Neutrosophic Integer Programming Problems

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Abstract

In this chapter, we introduce the integer programming in neutrosophic environment, by considering coefficients of problem as a triangular neutrosophic numbers. The degrees of acceptance, indeterminacy and rejection of objectives are simultaneously considered. The Neutrosophic Integer Programming Problem (NIP) is transformed into a crisp programming model, using truth membership (T), indeterminacy membership (I), and falsity membership (F) functions as well as single valued triangular neutrosophic numbers. To measure the efficiency of our proposed model we solved several numerical examples.

Keywords

Neutrosophic; integer programming; single valued triangular neutrosophic number.

1 Introduction

In linear programming models, decision variables are allowed to be fractional. For example, it is reasonable to accept a solution giving an hourly production of automobiles at 64.5, if the model were based upon average hourly production. However, fractional solutions are not realistic in many situations and to deal with this matter, integer programming problems are introduced. We can define integer programming problem as a linear programming problem with integer restrictions on decision variables. When some, but not all decision variables are restricted to be integer, this problem called a mixed integer problem and when all decision variables are integers, it's a pure integer program. Integer programming plays an important role in supporting managerial decisions. In integer programming problems, the decision maker may not be able to specify the objective function and/or constraints functions precisely. In 1995, Smarandache [1-3] introduce neutrosophy which is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. [4] Neutrosophic sets characterized by three independent degrees as in Fig. 1., namely truth-membership degree (*T*), indeterminacy-membership degree(*I*), and falsity-membership degree (*F*), where *T,I,F* are standard or non-standard subsets of $]^-0, 1^+[$. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership.

The structure of the chapter is as follows: the next section is a preliminary discussion; the third section describes the formulation of integer programing problem using the proposed model; the fourth section presents some illustrative examples to put on view how the approach can be applied; the last section summarizes the conclusions and gives an outlook for future research.

2 Preliminaries

2.1 Neutrosophic Set [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T(x)$, an indeterminacy-membership function $I(x)$ and a falsity-membership function (x). $T(x)$, $I(x)$ and (x) are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x):X \rightarrow]0^-,1^+[$, $I_A(x):X \rightarrow]0^-,1^+[$ and $F_A(x):X \rightarrow]0^-,1^+[$. There is no restriction on the sum of $T(x)$, $I(x)$ and $F(x)$, so $0- \leq \sup(x) \leq \sup I_A(x) \leq F_A(x) \leq 3+$.

2.2 Single Valued Neutrosophic Sets (SVNS) [3-4]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A = \{\langle x, T(x), I_A(x), F_A(x)\rangle : x \in X\}$, where $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X \rightarrow [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T(x)$, $I(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

In the following, we write SVN numbers instead of single valued neutrosophic numbers. For convenience, a SVN number is denoted by A= (a, b, c), where a, b, $c\in$ [0, 1] and a+b+$c\leq$3.

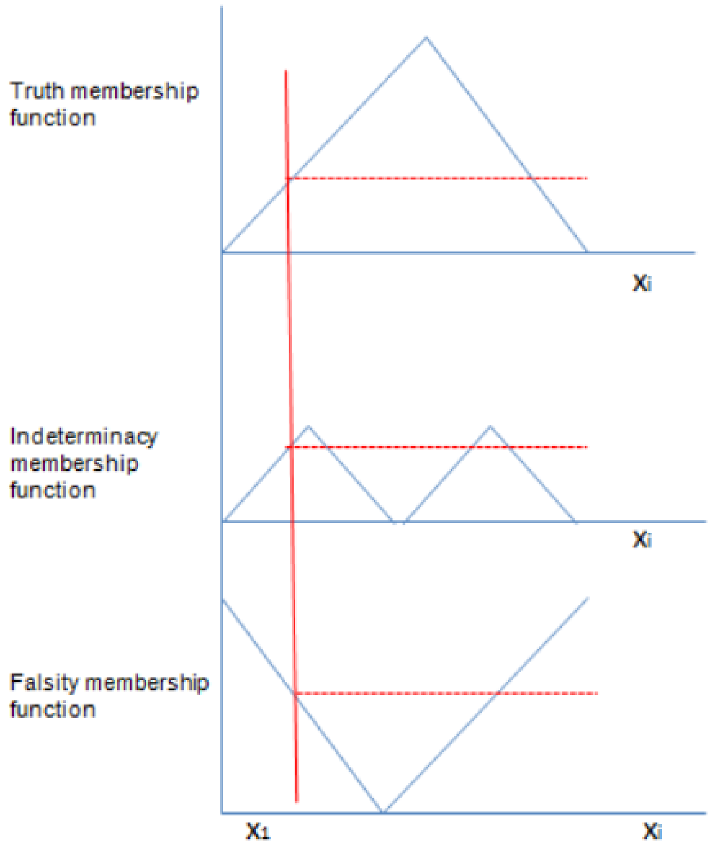

Fig.1: Neutrosophication process

2.3 Complement [5]

The complement of a single valued neutrosophic set A is denoted by $_{C}$ (A) and is defined by

$$T_c(A)(x) = F(A)(x)\,,$$

$$I_c(A)(x) \;= 1 - I(A)(x)\,,$$

$$F_c(A)(x) = T(A)(x) \qquad \text{for all } x \text{ in } X$$

2.4 Union [5]

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A∪B, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$T(C)(x) = max\ (\ T(A)(x)\ ,T(B)(x)\ )$,

$I(C)(x) = max\ (\ I(A)(x)\ ,I(B)(x)\ )$,

$F(C)(x) = \ min((A)(x)\ ,F(B)(x)\ )$ for all x in X

2.5 Intersection [5]

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as

C = A∩B, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$T(C)(x) = min\ (\ T(A)(x)\ ,T(B)(x)\ )$,

$I(C)(x) = min\ (\ I(A)(x)\ ,I(B)(x)\ )$,

$F(C)(x) = \ max((A)(x)\ ,F(B)(x)\ )$ for all x in X

3 Neutrosophic Integer Programming Problems

Integer programming problem with neutrosophic coefficients (NIPP) is defined as the following:

Maximize Z= $\sum_{j=1}^{n} \tilde{c_j} x_j$

Subject to

$$\sum_{j=1}^{n} \mathrm{a}_{\mathrm{ij}}^{\sim n} x_j \leq b_{\mathrm{i}} \quad i = 1, \dots, m\ , \tag{1}$$

$x_j \geq 0, \qquad j = 1, \dots n\ ,$

x_j Integer for $j \in \{0,1, \dots n\}$.

where $\tilde{c_j}$, $\mathrm{a}_{\mathrm{ij}}^{\sim n}$ are neutrosophic numbers.

The single valued neutrosophic number ($\mathrm{a}_{\mathrm{ij}}^{\sim n}$) is donated by A=(a,b,c) where a,b,c ∈ [0,1] And a,b,c ≤ 3

The truth- membership function of neutrosophic number $\mathrm{a}_{\mathrm{ij}}^{\sim n}$ is defined as:

$$\mathrm{T}\ \mathrm{a}_{\mathrm{ij}}^{\sim n}(\mathrm{x}) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_2-x}{a_{3-a_2}} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \tag{2}$$

The indeterminacy- membership function of neutrosophic number a_{ij}^{n} is defined as:

$$\mathrm{I}\ a_{ij}^{\sim n}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & b_1 \le x \le b_2 \\ \frac{b_2-x}{b_3-b_2} & b_2 \le x \le b_3 \\ 0 & otherwise \end{cases} \tag{3}$$

And its falsity- membership function of neutrosophic number $a_{ij}^{\sim n}$ is defined as:

$$\mathrm{F}\ a_{ij}^{\sim n}(x) = \begin{cases} \frac{x-C_1}{C_2-C_1} & C_1 \le x \le C_2 \\ \frac{c_2-x}{c_3-c_2} & C_2 \le x \le C_3 \\ 1 & otherwise \end{cases} \tag{4}$$

Then we find the maximum and minimum values of the objective function for truth-membership, indeterminacy and falsity membership as follows:

$f^{max} = \max\{f(x_i^*)\}$ and $f^{min} = \min\{f(x_i^*)\}$,

where $1 \le i \le k$

$f_{min}^{F} = f_{min}^{T}$ and $f_{max}^{F} = f_{max}^{T} - R(f_{max}^{T} - f_{min}^{T})$

$f_{max}^{I} = f_{max}^{I}$ and $f_{min}^{I} = f_{min}^{I} - S(f_{max}^{T} - f_{min}^{T})$,

where R, S are predetermined real number in (0, 1)

The truth membership, indeterminacy membership, falsity membership of objective function are as follows:

$$T^{f}(x) = \begin{cases} 1 & if\ \ f \le f^{min} \\ \frac{f^{max} - f(x)}{f^{max} - f^{min}} & if\ \ f^{min} < f(x) \le f^{max} \\ 0 & if\ f(x)\ > f^{max} \end{cases} \tag{5}$$

$$I^{f}(x) = \begin{cases} 0 & if\ \ f \le f^{min} \\ \frac{f(x) - f^{max}}{f^{max} - f^{min}} & if\ \ f^{min} < f(x) \le f^{max} \\ 0 & if\ f(x)\ > f^{max} \end{cases} \tag{6}$$

$$F^{f}(x) = \begin{cases} 0 & if\ \ f \le f^{min} \\ \frac{f(x) - f^{min}}{f^{max} - f^{min}} & if\ \ f^{min} < f(x) \le f^{max} \\ 1 & if\ f(x)\ > f^{max} \end{cases} \tag{7}$$

The neutrosophic set of the j^{th} decision variable x_j is defined as:

$$T_{x_j}^{(x)} = \begin{cases} 1 & if \quad x_j \leq 0 \\ \frac{d_j - x_j}{d_j} & if \quad 0 < x_j \leq d_j \\ 0 & if \quad x_j > d_j \end{cases} \tag{8}$$

$$F_{x_j}^{(x)} = \begin{cases} 0 & if \quad x_j \leq 0 \\ \frac{x_j}{d_j + b_j} & if \quad 0 < x_j \leq d_j \\ 1 & if \quad x_j > d_j \end{cases} \tag{9}$$

$$I_j^{(x)} = \begin{cases} 0 & if \quad x_j \leq 0 \\ \frac{x_j - d_j}{d_j + b_j} & if \quad 0 < x_j \leq d_j \\ 0 & if \quad x_j > d_j \end{cases} \tag{10}$$

where d_j, b_j are integer numbers.

4 Neutrosophic Optimization Model of Integer Programming Problem

In our neutrosophic model we want to maximize the degree of acceptance and minimize the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Neutrosophic optimization model can be defined as:

$maxT_{(x)}$

$minF_{(x)}$

$minI_{(x)}$

Subject to

$T_{(X)} \geq F_{(x)}$

$T_{(X)} \geq I_{(x)}$

$0 \leq T_{(X)} + I_{(x)} + F_{(x)} \leq 3$ (11)

$T_{(X)}, \quad I_{(X)}, \quad F_{(X)} \geq 0$

$x \geq 0$, integer,

where $T_{(x)}. F_{(x)}, I_{(x)}$denotes the degree of acceptance, rejection and indeterminacy of x respectively.

The above problem is equivalent to the following:

$max\ \alpha,\ min\ \beta\ ,min\ \theta$

Subject to

$\alpha \le T_{(x)}$

$\beta \le F_{(x)}$

$\theta \le I_{(x)}$

$\alpha \ge \beta$

$\alpha \ge \theta$

$0 \le \alpha + \beta + \theta \le 3$ (12)

$x \ge 0$, integer,

where α denotes the minimal acceptable degree, β denote the maximal degree of rejection and θ denote maximal degree of indeterminacy.

The neutrosophic optimization model can be changed into the following optimization model:

$max(\alpha - \beta - \theta)$

Subject to

$\alpha \le T_{(x)}$ (13)

$\beta \ge F_{(x)}$

$\theta \ge I_{(x)}$

$\alpha \ge \beta$

$\alpha \ge \theta$s

$0 \le \alpha + \beta + \theta \le 3$

$\alpha, \beta, \theta \ge 0$

$x \ge 0$, integer.

The previous model can be written as:

min (1- α) β θ

Subject to

$\alpha \le T_{(x)}$

$\beta \ge F_{(x)}$

$\theta \ge I_{(x)}$

$\alpha \geq \beta$

$\alpha \geq \theta$

$0 \leq \ \alpha + \ \beta + \ \theta \leq 3$ (14)

$x \geq 0$, integer.

5 The Algorithms for Solving Neutrosophic Integer Programming Problem (NIPP)

5.1 Neutrosophic Cutting Plane Algorithm

Step 1: Convert neutrosophic integer programming problem to its crisp model by using the following method:

By defining a method to compare any two single valued triangular neutrosophic numbers which is based on the score function and the accuracy function. Let $\tilde{a} = \langle (a_1, b_1, c_1), w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ be a single valued triangular neutrosophic number, then

$$S(\tilde{a}) = \frac{1}{16}[a + b + c] \times (2 + \mu_{\tilde{a}} - v_{\tilde{a}} - \lambda_{\tilde{a}}) \quad (15)$$

and

$$A(\tilde{a}) = \frac{1}{16}[a + b + c] \times (2 + \mu_{\tilde{a}} - v_{\tilde{a}} + \lambda_{\tilde{a}}) \quad (16)$$

It is called the score and accuracy degrees of $\tilde{a}$, respectively. The neutrosophic integer programming NIP can be represented by crisp programming model using truth membership, indeterminacy membership, and falsity membership functions and the score and accuracy degrees of ã, at equations (15) or (16).

Step 2: Create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3: Solve the problem as a linear programming problem and ignore integrality.

Step 4: If the optimal solution is integer, then it's right. Otherwise, go to the next step.

Step 5: Generate a constraint which is satisfied by all integer solutions and add this constraint to the problem.

Step 6: Go to step 1.

5.2 Neutrosophic Branch and Bound Algorithm

Step 1: Convert neutrosophic integer programming problem to its crisp model by using Eq.16.

Step 2: Create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3: At the first node let the solution of linear programming model with integer restriction as an upper bound and the rounded-down integer solution as a lower bound.

Step 4: For branching process, we select the variable with the largest fractional part. Two constrains are obtained after the branching process, one for $\leq$ and the other is $\geq$ constraint.

Step 5: Create two nodes for the two new constraints.

Step 6: Solve the model again, after adding new constraints at each node.

Step 7: The optimal integer solution has been reached, if the feasible integer solution has the largest upper bound value of any ending node. Otherwise return to step 4.

The previous algorithm is for a maximization model. For a minimization model, the solution of linear programming problem with integer restrictions are rounded up and upper and lower bounds are reversed.

6 Numerical Examples

To measure the efficiency of our proposed model we solved many numerical examples.

6.1 Illustrative Example #1

$$
\begin{aligned}
&max \quad \tilde{5}x_1 + \tilde{3}x_2 \\
&subject\ to \\
&\qquad\qquad \tilde{4}x_1 + \tilde{3}x_2 \leq \widetilde{12} \\
&\qquad\qquad \tilde{1}x_1 + \tilde{3}x_2 \leq \tilde{6} \\
&\qquad\qquad x_1, x_2 \geq 0\ and\ integer \\
&\qquad where \\
&\qquad\quad \tilde{5} = \langle (4,5,6\ ), 0.8, 0.6, 0.4\ \rangle \\
&\qquad\quad \tilde{3} = \langle (2.5,3,3.5\ ), 0.75, 0.5, 0.3\ \rangle \\
&\qquad\quad \tilde{4} = \langle (3.5,4,4.1\ ), 1, 0.5, 0.0\ \rangle \\
&\qquad\quad \tilde{3} = \langle (2.5,3,3.5\ ), 0.75, 0.5, 0.25\ \rangle \\
&\qquad\quad \tilde{1} = \langle (0,1,2\ ), 1, 0.5, 0\ \rangle
\end{aligned}
$$

$\tilde{3} = \langle(2.8,3,3.2), 0.75, 0.5, 0.25\rangle$
$\widetilde{12} = \langle(11,12,13), 1, 0.5, 0\rangle$
$\tilde{6} = \langle(5.5,6,7.5), 0.8, 0.6, 0.4\rangle$

Then the neutrosophic model converted to the crisp model by using Eq.16.as follows:

$$
\begin{aligned}
&max \quad 5.6875x_1 + 3.5968x_2 \\
&\text{subject to} \\
&\qquad 4.3125x_1 + 3.625x_2 \le 14.375 \\
&\qquad 0.2815x_1 + 3.925x_2 \le 7.6375 \\
&\qquad x_1, x_2 \ge 0 \; and\ integer
\end{aligned}
$$

The optimal solution of the problem is $x^* = (3,0)$ with optimal objective value 17.06250.

6.2 Illustrative Example #2

$$
\begin{aligned}
&max \quad \widetilde{25}x_1 + \widetilde{48}x_2 \\
&subject\ to \\
&\quad 15x_1 + 30x_2 \le 45000 \\
&\quad 24x_1 + 6x_2 \le 24000 \\
&\quad 21x_1 + 14x_2 \le 28000 \\
&\quad x_1, x_2 \ge 0 \; and\ integer \\
&\qquad where \\
&\quad \widetilde{25} = \langle(19,25,33), 0.8,0.5,0\rangle; \\
&\quad \widetilde{48} = \langle(44,48,54), 0.9,0.5,0\rangle
\end{aligned}
$$

Then the neutrosophic model converted to the crisp model as:

$$
\begin{aligned}
&\max \quad 27.8875x1 + 55.3x_2 \\
&\text{subject to} \\
&\qquad 15x_1 + 30x_2 \le 45000 \\
&\qquad 24x_1 + 6x_2 \le 24000 \\
&\qquad 21x_1 + 14x_2 \le 28000 \\
&\qquad x_1, x_2 \ge 0 \; and\ integer
\end{aligned}
$$

The optimal solution of the problem is $x^* = (500,1250)$ with optimal objective value 83068.75.

6.3 Illustrative Example #3

The owner of a machine shop is planning to expand by purchasing some new machines - presses and lathes. The owner has estimated that each press purchased will increase profit by \$100 per day and each lathe will increase profit by \$150 daily.

The number of machines the owner can purchase is limited by the cost of the machines and the available floor space in the shop. The machine purchase prices and space requirements are as follows.

Table 1. Requirements of machines

Machine	Required Floor Space (ft^2)	Purchase Price
Press	$\widetilde{40}$	\$8,000
Lathe	$\widetilde{70}$	\$4,000

The owner has a budget of \$40,000 for purchasing machines and 200 square feet of available floor space. The owner wants to know how many of each type of machine to purchase to maximize the daily increase in profit.

The problem can be formulated as follows:

max $100x_1 + 150x_2$

Subject to

$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$

$\widetilde{40}x_1 + \widetilde{70}x_2 \leq 40{,}000$

$x_1, x_2 \geq 0$ *and integer*

Since $\widetilde{40}$= $\{(30,\ 40,\ 50);(0.7,\ 0.4,\ 0.3)\}$

Since $\widetilde{70}$= $\{(50,\ 70,\ 120);(0.7,\ 0.4,\ 0.3)\}$

By using Neutrosophic Branch and Bound Algorithm, then by converting neutrosophic integer programming parameter to its crisp values by using Eq.16 then,

max $100x_1 + 150x_2$

Subject to

$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$

$15x_1 + 30x_2 \leq 40{,}000$

$x_1, x_2 \geq 0$ *and integer*

We began the branch and bound method by first solving the problem as a regular linear programming model without integer restrictions, the result as follows:

$x_1 = 2.22, x_2 = 5.56$, And optimal objective value = 1,055.56.

By applying branch and bound steps then, the upper and lower bounds at each node presented in Fig.2:

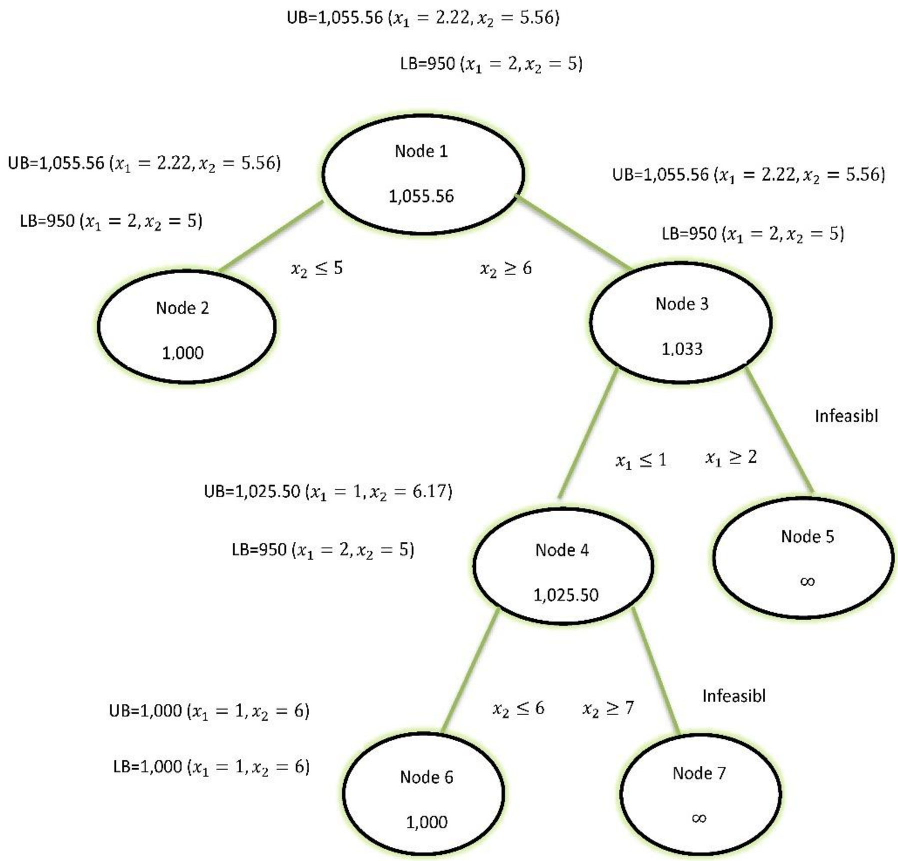

Fig.2. Branch and bound diagram with optimal solution at node 6

The previous branch and bound diagram indicates that the optimal integer solution $x_1 = 1, x_2 = 6$, has been reached at node 6 with optimal value =1000.

7 Conclusions and Future Work

In this chapter, we proposed an integer programming model based on neutrosophic environment, simultaneously considering the degrees of acceptance, indeterminacy and rejection of objectives, by proposed model for solving neutrosophic integer programming problems (NIPP). In the proposed model, we maximized the degrees of acceptance and minimized indeterminacy and rejection of objectives. NIPP was transformed into a crisp programming model using truth membership, indeterminacy membership, falsity membership and score functions. We also gave numerical examples to show the efficiency of the proposed method. As far as future directions are concerned, these will include studying the duality theory of integer programming problems based on Neutrosophics.

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References

[1] Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. Infinite Study, 2005.

[2] I. M. Hezam, M. Abdel-Baset, F. Smarandache. Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem. In: Neutrosophic Sets and Systems, An International Journal in Information Science and Engineering, Vol. 10, 2015, pp. 39-45.

[3] Abdel-Baset, M., Hezam, I. M., & Smarandache, F. (2016). Neutrosophic Goal Programming. Neutrosophic Sets & Systems, vol. 11.

[4] El-Hefenawy, N., Metwally, M. A., Ahmed, Z. M., & El-Henawy, I. M. A Review on the Applications of Neutrosophic Sets. Journal of Computational and Theoretical Nanoscience, 13(1), (2016), pp. 936-944.

[5] Abdel-Basset, M., Mohamed, M. & Sangaiah, A.K. J Ambient Intell Human Comput (2017). DOI: https://doi.org/10.1007/s12652-017-0548-7