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NC-TODIM-Based MAGDM under a Neutrosophic Cubic Set Environment

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Abstract: A neutrosophic cubic set is the hybridization of the concept of a neutrosophic set and an interval neutrosophic set. A neutrosophic cubic set has the capacity to express the hybrid information of both the interval neutrosophic set and the single valued neutrosophic set simultaneously. As newly defined, little research on the operations and applications of neutrosophic cubic sets has been reported in the current literature. In the present paper, we propose the score and accuracy functions for neutrosophic cubic sets and prove their basic properties. We also develop a strategy for ranking of neutrosophic cubic numbers based on the score and accuracy functions. We firstly develop a TODIM (Tomada de decisao interativa e multicritério) in the neutrosophic cubic set (NC) environment, which we call the NC-TODIM. We establish a new NC-TODIM strategy for solving multi attribute group decision making (MAGDM) in neutrosophic cubic set environment. We illustrate the proposed NC-TODIM strategy for solving a multi attribute group decision making problem to show the applicability and effectiveness of the developed strategy. We also conduct sensitivity analysis to show the impact of ranking order of the alternatives for different values of the attenuation factor of losses for multi-attribute group decision making strategies.

Keywords: neutrosophic cubic set; single valued neutrosophic set; interval neutrosophic set; multi attribute group decision making; TODIM strategy; NC-TODIM

1. Introduction

While modelling multi attribute decision making (MADM) and multi attribute group decision making (MAGDM), it is often observed that the parameters of the problem are not precisely known. The parameters often involve uncertainty. To deal with uncertainty, Zadeh [1] left an important mark to represent and compute with imperfect information by introducing the fuzzy set. The fuzzy set fostered a broad research community, and its impact has also been clearly felt at the application level in MADM [2–4] and MAGDM [5–9].

Atanassov [10] incorporated the non-membership function as an independent component and defined the intuitionistic fuzzy set (IFS) at first to express uncertainty in a more meaningful way. IFSs have been applied in many MADM problems [11–13]. Smarandache [14] proposed the notion of the neutrosophic set (NS) by introducing indeterminacy as an independent component. Wang et al. [15] grounded the concept of the single valued neutrosophic set (SVNS), an instance of the neutrosophic set, to deal with incomplete, inconsistent, and indeterminate information in a realistic way. Wang et al. [16] proposed the interval neutrosophic set (INS) as a subclass of neutrosophic sets in which the values of truth, indeterminacy, and falsity membership degrees are interval numbers. Theoretical development

and applications of SVN_Ss and IN_Ss are found in [17–37] for MADM or MAGDM. Some studies on MADM in single valued neutrosophic hesitant fuzzy set environments are found in [38–41].

NS and INS are both capable of handling uncertainty and incomplete information. By fusing NS and INS, Ali et al. [42] proposed the neutrosophic cubic set (NCS) and defined external and internal neutrosophic cubic sets, and established some of their properties. In the same study, Ali et al. [42] proposed an adjustable strategy to NCS-based decision making. Jun et al. [43] also defined NCS by combining NS and INS. In decision making process, the advantage of NCSs is that the decision makers can employ the hybrid information comprising of IN_Ss and SVN_Ss for evaluating and rating of the alternatives with respect to their predefined attributes. However, there are only a few studies in the literature to deal with MADM and MAGDM in the NCS environment. Banerjee et al. [44] established grey relational analysis (GRA) [45–47] based on the new MADM strategy in the NCS environment. In the same study, Banerjee et al. [44] proposed the Hamming distances for weighted grey relational coefficients and ideal grey relational coefficients, and offered the concept of relative closeness coefficients for presenting the ranking order of the alternatives based on the descending order of their relative closeness coefficients.

Similarity measure is an important mathematical tool in decision-making problems. Pramanik et al. [48] at first defined similarity measure for NCSs and proved its basic properties. In the same study, Pramanik et al. [48] developed a new MAGDM strategy in the NCS environment. Lu and Ye [49] proposed cosine measures between NCSs and established their basic properties. In the same study, Lu and Ye [49] proposed three new cosine measures-based MADM strategies under a NCS environment.

Due to little research on the operations and application of NCSs, Pramanik et al. [50] proposed several operational rules on NCSs, and defined Euclidean distances and arithmetic average operators of NCSs. In the same study, Pramanik et al. [50] also employed the information entropy scheme to calculate the unknown weights of the attributes, and developed a new extended TOPSIS strategy for MADM under the NCS environment. Zhan et al. [51] proposed a new algorithm for multi-criteria decision making (MCDM) in an NCS environment based on a weighted average operator and a weighted geometric operator. Ye [52] established the concept of a linguistic neutrosophic cubic number (LNCN). In the same study, Ye [52] developed a new MADM strategy based on a LNCN weighted arithmetic averaging (LNCNWAA) operator and a LNCN weighted geometric averaging (LNCNWGA) operator under a linguistic NCS environment.

In the literature, there are only six strategies [44,48–52] for MADM and MAGDM in NCS environment. However, we say that none of them is generally superior to all others. So, new strategies for MADM and MAGDM should be explored under the NCS environment for the development of neutrosophic studies.

TODIM (an acronym in Portuguese for interactive multi-criteria decision making strategy named Tomada de decisao interativa e multicritério) is an important MADM strategy, since it considers the decision makers' bounded rationality. Firstly, Gomes and Lima [53] introduced the TODIM strategy based on prospect theory [54]. Krohling and Souza [55] defined the fuzzy TODIM strategy to solve MCDM problems. Several researchers applied the TODIM strategy in various fuzzy MADM or MAGDM problems [56–58]. Fan et al. [59] introduced the extended TODIM strategy to deal with the hybrid MADM problems. Krohling et al. [60] extended the TODIM strategy from fuzzy environment to intuitionistic fuzzy environment to process the intuitionistic fuzzy information. Wang [61] introduced TODIM strategy into multi-valued neutrosophic set environment. Zhang et al. [62] proposed the TODIM strategy for MAGDM problems under a neutrosophic number environment. Ji et al. [63] proposed the TODIM strategy under a multi-valued neutrosophic environment and employed it to solve personal selection problems. In 2017, Xu et al. [64] developed the TODIM strategy in a single valued neutrosophic setting and extended it into interval neutrosophic setting. Neutrosophic TODIM [64] is capable of dealing with only single-valued neutrosophic information or interval neutrosophic information.

NCS can be used to express the interval neutrosophic information and neutrosophic information in the process of MADGM. It seems that TODIM in NCSs has an enormous chance of success to deal with group decision making problems. In the NCS environment, the TODIM strategy is yet to appear. Motivated by these, we initiated the study of TODIM in the NCS environment, which we call NC-TODIM.

However, NCSs comprise of hybrid information of INSs and SVNNSs simultaneously, which are more flexible and elegant for expressing neutrosophic cubic information. To apply NCSs to MADGM problems, we introduce some basic operations of neutrosophic cubic (NC) numbers and the score, and accuracy functions of NC numbers, and the ranking strategy of NC numbers.

In this paper we develop a TODIM strategy (for short, NC-TODIM strategy) for MAGDM in the NCS environment. The proposed NC-TODIM strategy was proven to be capable of successfully dealing with MAGDM problems by solving an illustrative example. What is more, a comparative analysis ensured the feasibility of the proposed NC-TODIM strategy.

The remainder of the paper is divided into seven sections that are organized as follows: Section 2 presents some basic definitions of NS, SVNNS, INS, and NCS. Section 3 presents comparison strategy of two NC-numbers. Section 4 is devoted to present the proposed NC-TODIM strategy. Section 5 presents an illustrative numerical example of MAGDM in the NCS environment. Section 6 is devoted to analyzing the ranking order with different values of attenuation factors of losses. Section 7 presents a comparative analysis between the developed strategy and other existing strategies in the NCS environment. Section 8 presents the conclusion and the future scope of research.

2. Preliminaries

In this section, we review some basic definitions which are important to develop the paper.

Definition 1. [14] NS. Let U be a space of points (objects) with a generic element in U denoted by u , i.e., $u \in U$. A neutrosophic set R in U is characterized by truth-membership function t_R , indeterminacy-membership function i_R , and falsity-membership function f_R , where t_R, i_R, f_R are the functions from U to $]^{-0}, 1^+[$ i.e., $t_R, i_R, f_R: U \rightarrow]^{-0}, 1^+[$ that means $t_R(u), i_R(u), f_R(u)$ are the real standard or non-standard subset of $]^{-0}, 1^+[$. The neutrosophic set can be expressed as $R = \{ \langle u; (t_R(u), i_R(u), f_R(u)) \rangle : \forall u \in U \}$. Since $t_R(u), i_R(u), f_R(u)$ are the subset of $]^{-0}, 1^+[$, there the sum $(t_R(u) + i_R(u) + f_R(u))$ lies between $^{-0}$ and 3^+ , where $^{-0} = 0 - \epsilon$ and $3^+ = 3 + \epsilon, \epsilon > 0$.

Example 1. Suppose that $U = \{u_1, u_2, u_3, \dots\}$ is the universal set. Let R_1 be any neutrosophic set in U . Then R_1 expressed as $R_1 = \{ \langle u_1; (0.6, 0.3, 0.4) \rangle : u_1 \in U \}$.

Definition 2. [15] SVNNS. Let U be a space of points (objects) with a generic element in U denoted by u . A single valued neutrosophic set H in U is expressed by $H = \{ \langle u, (t_H(u), i_H(u), f_H(u)) \rangle : u \in U \}$, where $t_H(u), i_H(u), f_H(u) \in [0, 1]$. Therefore for each $u \in U, t_H(u), i_H(u), f_H(u) \in [0, 1]$ and $0 \leq t_H(u) + i_H(u) + f_H(u) \leq 3$.

Definition 3. [16] INS. Let G be a non-empty set. An interval neutrosophic set \tilde{G} in G is characterized by truth-membership function $t_{\tilde{G}}$, the indeterminacy membership function $i_{\tilde{G}}$ and falsity membership function $f_{\tilde{G}}$. For each $g \in G, t_{\tilde{G}}(g), i_{\tilde{G}}(g), f_{\tilde{G}}(g) \subseteq [0, 1]$ and \tilde{G} defined as

$$\tilde{G} = \{ \langle g; [t_{\tilde{G}}^-(g), t_{\tilde{G}}^+(g)], [i_{\tilde{G}}^-(g), i_{\tilde{G}}^+(g)], [f_{\tilde{G}}^-(g), f_{\tilde{G}}^+(g)] : \forall g \in G \}.$$

Here,

$$t_{\tilde{G}}^-(g), t_{\tilde{G}}^+(g), i_{\tilde{G}}^-(g), i_{\tilde{G}}^+(g), f_{\tilde{G}}^-(g), f_{\tilde{G}}^+(g) : G \rightarrow]^{-0}, 1^+[,$$

and

$$^{-0} \leq \sup t_{\tilde{G}}^+(g) + \sup i_{\tilde{G}}^+(g) + \sup f_{\tilde{G}}^+(g) \leq 3^+.$$

In real problems it is difficult to express the truth-membership function, indeterminacy-membership function and falsity-membership function in the form of

$$t_{\tilde{G}}^-(g), t_{\tilde{G}}^+(g), i_{\tilde{G}}^-(g), i_{\tilde{G}}^+(g), f_{\tilde{G}}^-(g), f_{\tilde{G}}^+(g) : G \rightarrow]^{-0}, 1^+[.$$

Here,

$$t_{\tilde{G}}^-(g), t_{\tilde{G}}^+(g), i_{\tilde{G}}^-(g), i_{\tilde{G}}^+(g), f_{\tilde{G}}^-(g), f_{\tilde{G}}^+(g) : G \rightarrow [0, 1].$$

Example 2. Suppose that $G = \{g_1, g_2, g_3, \dots, g_n\}$ is a non-empty set. Let \tilde{G}_1 be an INS. Then \tilde{G}_1 is expressed as

$$\tilde{G}_1 = \{ \langle g_1; [0.39, 0.47], [0.17, 0.43], [0.18, 0.36] : g_1 \in G \}.$$

Definition 4. [42,43] NCS. A NCS in a non-empty set G is defined as $\odot = \langle g; \tilde{G}(g), R(g) \rangle : \forall g \in G$, where \tilde{G} and R are the INS and NS in G respectively. NCS can be presented as an order pair $\odot = \langle \tilde{G}, R \rangle$, then we call it as a neutrosophic cubic (NC) number.

Example 3. Suppose that $G = \{g_1, g_2, g_3, \dots, g_n\}$ is a non-empty set. Let \odot_1 be any NC-number. Then \odot_1 can be expressed as $\odot_1 = \langle g_1; [0.39, 0.47], [0.17, 0.43], [0.18, 0.36], (0.6, 0.3, 0.4) \rangle : g_1 \in G$.

Some operations of NC-numbers:

i. Union of any two NC-numbers

Let $\odot_1 = \langle \tilde{G}_1, R_1 \rangle$ and $\odot_2 = \langle \tilde{G}_2, R_2 \rangle$ be any two NC-numbers in a non-empty set G . Then the union of \odot_1 and \odot_2 denoted by $\odot_1 \cup \odot_2$ and defined as

$$\odot_1 \cup \odot_2 = \langle \tilde{G}_1(g) \cup \tilde{G}_2(g), R_1(g) \cup R_2(g) \forall g \in G \rangle,$$

where $\tilde{G}_1(g) \cup \tilde{G}_2(g) = \langle g, [\max \{t_{\tilde{G}_1}^-(g), t_{\tilde{G}_2}^-(g)\}, \max \{t_{\tilde{G}_1}^+(g), t_{\tilde{G}_2}^+(g)\}], [\max \{i_{\tilde{G}_1}^-(g), i_{\tilde{G}_2}^-(g)\}, \max \{i_{\tilde{G}_1}^+(g), i_{\tilde{G}_2}^+(g)\}], [\min \{f_{\tilde{G}_1}^-(g), f_{\tilde{G}_2}^-(g)\}, \min \{f_{\tilde{G}_1}^+(g), f_{\tilde{G}_2}^+(g)\}] \rangle : g \in G$ and $R_1(g) \cup R_2(g) = \langle g, \max \{t_{R_1}(g), t_{R_2}(g)\}, \max \{i_{R_1}(g), i_{R_2}(g)\}, \min \{f_{R_1}(g), f_{R_2}(g)\} \rangle : \forall g \in U$.

Example 4. Let \odot_1 and \odot_2 be two NC-numbers in G presented as follows:

$$\odot_1 = \langle [0.39, 0.47], [0.17, 0.43], [0.18, 0.36], (0.6, 0.3, 0.4) \rangle$$

and

$$\odot_2 = \langle [0.56, 0.70], [0.27, 0.42], [0.15, 0.26], (0.7, 0.3, 0.6) \rangle .$$

Then

$$\odot_1 \cup \odot_2 = \langle [0.56, 0.70], [0.27, 0.43], [0.15, 0.26], (0.7, 0.3, 0.4) \rangle .$$

ii. Intersection of any two NC-numbers

Intersection of two NC-numbers denoted and defined as follows:

$$\odot_1 \cap \odot_2 = \langle \tilde{G}_1(g) \cap \tilde{G}_2(g), R_1(g) \cap R_2(g) \forall g \in G \rangle,$$

where $\tilde{G}_1(g) \cap \tilde{G}_2(g) = \langle g, [\min \{t_{\tilde{G}_1}^-(g), t_{\tilde{G}_2}^-(g)\}, \min \{t_{\tilde{G}_1}^+(g), t_{\tilde{G}_2}^+(g)\}], [\min \{i_{\tilde{G}_1}^-(g), i_{\tilde{G}_2}^-(g)\}, \min \{i_{\tilde{G}_1}^+(g), i_{\tilde{G}_2}^+(g)\}], [\max \{f_{\tilde{G}_1}^-(g), f_{\tilde{G}_2}^-(g)\}, \max \{f_{\tilde{G}_1}^+(g), f_{\tilde{G}_2}^+(g)\}] \rangle : g \in G$ and $R_1(g) \cap R_2(g) = \langle g, \min \{t_{R_1}(g), t_{R_2}(g)\}, \min \{i_{R_1}(g), i_{R_2}(g)\}, \max \{f_{R_1}(g), f_{R_2}(g)\} \rangle : \forall g \in U$.

Example 5. Let \odot_1 and \odot_2 be any two NC-numbers in G presented as follows:

$$\odot_1 = \langle [0.45, 0.57], [0.27, 0.33], [0.18, 0.46], (0.7, 0.3, 0.5) \rangle$$

and

$$\odot_2 = \langle [0.67, 0.75], [0.22, 0.44], [0.17, 0.21], (0.8, 0.4, 0.4) \rangle.$$

Then

$$\odot_1 \cap \odot_2 = \langle [0.45, 0.57], [0.22, 0.33], [0.18, 0.46], (0.7, 0.3, 0.4) \rangle.$$

iii. Compliment of a NC-number

Let $\odot_1 = \langle \tilde{G}_1, R_1 \rangle$ be a NCS in G . Then, the compliment of $\odot_1 = \langle \tilde{G}_1, R_1 \rangle$ denoted by $\odot_1^c = \langle g, \tilde{G}_1^c(g), R_1^c(g) \rangle: \forall g \in G$.

Here, $\tilde{G}_1^c = \langle g, [t_{\tilde{G}_1^c}^-(g), t_{\tilde{G}_1^c}^+(g)], [i_{\tilde{G}_1^c}^-(g), i_{\tilde{G}_1^c}^+(g)], [f_{\tilde{G}_1^c}^-(g), f_{\tilde{G}_1^c}^+(g)] \rangle: \forall g \in G$, where, $t_{\tilde{G}_1^c}^-(g) = f_{\tilde{G}_1}^-(g)$, $t_{\tilde{G}_1^c}^+(g) = f_{\tilde{G}_1}^+(g)$, $i_{\tilde{G}_1^c}^-(g) = \{1\} - i_{\tilde{G}_1}^-(g)$, $i_{\tilde{G}_1^c}^+(g) = \{1\} - i_{\tilde{G}_1}^+(g)$, $f_{\tilde{G}_1^c}^-(g) = t_{\tilde{G}_1}^-(g)$, $f_{\tilde{G}_1^c}^+(g) = t_{\tilde{G}_1}^+(g)$ and $t_{R_1^c}(g) = f_{R_1}(g)$, $i_{R_1^c}(g) = \{1^+\} - i_{R_1}(g)$, $f_{R_1^c}(g) = t_{R_1}(g)$.

Example 6. Assume that \odot_1 be any NC-number in G in the form:

$$\odot_1 = \langle [0.45, 0.57], [0.27, 0.33], [0.18, 0.46], (0.7, 0.3, 0.5) \rangle$$

Then compliment of \odot_1 is obtained as

$$\odot_1^c = \langle [0.18, 0.46], [0.73, 0.67], [0.45, 0.57], (0.5, 0.7, 0.7) \rangle.$$

Definition 5. Score function. Let \odot_1 be a NC-number in a non-empty set G . Then, a score function of \odot_1 , denoted by $Sc(\odot_1)$ is defined as:

$$Sc(\odot_1) = \frac{1}{2} \left[\left(\frac{2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2}{4} \right) + \left(\frac{1 + a - 2b - c}{2} \right) \right] \tag{1}$$

where, $\odot_1 = \langle [a_1, a_2], [b_1, b_2], [c_1, c_2], (a, b, c) \rangle$ and $Sc(\odot_1) \in [-1, 1]$.

Proposition 1. Score function of two NC-numbers lies between -1 to 1 .

Proof. Using the definition of INS and NS, we have all $a_1, a_2, b_1, b_2, c_1, c_2, a, b$, and $c \in [0, 1]$.

Since,

$$\begin{aligned} 0 \leq a_1 \leq 1, 0 \leq a_2 \leq 1 \\ 0 \leq a_1 + a_2 \leq 2, \\ \Rightarrow 2 \leq 2 + a_1 + a_2 \leq 4 \end{aligned} \tag{2}$$

$$\begin{aligned} 0 \leq b_1 \leq 1 \Rightarrow 0 \leq 2b_1 \leq 2 \text{ and } 0 \leq b_2 \leq 1 \Rightarrow 0 \leq 2b_2 \leq 2 \\ \Rightarrow -2 \leq -2b_1 \leq 0 \\ \Rightarrow -2 \leq -2b_2 \leq 0 \\ \Rightarrow -4 \leq -2b_1 - 2b_2 \leq 0 \end{aligned} \tag{3}$$

$$\begin{aligned} 0 \leq c_1 \leq 1 \Rightarrow -1 \leq -c_1 \leq 0 \\ 0 \leq c_2 \leq 1 \Rightarrow -1 \leq -c_2 \leq 0 \\ \Rightarrow -2 \leq -c_1 - c_2 \leq 0 \end{aligned} \tag{4}$$

Adding Equations (2)–(4), we obtain

$$\begin{aligned} \Rightarrow -4 &\leq 2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2 \leq 4, \\ \Rightarrow -1 &\leq \frac{2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2}{4} \leq 1 \end{aligned} \tag{5}$$

Again,

$$0 \leq a \leq 1 \Rightarrow 1 \leq 1 + a \leq 2, \tag{6}$$

$$0 \leq b \leq 1 \Rightarrow 0 \leq 2b \leq 2,$$

$$0 \leq c \leq 1, \tag{7}$$

$$\Rightarrow 0 \leq 2b + c \leq 3,$$

$$\Rightarrow -3 \leq -2b - c \leq 0$$

Adding (6) and (7), we obtain

$$\begin{aligned} -2 &\leq 1 + a - 2b - c \leq 2, \\ \Rightarrow -1 &\leq \frac{1 + a - 2b - c}{2} \leq 1 \end{aligned} \tag{8}$$

Adding (5) and (8) and dividing by 2, we obtain

$$-1 \leq \frac{1}{2} \left[\left(\frac{2 + a_1 + a_2 - 2b_1 - 2b_2 - c_1 - c_2}{4} \right) + \left(\frac{1 + a - 2b - c}{2} \right) \right] \leq 1$$

$$\text{Sc}(\odot_1) \in [-1, 1].$$

Hence the proof is complete. \square

Example 7. Let \odot_1 and \odot_2 be two NC-numbers in G , presented as follows:

$$\odot_1 = \langle [0.39, 0.47], [0.17, 0.43], [0.18, 0.36], (0.6, 0.3, 0.4) \rangle$$

and

$$\odot_2 = \langle [0.56, 0.70], [0.27, 0.42], [0.15, 0.26], (0.7, 0.3, 0.6) \rangle .$$

Then, by applying Definition 5, we obtain $\text{Sc}(\odot_1) = -0.01$ and $\text{Sc}(\odot_2) = 0.07$, In this case, we can say that $\odot_2 > \odot_1$.

Definition 6. Accuracy function. Let \odot_1 be a NC-number in a non-empty set G , an accuracy function of \odot_1 is defined as:

$$\text{Ac}(\odot_1) = 1/2 [1/2(a_1 + a_2 - b_2(1 - a_2) - b_1(1 - a_1) - c_2(1 - b_1) - c_1(1 - b_2)) + a - b(1 - a) - c(1 - b)] \tag{9}$$

Here, $\text{Ac}(\odot_1) \in [-1, 1]$.

When the value of $\text{Ac}(\odot_1)$ increases, we say that the degree of accuracy of the NC-number \odot_1 increases.

Proposition 2. Accuracy function of two NC-numbers lies between -1 to 1 .

Proof. The values of accuracy function depend upon

$$\left\{ \frac{1}{2}(a_1 + a_2 - b_2(1 - a_2) - b_1(1 - a_1) - c_2(1 - b_1) - c_1(1 - b_2)) \right\} \text{ and } \{ a - b(1 - a) - c(1 - b) \}$$

The values of

$$\left\{ \frac{1}{2}(a_1 + a_2 - b_2(1 - a_2) - b_1(1 - a_1) - c_2(1 - b_1) - c_1(1 - b_2)) \right\}$$

and

$$\{ a - b(1 - a) - c(1 - b) \}$$

lie between -1 to 1 from [37].

Thus, $-1 \leq Ac(\odot_1) \leq 1$.

Hence the proof is completed. \square

Example 8. Let \odot_1 and \odot_2 be two NC-numbers in G presented as follows:

$$\odot_1 = \langle [0.41, 0.52], [0.10, 0.18], [0.06, 0.17], (0.48, 0.11, 0.11) \rangle$$

and

$$\odot_2 = \langle [0.40, 0.51], [0.10, 0.20], [0.10, 0.19], (0.50, 0.11, 0.11) \rangle .$$

Then, by applying Definition 6, we obtain $Ac(\odot_1) = 0.14$ and $Ac(\odot_2) = 0.30$. In this case, we can say that alternative \odot_2 is better than \odot_1 .

With respect to the score function Sc and the accuracy function Ac , a strategy for comparing NC-numbers can be defined as follows:

3. Comparison Strategy of Two NC-Numbers

Let \odot_1 and \odot_2 be any two NC-numbers. Then we define comparison strategy as follows:

i. If
$$Sc(\odot_1) > Sc(\odot_2), \text{ then } \odot_1 > \odot_2. \tag{10}$$

ii. If
$$Sc(\odot_1) = Sc(\odot_2) \text{ and } Ac(\odot_1) > Ac(\odot_2), \text{ then } \odot_1 > \odot_2. \tag{11}$$

iii. If
$$Sc(\odot_1) = Sc(\odot_2) \text{ and } Ac(\odot_1) = Ac(\odot_2), \text{ then } \odot_1 = \odot_2. \tag{12}$$

Example 9. Let \odot_1 and \odot_2 be two NC-numbers in G , presented as follows:

$$\odot_1 = \langle [0.23, 0.29], [0.37, 0.46], [0.34, 0.42], (0.26, 0.26, 0.26) \rangle$$

and

$$\odot_2 = \langle [0.25, 0.31], [0.35, 0.44], [0.35, 0.44], (0.28, 0.28, 0.28) \rangle .$$

Then, applying Definition 5, we obtain $Sc(\odot_1) = 0.13$ and $Sc(\odot_2) = 0.13$. Applying Definition 6, we obtain $Ac(\odot_1) = -0.20$ and $Ac(\odot_2) = -0.18$. In this case, we say that alternative $\odot_2 > \odot_1$. (Score values and Accuracy values taking correct up to two decimal places).

Definition 7. Let \odot_1 and \odot_2 be any two NC-numbers, then the distance between them is defined by

$$\partial(\odot_1, \odot_2) = \frac{1}{9} [|a_1 - d_1| + |a_2 - d_2| + |b_1 - e_1| + |b_2 - e_2| + |c_1 - f_1| + |c_2 - f_2| + |a - d| + |b - e| + |c - f|] \tag{13}$$

where, $\odot_1 = \langle [a_1, a_2], [b_1, b_2], [c_1, c_2], (a, b, c) \rangle$ and $\odot_2 = \langle [d_1, d_2], [e_1, e_2], [f_1, f_2], (d, e, f) \rangle$.

Example 10. Let \odot_1 and \odot_2 be two NC-numbers in G presented as follows:

$$\odot_1 = \langle [0.66, 0.75], [0.25, 0.32], [0.17, 0.34], (0.53, 0.17, 0.22) \rangle$$

and

$$\odot_2 = \langle [0.35, 0.55], [0.12, 0.25], [0.12, 0.20], (0.60, 0.23, 0.43) \rangle$$

Then, applying Definition 7, we obtain $\partial(\odot_1, \odot_2) = 0.12$.

Definition 8. Let $\odot_{ij} = \{ \langle [t_{ij}^-, t_{ij}^+], [i_{ij}^-, i_{ij}^+], [f_{ij}^-, f_{ij}^+], (t, i, f) \rangle \}$ be any neutrosophic cubic value. \odot_{ij} used to evaluate i -th alternative with respect to j -th criterion. The normalized form of \odot_{ij} is defined as follows:

$$\begin{aligned} \odot_{ij}^{\otimes} = \{ \langle & \left[\frac{t_{ij}^-}{\left(\sum_{i=1}^m (t_{ij}^-)^2 + (t_{ij}^+)^2\right)^{\frac{1}{2}}}, \frac{t_{ij}^+}{\left(\sum_{i=1}^m (t_{ij}^-)^2 + (t_{ij}^+)^2\right)^{\frac{1}{2}}} \right], \\ & \left[\frac{i_{ij}^-}{\left(\sum_{i=1}^m (i_{ij}^-)^2 + (i_{ij}^+)^2\right)^{\frac{1}{2}}}, \frac{i_{ij}^+}{\left(\sum_{i=1}^m (i_{ij}^-)^2 + (i_{ij}^+)^2\right)^{\frac{1}{2}}} \right], \\ & \left[\frac{f_{ij}^-}{\left(\sum_{i=1}^m (f_{ij}^-)^2 + (f_{ij}^+)^2\right)^{\frac{1}{2}}}, \frac{f_{ij}^+}{\left(\sum_{i=1}^m (f_{ij}^-)^2 + (f_{ij}^+)^2\right)^{\frac{1}{2}}} \right] \rangle, \\ & \left[\frac{t_{ij}}{\left(\sum_{i=1}^m (t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2\right)^{\frac{1}{2}}}, \frac{i_{ij}}{\left(\sum_{i=1}^m (t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2\right)^{\frac{1}{2}}}, \frac{f_{ij}}{\left(\sum_{i=1}^m (t_{ij})^2 + (i_{ij})^2 + (f_{ij})^2\right)^{\frac{1}{2}}} \right] \rangle \}. \end{aligned} \tag{14}$$

A conceptual model of the evolution of the neutrosophic cubic set is shown in Figure 1.

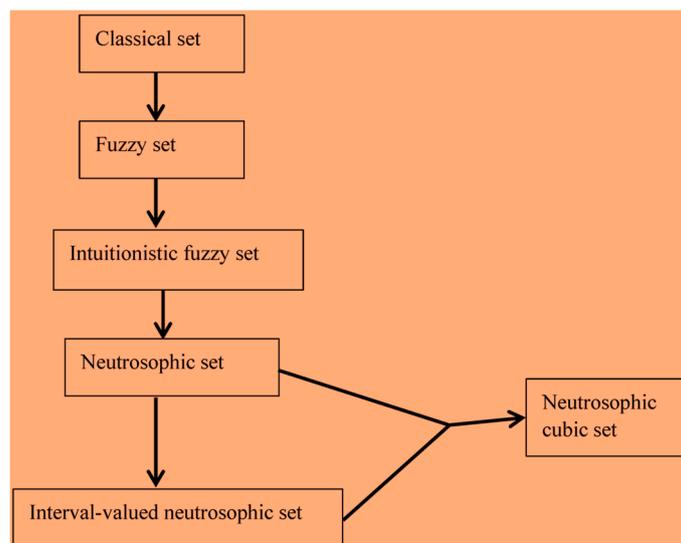


Figure 1. Evolution of the neutrosophic cubic set.

4. NC-TODIM Based MAGDM under a NCS Environment

Assume that $A = \{A_1, A_2, \dots, A_m\}$ ($m \geq 2$) and $C = \{C_1, C_2, \dots, C_n\}$ ($n \geq 2$) are the discrete set of alternatives and attributes respectively. $W = \{W_1, W_2, \dots, W_n\}$ is the weight vector of attributes C_j ($j = 1, 2, \dots, n$), where $W_j > 0$ and $\sum_{j=1}^n W_j = 1$. Let $E = \{E_1, E_2, \dots, E_r\}$ be the set of decision makers and $\gamma = \{\gamma_1, \gamma_2, \dots, \gamma_r\}$ be the weight vector of decision makers, where $\gamma_k > 0$ and $\sum_{k=1}^r \gamma_k = 1$.

NC-TODIM Strategy

Now, we describe the NC-TODIM strategy to solve the MAGDM problems with NC-numbers. The NC-TODIM strategy consists of the following steps:

Step 1. Formulate the decision matrix

Assume that $M^k = (\odot_{ij}^k)_{m \times n}$ be the decision matrix, where $\odot_{ij}^k = \langle \widehat{G}_{ij}^k, R_{ij}^k \rangle$ is the rating value provided by the k-th (E_k) decision maker for alternative A_i , with respect to attribute C_j . The matrix form of M^k is presented as:

$$M^k = \begin{pmatrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \odot_{11}^k & \odot_{12}^k & \dots & \odot_{1n}^k \\ A_2 & \odot_{21}^k & \odot_{22}^k & & \odot_{2n}^k \\ \cdot & \cdot & \dots & \cdot & \\ A_m & \odot_{m1}^k & \odot_{m2}^k & \dots & \odot_{mnj}^k \end{pmatrix} \tag{15}$$

Step 2. Normalize the decision matrix

The MAGDM problem generally consists of cost criteria and benefit criteria. So, the decision matrix needs to be normalized. For cost criterion C_j , we use the Definition 8 to normalize the decision matrix (Equation (15)) provided by the decision makers. For benefit criterion C_j we don't need to normalize the decision matrix. When C_j is a cost criterion, the normalized form of decision matrix (see Equation (15)) is presented below.

$$M^{\otimes k} = \begin{pmatrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \odot_{11}^{\otimes k} & \odot_{12}^{\otimes k} & \dots & \odot_{1n}^{\otimes k} \\ A_2 & \odot_{21}^{\otimes k} & \odot_{22}^{\otimes k} & & \odot_{2n}^{\otimes k} \\ \cdot & \cdot & \dots & \cdot & \\ A_m & \odot_{m1}^{\otimes k} & \odot_{m2}^{\otimes k} & \dots & \odot_{mnj}^{\otimes k} \end{pmatrix} \tag{16}$$

Here $\odot_{ij}^{\otimes k}$ is the normalized form of the NC-number.

Step 3. Determine the relative weight of each criterion

The relative weight W_{ch} of each criterion is obtained by the following equation.

$$W_{ch} = \frac{W_C}{W_h} \tag{17}$$

where, $W_h = \max \{W_1, W_2, \dots, W_n\}$.

Step 4. Calculate score values

Using Equation (1), calculate the score value $Sc(\odot_{ij}^{\otimes k})$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) of $\odot_{ij}^{\otimes k}$ if C_j is a cost criterion. Using Equation (1), calculate the score value $Sc((c)_{ij}^k)$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) of \odot_{ij}^k if C_j is a benefit criterion.

Step 5. Calculate accuracy values

Using Equation (9), calculate the accuracy value $Ac(\odot_{ij}^{\otimes k})$ ($I = 1, 2, \dots, m; j = 1, 2, \dots, n$) of $\odot_{ij}^{\otimes k}$ if C_j is a cost criterion. Using Equation (9), calculate the accuracy value $Ac(\odot_{ij}^k)$ ($I = 1, 2, \dots, m; j = 1, 2, \dots, n$) of \odot_{ij}^k if C_j is a benefit criterion.

Step 6. Formulate the dominance matrix

Calculate the dominance of each alternative A_i over each alternative A_j with respect to the criteria $C(C_1, C_2, \dots, C_n)$, of the k-th decision maker E_k by the following Equations (18) and (19).

(For cost criteria)

$$\left. \begin{aligned} \Psi_c^k(A_i, A_j) &= \sqrt{\left(\frac{W_{Ch}}{\sum_{c=1}^n W_{ch}} \partial(\odot_{ic}^{\otimes k}, \odot_{jc}^{\otimes k})\right)}, \text{ if } \odot_{ic}^{\otimes k} > \odot_{jc}^{\otimes k} \\ &= 0, \text{ if } \odot_{ic}^{\otimes k} = \odot_{jc}^{\otimes k} \\ &= -\frac{1}{\alpha} \sqrt{\left(\frac{\sum_{c=1}^n W_{ch}}{W_{Ch}} \partial(\odot_{ic}^{\otimes k}, \odot_{jc}^{\otimes k})\right)}, \text{ if } \odot_{ic}^{\otimes k} < \odot_{jc}^{\otimes k} \end{aligned} \right\} \quad (18)$$

(For benefit criteria)

$$\left. \begin{aligned} \Psi_c^k(A_i, A_j) &= \sqrt{\left(\frac{W_{Ch}}{\sum_{c=1}^n W_{ch}} \partial(\odot_{ic}^k, \odot_{jc}^k)\right)}, \text{ if } \odot_{ic}^k > \odot_{jc}^k \\ &= 0, \text{ if } \odot_{ic}^k = \odot_{jc}^k \\ &= -\frac{1}{\alpha} \sqrt{\left(\frac{\sum_{c=1}^n W_{ch}}{W_{Ch}} \partial(\odot_{ic}^k, \odot_{jc}^k)\right)}, \text{ if } \odot_{ic}^k < \odot_{jc}^k \end{aligned} \right\} \quad (19)$$

where, parameter α represents the attenuation factor of losses and α must be positive.

Step 7. Formulate the individual overall dominance matrix

Using Equation (20), calculate the individual total dominance matrix of each alternative A_i over each alternative A_j under the criterion C_j .

$$\phi^k = (A_i, A_j) = \sum_{c=1}^n \Psi_c^k(A_i, A_j) \quad (20)$$

Step 8. Aggregate the dominance matrix

Using Equation (21), calculate the collective overall dominance of alternative A_i over each alternative A_j .

$$\phi(A_i, A_j) = \sum_{k=1}^m \gamma_k \lambda^k(A_i, A_j) \quad (21)$$

Step 9. Calculate global values

We present the global value of each alternative as follows:

$$\Omega_i = \frac{\sum_{j=1}^n \phi(A_i, A_j) - \min_{1 \leq i \leq m} (\sum_{j=1}^n \phi(A_i, A_j))}{\max_{1 \leq i \leq m} (\sum_{j=1}^n \phi(A_i, A_j)) - \min_{1 \leq i \leq m} (\sum_{j=1}^n \phi(A_i, A_j))} \quad (22)$$

Step 10. Rank the priority

Sorting the values of Ω_i provides the rank of each alternative. A set of alternatives can be preference-ranked according to the descending order of Ω_i . The highest global value corresponds to the best alternative.

A conceptual model of the NC-TODIM strategy is shown in Figure 2.

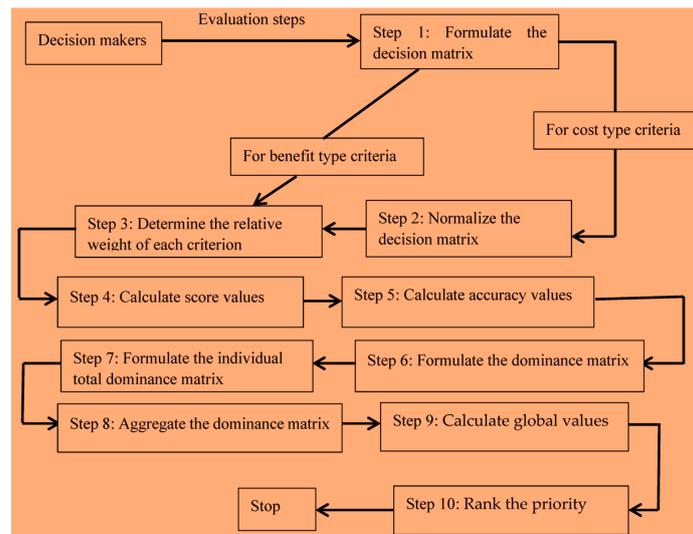


Figure 2. A flow chart of the proposed neutrosophic cubic set (NC)-TODIM strategy.

5. Illustrative Example

In this section, a MAGDM problem is adapted from the study [18] under the NCS environment. An investment company wants to select the best alternative among the set of feasible alternatives. The feasible alternatives are

1. Car company (A_1)
2. Food company (A_2)
3. Computer company (A_3)
4. Arms company (A_4).

The best alternative is selected based on the following criteria:

1. Risk analysis (C_1)
2. Growth analysis (C_2)
3. Environmental impact analysis (C_3).

An investment company forms a panel of three decision makers $\{E_1, E_2, E_3\}$ who evaluate four alternatives in decision making process. The weight vector of attributes and decision makers are considered as $W = (0.4, 0.35, 0.25)^T$ $\gamma = (0.32, 0.33, 0.35)^T$ respectively.

The proposed strategy is presented using the following steps:

Step 1. Formulate the decision matrix

Formulate the decision matrices M^k ($k = 1, 2, 3$) using the rating values of alternatives with respect to three criteria provided by the three decision makers in terms of NC-numbers. Assume that the NC-numbers $\odot_{ij}^k = \langle \tilde{G}_{ij}^k, R_{ij}^k \rangle$ present the rating value provided by the decision maker E_k for alternative A_i with respect to attribute C_j . Using these rating values \odot_{ij}^k ($k = 1, 2, 3; i = 1, 2, 3, 4; j = 1, 2, 3$), three decision matrices $M^k = (\odot_{ij}^k)_{4 \times 3}$ ($k = 1, 2, 3$) are constructed (see Equations (23)–(25)).

Decision matrix for E_1

$$M^1 = \begin{pmatrix} A_1 < [0.41, 0.52], [0.10, 0.18], [0.06, 0.17], (0.48, 0.11, 0.11) > < [0.40, 0.51], [0.10, 0.20], [0.10, 0.19], (0.50, 0.11, 0.11) > < [0.22, 0.27], [0.41, 0.52], [0.41, 0.52], (0.31, 0.31, 0.31) > \\ A_2 < [0.35, 0.46], [0.18, 0.27], [0.17, 0.34], (0.43, 0.16, 0.21) > < [0.22, 0.28], [0.40, 0.50], [0.39, 0.48], (0.28, 0.28, 0.28) > < [0.38, 0.49], [0.10, 0.21], [0.10, 0.21], (0.57, 0.12, 0.12) > \\ A_3 < [0.23, 0.29], [0.36, 0.45], [0.34, 0.42], (0.26, 0.26, 0.26) > < [0.34, 0.45], [0.20, 0.30], [0.19, 0.39], (0.44, 0.16, 0.22) > < [0.22, 0.27], [0.41, 0.52], [0.41, 0.52], (0.31, 0.31, 0.31) > \\ A_4 < [0.17, 0.23], [0.45, 0.55], [0.42, 0.59], (0.21, 0.32, 0.37) > < [0.22, 0.28], [0.40, 0.50], [0.39, 0.48], (0.28, 0.28, 0.28) > < [0.38, 0.49], [0.10, 0.21], [0.10, 0.21], (0.57, 0.12, 0.12) > \end{pmatrix} \quad (23)$$

Decision matrix for E_2

$$M^2 = \begin{pmatrix} A_1 < [0.17, 0.23], [0.46, 0.55], [0.42, 0.59], (0.21, 0.32, 0.37) > < [0.25, 0.31], [0.35, 0.44], [0.35, 0.44], (0.28, 0.28, 0.28) > < [0.34, 0.43], [0.13, 0.27], [0.13, 0.27], (0.49, 0.11, 0.11) > \\ A_2 < [0.23, 0.29], [0.37, 0.46], [0.34, 0.42], (0.26, 0.26, 0.26) > < [0.25, 0.31], [0.35, 0.44], [0.35, 0.44], (0.28, 0.28, 0.28) > < [0.34, 0.43], [0.13, 0.27], [0.13, 0.27], (0.49, 0.11, 0.11) > \\ A_3 < [0.41, 0.52], [0.10, 0.18], [0.10, 0.17], (0.48, 0.11, 0.11) > < [0.44, 0.57], [0.10, 0.17], [0.10, 0.17], (0.51, 0.11, 0.11) > < [0.19, 0.24], [0.53, 0.67], [0.53, 0.67], (0.27, 0.27, 0.27) > \\ A_4 < [0.35, 0.46], [0.20, 0.28], [0.17, 0.34], (0.42, 0.16, 0.21) > < [0.25, 0.31], [0.35, 0.44], [0.35, 0.44], (0.28, 0.28, 0.28) > < [0.34, 0.43], [0.13, 0.27], [0.13, 0.27], (0.49, 0.11, 0.11) > \end{pmatrix} \quad (24)$$

Decision matrix for E_3

$$M^3 = \begin{pmatrix} A_1 < [0.22, 0.27], [0.42, 0.52], [0.42, 0.52], (0.28, 0.28, 0.28) > < [0.22, 0.28], [0.40, 0.50], [0.39, 0.48], (0.28, 0.28, 0.28) > < [0.41, 0.52], [0.10, 0.18], [0.10, 0.17], (0.48, 0.11, 0.11) > \\ A_2 < [0.22, 0.27], [0.42, 0.52], [0.42, 0.52], (0.28, 0.28, 0.28) > < [0.40, 0.51], [0.10, 0.20], [0.10, 0.19], (0.50, 0.11, 0.11) > < [0.23, 0.29], [0.36, 0.45], [0.34, 0.42], (0.26, 0.26, 0.26) > \\ A_3 < [0.38, 0.49], [0.10, 0.21], [0.10, 0.21], (0.50, 0.11, 0.11) > < [0.34, 0.45], [0.20, 0.30], [0.19, 0.39], (0.44, 0.16, 0.22) > < [0.38, 0.49], [0.10, 0.21], [0.10, 0.21], (0.50, 0.11, 0.11) > \\ A_4 < [0.38, 0.49], [0.10, 0.21], [0.10, 0.21], (0.50, 0.11, 0.11) > < [0.22, 0.28], [0.40, 0.50], [0.39, 0.48], (0.28, 0.28, 0.28) > < [0.17, 0.23], [0.45, 0.54], [0.42, 0.59], (0.21, 0.32, 0.37) > \end{pmatrix} \quad (25)$$

Step 2. Normalize the decision matrix

Since all the criteria are benefit type, we do not need to normalize the decision matrix.

Step 3. Determine the relative weight of each criterion

Using Equation (17), we obtain the relative weight vector W_{ch} of criteria as follows:

$$W_{ch} = (1, 0.875, 0.625)^T.$$

Step 4. Calculate score values

The score values of each alternative relative to each criterion obtained by Equation (1) are presented in the Tables 1–3.

Table 1. Score values for M^1 .

	C_1	C_2	C_2
A_1	0.56	0.54	0.06
A_2	0.40	0.09	0.54
A_3	0.50	0.38	0.06
A_4	−0.03	0.09	0.54

Table 2. Score values for M^2 .

	C_1	C_2	C_2
A_1	−0.03	0.13	0.49
A_2	0.13	0.13	0.49
A_3	0.56	0.60	−0.04
A_4	0.39	0.13	0.49

Table 3. Score values for M^3 .

	C_1	C_2	C_2
A_1	0.07	0.09	0.56
A_2	0.07	0.52	0.13
A_3	0.51	0.37	0.39
A_4	0.51	0.09	−0.03

Step 5. Calculate accuracy values

The accuracy values of each alternative relative to each criterion obtained by Equation (9) are presented in Tables 4–6.

Table 4. Accuracy values for M^1 .

	C_1	C_2	C_2
A_1	0.14	0.30	−0.24
A_2	0.12	−0.23	0.32
A_3	−0.20	0.09	−0.24
A_4	−0.38	−0.23	0.32

Table 5. Accuracy values for M^2 .

	C_1	C_2	C_2
A_1	-0.38	-0.18	0.21
A_2	-0.20	-0.18	0.21
A_3	0.14	0.36	-0.21
A_4	0.12	-0.18	0.21

Table 6. Accuracy values for M^3 .

	C_1	C_2	C_2
A_1	-0.24	-0.23	0.41
A_2	-0.24	0.30	-0.20
A_3	0.26	0.09	0.12
A_4	0.26	-0.23	-0.38

Step 6. Formulate the dominance matrix

Using Equation (19), we construct dominance matrix for $\alpha = 1$. The dominance matrices are represented in matrix form (See Equations (26)–(34)).

The dominance matrix Ψ_1^1 , the dominance matrix Ψ_2^1

$$\Psi_1^1 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & 0.18 & 0.30 & 0.35 \\ A_2 & -0.46 & 0 & -0.58 & 0.30 \\ A_3 & -0.74 & 0.23 & 0 & 0.19 \\ A_4 & -0.88 & -0.74 & -0.47 & 0 \end{pmatrix} \tag{26}$$

$$\Psi_2^1 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & 0.29 & 0.18 & 0.28 \\ A_2 & -0.82 & 0 & -0.69 & 0 \\ A_3 & -0.51 & 0.24 & 0 & 0.29 \\ A_4 & -0.81 & 0 & -0.65 & 0 \end{pmatrix} \tag{27}$$

The dominance matrix Ψ_3^1 , the dominance matrix Ψ_1^2

$$\Psi_3^1 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -1 & 0 & -1 \\ A_2 & 0.25 & 0 & 0.26 & 0 \\ A_3 & 0 & -1 & 0 & -1 \\ A_4 & 0.25 & 0 & 0.26 & 0 \end{pmatrix} \tag{28}$$

$$\Psi_1^2 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.46 & -0.88 & -0.74 \\ A_2 & 0.18 & 0 & -0.75 & -0.58 \\ A_3 & 0.35 & 0.09 & 0 & 0.04 \\ A_4 & 0.30 & 0.23 & 0.19 & 0 \end{pmatrix} \tag{29}$$

The dominance matrix Ψ_2^2 , the dominance matrix Ψ_3^2

$$\Psi_2^2 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & 0 & -0.84 & 0 \\ A_2 & 0 & 0 & 0.84 & 0 \\ A_3 & 0.29 & 0.29 & 0 & 0.29 \\ A_4 & 0 & 0 & -0.84 & 0 \end{pmatrix} \tag{30}$$

$$\Psi_3^2 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & 0 & 0.26 & 0 \\ A_2 & 0 & 0 & 0.26 & 0 \\ A_3 & -1 & -1 & 0 & -1 \\ A_4 & 0 & 0 & 0.26 & 0 \end{pmatrix} \tag{31}$$

The dominance matrix Ψ_1^3 , the dominance matrix Ψ_2^3

$$\Psi_1^3 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & 0 & 0.78 & 0.78 \\ A_2 & 0 & 0 & 0.78 & 0.78 \\ A_3 & 0.31 & 0.31 & 0 & 0 \\ A_4 & 0.31 & 0.31 & 0 & 0 \end{pmatrix} \tag{32}$$

$$\Psi_2^3 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.83 & 0.65 & 0 \\ A_2 & 0.29 & 0 & 0.18 & 0.29 \\ A_3 & 0.23 & -0.51 & 0 & 0.23 \\ A_4 & 0 & -0.83 & -0.65 & 0 \end{pmatrix} \tag{33}$$

The dominance matrix Ψ_3^3

$$\Psi_3^3 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.94 & 0.59 & -1.1 \\ A_2 & 0.23 & 0 & -0.73 & 0.15 \\ A_3 & 0.59 & 0.18 & 0 & 0.23 \\ A_4 & -1.1 & 0.58 & -0.94 & 0 \end{pmatrix} \tag{34}$$

Step 7. Formulate the individual overall dominance matrix

The individual overall dominance matrix is calculated by the Equation (20) and the dominance matrices are represented in matrix form (see Equations (35)–(37)).

First decision maker’s overall dominance matrix ϕ^1

$$\phi^1 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.53 & 0.47 & -0.37 \\ A_2 & -1 & 0 & -1 & 0.30 \\ A_3 & -1.3 & 0.53 & 0 & 0.52 \\ A_4 & -1.5 & 0.74 & -0.86 & 0 \end{pmatrix} \tag{35}$$

Second decision maker’s overall dominance matrix ϕ^2

$$\phi^2 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.46 & -1.5 & -0.74 \\ A_2 & 0.18 & 0 & -1.3 & -0.58 \\ A_3 & -0.36 & -0.62 & 0 & 0.67 \\ A_4 & 0.30 & 0.23 & -0.39 & 0 \end{pmatrix} \tag{36}$$

Third decision maker’s overall dominance matrix ϕ^3

$$\phi^3 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -1.8 & -2 & 1.9 \\ A_2 & 0.52 & 0 & -1.3 & -0.34 \\ A_3 & -0.05 & -0.02 & 0 & 0.46 \\ A_4 & -0.79 & -1.1 & -1.6 & 0 \end{pmatrix} \tag{37}$$

Step 8. Aggregate the dominance matrix

Using Equation (21), the aggregate dominance matrix ϕ is constructed (see Equation (38)) as follows:

$$\phi = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 0 & -0.94 & -1.1 & -0.53 \\ A_2 & -0.10 & 0 & -1.23 & -0.22 \\ A_3 & -0.54 & -0.38 & 0 & -0.23 \\ A_4 & -0.64 & -0.55 & -0.96 & 0 \end{pmatrix} \tag{38}$$

Step 9. Calculate global values

Using Equation (22), we calculate the values of Ω_i ($i = 1, 2, 3, 4$) and represented in Table 7.

Table 7. Global values of alternatives.

A_i	A_1	A_2	A_3	A_4
Ω_i	0.49	0.61	1	0

Step 10. Rank the priority

Since $\Omega_3 > \Omega_2 > \Omega_1 > \Omega_4$, alternatives are then preference ranked as follows: $A_3 > A_2 > A_1 > A_4$. Hence A_3 is the best alternative.

From the illustrative example, we see that the proposed NC-TODIM strategy is more suitable for real scientific and engineering applications because it can handle hybrid information consisting of INS and SVNS information simultaneously to cope with indeterminate and inconsistent information. Thus, NC-TODIM extends the existing decision-making strategies and provides a sophisticated mathematical tool for decision makers.

6. Rank of Alternatives with Different Values of α

Table 8 shows that the ranking order of alternatives depends on the values of the attenuation factor, which reflects the importance of the attenuation factor in the NC-TODIM strategy.

Table 8. Global values and ranking of alternatives for different values of α .

Values of α	Global Values of Alternative (Ω_i)	Rank Order of A_i
0.5	$\Omega_1 = 0, \Omega_2 = 0.89, \Omega_3 = 1, \Omega_4 = 0.46$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$
1	$\Omega_1 = 0.49, \Omega_2 = 0.61, \Omega_3 = 1, \Omega_4 = 0$ $\Omega_3 > \Omega_2 > \Omega_1 > \Omega_4$	$A_3 > A_2 > A_1 > A_4$
1.5	$\Omega_1 = 0, \Omega_2 = 0.72, \Omega_3 = 1, \Omega_4 = 0.44$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$
2	$\Omega_1 = 0, \Omega_2 = 1, \Omega_3 = 0.81, \Omega_4 = 0.38$ $\Omega_2 > \Omega_3 > \Omega_4 > \Omega_1$	$A_2 > A_3 > A_4 > A_1$
3	$\Omega_1 = 0, \Omega_2 = 0.56, \Omega_3 = 1, \Omega_4 = 0.45$ $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$	$A_3 > A_2 > A_4 > A_1$

Analysis on Influence of the Parameter α to Ranking Order

The impact of parameter α on ranking order is examined by comparing the ranking orders taken with varying the different values of α . When $\alpha = 0.5, 1, 1.5, 2, 3$, ranking order are presented in Table 8. We draw Figures 3 and 4 to compare the ranking order for different values of α . When $\alpha = 0.5, \alpha = 1.5$ and $\alpha = 3$, the ranking order is unchanged and A_3 is the best alternative, while A_1 is the worst alternative. When $\alpha = 1$, the ranking order is changed and A_3 is the best alternative and A_4 is the worst alternative. For $\alpha = 2$, the ranking order is changed and A_2 is the best alternative and A_1 is the worst alternative. From Table 8, we see that A_3 is the best alternative in four cases and A_1 is the worst for four cases. We can say that ranking order depends on parameter α .

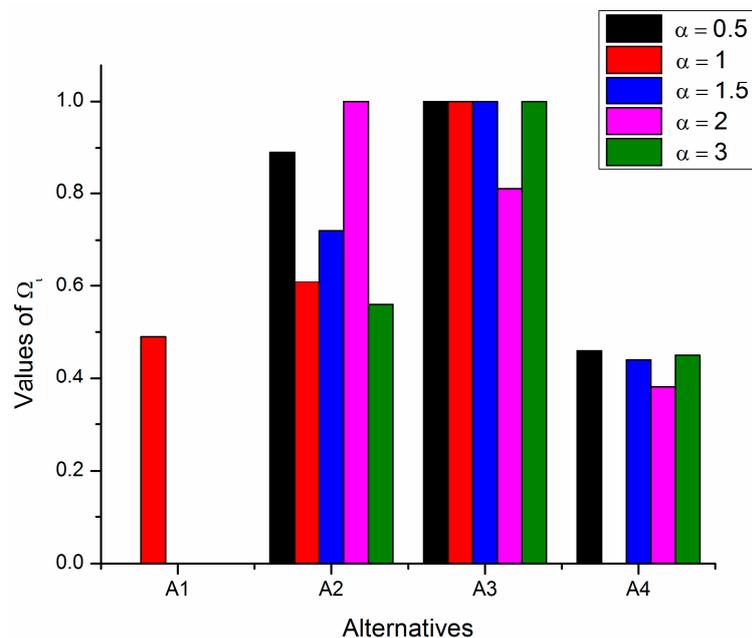


Figure 3. Global values of the alternatives for different values of attenuation factor $\alpha = 0.5, 1, 1.5, 2, 3$.

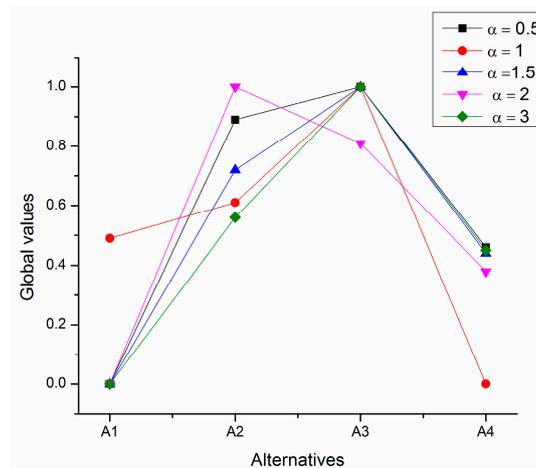


Figure 4. Ranking of the alternatives for $\alpha = 0.5, 1, 1.5, 2, 3$.

7. Comparative Analysis and Discussion

On comparing with the existing neutrosophic decision making strategies [26–29,33–35,64–69], we see that the decision information used in the proposed NC-TODIM strategy is NC numbers, which comprises of interval neutrosophic information and single-valued neutrosophic information simultaneously; whereas the decision information in the existing literature is either SVN or IN. Since NC numbers comprises of much more information, the NC numbers based on the TODIM strategy proposed in this paper is more elegant, typical and more general in applications, while the existing neutrosophic decision-making strategies cannot deal with the NC number decision-making problem developed in this paper.

The first decision making paper in NCS environment was studied by Banerjee et al. [44]. On comparison with existing GRA-based NCS decision making strategies [44], we observe that the proposed NC-TODIM strategy uses the score, and accuracy functions, while the decision making-strategy in [44] uses Hamming distances for weighted grey relational coefficients and standard (ideal) grey relational coefficients, and ranks the alternatives based on the relative closeness coefficients. Hence, the proposed NC-TODIM strategy is relatively simple in the decision making process.

On comparing with cosine measures of NCSs [49], we observe that the proposed NC-TODIM involves multiple decision makers, while in [49] only a single decision maker is involved. This shows that [49] cannot deal with group decision making, while the proposed NC-TODIM strategy is more sophisticated as it can deal with single as well as group decision making in the NCS environment.

On comparison with extended TOPSIS [50] with neutrosophic cubic information, we observe that nine components are present in NCSs. Therefore, by calculation of a weighted decision matrix, a neutrosophic cubic positive ideal solution (NCPIS), and a neutrosophic cubic negative ideal solution, the distance measures of alternatives from NCPIS and NCNIS (NCNIS,) and entropy weight, and use of an aggregation operator are lengthy, time consuming, and hence expensive. The proposed NC-TODIM strategy is free from different kinds of typical aggregation operators. The calculations required for the proposed strategy are relatively straightforward and time-saving. Therefore, the final ranking obtained by the proposed strategy is more conclusive than those produced by the other strategies, and it is evident that the proposed strategy is accurate and reliable.

On comparison with the strategy proposed by Zhan et al. [51], we see that they employ score, accuracy, and certainty functions, and a weighted average operator and weighted geometric operator of NCSs for decision making problem involving only a single decision maker. This reflects that the strategy introduced by Zhan et al. [51] is only applicable for decision making problems involving single decision maker. However, our proposed NC-TODIM strategy is more general as it is capable of dealing with group decision-making problems.

A comparative study is conducted with the existing strategy [48] for group decision making under a NCS environment (See Table 9). Since the philosophy of two strategies are different, the obtained results (ranking order) are different. At a glance, it cannot be said which strategy is superior to the other. However, on comparison with similarity measure-based strategies studied in [48], we observed that ideal solutions are needed for ranking of alternatives but in a real world ideal solution, this is an imaginary case, which means that an indeterminacy arises automatically, whereas in our proposed NC-TODIM strategy we can calculate the rank of the alternatives based on global values of alternatives. So, the proposed NC-TODIM strategy is relatively easy to implement and apply for solving MAGDM problems.

Table 9. Ranking order of alternatives using three different decision making strategies in the neutrosophic cubic set (NCS) environment.

Proposed NC-TODIM Strategy	Similarity Measure [48]
$\Omega_1 = 0, \Omega_2 = 0.89, \Omega_3 = 1, \Omega_4 = 0.46$ Ranking order: $A_3 > A_2 > A_4 > A_1$	$\rho_1 = 0.20, \rho_2 = 0.80, \rho_3 = 0.22, \rho_4 = 0.19$ Ranking order: $A_2 > A_3 > A_1 > A_4$

8. Conclusions

NCSs can better describe hybrid information comprising of INs and NSs. In this study, we proposed a score function and an accuracy function, and established their properties. We developed a NC-TODIM strategy, which is capable for tackling MAGDM problems affected by uncertainty and indeterminacy represented by NC numbers. The standard TODIM, in its original formulation, is only applicable to a crisp environment. Existing neutrosophic TODIM strategies deal with single valued neutrosophic information or interval neutrosophic information. Therefore, proposed NC-TODIM strategy demonstrates the advantages of presenting and manipulating MAGDM problems with NCSs comprising of the hybrid information of INs and NSs. Furthermore, NC-TODIM strategy that considers the risk preferences of decision makers, is significant to solve MAGDM problems. The proposed NC-TODIM strategy was verified to be applicable, feasible, and effective by solving an illustrative example regarding the selection problem of investment alternatives. In addition, we investigated the influence of attenuation factor of losses α on ranking the order of alternatives.

The contribution of this study can be concluded as follows. First, this study utilized NCSs to present the interval neutrosophic information and neutrosophic information in the MAGDM process. Second, the NC-TODIM strategy established in this paper is simpler and easier than the existing strategy proposed by Pramanik et al. [48] for group decision making with neutrosophic cubic information based on similarity measure and demonstrates the main advantage of its simple and easy group decision making process. Third, TODIM strategy was extended to the NCS environment. Fourth, we defined the NC number. Fifth, we defined the score and accuracy functions and proved their basic properties. Sixth, we developed the ranking of NC numbers using score and accuracy functions. Therefore, two functions namely, score function, accuracy function, and proofs of their basic properties, ranking of NC numbers, and NC-TODIM strategy for MAGDM are the main contributions of the paper.

Several directions for future research are generated from this study. First, this study employs the NC-TODIM strategy to deal with MAGDM. In addition to MAGDM, MAGDM problems in a variety of other fields can be solved using the NC-TODIM strategy, including logistics center selection, personnel selection, teacher selection, renewable energy selection, medical diagnosis, image processing, fault diagnosis, etc. Second, this study considers the risk preferences of decision makers i.e., the essence of TODIM, while the interrelationship between criteria are ignored. In future research, the NC-TODIM strategy will be improved to address this deficiency. Third, the proposed strategy can only deal with crisp weights of attributes and decision makers, rather than NCS, which reflects its main limitation. This limitation will be effectively addressed in our future research. Fourth, in our illustrative

example, three criteria are considered as an example. However, in real world group decision making problems, many other criteria should be included. A comprehensive framework for MAGDM problem comprising of all relevant criteria should be designed based on prior studies and the proposed NC-TODIM strategy in future research. Finally, we conclude that the developed NC-TODIM strategy offers a novel and effective strategy for decision makers under the NCS environment, and will open up a new avenue of research into the neutrosophic hybrid environment.

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