, p, 0) = \left\{ \begin{array}{l}
\{[0.1, 0.3]; [0.2, 0.5]; [0.1, 0.3]\} \\
\{[0.1, 0.3]; [0.2, 0.5]; [0.1, 0.3]\}
\end{array} \right\}
\]

\[
F(e_1, q, 0) = \left\{ \begin{array}{l}
\{[0.1, 0.3]; [0.2, 0.5]; [0.1, 0.3]\} \\
\{[0.1, 0.3]; [0.2, 0.5]; [0.1, 0.3]\}
\end{array} \right\}
\]
Example 3.7. Consider Example 3.2. Then the agree-neutrosophic vague soft expert set \((F, A)\) over \(U\) is a neutrosophic vague soft expert subset of \((F, A)\) defined as follows:

\[
(F, A)_1 = \{ (e_1, p, 1), (e_1, q, 1), (e_2, p, 1) \}
\]

Definition 3.8. A disagree-neutrosophic vague soft expert set \((F, A)_0\) over \(U\) is a neutrosophic vague soft expert subset of \((F, A)\) defined as follows:

\[
(F, A)_0 = \{ (e_1, q, 0), (e_2, q, 0), (e_1, q, 0) \}
\]
vague soft expert set \((F, A)\) is denoted by \((F, A)\).

**Example 3.9.** Consider Example 3.2. The disagreement neutrosophic vague soft expert set \((F, A)_0\) over \(U\) is

\((F, A)_0 = \{ (c_1, p, 0) \} \)

\[
\begin{align*}
\mu_1 &= \left[ (0.2, 0.8); (0.3, 0.9); (0.5, 0.8); (0.1, 0.7) \right] \cdot \left[ (0.3, 0.9); (0.5, 0.8); (0.1, 0.7) \right], \\
\mu_2 &= \left[ (0.4, 0.5); (0.3, 0.7); (0.5, 0.6) \right] \cdot \left[ (0.8, 0.9); (0.8, 0.1) \right], \\
\mu_3 &= \left[ (0.1, 0.2); (0.6, 0.7); (0.8, 0.9) \right] \cdot \left[ (0.5, 1); (0.3, 0.5); (0.0, 0.5) \right], \\
\mu_4 &= \left[ (0.1, 0.1); (0.3, 0.9); (0.6, 0.9) \right] \cdot \left[ (0.4, 0.4); (0.6, 0.8); (0.2, 0.4) \right].
\end{align*}
\]

4. Basic operations on neutrosophic vague soft expert sets

In this section, we introduce some basic operations on neutrosophic vague soft expert sets, namely the complement, union and intersection of neutrosophic vague soft expert sets, derive their properties and give some examples.

We define the complement operation for neutrosophic vague soft expert set and give an illustrative example and proved proposition.

**Definition 4.1.** The complement of a neutrosophic vague soft expert set \((F, A)\) is denoted by \((F, A)^c\) and is defined by \((F, A)^c = (F^c, A)\) where \(F^c : A \rightarrow NV^U\) is a mapping given by

\[F^c(\alpha) = \overline{c}(F(\alpha)), \forall \alpha \in A\]

where \(\overline{c}\) is a neutrosophic vague complement.

**Example 4.2.** Consider Example 3.2. By using the basic neutrosophic vague complement, we have

\[(F, Z^c) = \{ (c_1, p, 1) \} \]

\[
\begin{align*}
\mu_1 &= \left[ (0.2, 0.8); (0.7, 0.9); (0.2, 0.8) \right] \cdot \left[ (0.3, 0.9); (0.5, 0.8); (0.1, 0.7) \right], \\
\mu_2 &= \left[ (0.4, 0.5); (0.3, 0.7); (0.5, 0.6) \right] \cdot \left[ (0.2, 0.6); (0.8, 0.9); (0.8, 1) \right], \\
\mu_3 &= \left[ (0.1, 0.2); (0.6, 0.7); (0.8, 0.9) \right] \cdot \left[ (0.5, 1); (0.3, 0.5); (0.0, 0.5) \right], \\
\mu_4 &= \left[ (0.1, 0.1); (0.3, 0.9); (0.6, 0.9) \right] \cdot \left[ (0.4, 0.4); (0.6, 0.8); (0.2, 0.4) \right].
\end{align*}
\]

**Proposition 4.3.** If \((F, A)\) is a neutrosophic vague soft expert set over \(U\), then \(((F, A)^c)^c = (F, A)\)

**Proof.** From Definition 4.1. We have \((F, A)^c = (F^c, A)\), where \(F^c(\alpha) = 1 - F(\alpha), \forall \alpha \in A\). Now, \(((F, A)^c)^c = ((F^c)^c, A)\), where \((F^c)^c(\alpha) = 1 - (1 - F(\alpha)), \forall \alpha \in A = F(\alpha), \forall \alpha \in A\).
We define the union of two neutrosophic vague soft expert sets and give an illustrative example.

**Definition 4.4.** The union of two neutrosophic vague soft expert sets \((F, A)\) and \((G, B)\) over \(U\), denoted by \((F, A) \cup (G, B)\), is a neutrosophic vague soft expert set \((H, C)\), where \(C = A \cup B\) and \(\forall \varepsilon \in C\),

\[
(H, C) = \begin{cases}
F(\varepsilon), & \text{if } \varepsilon \in A - B, \\
G(\varepsilon), & \text{if } \varepsilon \in B - A, \\
F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B.
\end{cases}
\]

where \(\cup\) denote the neutrosophic vague set union.

**Example 4.5.** Consider Example 3.2. Suppose that the company takes the opinion of the experts twice again over a time after using the products. Let \(A = \{(e_1, p, 1), (e_1, q, 0), (e_1, p, 0)\}\) and \(B = \{(e_1, p, 1), (e_1, q, 0), (e_2, p, 1)\}\).

Suppose \((F, A)\) and \((G, B)\) are two neutrosophic vague soft expert sets over \(U\) such that:

\[
(F, A) = \left\{ \begin{array}{l}
\left( e_1, p, 1, \begin{array}{c}
u_1 \\
[0.7, 0.8]; [0.1, 0.3]; [0.2, 0.3] \\
[0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9]
\end{array}, \nu_2 \\
[0.4, 0.7]; [0.2, 0.5]; [0.3, 0.6]
\right), \\
\left( e_1, q, 0, \begin{array}{c}
u_1 \\
[0.1, 0.9]; [0.2, 0.4]; [0.1, 0.9]
\end{array}, \nu_2 \\
[0.4, 0.8]; [0.3, 0.5]; [0.2, 0.6]
\right), \\
\left( e_1, p, 0, \begin{array}{c}
u_1 \\
[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]
\end{array}, \nu_2 \\
[0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5]
\right)
\end{array} \right\},
\]

\[
(G, B) = \left\{ \begin{array}{l}
\left( e_1, p, 1, \begin{array}{c}
u_1 \\
[0.2, 0.3]; [0.7, 0.9]; [0.7, 0.8]
\end{array}, \nu_2 \\
[0.3, 0.6]; [0.5, 0.8]; [0.4, 0.7]
\right), \\
\left( e_1, q, 0, \begin{array}{c}
u_1 \\
[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]
\end{array}, \nu_2 \\
[0.5, 0.6]; [0.8, 0.9]; [0.4, 0.5]
\right)
\end{array} \right\}.
\]

By using basic neutrosophic vague union, we have \((F, A) \cup (G, B) = (H, C)\), where

\[
(H, C) = \left\{ \begin{array}{l}
\left( e_1, p, 1, \begin{array}{c}
u_1 \\
[0.7, 0.8]; [0.1, 0.3]; [0.2, 0.3] \\
[0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9]
\end{array}, \nu_2 \\
[0.4, 0.7]; [0.2, 0.5]; [0.3, 0.6]
\right), \\
\left( e_1, q, 0, \begin{array}{c}
u_1 \\
[0.1, 0.9]; [0.2, 0.4]; [0.1, 0.9]
\end{array}, \nu_2 \\
[0.4, 0.8]; [0.3, 0.5]; [0.2, 0.6]
\right), \\
\left( e_1, p, 0, \begin{array}{c}
u_1 \\
[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]
\end{array}, \nu_2 \\
[0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5]
\right)
\end{array} \right\}.
\]

We define the intersection of two neutrosophic vague soft expert sets and give an illustrative example.

**Definition 4.6.** The *intersection* of two neutrosophic vague soft expert sets \((F, A)\) and \((G, B)\) over a universe \(U\), is a neutrosophic vague soft expert set \((H, C)\), denoted by \((F, A) \cap (G, B)\), such that \(C = A \cap B\) and \(\forall \varepsilon \in c\)

\[
(H, C) = \begin{cases}
F(\varepsilon), & \text{if } \varepsilon \in A - B, \\
G(\varepsilon), & \text{if } \varepsilon \in B - A, \\
F(\varepsilon) \cap G(\varepsilon), & \text{if } \varepsilon \in A \cap B.
\end{cases}
\]

where \(\cap\) denote the neutrosophic vague set intersection.

**Example 4.7.** Consider Example 4.5. By using basic neutrosophic vague intersection, we have \((F, A) \cap (G, B) = (H, C)\), where

\[
(H, C) = \left\{ \begin{array}{l}
\left( e_1, p, 1, \begin{array}{c}
u_1 \\
[0.3, 0.6]; [0.1, 0.4]; [0.4, 0.7]
\end{array}, \nu_2 \\
[0.5, 0.7]; [0.2, 0.5]; [0.3, 0.5]
\right), \\
\left( e_1, q, 0, \begin{array}{c}
u_1 \\
[0.1, 0.9]; [0.2, 0.3]; [0.1, 0.9]
\end{array}, \nu_2 \\
[0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2]
\right), \\
\left( e_1, p, 1, \begin{array}{c}
u_1 \\
[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.1]
\end{array}, \nu_2 \\
[0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2]
\right)
\end{array} \right\}.
\]
(H, C) = \{ (e_1, p, 1),
\begin{align*}
\{ & \{ u_1, u_2 \mid \begin{array}{l}
[0.3, 0.6], [0.1, 0.4], [0.4, 0.7], \end{array} \\
[0.1, 0.2], [0.6, 0.7], [0.8, 0.9] \} \\
& \{ u_3, u_4 \mid \begin{array}{l}
[0.4, 0.5], [0.1, 0.2], [0.5, 0.6] \end{array} \} \\
& \end{align*}
\} \cdot \{ (e_1, q, 0),
\begin{align*}
\{ & \{ u_1, u_2 \mid \begin{array}{l}
[0.1, 0.9], [0.2, 0.4], [0.1, 0.9] \end{array} \\
[0.4, 0.8], [0.3, 0.5], [0.2, 0.6] \} \\
& \{ u_3, u_4 \mid \begin{array}{l}
[0.5, 0.7], [0.4, 0.6], [0.3, 0.5] \end{array} \} \\
& \end{align*}
\} \cdot \{ (e_2, p, 1),
\begin{align*}
\{ & \{ u_1, u_2 \mid \begin{array}{l}
[0.2, 0.3], [0.7, 0.9], [0.7, 0.8] \end{array} \\
[0.3, 0.4], [0.1, 0.2] \} \\
& \{ u_3, u_4 \mid \begin{array}{l}
[0.5, 0.6], [0.8, 0.9], [0.4, 0.5] \end{array} \} \\
& \end{align*}
\} \cdot \{ (e_2, q, 0),
\begin{align*}
\{ & \{ u_1, u_2 \mid \begin{array}{l}
[0.1, 0.4], [0.2, 0.4], [0.6, 0.9] \end{array} \\
[0.5, 0.8], [0.3, 0.5], [0.2, 0.5] \} \\
& \{ u_3, u_4 \mid \begin{array}{l}
[0.5, 0.6], [0.5, 0.6], [0.4, 0.5] \end{array} \} \\
& \end{align*}
\} \).

5. AND and OR operations

In this section, we introduce the definitions of AND and OR operations for neutrosophic vague soft expert set and derive their properties.

Definition 5.1. Let \((F, A)\) and \((G, B)\) be any two neutrosophic vague soft expert sets over a soft universe \((U, Z)\).

Then “(F, A) AND (G, B)” denoted \((F, A) \wedge (G, B)\) is defined by \((F, A) \wedge (G, B) = (H, A \times B)\), where \((H, A \times B) = H(\alpha, \beta)\) such that \(H(\alpha, \beta) = F(\alpha) \cap G(\beta)\), for all \((\alpha, \beta) \in A \times B\), where \(\cap\) represents the basic intersection.

Definition 5.2. Let \((F, A)\) and \((G, B)\) be any two neutrosophic vague soft expert sets over a soft universe \((U, Z)\).

Then “(F, A) OR (G, B)” denoted \((F, A) \vee (G, B)\) is defined by \((F, A) \vee (G, B) = (H, A \times B)\), where \((H, A \times B) = H(\alpha, \beta)\) such that \(H(\alpha, \beta) = F(\alpha) \cup G(\beta)\), for all \((\alpha, \beta) \in A \times B\), where \(\cup\) represents the basic union.

Proposition 5.3. If \((F, A)\) and \((G, B)\) are two neutrosophic vague soft expert sets over a soft universe \((U, Z)\). Then,

1. \( ((F, A) \wedge (G, B))^c = (F, A)^c \wedge (G, B)^c \)
2. \( ((F, A) \vee (G, B))^c = (F, A)^c \vee (G, B)^c \)

Proof. (1) Suppose that \((F, A)\) and \((G, B)\) are two neutrosophic vague soft expert sets over a soft universe \((U, Z)\) defined as:
\((F, A) = F(\alpha)\) for all \(\alpha \in A \subseteq Z\) and \((G, B) = G(\beta)\) for all \(\beta \in B \subseteq Z\). By definitions 4.8 and 4.9 it follows that:

\( ((F, A) \wedge (G, B))^c = ((F(\alpha) \cap G(\beta))^c = \tilde{c}(F(\alpha) \cap G(\beta)) = \tilde{c}(F(\alpha)) \cup \tilde{c}(G(\beta)) = (F(\alpha))^c \vee (G(\beta))^c = (F, A)^c \vee (G, B)^c. \)

(2) The proof is similar to that in part(1) and therefore is omitted.

6. Application of NVSES in a decision making problem

In this section, we introduce a generalized algorithm which will be applied to the NVSES model introduced in Section 3 and used to solve a hypothetical decision making problem.

Example 6.1. Suppose that company Y is looking to hire a person to fill the vacancy for a position in their company. Out of all the people who applied for the position, two candidates were shortlisted and these two candidates form the universe of elements, \(U = \{ u_1, u_2 \}\). The hiring committee consists of the hiring manager and head of department and this committee is represented by the set \(X = \{ p, q \} \) (a set of experts) while the set \(Q = \{ 1 = agree, 0 = disagree \} \) represents the set of opinions of the hiring committee members. The hiring committee considers a set of parameters, \(E = \{ e_1, e_2, e_3 \} \), where the parameters \(e_i \) (\(i = 1, 2, 3\)) represent the characteristics or qualities that the candidates are assessed on, namely, “relevan job experience”, “excellent academic qualifications in the relevant field” and “attitude and level of professionalism”, respectively. After interviewing the two candidates and going through their certificates and other supporting documents, the hiring committee constructs the following NVSES:

\( (F, A) = \{ (e_1, p, 1) = \left\{ u_1 \mid \begin{array}{l}
[0.3, 0.7], [0.5, 0.9], [0.3, 0.7] \end{array} \} \right\} \)
The algorithm given below is employed by the hiring committee to determine the best or most suitable candidate to be hired for the position. The generalized algorithm is as follows:

**Algorithm**

1. Input the NVSES $(F, A)$.

2. Find the values of $\alpha_{F(a_j)}(u_i) = T_{F(a_j)}(u_i) - F_{F(a_j)}(u_i)$ for interval truth-membership part $[T_{F(a_j)}(u_i), T_{F(a_j)}(u_i)]$, where $T_{F(a_j)}(u_i) = 1 - F_{F(a_j)}(u_i)$, for each element $u_i \in U$.

3. Take the arithmetic average $\beta_{F(a_j)}(u_i)$ of the end points of the interval indeterminacy-membership part $[I_{F(a_j)}(u_i), I_{F(a_j)}(u_i)]$, for each element $u_i \in U$.

4. Find the values of $\gamma_{F(a_j)}(u_i) = F_{F(a_j)}(u_i) - T_{F(a_j)}(u_i)$ for interval falsity-membership part $[F_{F(a_j)}(u_i), F_{F(a_j)}(u_i)]$, where $F_{F(a_j)}(u_i) = 1 - T_{F(a_j)}(u_i)$, for each element $u_i \in U$.

5. Find the values of $\alpha_{F(a_j)}(u_i) - \beta_{F(a_j)}(u_i) - \gamma_{F(a_j)}(u_i)$ for each element $u_i \in U$.

6. Find the highest numerical grade for the agree-NVSES and disagree-NVSES.

7. Compute the score of each element $u_i \in U$ by taking the sum of the products of the numerical grade of each element for the agree-NVSES and disagree-NVSES, denoted by $A_i$ and $D_i$, respectively.

8. Find the values of the score $r_i = A_i - D_i$ for each element $u_i \in U$.

9. Determine the value of the highest score $s = \max_{u_i \in U}\{ r_i \}$. The decision is to choose element $u_i$ as the optimal or best solution to the problem. If there are more than one element with the highest $r_i$ score, then any one of those elements can be chosen as the optimal solution.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td>$a_4$</td>
</tr>
<tr>
<td>$u_1$</td>
<td>$u_2$</td>
<td>$u_3$</td>
<td>$u_4$</td>
</tr>
<tr>
<td>$(0.6, 0.9)$</td>
<td>$(0.2, 0.5)$</td>
<td>$(0.1, 0.4)$</td>
<td>$(0.3, 0.7)$</td>
</tr>
</tbody>
</table>

Next, the NVSES $(F, A)$ is used together with a generalized algorithm to solve the decision making problem stated at the beginning of this section.
respectively. These values are given in Table 4.

The committee is advised to hire candidate NVSES, respectively.

Vacant position.

\[
\text{Table 4} \quad \begin{array}{c|c|c|c|c|c}
\hline
 & u_1 & u_2 & u_1 & u_2 \\
\hline
(e_1, p, 1) & (0, 0.7, 0) & (0.5, 0.35, -0.5) & (e_1, p, 0) & (0.3, 0) & (-0.5, 0.65, 0.5) \\
\hline
(e_2, p, 1) & (0.2, 0.15, -0.2) & (-0.5, 0.85, 0.5) & (e_2, p, 0) & (-0.2, 0.85, 0.2) & (0.5, 0.15, -0.5) \\
\hline
(e_3, p, 1) & (0.2, 0.2, -0.2) & (-0.1, 0.8, 0.1) & (e_3, p, 0) & (-0.2, 0.8, 0.2) & (0.1, 0.2, -0.1) \\
\hline
(e_1, q, 1) & (0.2, 0.65, -0.2) & (0.1, 0.5, -0.1) & (e_1, q, 0) & (-0.2, 0.35, 0.2) & (-0.1, 0.5, 0.1) \\
\hline
(e_2, q, 1) & (0.5, 0.5, -0.5) & (-0.5, 0.55, 0.5) & (e_2, q, 0) & (-0.5, 0.5, 0.5) & (0.5, 0.45, -0.5) \\
\hline
(e_3, q, 1) & (0.3, 0.55, -0.3) & (0.1, 0.4, -0.1) & (e_3, q, 0) & (-0.3, 0.45, 0.3) & (-0.1, 0.6, 0.1) \\
\hline
\end{array}
\]

To illustrate the advantages of our proposed method using NVSES as compared to that of vague soft expert set as proposed by Hassan and Alhazaymeh [41], let us consider Example 6.1 above. The vague soft expert set can describe this problem as follows.

\[
(F_{ul}, Z) = \{ (e_1, p, 1) \}
\]

\[
= \left\{ \left( \frac{u_1}{([0.3, 0.7])}, \left( \frac{u_2}{([0.6, 0.9], 1])} \right) \right\}, \ldots
\]

Note that the NVSES is a generalization of vague soft expert set. Thus as shown in Example 6.1 above, the NVSES can explain the universal \( U \) in more detail with three membership functions, especially when there are many parameters involved, whereas vague soft expert set can tell us a limited information about the universal \( U \). It can only handle the incomplete information considering both the truth-membership and falsity-membership values, while NVSES can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than vague soft expert set.

### 7. Conclusion

In this paper, we reviewed the basic concepts of neutrosophic vague set and neutrosophic soft expert set, and gave some basic operations on both neutrosophic vague set and neutrosophic soft expert set, before establishing the concept of neutrosophic vague soft expert set. The basic operations on neutrosophic vague soft expert set, namely complement, union, intersection, AND, and OR operations, were defined. Subsequently, the basic properties of these operations such as De Morgan’s laws and other relevant laws pertaining to the concept of neutrosophic vague...
soft expert set are proved. Finally, a generalized algorithm is introduced and applied to the NVSES model to solve a hypothetical decision making problem. This new extension will provide a significant addition to existing theories for handling indeterminacy, and spurs more developments of further research and pertinent applications.

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References


