NEUTROSOPHIC TOPOLOGIES IN CRISP APPROXIMATION SPACES

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Abstract: In this paper we introduce a new type of neutrosophic topology and we investigate the topological structures neutrosophic rough sets. Further we examine the reflexivity and transitivity of neutrosophic rough sets and obtain some of its properties.

Keywords: Approximation Operators, Neutrosophic Rough Topological spaces, Neutrosophic Rough sets.

Introduction: Topology is a branch of mathematics, which has application not only every other branch of mathematics but also in many real time problems. The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh [12] and intuitionistic fuzzy set by Atanassov [4]. Rough set theory presents still another attempt to tackle for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh [12] and intuitionistic fuzzy set by Atanassov [4]. Rough set theory presents still another attempt to

Preliminaries:

Definition 2.1[10]: A neutrosophic set A on the universe of discourse X is defined as
\[ A = \{(x, T(x), I(x), F(x)), x \in X\} \]
where \( T, I, F: X \to [0, 1] \) and
\[ 0 \leq T(x) + I(x) + F(x) \leq 3 \].

Definition 2.2[1]: A neutrosophic relation R is a neutrosophic set
\[ R = \{< x, y >, T_R(x, y), I_R(x, y), F_R(x, y), x, y \in U\} \]
where \( T_R: U \times U \to [0, 1], I_R: U \times U \to [0, 1], F_R: U \times U \to [0, 1] \) and satisfies
\[ 0 \leq T_R(x, y) + I_R(x, y) + F_R(x, y) \leq 3, \]
for all \((x, y) \in U \times U\).

Definition 2.3[3]: Let \( U \) be a nonempty universe of discourse which may be infinite. A subset \( R \in P(U \times U) \) is referred to as a (crisp) binary relation on \( U \). The relation \( R \) is referred to as reflexive if for all \( x \in U \), \((x, x) \in R\); \( R \) is referred to as symmetric if for all \( x, y \in U \), \((x, y) \in R \) implies \((y, x) \in R\); \( R \) is referred to as transitive if for all \( x, y, z \in U \), \((x, y) \in R \) and \((y, z) \in R \) implies \((x, z) \in R\); \( R \) is referred to as a similarity relation if \( R \) is reflexive and symmetric; \( R \) is referred to as a preorder if \( R \) is reflexive and transitive; and \( R \) is referred to as an equivalence relation if \( R \) is reflexive, symmetric and transitive.

Definition 2.4[1]: For an arbitrary crisp relation \( R \) on \( U \), we can define a set-valued mapping \( R_S: U \to P(U) \) by:
\[ R_S(x) = \{y \in U|(x, y) \in R\}, x \in U. \]
\( R_S(x) \) is called the successor neighborhood of \( x \) with respect to \( R \).

Definition 2.5[2]: A neutrosophic set \( A \) is contained in another neutrosophic set \( B \), (i.e \( A \subseteq B \iff T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x) \), \( \forall x \in X \).

Definition 2.7[2]: The complement of a neutrosophic set \( F(A) \) denoted by \((F, A)^C \) and is defined as \((F, A)^C = (F^C, A)\) where
A neutrosophic rough set is the approximation of a neutrosophic set with respect to a crisp approximation space, and each approximating operator satisfies the following axioms:

\( \text{Definition 3.1:}
\)

Let \( (U, \tau) \) be a neutrosophic topological space, A in \( \tau \) is called a neutrosophic open set in \( (U, \tau) \). The complement of neutrosophic open set in the neutrosophic topological space \( (U, \tau) \) is called a neutrosophic closed set in \( (U, \tau) \).

\( \text{Definition 4.2:}
\)

Let \( (U, \tau) \) be a neutrosophic topological space and \( A \in \tau \). Then the neutrosophic interior and neutrosophic closure of \( A \) are, respectively, defined as follows:

\( \text{Definition 4.4:}
\)

Let \( (U, \tau) \) be a neutrosophic topological space, for any \( A \in \tau \), the upper and lower approximations of \( A \) w.r.t. \( (U, \tau) \) are defined as follows: we define the upper and lower approximation of \( A \) with respect to \( (U, \tau) \) are respectively, denoted by \( R(A) \) and \( \overline{R}(A) \).

\( \text{Definition 4.4:}
\)

Let \( (U, \tau) \) be a neutrosophic topological space, for a family \( \mathcal{A} \) of neutrosophic open sets in \( U \), the complement of \( \mathcal{A} \) in \( U \) is called a neutrosophic closed set in \( (U, \tau) \).
A neutrosophic topology \( \tau \) on \( U \) is called a neutrosophic Alexandrov topology if the intersection of arbitrarily many neutrosophic open sets is still open, or equivalently, the union of arbitrarily many neutrosophic closed sets is still closed. A neutrosophic topological space \((U, \tau)\) is said to be a neutrosophic Alexandrov space if \( \tau \) is a neutrosophic Alexandrov topology on \( U \).

A mapping \( \text{cl} : N(U) \rightarrow N(U) \) is referred to as a neutrosophic closure operator if it satisfies axioms (C1)-(C4).

Definition 4.4.
A mapping \( \text{int} : N(U) \rightarrow N(U) \) is referred to as a neutrosophic interior operator if it satisfies axioms (I1)-(I4).

It is easy to show that a neutrosophic interior operator determines a neutrosophic topology, \( \tau_{\text{Int}} = \{ A \in N(U) | \text{int}(A) = A \} \).

So, the neutrosophic open sets are the fixed points of \( \text{int} \). Dually, from a neutrosophic closure we can obtain a neutrosophic topology on \( U \) by setting \( \tau_{\text{Cl}} = \{ A \in N(U) | \text{cl}(\sim A) = \sim A \} \).

Definition 4.5.
A neutrosophic topology \( \tau \) on \( U \) is called a neutrosophic Alexandrov topology on \( U \) by setting \( \tau_{\text{Cl}} = \{ A \in N(U) | \text{cl}(A) = A \} \).

Properties:
\begin{align*}
(\text{I1}) \text{int}(\alpha, \beta, \gamma) &= \alpha, \beta, \gamma & \forall (\alpha, \beta, \gamma) \in [0,1]. \\
(\text{I2}) \text{int}(A \cap B) &= \text{int}(A) \cap \text{int}(B) & \forall A, B \in N(U). \\
(\text{I3}) \text{int}(\text{int}(A)) &= \text{int}(A) & \forall A \in N(U). \\
(\text{I4}) \text{int}(A) \subseteq A & \forall A \in N(U).
\end{align*}

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