Neutrosophic Vague Generalized Pre-Closed Sets in Neutrosophic Vague Topological Spaces

Research Article

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Abstract: The aim of this paper is to introduce and develop a new class of sets namely neutrosophic vague generalized pre-closed sets in neutrosophic vague topological space. Further we have analyse the properties of neutrosophic vague generalized pre-open sets. Also some applications namely neutrosophic vague $T_{1/2}$ space, neutrosophic vague $pT_{1/2}$ space and neutrosophic vague $gpT_{1/2}$ space are introduced.

Keywords: Neutrosophic vague topological space, neutrosophic vague generalized pre-closed sets, neutrosophic vague $T_{1/2}$ space, neutrosophic vague $pT_{1/2}$ space and neutrosophic vague $gpT_{1/2}$ space.

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1. Introduction

In 1970, Levine [7] initiated the study of generalized closed sets. Zadeh [16] introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set as a mathematical tool to solve problems and vagueness in everyday life. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The theory of fuzzy topology was introduced by C.L.Chang [4] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Atanassov [3] introduced the degree of nonmembership/falsehood (F) in 1986 and defined the intuitionistic fuzzy set as a generalization of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehre [6] as an extension of fuzzy set theory in 1993. Then, Smarandache [14] introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and defined the neutrosophic set. He has coined the words neutrosophy and neutrosophic. Neutrosophic set is a generalization of fuzzy set theory and intuitionistic fuzzy sets. Shawkat Alkhazaleh [13] in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. In this paper we introduce the concept of neutrosophic vague generalized pre-closed sets and neutrosophic vague generalized pre-open sets and their properties are obtained. Also its relationship with other existing sets are compared and discussed with examples.

2. Preliminaries

Definition 2.1 ([13]). A neutrosophic vague set $A_{NV}$ (NVS in short) on the universe of discourse $X$ written as $A_{NV} = \left\{ \left( x; \hat{T}_{A_{NV}} (x); \hat{I}_{A_{NV}} (x); \hat{F}_{A_{NV}} (x) \right) ; x \in X \right\}$, whose truth membership, indeterminacy membership and false membership
functions is defined as:
\[ \hat{T}_{\lambda_{NV}}(x) = [T^-, T^+] \], \[ \hat{I}_{\lambda_{NV}}(x) = [I^-, I^+] \], \[ \hat{F}_{\lambda_{NV}}(x) = [F^-, F^+] \]

where,

(1). \( T^+ = 1 - F^- \)

(2). \( F^+ = 1 - T^- \) and

(3). \(-0 \leq T^- + I^- + F^- \leq 2^+ \).

Definition 2.2 ([13]). Let \( A_{NV} \) and \( B_{NV} \) be two NVSs of the universe \( U \). If \( \forall u_i \in U, \hat{T}_{A_{NV}} (u_i) \leq \hat{T}_{B_{NV}} (u_i); \hat{I}_{A_{NV}} (u_i) \geq \hat{I}_{B_{NV}} (u_i); \hat{F}_{A_{NV}} (u_i) \geq \hat{F}_{B_{NV}} (u_i) \), then the NVS \( A_{NV} \) is included by \( B_{NV} \), denoted by \( A_{NV} \subseteq B_{NV} \), where \( 1 \leq i \leq n \).

Definition 2.3 ([13]). The complement of NVS \( A_{NV} \) is denoted by \( A^c_{NV} \) and is defined by
\[ \hat{T}_{\lambda_{NV}}(x) = [1 - T^+, 1 - T^-] \], \[ \hat{I}_{\lambda_{NV}}(x) = [1 - I^+, 1 - I^-] \], \[ \hat{F}_{\lambda_{NV}}(x) = [1 - F^+, 1 - F^-] \].

Definition 2.4 ([13]). Let \( A_{NV} \) be NVS of the universe \( U \) where \( \forall u_i \in U, \hat{T}_{A_{NV}} (x) = [1, 1]; \hat{I}_{A_{NV}} (x) = [0, 0]; \hat{F}_{A_{NV}} (x) = [0, 0] \). Then \( A_{NV} \) is called unit NVS, where \( 1 \leq i \leq n \).

Definition 2.5 ([13]). Let \( A_{NV} \) be NVS of the universe \( U \) where \( \forall u_i \in U, \hat{T}_{A_{NV}} (x) = [0, 0]; \hat{I}_{A_{NV}} (x) = [1, 1]; \hat{F}_{A_{NV}} (x) = [1, 1] \). Then \( A_{NV} \) is called zero NVS, where \( 1 \leq i \leq n \).

Definition 2.6 ([13]). The union of two NVSs \( A_{NV} \) and \( B_{NV} \) is NVS \( C_{NV} \), written as \( C_{NV} = A_{NV} \cup B_{NV} \), whose truth-membership, indeterminacy-membership and false-membership functions are related to those of \( A_{NV} \) and \( B_{NV} \) given by,
\[
\hat{T}_{C_{NV}} (x) = \left[ \max \left( T_{A_{NV}}, T_{B_{NV}} \right) \right], \hat{I}_{C_{NV}} (x) = \left[ \min \left( I_{A_{NV}}, I_{B_{NV}} \right) \right], \hat{F}_{C_{NV}} (x) = \left[ \min \left( F_{A_{NV}}, F_{B_{NV}} \right) \right].
\]

Definition 2.7 ([13]). The intersection of two NVSs \( A_{NV} \) and \( B_{NV} \) is NVS \( C_{NV} \), written as \( C_{NV} = A_{NV} \cap B_{NV} \), whose truth-membership, indeterminacy-membership and false-membership functions are related to those of \( A_{NV} \) and \( B_{NV} \) given by,
\[
\hat{T}_{C_{NV}} (x) = \left[ \min \left( T_{A_{NV}}, T_{B_{NV}} \right) \right], \hat{I}_{C_{NV}} (x) = \left[ \max \left( I_{A_{NV}}, I_{B_{NV}} \right) \right], \hat{F}_{C_{NV}} (x) = \left[ \max \left( F_{A_{NV}}, F_{B_{NV}} \right) \right].
\]

Definition 2.8 ([13]). Let \( A_{NV} \) and \( B_{NV} \) be two NVSs of the universe \( U \). If \( \forall u_i \in U, \hat{T}_{A_{NV}} (u_i) = \hat{T}_{B_{NV}} (u_i); \hat{I}_{A_{NV}} (u_i) = \hat{I}_{B_{NV}} (u_i); \hat{F}_{A_{NV}} (u_i) = \hat{F}_{B_{NV}} (u_i) \), then the NVS \( A_{NV} \) and \( B_{NV} \) are called equal, where \( 1 \leq i \leq n \).

Definition 2.9. Let \((X, \tau)\) be topological space. A subset \( A \) of \( X \) is called:

(1). semi closed set (SCS in short) [8] if \( \text{int}(\text{cl}(A)) \subseteq A \).
Definition 2.10. Let \((X, \tau)\) be topological space. A subset \(A\) of \(X\) is called:

1. generalized closed (briefly, g-closed) \([7]\) if \(\text{cl}(A) \subseteq U\), whenever \(A \subseteq U\) and \(U\) is open in \(X\).

2. generalized semi closed (briefly, gs-closed) \([2]\) if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).

3. a-generalized closed (briefly, ag-closed) \([9]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).

4. generalized pre-closed (briefly, gp-closed) \([10]\) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).

5. generalized semi-pre closed (briefly, gsp-closed) \([5]\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).

3. Neutrosophic Vague Topological Space

In this section we introduce neutrosophic vague topology.

Definition 3.1. A neutrosophic vague topology (NVT in short) on \(X\) is a family \(\tau\) of neutrosophic vague sets (NVS in short) in \(X\) satisfying the following axioms:

1. \(0_{NV}, 1_{NV} \in \tau\)

2. \(G_1 \cap G_2 \in \tau\) for any \(G_1, G_2 \in \tau\)

3. \(\cup G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau\).

In this case the pair \((X, \tau)\) is called neutrosophic vague topological space (NVTS in short) and any NVS in \(\tau\) is known as neutrosophic vague open set (NVOS in short) in \(X\). The complement \(A^c\) of NVOS in NVTS \((X, \tau)\) is called neutrosophic vague closed set (NVCS in short) in \(X\).

Definition 3.2. Let \((X, \tau)\) be NVTS and \(A = \{x, [\hat{T}_A, \hat{I}_A, \hat{F}_A]\}\) be NVS in \(X\). Then the neutrosophic vague interior and neutrosophic vague closure are defined by

1. \(\text{NVint}(A) = \cup\{G/G\text{ is a NVOS in }X \text{ and } G \subseteq A\}\),

2. \(\text{NVcl}(A) = \cap\{K/K\text{ is a NVCS in }X \text{ and } A \subseteq K\}\).

Note that for any NVS \(A\) in \((X, \tau)\), we have \(\text{NVcl}(A^c) = (\text{NVint}(A))^c\) and \(\text{NVint}(A^c) = (\text{NVcl}(A))^c\). It can be also shown that \(\text{NVcl}(A)\) is NVCS and \(\text{NVint}(A)\) is NVOS in \(X\).

1. \(A\) is NVCS in \(X\) if and only if \(\text{NVcl}(A) = A\).

2. \(A\) is NVOS in \(X\) if and only if \(\text{NVint}(A) = A\).

Proposition 3.3. Let \(A\) be any NVS in \(X\). Then

1. \(\text{NVint}(1 - A) = 1 - (\text{NVcl}(A))\) and
(2). \( NV\text{cl} \( 1 - A \)) = 1 - (NV\text{int} \( A \)).

**Proof.**

(1). By definition \( NV\text{cl} \( A \)) = \( \cap \{ K/K\text{isaNVCSin}X \text{and} A \subseteq K \} \).

\[
1 - (NV\text{cl} \( A \)) = 1 - \cap \{ K/K\text{isaNVCSin}X \text{and} A \subseteq K \} \\
= \cup \{ 1 - K/K\text{isaNVCSin}X \text{and} A \subseteq K \} \\
= \cup \{ G/G\text{isNVOSin}X \text{and} G \subseteq 1 - A \} \\
= NV\text{int} (1 - A)
\]

(2). The proof is similar to (1).

**Proposition 3.4.** Let \( (X, \tau) \) be a NVTS and \( A, B \) be NVSs in \( X \). Then the following properties hold:

(a). \( NV\text{int} \( A \)) \subseteq A \, , \quad (a'). \ A \subseteq NV\text{cl} \( A \)

(b). \( A \subseteq B \Rightarrow NV\text{int} \( A \)) \subseteq NV\text{int} \( B \) \,\, , \quad (b'). \ A \subseteq B \Rightarrow NV\text{cl} \( A \)) \subseteq NV\text{cl} \( B \)

(c). \( NV\text{int} (NV\text{int} \( A \)) = NV\text{int} \( A \) \, , \quad (c'). \ NV\text{cl} (NV\text{cl} \( A \)) = NV\text{cl} \( A \)

(d). \( NV\text{int} (A \cap B) = NV\text{int} \( A \) \cap NV\text{int} \( B \) \, , \quad (d'). \ NV\text{cl} (A \cup B) = NV\text{cl} \( A \) \cup NV\text{cl} \( B \)

(e). \( NV\text{int} (1_{NV} ) = 1_{NV} \, , \quad (e'). \ NV\text{cl} (0_{NV} ) = 0_{NV}. \)

**Proof.** (a), (b) and (e) are obvious, (c) follows from (a).

(d) From \( NV\text{int} (A \cap B) \subseteq NV\text{int} \( A \) \) and \( NV\text{int} (A \cap B) \subseteq NV\text{int} \( B \) \) we obtain \( NV\text{int} (A \cap B) \subseteq NV\text{int} \( A \) \cap NV\text{int} \( B \) \).

On the other hand, from the facts \( NV\text{int} \( A \) \subseteq A \) and \( NV\text{int} \( B \) \subseteq B \Rightarrow NV\text{int} (A \cap NV\text{int} \( B \) \subseteq A \cap B \) and \( NV\text{int} \( A \) \cap NV\text{int} \( B \) \) \in \( \tau \) we see that \( NV\text{int} \( A \) \cap NV\text{int} \( B \) \subseteq NV\text{int} (A \cap B) \), for which we obtain the required result.

(a')-(e') They can be easily deduced from (a)-(e).

**Definition 3.5.** A NVS \( A = \{ \{ x, [\hat{I}_x, \hat{I}_A, \hat{F}_A] \} \} \) in NVTS \( (X, \tau) \) is said to be

(1). Neutrosophic Vague semi closed set (NVSCS in short) if \( NV\text{int} (NV\text{cl} \( A \)) \subseteq A \),

(2). Neutrosophic Vague semi open set (NVSOS in short) if \( A \subseteq NV\text{cl} \( NV\text{int} \( A \)) \),

(3). Neutrosophic Vague pre-closed set (NVPCS in short) if \( NV\text{cl} (NV\text{int} \( A \)) \subseteq A \),

(4). Neutrosophic Vague pre-open set (NVPOS in short) if \( A \subseteq NV\text{int} (NV\text{cl} \( A \)) \),

(5). Neutrosophic Vague \( \alpha \)-closed set (NV \( \alpha \) CS in short) if \( NV\text{cl} (NV\text{cl} \( NV\text{int} \( A \)) \)) \subseteq A \),

(6). Neutrosophic Vague \( \alpha \)-open set (NV \( \alpha \) OS in short) if \( A \subseteq NV\text{int} (NV\text{cl} \( NV\text{int} \( A \)) \),

(7). Neutrosophic Vague semi pre- closed set (NVSPCS in short) if \( NV\text{int} (NV\text{cl} (NV\text{int} \( A \)) \)) \subseteq A \),

(8). Neutrosophic Vague semi pre-open set (NVSPOS in short) if \( A \subseteq NV\text{cl} (NV\text{int} (NV\text{cl} \( A \)) \),

(9). Neutrosophic Vague regular open set (NVROS in short) if \( A = NV\text{int} (NV\text{cl} \( A \)) \),

(10). Neutrosophic Vague regular closed set (NVRCS in short) if \( A = NV\text{cl} (NV\text{int} \( A \)) \).
Definition 3.6. Let $A$ be NVS of a NVTS $(X, \tau)$. Then the neutrosophic vague semi interior of $A$ (NVsint $(A)$ in short) and neutrosophic vague semi closure of $A$ (NV scl $(A)$ in short) are defined by

1. $\text{NVsint } (A) = \cup \{G/G\text{isaNVOSin }X\text{and } G \subseteq A\}$,
2. $\text{NV scl } (A) = \cap \{K/K\text{isaNVSCSin }X\text{and } A \subseteq K\}$.

Result 3.7. Let $A$ be NVS of a NVTS $(X, \tau)$, then

1. $\text{NV scl } (A) = A \cup \text{NV int } (\text{NV cl } (A))$,
2. $\text{NVsint } (A) = A \cap \text{NV cl } (\text{NV int } (A))$.

Definition 3.8. Let $A$ be NVS of a NVTS $(X, \tau)$. Then the neutrosophic vague alpha interior of $A$ (NV $\alpha$ int $(A)$ in short) and neutrosophic vague alpha closure of $A$ (NV $\alpha$ cl $(A)$ in short) are defined by

1. $\text{NV $\alpha$ int } (A) = \cup \{G/G\text{isaNV$\alpha$OSin }X\text{and } G \subseteq A\}$,
2. $\text{NV $\alpha$ cl } (A) = \cap \{K/K\text{isaNV$\alpha$CSin }X\text{and } A \subseteq K\}$.

Result 3.9. Let $A$ be NVS of a NVTS $(X, \tau)$, then

1. $\text{NV $\alpha$ cl } (A) = A \cup \text{NV int } (\text{NV cl } (A))$,
2. $\text{NV $\alpha$ int } (A) = A \cap \text{NV cl } (\text{NV int } (A))$.

Definition 3.10. Let $A$ be NVS of a NVTS $(X, \tau)$. Then the neutrosophic vague semi-pre interior of $A$ (NV $\text{sp}$ int $(A)$ in short) and neutrosophic vague semi-pre closure of $A$ (NV $\text{sp}$ cl $(A)$ in short) are defined by

1. $\text{NV$\text{sp}$ int } (A) = \cup \{G/G\text{isaNV$\text{sp}$OSin }X\text{and } G \subseteq A\}$,
2. $\text{NV$\text{sp}$ cl } (A) = \cap \{K/K\text{isaNV$\text{sp}$CSin }X\text{and } A \subseteq K\}$.

Definition 3.11. A NVS $A$ of a NVTS $(X, \tau)$ is said to be neutrosophic vague generalized closed set (NVGCS in short) if $\text{NV cl } (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is NVOS in $X$.

Definition 3.12. A NVS $A$ of a NVTS $(X, \tau)$ is said to be neutrosophic vague generalized semi closed set (NVGCS in short) if $\text{NV scl } (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is NVOS in $X$.

Definition 3.13. A NV $A$ of a NVTS $(X, \tau)$ is said to be neutrosophic vague generalized closed set (NVGCS in short) if $\text{NV scl } (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is NVOS in $X$.

Definition 3.14. A NVS $A$ of a NVTS $(X, \tau)$ is said to be neutrosophic vague generalized semi-pre closed set (NVGSCS in short) if $\text{NV $\text{sp}$ cl } (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is NVOS in $X$.

Definition 3.15. Let $(X, \tau)$ be a NVTS and $A = \left\{ [x, \hat{T}_A, \hat{I}_A, \hat{F}_A] \right\}$ be a NVS in $X$. The neutrosophic vague pre interior of $A$ and denoted by NV $\text{pint}$ $(A)$ is defined to be the union of all neutrosophic vague pre-open sets of $X$ which are contained in $A$. The intersection of all neutrosophic vague pre-closed sets containing $A$ is called the neutrosophic pre-closure of $A$ and is denoted by NV $\text{pcl}$ $(A)$.

1. $\text{NV $\text{pint}$ } (A) = \cup \{G/G\text{isaNV$\text{POS}$in }X\text{and } G \subseteq A\}$,
2. $\text{NV $\text{pcl}$ } (A) = \cap \{K/K\text{isaNV$\text{PC}$Sin }X\text{and } A \subseteq K\}$.
Result 3.16. Let $A$ be NVS of a NVTS $(X, \tau)$ , then

(1) $NVpc (A) = A \cup NVcl (NVint (A)),$

(2) $NVpint (A) = A \cap NVcl (NVcl (A)).$

4. Neutrosophic Vague Generalized Pre-closed Sets

In this section we introduce neutrosophic vague generalized pre-closed set and their properties are analysed.

Definition 4.1. A NVS $A$ is said to be neutrosophic vague generalized pre-closed set (NVGPCS in short) in $(X, \tau)$ if $NVpc (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is NVS in $X$. The family of all NVGPCSs of a NVTS $(X, \tau)$ is denoted by $NVGPC (X)$.

Example 4.2. Let $X = \{a, b\}$ and let $\tau = \{0, G, 1\}$ is a NVT on $X$, where $G = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, Then the NVT $A = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$ is NVGPC in $X$.

Theorem 4.3. Every NVCS is NVGCS but not conversely.

Proof. Let $A$ be NVCS in $X$. Suppose $U$ is NVOS in $X$, such that $A \subseteq U$. Then $NVcl (A) = A \subseteq U$. Hence $A$ is NVGCS in $X$.

Example 4.4. Let $X = \{a, b, c\}$ and let $\tau = \{0, G, 1\}$ be a NVT on $X$, where $G = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, Then the NVT $A = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$ is NVGCS in $X$ but not NVCS in $X$. Since $NVcl (A) = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\} \neq A$.

Theorem 4.5. Every NVCS is NV $\alpha$ CS but not conversely.

Proof. Let $A$ be NVCS in $X$. Since $NVint (A) \subseteq A$, and $NVcl (A) = A$, which implies $NVint (NVcl (A)) \subseteq NVcl (A)$, so $NVcl (NVint (NVcl (A))) \subseteq A$. Hence $A$ is NV $\alpha$ CS in $X$.

Example 4.6. Let $X = \{a, b, c\}$ and let $\tau = \{0, G, 1\}$ be a NVT on $X$, where $G_1 = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, $G_2 = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, Then the NVT $A = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$ is NV $\alpha$ CS in $X$ but not NVCS in $X$. Since $NVcl (A) = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, $\neq A$.

Theorem 4.7. Every NVCS is NVPCPS but not conversely.

Proof. Suppose $A$ is NVCS in $X$. Since $NVint (A) \subseteq A$, $NVcl (NVint (A)) \subseteq NVcl (A) = A$, which implies $NVcl (NVint (A)) \subseteq A$. Thus $A$ is NVPCPS in $X$.

Example 4.8. Let $X = \{a, b\}$ and let $\tau = \{0, G, 1\}$ be NVT on $X$, where $G_1 = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$, $G_2 = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$. Then the NVT $A = \{x, \tau = [0.6, 0.8], [0.5, 0.8], [0.2, 0.4], [0.2, 0.4]\}$ is NVPCPS in $X$ but not NVCS in $X$.

Theorem 4.9. Every NV $\alpha$ CS is NVPCPS but not conversely.
Example 4.10. Let $X = \{a, b, c\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVT on $X$, where $G_1 = \{x, \{0.7, 0.9\}; \{0.5, 0.9\}; \{0.1, 0.3\}\}$, $G_2 = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$, $G_{cl} = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$. Then the NVS $A = \{x, \{0.3, 0.6\}; \{0.2, 0.4\}; \{0.1, 0.2\}\}$ is NVPCS in $X$ but not NVCS in $X$. Since $NVcl(NVint(NVcl(A))) = 1 \not\subset A$.

Theorem 4.11. Every NVRCS is NVCS but not conversely.

Proof. Let $A$ be NVRCS in $X$. Then $A = NVcl(NVint(A)) \Rightarrow NVcl(A) = NVcl(NVint(A))$. Therefore $NVcl(A) = A$. Hence, $A$ is NVCS in $X$.

Example 4.12. Let $X = \{a, b, c\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVT on $X$, where $G_1 = \{x, \{0.7, 0.9\}; \{0.5, 0.9\}; \{0.1, 0.3\}\}$, $G_2 = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$, $G_{cl} = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$. Then the NVS $A = \{x, \{0.3, 0.6\}; \{0.2, 0.4\}; \{0.1, 0.2\}\}$ is a NVCS in $X$ but not NVRCS in $X$.

Theorem 4.13. Every NV $\alpha$ CS is NVSCS but not conversely.

Proof. Let $A$ be NV $\alpha$ CS in $X$. Then $NVcl(NVint(NVcl(A))) \subseteq A$. Since $A \subseteq NVcl(A)$, so $NVint(NVcl(A)) \subseteq A$. Hence, $A$ is neutrosophic vague semi closed set in $X$.

Example 4.14. Let $X = \{a, b, c\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVT on $X$, where $G_1 = \{x, \{0.7, 0.9\}; \{0.5, 0.9\}; \{0.1, 0.3\}\}$, $G_2 = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$, $G_{cl} = \{x, \{0.5, 0.7\}; \{0.4, 0.8\}; \{0.3, 0.5\}\}$. Then the NVS $A = \{x, \{0.3, 0.6\}; \{0.2, 0.4\}; \{0.1, 0.2\}\}$ is NVCS in $X$ but not NV $\alpha$ CS in $X$.

Theorem 4.15. Every NVPCS is NVSPCS but not conversely.

Proof. Let $A$ be NVPCS in $X$. By hypothesis $NVcl(NVint(A)) \subseteq A$. Therefore $NVint(NVcl(NVcl(A))) \subseteq NVint(A) \subseteq A$. Therefore $NVint(NVcl(NVcl(A))) \subseteq A$. Hence, $A$ is NVSPCS in $X$.

Example 4.16. Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVT on $X$, where $G_1 = \{x, \{0.9, 1.1\}; \{0.2, 0.3\}; \{0.1, 0.3\}\}$, $G_2 = \{x, \{0.2, 0.4\}; \{0.3, 0.5\}; \{0.1, 0.3\}\}$, $G_{cl} = \{x, \{0.2, 0.4\}; \{0.3, 0.5\}; \{0.1, 0.3\}\}$. Then the NVS $A = \{x, \{0.4, 0.7\}; \{0.8, 0.9\}; \{0.3, 0.6\}\}$ is NVPCS in $X$ but not NVPCS in $X$.

Theorem 4.17. Every NVCS is NVGPCS but not conversely.

Proof. Let $A$ be NVCS in $X$ and let $A \subseteq U$ and $U$ be NVOS in $X$. Since $NVpcl(A) \subseteq NVcl(A)$ and $A$ is NVCS in $X$, $NVpcl(A) \subseteq NVcl(A) = A \subseteq U$. Therefore, $A$ is NVGPCS in $X$.

Example 4.18. Let $X = \{a, b\}$ and let $\tau = \{0, G, 1\}$ be NVT on $X$, where $G = \{x, \{0.2, 0.4\}; \{0.3, 0.6\}; \{0.1, 0.3\}\}$. Then the NVS $A = \{x, \{0.4, 0.7\}; \{0.8, 0.9\}; \{0.3, 0.6\}\}$ is NVGPCS in $X$ but not NVCS in $X$.

Theorem 4.19. Every NVGS is NVGPCS but not conversely.

Proof. Let $A$ be NVGS in $X$ and let $A \subseteq U$ and $U$ is NVOS in $(X, \tau)$. Since $NVpcl(A) \subseteq NVcl(A)$ and by hypothesis, $NVpcl(A) \subseteq U$. Therefore, $A$ is NVGPCS in $X$.
Example 4.20. Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVTS on $X$, where $G_1 = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$ and $G_2 = \left\{ x, \frac{0.1}{[0.7, 0.9]}, \frac{0.2}{[0.7, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$. Then the NVS $A = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$ is NVGPCS in $X$ but not NVGCS in $X$ since NVcl ($A$) $= \emptyset$.

Theorem 4.21. Every NV $\alpha$ CS is NVGPCS but not conversely.

Proof. Let $A$ be NV $\alpha$ CS in $X$ and let $A \subseteq U$ and $U$ be NVOS in $X$. By hypothesis, NVcl ($NVint (NVcl (A))$) $\subseteq A$. Since $A \subseteq NVcl (A)$, NVcl ($NVint (A)$) $\subseteq NVcl (NVint (NVcl (A)))$ $\subseteq A$. Hence NVpcl ($A$) $\subseteq A \subseteq U$. Therefore $A$ is NVGPCS in $X$.

Example 4.22. Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVTS on $X$, where $G_1 = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$ and $G_2 = \left\{ x, \frac{0.1}{[0.7, 0.9]}, \frac{0.2}{[0.7, 0.9]}, \frac{0.3}{[0.7, 0.9]} \right\}$. Then the NVS $A = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$ is NVGPCS in $X$ but not NV $\alpha$ CS in $X$ since NVcl ($NVint (NVcl (A))$) $= \emptyset$.

Theorem 4.23. Every NVRCS is NVGPCS but not conversely.

Proof. Let $A$ be a NVRCS in $X$. By Definition 3.5, $A = NVcl (NVint (A))$. This implies NVcl ($A$) $= NVcl (NVint (A))$. Therefore NVcl ($A$) $= A$. That is $A$ is NVCS in $X$. By Theorem 4.17, $A$ is NVGPCS in $X$.

Example 4.24. Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVTS on $X$, where $G_1 = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\}$ and $G_2 = \left\{ x, \frac{0.1}{[0.7, 0.9]}, \frac{0.2}{[0.7, 0.9]}, \frac{0.3}{[0.7, 0.9]} \right\}$. Then the NVS $A = \left\{ x, \frac{0.1}{[0.6, 0.9]}, \frac{0.2}{[0.6, 0.9]}, \frac{0.3}{[0.6, 0.9]} \right\} \neq A$.

Theorem 4.25. Every NVGPCS is NVGPCS but not conversely.

Proof. Let $A$ be NVPCS in $X$ and let $A \subseteq U$ and $U$ is NVOS in $X$. By Definition 3.5, NVcl ($NVint (A)$) $\subseteq A$. This implies NVpcl ($A$) $= A \cup NVcl (NVint (A)) \subseteq A$. Therefore NVpcl ($A$) $\subseteq U$. Hence $A$ is NVGPCS in $X$.

Example 4.26. Let $X = \{a, b, c\}$ and let $\tau = \{0, G_1, 1\}$ be NVTS on $X$, where $G = \left\{ x, \frac{0.1}{[0.6, 0.7]}, \frac{0.2}{[0.6, 0.7]}, \frac{0.3}{[0.6, 0.7]} \right\}$. Then the NVS $A = \left\{ x, \frac{0.1}{[0.6, 0.7]}, \frac{0.2}{[0.6, 0.7]}, \frac{0.3}{[0.6, 0.7]} \right\}$ is NVGPCS in $X$ but not NVPCS in $X$ since NVcl ($NVint (A)$) $= \emptyset$.

Theorem 4.27. Every NV $\alpha$ GCS is NVGPCS but not conversely.

Proof. Let $A$ be NV $\alpha$ GCS in $X$ and let $A \subseteq U$ and $U$ is NVOS in $(X, \tau)$. By Result 3.9, $A \cup NVcl (NVint (NVcl (A))) \subseteq U$. This implies NVcl ($NVint (NVcl (A))$) $\subseteq U$ and NVcl ($NVint (A)$) $\subseteq U$. Thus NVpcl ($A$) $= A \cup NVcl (NVint (A)) \subseteq U$. Hence $A$ is NVGPCS in $X$.

Example 4.28. Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, 1\}$ be NVTS on $X$, where $G_1 = \left\{ x, \frac{0.1}{[0.6, 0.8]}, \frac{0.2}{[0.6, 0.8]}, \frac{0.3}{[0.6, 0.8]} \right\}$ and $G_2 = \left\{ x, \frac{0.1}{[0.7, 0.9]}, \frac{0.2}{[0.7, 0.9]}, \frac{0.3}{[0.7, 0.9]} \right\}$. Then the NVS $A = \left\{ x, \frac{0.1}{[0.6, 0.8]}, \frac{0.2}{[0.6, 0.8]}, \frac{0.3}{[0.6, 0.8]} \right\}$ is NVGPCS in $X$ but not NV $\alpha$ GCS in $X$ since NVcl ($A$) $= \emptyset$.

Theorem 4.29. Every NVGPCS is NVSPCS but not conversely.

Proof. Let $A$ be NVGPCS in $X$, this implies NVpcl ($A$) $\subseteq U$ whenever $A \subseteq U$ and $U$ is NVOS in $X$. By hypothesis $NVcl (NVint (A)) \subseteq A$. Therefore $NVint (NVcl (NVint (A))) \subseteq NVint (A) \subseteq A$. Therefore $NVint (NVcl (NVint (A))) \subseteq A$. Hence $A$ is NVSPCS in $X$.
Example 4.30. Let \( X = \{a, b, c\} \) and let \( \tau = \{0, G, 1\} \) is NVT on \( X \), where \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = G \) is NVSPCS in \( X \) but not NVGPCS in \( X \).

Theorem 4.31. Every NVGPCS is NVGSPCS but not conversely.

Proof. Let \( A \) be NVGPCS in \( X \) and let \( A \subseteq U \) and \( U \) is NVS in \( X \). By hypothesis \( \text{NVcl}(\text{NVint}(A)) \subseteq A \subseteq U \).

Every NVGPCS is NVGSPCS but not conversely.

Example 4.32. Let \( X = \{a, b, c\} \) and let \( \tau = \{0, G, 1\} \) be NVT on \( X \), where \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = G \) is NVGSPCS in \( X \) but not NVGPCS in \( X \).

Proposition 4.33. NVS CS and NVGPCS are independent of each other.

Example 4.34. Let \( X = \{a, b, c\} \) and let \( \tau = \{0, G, 1\} \) be NVT on \( X \), where \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = G \) is NVS CS in \( X \) but not NVGPCS in \( X \).

Example 4.35. Let \( X = \{a, b, c\} \) and let \( \tau = \{0, G, 1\} \) be NVT on \( X \), where \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \) is NVGPCS in \( X \) but not NVS CS in \( X \) since \( \text{NVint}(\text{NVcl}(A)) = 1 \not\subset A \).

Proposition 4.36. NVS CS and NVGPCS are independent to each other.

Example 4.37. Let \( X = \{a, b, c\} \) and let \( \tau = \{0, G, 1\} \) be NVT on \( X \), where \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = G \) is NVS CS in \( X \) but not NVGPCS in \( X \) since \( A \subseteq G \) but \( \text{NVcl}(A) = 1 \not\subseteq G \).

Example 4.38. Let \( X = \{a, b\} \) and let \( \tau = \{0, G_1, G_2, 1\} \) be NVT on \( X \), where \( G_1 = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \) and \( G_2 = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \). Then the NVS \( A = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \) is NVGPCS in \( X \) but not NVS CS in \( X \).

Remark 4.39. We have the following implications by summing up the above theorems.

Remark 4.40. The union of any two NVGPCSs is not NVGPCS in general as seen in the following example.

Example 4.41. Let \( X = \{a, b, c\} \) and let \( G = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \) be NVT on \( X \) and the NVSs \( A = \left\{ x, \begin{bmatrix} 0 & 0.3 & 0.6 \\ 0 & 0.4 & 0.5 \\ 0 & 0.6 & 0.7 \end{bmatrix} \right\} \) are NVGPCSs in \( X \) but \( A \cup B \) is not NVGPCS in \( X \).
5. Neutrosophic Vague Generalized Pre-open Set

In this section we introduce neutrosophic vague generalized pre-open set and their properties are deliberated.

**Definition 5.1.** A NVS $A$ is said to be neutrosophic vague generalized pre-open set (NVGPOS in short) in $(X, \tau)$ if the complement $A^c$ is NVGPCS in $(X, \tau)$. The family of all NVGPOSs of NVTS $(X, \tau)$ is denoted by NVGO $(X)$.

**Example 5.2.** Let $X = \{a, b, c\}$ and let $\tau = \{0, G, 1\}$ is NVT on $X$, where $G = \{x, \{0.7, 0.8\} \cup \{0.1, 0.2\} \cup \{0.5, 0.6\}\}$. Then the NVS $A = \{x, \{0.3, 0.5\} \cup \{0.1, 0.4\} \cup \{0.5, 0.7\}\}$ is NVGPOS in $X$.

**Theorem 5.3.** For any NVTS $(X, \tau)$, we have the following results.

1. Every NVOS is NVGPOS but not conversely.
2. Every NVROS is NVGPOS but not conversely.
3. Every NV $\alpha$ OS is NVGPOS but not conversely.
4. Every NVPOS is NVGPOS but not conversely.

The converse of the above theorem need not be true which can be seen from the following examples.

**Example 5.4.** Let $X = \{a, b\}$ and $G_1 = \{x, \{0.2, 0.4\} \cup \{0.3, 0.5\}\}$. Then $\tau = \{0, G_1, G_2, 1\}$ is NVLT on $X$. The NVS $A = \{x, \{0.3, 0.5\} \cup \{0.4, 0.7\}\}$ is NVGPOS in $X$ but not NVOS in $X$.

**Example 5.5.** Let $X = \{a, b\}$ and $G_1 = \{x, \{0.7, 0.8\} \cup \{0.1, 0.3\}\}$. Then $\tau = \{0, G_1, G_2, 1\}$ is NVLT on $X$. The NVS $A = \{x, \{0.3, 0.5\} \cup \{0.4, 0.7\}\}$ is NVGPOS in $X$ but not NVROS in $X$.

**Example 5.6.** Let $X = \{a, b, c\}$ and $G_1 = \{x, \{0.2, 0.4\} \cup \{0.3, 0.5\}\}$. Then $\tau = \{0, G_1, G_2, 1\}$ is a NVT on $X$. The NVS $A = \{x, \{0.3, 0.5\} \cup \{0.4, 0.7\}\}$ is NVGPOS in $X$ but not NV $\alpha$ OS in $X$.

**Example 5.7.** Let $X = \{a, b, c\}$ and $G = \{x, \{0.7, 0.8\} \cup \{0.1, 0.2\} \cup \{0.5, 0.6\}\}$. Then $\tau = \{0, G, 1\}$ is NVT on $X$. The NVS $A = \{x, \{0.3, 0.5\} \cup \{0.4, 0.7\}\}$ is NVGPOS in $X$ but not NVPOS in $X$.

**Remark 5.8.** The intersection of any two NVPOSs is not NVPOS in general and it is shown in the following example.

**Example 5.9.** Let $X = \{a, b, c\}$ and let $G = \{x, \{0.3, 0.5\} \cup \{0.6, 0.7\}\}$. Then $\tau = \{0, G, 1\}$ is NVT on $X$ and the NVSs $A = \{x, \{0.4, 0.6\} \cup \{0.1, 0.2\} \cup \{0.5, 0.7\}\}$ and $B = \{x, \{0.4, 0.6\} \cup \{0.3, 0.5\}\}$ are NVPOSs in $X$ but $A \cap B$ is not NVPOS in $X$.

**Theorem 5.10.** Let $(X, \tau)$ be NVTS. If $A \in$ NVGO $(X)$ then $V \subseteq$ NVint $(NVcl (A))$ whenever $V \subseteq A$ and $V$ is NVCS in $X$. 
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Proof. Let $A \in \text{NVGPO}(X)$. Then $A^c$ is NVGPCS in $X$. Therefore $\text{NVpcl}(A^c) \subseteq U$ whenever $A^c \subseteq U$ and $U$ is NVOS in $X$. That is $\text{NVcl}(\text{NVint}(A^c)) \subseteq U$. This implies $U^c \subseteq \text{NVint}(\text{NVcl}(A))$ whenever $U^c \subseteq A$ and $U^c$ is NVCS in $X$. Replacing $U^c$ by $V$, we get $V \subseteq \text{NVint}(\text{NVcl}(A))$ whenever $V \subseteq A$ and $V$ is NVCS in $X$. \hfill \Box

**Theorem 5.11.** Let $(X, \tau)$ be NVTS. Then for every $A \in \text{NVGPO}(X)$ and for every $B \in \text{NVS}(X)$, $\text{NVpint}(A) \subseteq B \subseteq A$ implies $B \in \text{NVGPO}(X)$.

**Proof.** By hypothesis $A^c \subseteq B^c \subseteq (\text{NVpint}(A))^c$. Let $B^c \subseteq U$ and $U$ be NVOS. Since $A^c \subseteq B^c$, $A^c \subseteq U$. But $A^c$ is NVGPCS, $\text{NVpcl}(A^c) \subseteq U$. Also $B^c \subseteq (\text{NVpint}(A))^c = \text{NVpcl}(A^c)$. Therefore $\text{NVpcl}(B^c) \subseteq \text{NVpcl}(A^c) \subseteq U$. Hence $B^c$ is NVGPCS. Which implies $B$ is NVGPOS of $X$.

**Theorem 5.12.** A NVS $A$ of NVTS $(X, \tau)$ is NVPOS if and only if $F \subseteq \text{NVpint}(A)$ whenever $F$ is NVCS and $F \subseteq A$.

**Proof.** Necessity: Suppose $A$ is NVGPOS in $X$. Let $F$ be NVCS and $F \subseteq A$. Then $F^c$ is NVOS in $X$ such that $A^c \subseteq F^c$. Since $A^c$ is NVGPCS, we have $\text{NVpcl}(A^c) \subseteq F^c$. Hence $(\text{NVpint}(A))^c \subseteq F^c$. Therefore $F \subseteq \text{NVpint}(A)$.

Sufficiency: Let $A$ be NVS of $X$ and let $F \subseteq \text{NVpint}(A)$ whenever $F$ is NVCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and $F^c$ is NVOS. By hypothesis, $(\text{NVpint}(A))^c \subseteq F^c$. Which implies $\text{NVpcl}(A^c) \subseteq F^c$. Therefore $A^c$ is NVGPCS of $X$. Hence $A$ is NVGPOS of $X$.

**Corollary 5.13.** A NVS $A$ of a NVTS $(X, \tau)$ is NVGPOS if and only if $F \subseteq \text{NVint}(\text{NVcl}(A))$ whenever $F$ is NVCS and $F \subseteq A$.

**Proof.** Necessity: Suppose $A$ is NVGPOS in $X$. Let $F$ be NVCS and $F \subseteq A$. Then $F^c$ is NVOS in $X$ such that $A^c \subseteq F^c$. Since $A^c$ is NVGPCS, we have $\text{NVpcl}(A^c) \subseteq F^c$. Therefore $\text{NVcl}(\text{NVint}(A^c)) \subseteq F^c$. Hence $(\text{NVint}(\text{NVcl}(A)))^c \subseteq F^c$. This implies $F \subseteq \text{NVint}(\text{NVcl}(A))$.

Sufficiency: Let $A$ be NVS of $X$ and let $F \subseteq \text{NVint}(\text{NVcl}(A))$ whenever $F$ is NVCS and $F \subseteq A$. Then $A^c \subseteq F^c$ and $F^c$ is NVOS. By hypothesis, $(\text{NVint}(\text{NVcl}(A)))^c \subseteq F^c$. Hence $\text{NVcl}(\text{NVint}(A^c)) \subseteq F^c$, which implies $\text{NVpcl}(A^c) \subseteq F^c$. Hence $A$ is NVGPOS of $X$.

**Theorem 5.14.** For a NVS, $A$ is NVOS and NVGPCS in $X$ if and only if $A$ is NVROS in $X$.

**Proof.** Necessity: Let $A$ be NVOS and NVGPCS in $X$. Then $\text{NVpcl}(A) \subseteq A$. This implies $\text{NVcl}(\text{NVint}(A)) \subseteq A$. Since $A$ is NVOS, it is NVPOS. Hence $A \subseteq \text{NVint}(\text{NVcl}(A))$. Therefore $A = \text{NVint}(\text{NVcl}(A))$. Hence $A$ is NVROS in $X$.

Sufficiency: Let $A$ be NVROS in $X$. Therefore $A = \text{NVint}(\text{NVcl}(A))$. Let $A \subseteq U$ and $U$ is NVOS in $X$. This implies $\text{NVpcl}(A) \subseteq A$. Hence $A$ is NVGPCS in $X$.

### 6. Applications of Neutrosophic Vague Generalized Pre-closed Sets

In this section we provide some applications of neutrosophic vague generalized pre-closed sets.

**Definition 6.1.** A NVTS $(X, \tau)$ is said to be neutrosophic vague $T_{1/2}$ space ($\text{NVT}_{1/2}$ in short) if every NVGCS in $X$ is NVCS in $X$.

**Definition 6.2.** A NVTS $(X, \tau)$ is said to be neutrosophic vague $pT_{1/2}$ space ($\text{NVP}_{1/2}$ in short) if every NVPCS in $X$ is NVCS in $X$. 


Definition 6.3. A NVTS \((X, \tau)\) is said to be neutrosophic vague \(gpT_{1/2}\) space (NV\(gpT_{1/2}\) in short) if every NVGPCS in \(X\) is NVCS in \(X\).

Definition 6.4. A NVTS \((X, \tau)\) is said to be a neutrosophic vague \(gpT_p\) space (NV\(gpT_p\) in short) if every NVGPCS in \(X\) is NVPCS in \(X\).

Theorem 6.5. Every NV \(T_{1/2}\) space is NV\(gpT_p\) space. But the converse is not true in general.

\textbf{Proof.} Let \(X\) be NV \(T_{1/2}\) space and let \(A\) be NVGCS in \(X\), we know that every NVGCS is NVGPCS, hence \(A\) is NVGPCS in \(X\). By hypothesis \(A\) is NVCS in \(X\). Since every NVCS is NVPCS, \(A\) is NVPCS in \(X\). Hence \(X\) is NV\(gpT_p\) space.

Example 6.6. Let \(X = \{a, b\}\) and \(G = \left\{ x, \begin{array}{l} a \in [0,0.6,0.7], \frac{a}{0,1,0.3}, \frac{a}{0,3,6,4} \end{array}, \begin{array}{l} b \in [0,0.8,0.9], \frac{b}{0,2,0.4}, \frac{b}{0,1,0.2} \end{array} \right\} \). Then \(\tau = \{0,G,1\}\) is NVT on \(X\). Let \(A = \left\{ x, \begin{array}{l} a \in [0,0.7,0.8], \frac{a}{0,2,0.3}, \frac{a}{0,3,0.9}, \frac{a}{0,6,0.1}, \frac{a}{0,3,0.1} \end{array}, \begin{array}{l} b \in [0,3,0.9], \frac{b}{0,6,0.1} \end{array} \right\} \). Then \((X, \tau)\) is NV\(gpT_p\) space. But it is not NV \(T_{1/2}\) space since \(A\) is NVGCS but not NVCS in \(X\).

Theorem 6.7. Every NV\(gpT_{1/2}\) space is NV\(gpT_p\) space. But the converse is not true in general.

\textbf{Proof.} Let \(X\) be NV\(gpT_{1/2}\) space and let \(A\) be NVGPCS in \(X\). By hypothesis \(A\) is NVCS in \(X\). Since every NVCS is NVPCS, \(A\) is NVPCS in \(X\). Hence \(X\) is NV\(gpT_p\) space.

Example 6.8. Let \(X = \{a, b, c\}\) and \(G = \left\{ x, \begin{array}{l} a \in [0,0.5,0.7], \frac{a}{0,2,0.4}, \frac{a}{0,3,0.9}, \frac{a}{0,6,0.1}, \frac{a}{0,3,0.1} \end{array}, \begin{array}{l} b \in [0,0.3,0.9], \frac{b}{0,1,0.3}, \frac{b}{0,2,0.7} \end{array}, \begin{array}{l} c \in [0,4,0.7], \frac{c}{0,2,0.6}, \frac{c}{0,3,0.9}, \frac{c}{0,4,0.9} \end{array} \right\} \). Then \(\tau = \{0,G,1\}\) is NVT on \(X\). Let \(A = \left\{ x, \begin{array}{l} a \in [0,0.4,0.8], \frac{a}{0,2,0.8}, \frac{a}{0,2,0.6}, \frac{a}{0,3,0.9}, \frac{a}{0,4,0.9} \end{array}, \begin{array}{l} b \in [0,2,0.6], \frac{b}{0,1,0.4}, \frac{b}{0,5,0.9} \end{array}, \begin{array}{l} c \in [0,0.1,0.6], \frac{c}{0,2,0.5}, \frac{c}{0,4,0.9} \end{array} \right\} \). Then \(X\) is NV\(gpT_{1/2}\) space. But it is not NV\(gpT_p\) space since \(A\) is NVGCS but not NVCS in \(X\).

Theorem 6.9. Let \((X, \tau)\) be NVTS and \(X\) is NV\(gpT_{1/2}\) space then,

1. Any union of NVGPCS is NVGCS
2. Any intersection of NVGPOS is NVGPOS.

\textbf{Proof.}

1. Let \(\{A_i\}_{i \in A}\) is a collection of NVGPCSs in NV\(gpT_{1/2}\) space \((X, \tau)\). Therefore every NVGPCS is NVCS. But the union of NVCS is NVCS. Hence the union of NVGPCS is NVGCS in \(X\).

2. It can be proved by taking complement of (1).

Theorem 6.10. A NVTS \(X\) is NV\(gpT_{1/2}\) space if and only if NVGPO \((X)\) = NVPO \((X)\).

\textbf{Proof.} Necessity: Let \(A\) be NVGPOS in \(X\), then \(A^c\) is NVGPCS in \(X\). By hypothesis \(A^c\) is NVGPCS in \(X\). Therefore \(A\) is NVPOS in \(X\). Hence NVGPO \((X)\) = NVPO \((X)\).

Sufficiency: Let \(A\) be NVGPCS in \(X\). Then \(A^c\) is NVGPOS in \(X\). By hypothesis \(A^c\) is NVGPOS in \(X\). Therefore \(A\) is NVPCS in \(X\). Hence \(X\) is NV\(gpT_p\) space.

References


