Neutrosophic Linear Programming Problem

Abdel-Nasser Hussien¹, Mai Mohamed², Mohamed Abdel-Baset²*, and Florentin Smarandache³

¹Department of Computer Science, Faculty of Computers and Informatics, Zagazig University, Egypt.
²Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt.
³Math & Science Department, University of New Mexico, Gallup, NM 87301, USA.


Abstract: Smarandache presented neutrosophic theory as a tool for handling undetermined information, and together with Wang et al. introduced single valued neutrosophic sets that is a special neutrosophic set and can be used expeditiously to deal with real-world problems, especially in decision support. In this paper, we propose linear programming problems based on neutrosophic environment. Neutrosophic sets characterized by three independent parameters, namely truth-membership degree (T), indeterminacy-membership degree (I) and falsity-membership degree (F), which is more capable to handle imprecise parameters. We also transform the neutrosophic linear programming problem into a crisp programming model by using neutrosophic set parameters. To measure the efficiency of our proposed model we solved several numerical examples.

Keywords: linear programming problem; neutrosophic; neutrosophic sets.

1 Introduction

Linear programming is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Decision making is a process of solving the problem and achieving goals under the conditions, and it is very difficult in some cases due to incomplete and imprecise information. And in linear programming problems the decision maker may not be able to specify the objective function and/or constraints functions precisely. In 1995, Smarandache [5-7] introduced neutrosophy which is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information.[2,5-7] Neutrosophic sets characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F), where T,I,F are standard or non-standard subsets of ]0−,1+. That is T, I, F are real standard or real nonstandard subsets of ]0−,1+]. There is no restriction on the sum of (x), (x) and (x), so

\[0 \leq \text{sup}(T_x) + \text{sup}(I_x) + \text{sup}(F_x) \leq 3+.\]

In the following, we adopt the notations µ(x), σ(x) and ν(x) instead of T_x, I_x and F_x, respectively. Also we write SVN numbers instead of single valued neutrosophic numbers.

*Corresponding author E-mail: analyst_mohamed@yahoo.com

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2.2 Single Valued Neutrosophic Sets (SVNS)[2,7]

Let \( X \) be a universe of discourse. A single valued neutrosophic set \( A \) over \( X \) is an object having the form

\[
A = \{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \}
\]

where \( \mu_A(x) : X \rightarrow [0,1] \), \( \sigma_A(x) : X \rightarrow [0,1] \) and \( \nu_A(x) : X \rightarrow [0,1] \) with \( 0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3 \) for all \( x \in X \). The intervals \( \mu(x), \sigma(x) \) and \( \nu(x) \) denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of \( x \) to \( A \), respectively.

For convenience, a SVN number is denoted by \( A = (a, \sigma, \nu) \), where \( a, \sigma, \nu \in [0,1] \) and \( a + \sigma + \nu \leq 3 \).

2.3 Complement [3]

The complement of a single valued neutrosophic set \( A \) is denoted by \( \overline{A} \) and is defined by

\[
\overline{A} = \{ (x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \nu_A(x)) : x \in X \}
\]

for all \( x \in X \).

2.4 Union [3]

The union of two single valued neutrosophic sets \( A \) and \( B \) is a single valued neutrosophic set \( C \), written as \( C = A \cup B \), whose truth-membership, indeterminacy membership and falsity-membership functions are given by

\[
T_C(x) = \max \{ T_A(x), T_B(x) \},
I_C(x) = \max \{ I_A(x), I_B(x) \},
F_C(x) = \min \{ (A(x), F_B(x)) \},
\]

for all \( x \) in \( X \).

2.5 Intersection [3]

The intersection of two single valued neutrosophic sets \( A \) and \( B \) is a single valued neutrosophic set \( C \), written as \( C = A \cap B \), whose truth-membership, indeterminacy membership and falsity-membership functions are given by

\[
T_C(x) = \min \{ T_A(x), T_B(x) \},
I_C(x) = \min \{ I_A(x), I_B(x) \},
F_C(x) = \max \{ (A(x), F_B(x)) \} \text{ for all } x \text{ in } X.
\]

3 Neutrosophic Linear Programming Problem

Linear programming problem with neutrosophic coefficients (NLPP) is defined as the following:

Maximize \( Z = \sum_{j=1}^{n} c_j x_j \)

Subject to

\[
\sum_{j=1}^{n} a_{ij}^{-n} x_j \leq b_i \quad 1 \leq i \leq m \tag{1}
\]

\( x_j \geq 0, \quad 1 \leq j \leq n \)

where \( a_{ij}^L \) is a neutrosophic number.

The single valued neutrosophic number \( (a_{ij}^L) \) is given by \( A=(a, \sigma, \nu) \) where \( a, \sigma, \nu \in [0,1] \) and \( a + \sigma + \nu \leq 3 \)

The truth-membership function of neutrosophic number \( a_{ij}^L \) is defined as:

\[
T a_{ij}^L(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}
\]

The indeterminacy-membership function of neutrosophic number \( a_{ij}^L \) is defined as:

\[
I a_{ij}^L(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & b_1 \leq x \leq b_2 \\ \frac{b_3-x}{b_3-b_2} & b_2 \leq x \leq b_3 \\ 0 & \text{otherwise} \end{cases}
\]

And its falsity-membership function of neutrosophic number \( a_{ij}^L \) is defined as:

\[
F a_{ij}^L(x) = \begin{cases} \frac{x-c_1}{c_2-c_1} & c_1 \leq x \leq c_2 \\ \frac{c_3-x}{c_3-c_2} & c_2 \leq x \leq c_3 \\ 1 & \text{otherwise} \end{cases}
\]

Then we find the upper and lower bounds of the objective function for truth-membership, indeterminacy and falsity membership as follows:

\[
Z^T_{IL} = \max \{ z(x_i^L) \} \text{ and } Z^T_{IL} = \min \{ z(x_i^L) \} \text{ where } 1 \leq i \leq k
\]

\[
z^T_{u1} = z^T_{u1} + z^T_{u1} - R(z^T_{u1} - z^T_{l1})
\]

\[
z^T_{u1} = z^T_{u1} \text{ and } z^T_{l1} = z^T_{u1} - S(z^T_{u1} - z^T_{l1})
\]

Where \( R, S \) are predetermined real numbers in \((0, 1)\)

The truth membership, indeterminacy membership, falsity membership of objective function as follows:

\[
T o(x) = \begin{cases} 1 & \text{if } z \geq z^T_{u1} \\ \frac{z-z^T_{l1}}{z^T_{u1}-z^T_{l1}} & \text{if } z^T_{l1} \leq z \leq z^T_{u1} \\ 0 & \text{if } z < z^T_{l1} \end{cases}
\]

\[
I o(x) = \begin{cases} 1 & \text{if } z \geq z^T_{u1} \\ \frac{z-z^T_{l1}}{z^T_{u1}-z^T_{l1}} & \text{if } z^T_{l1} \leq z \leq z^T_{u1} \\ 0 & \text{if } z < z^T_{l1} \end{cases}
\]

\[
F o(x) = \begin{cases} 1 & \text{if } z \geq z^T_{u1} \\ \frac{z-z^T_{l1}}{z^T_{u1}-z^T_{l1}} & \text{if } z^T_{l1} \leq z \leq z^T_{u1} \\ 0 & \text{if } z < z^T_{l1} \end{cases}
\]

The neutrosophic set of the \( i^{th} \) constraint \( c_i \) is defined as:
The above problem is equivalent to the following:

\begin{equation}
max(\alpha - \beta - \theta)
\end{equation}

Subject to

\begin{equation}
\alpha \leq T(x)
\beta \geq F(x)
\theta \geq I(x)
\alpha \geq \beta
\alpha \geq \theta
0 \leq \alpha + \beta + \theta \leq 3
\alpha, \beta, \theta \geq 0
x \geq 0
\end{equation}

The previous model can be written as:

\begin{equation}
min (1 - \alpha)\beta \theta
\end{equation}

Subject to

\begin{equation}
\alpha \leq T(x)
\beta \geq F(x)
\theta \geq I(x)
\alpha \geq \beta
\alpha \geq \theta
0 \leq \alpha + \beta + \theta \leq 3
\alpha, \beta, \theta \geq 0
x \geq 0
\end{equation}

\section{4 Neutrosophic Optimization Model}

In our neutrosophic model we want to maximize the degree of acceptance and minimize the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Neutrosophic optimization model can be defined as:

\begin{equation}
max T(x)
\end{equation}

\begin{equation}
min F(x)
\end{equation}

\begin{equation}
min I(x)
\end{equation}

Subject to

\begin{equation}
T(x) \geq F(x)
\end{equation}

\begin{equation}
T(x) \geq I(x)
\end{equation}

\begin{equation}
0 \leq T(x) + I(x) + F(x) \leq 3
\end{equation}

\begin{equation}
x \geq 0
\end{equation}

Where $T(x), F(x), I(x)$ denotes the degree of acceptance, rejection and indeterminacy of $x$ respectively.

The above problem is equivalent to the following:

\begin{equation}
max \alpha, \min \beta, \min \theta
\end{equation}

Subject to

\begin{equation}
\alpha \leq T(x)
\beta \leq F(x)
\theta \leq I(x)
\alpha \geq \beta
\alpha \geq \theta
\end{equation}

\section{5 The Algorithm for Solving Neutrosophic Linear Programming Problem (NLPP)}

\textbf{Step 1}: Solve the objective function subject to the constraints.

\textbf{Step 2}: Create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

\textbf{Step 3}: Declare goals and tolerance.

\textbf{Step 4}: Construct membership functions.

\textbf{Step 5}: Set $\alpha, \beta, \theta$ in the interval $]0, 1[+$ for each neutrosophic number.

\textbf{Step 6}: Find the upper and lower bound of objective function as we illustrated previously in section 3.

\textbf{Step 7}: Construct neutrosophic optimization model as in equation (13).

\section{6 Numerical Examples}

To measure the efficiency of our proposed model we solved four numerical examples.

\subsection{6.1 Illustrative Example#1}
\[ \text{max } 5x_1 + 3x_2 \]
\[ \text{subject to } \]
\[ 4x_1 + 3x_2 \leq 12 \]
\[ x_1 + 3x_2 \leq 6 \]
\[ x_1, x_2 \geq 0 \]

where
\[ c_1 = \{ (4,5,6),(0.5,0.8,0.3) \} ; \]
\[ c_2 = \{ (2,3,3), (0.6,0.4,0) \} ; \]
\[ a_{11} = 4 = \{ (3.5,4.1), (0.75,0.5,0.25) \} ; \]
\[ a_{12} = 3 = \{ (2.5,3.2), (0.2,0.8,0.4) \} ; \]
\[ a_{21} = 1 = \{ (0.1,2), (0.15,0.5,0) \} ; \]
\[ a_{22} = 3 = \{ (2.8,3.3,2), (0.75,0.5,0.25) \} ; \]
\[ b_1 = 12 = \{ (11,12,13), (0.2,0.6,0.5) \} ; \]
\[ b_2 = 6 = \{ (5.5,6,7,5), (0.8,0.6,0.4) \} . \]

The equivalent crisp formulation is:
\[ \text{max } 1.3125x_1 + 0.0158x_2 \]
\[ \text{subject to } \]
\[ 2.5375x_1 + 0.54375x_2 \leq 2.475 \]
\[ 0.3093x_1 + 1.125x_2 \leq 2.1375 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 0.9754; x_2 = 0; \) with optimal objective value 1.2802

### 6.2 Illustrative Example#2

\[ \text{max } 25x_1 + 48x_2 \]
\[ \text{subject to } \]
\[ 15x_1 + 30x_2 \leq 45000 \]
\[ 24x_1 + 6x_2 \leq 24000 \]
\[ 21x_1 + 14x_2 \leq 28000 \]
\[ x_1, x_2 \geq 0 \]

where
\[ c_1 = \{ (19,25,33), (0.8,0.1,0.4) \} ; \]
\[ c_2 = \{ (24,48,54), (0.75,0.25,0) \} . \]

The corresponding crisp linear programs given as follows:
\[ \text{max } 11.069x_1 + 22.8125x_2 \]
\[ \text{subject to } \]
\[ 15x_1 + 30x_2 \leq 45000 \]
\[ 24x_1 + 6x_2 \leq 24000 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 0; x_2 = 1500; \) with optimal objective value 34218.75

### 6.3 Illustrative Example#3

\[ \text{max } 25x_1 + 48x_2 \]
\[ \text{subject to } \]
\[ 15x_1 + 30x_2 \leq 45000 \]
\[ 24x_1 + 6x_2 \leq 24000 \]
\[ 21x_1 + 14x_2 \leq 28000 \]
\[ x_1, x_2 \geq 0 \]

where
\[ a_{11} = 15 = \{ (14,15,17), (0.75,0.5,0.25) \} ; \]
\[ a_{12} = 30 = \{ (25,30,34), (0.25,0.7,0.4) \} ; \]
\[ a_{21} = 24 = \{ (21,24,26), (0.4,0.6,0) \} ; \]
\[ a_{22} = 6 = \{ (4,6,8), (0.75,0.5,0.25) \} ; \]
\[ a_{31} = 21 = \{ (17,21,22), (1,0.25,0) \} ; \]
\[ a_{32} = 14 = \{ (12,14,19), (0.6,0.4,0) \} ; \]
\[ b_1 = 45000 = \{ (44980,45000,45030), (0.3,0.4,0.8) \} ; \]
\[ b_2 = 24000 = \{ (23980,24000,24050), (0.4,0.25,0.5) \} ; \]
\[ b_3 = 28000 = \{ (27990,28000,28030), (0.9,0.2,0) \} . \]

The associated crisp linear programs model will be:
\[ \text{max } 25x_1 + 48x_2 \]
\[ \text{subject to } \]
\[ 5.75x_1 + 6.397x_2 \leq 9282 \]
\[ 10.312x_1 + 6.187x_2 \leq 14178.37 \]
\[ x_1, x_2 \geq 0 \]

The optimal solution is \( x_1 = 0; x_2 = 1450.993; \) with optimal objective value 69647.65
6.4 Illustrative Example\#4

\[
\begin{align*}
\text{max} & \quad 7x_1 + 5x_2 \\
\text{s.t.} & \quad \sim x_1 + 2x_2 \leq 6 \\
& \quad \sim 4x_1 + 3x_2 \leq 12 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

where

\[
\begin{align*}
a_{11} &= \{0.5, 1, 2\}, \\
a_{12} &= \{2.5, 3, 3.2\}, \\
a_{21} &= \{3.5, 4, 4.1\}, \\
a_{22} &= \{2.5, 3, 3.2\}\end{align*}
\]

The associated crisp linear programs model will be:

\[
\begin{align*}
\text{max} & \quad 7x_1 + 5x_2 \\
\text{s.t.} & \quad 0.284x_1 + 1.142x_2 \leq 6 \\
& \quad 1.45x_1 + 1.36x_2 \leq 12 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

The optimal solution is \(x_1 = 4.3665; x_2 = 4.168\); with optimal objective value 63.91

The result of our NLP model in this example is better than the results obtained by intuitionistic fuzzy set [4].

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7 Conclusion and Future Work

Neutrosophic sets and fuzzy sets are two hot research topics. In this paper, we propose linear programming model based on neutrosophic environment, simultaneously considering the degrees of acceptance, indeterminacy and rejection of objectives, by proposed model for solving neutrosophic linear programming problems (NIPP). In the proposed model, we maximize the degrees of acceptance and minimize indeterminacy and rejection of objectives. NIPP was transformed into a crisp programming model using truth membership, indeterminacy membership, and falsity membership functions. We also give a numerical examples to show the efficiency of the proposed method.

As far as future directions are concerned, these will include studying the duality theory of linear programming problems based on neutrosophic environment.

References


Abdel Nasser H. Zaied, is prof. of Information Systems, Dean, Faculty of Computers and Informatics, Zagazig University, Egypt. He previously worked as an associate professor of Industrial Engineering, Zagazig University Egypt, an assistant professor of Technology Management, Arabian Gulf University, Bahrain; and as visiting professor at Oakland University, USA. He supervised 12 PhD. thesis and 45 MSc. thesis, and examined 8 PhD. thesis and 47 MSc thesis. He published 30 research papers in International and Regional Journals and 22 research papers in International and National conferences. His areas of research are: Systems Analysis and Design; Information Security; Knowledge Management; Quality Management Systems, Information Security and project Management, Electronic applications.
Mai Mohamed, received her BS degree and master degree from Zagazig University, faculty of computers and informatics, Egypt. She is currently research interest is computation intelligence, neural networks, and Neutrosophic logic.

Mohamed Abdel-Baset
Received his B.Sc., M.Sc and the Ph.D in information technology from Zagazig University. He is a lecturer in the operations Research Department, Faculty of Computers and Informatics, Zagazig University.

His current research interests are Optimization, Operations Research, Data Mining, Computational Intelligence, Applied Statistics and Decision support systems. He is also a reviewer in different international journals and conferences. He has published more than 100 articles in international journals and conference proceedings.

Florentin Smarandache
polymath, professor of mathematics

University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA.

http://fs.gallup.unm.edu/