Neutrosophic Linear Programming Problems

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Abstract

Smarandache presented neutrosophic theory as a tool for handling undetermined information. Wang et al. introduced a single valued neutrosophic set that is a special neutrosophic sets and can be used expediently to deal with real-world problems, especially in decision support. In this paper, we propose linear programming problems based on neutrosophic environment. Neutrosophic sets are characterized by three independent parameters, namely truth-membership degree (T), indeterminacy-membership degree (I) and falsity-membership degree (F), which are more capable to handle imprecise parameters. We also transform the neutrosophic linear programming problem into a crisp programming model by using neutrosophic set parameters. To measure the efficiency of our proposed model we solved several numerical examples.

Keywords

Linear Programming Problem; Neutrosophic; Neutrosophic Sets.

1 Introduction

Linear programming is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Decision making is a process of solving the problem and achieving goals under asset of constraints, and it is very difficult in some cases due to incomplete and imprecise information. And in Linear programming problems the decision maker may not be able to specify the objective function and/or constraints functions precisely. In 1995, Smarandache [5-7] introduce neutrosophy which is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. [2,5-7] Neutrosophic sets characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree(I), and falsity-membership degree (F), where T,I,F are standard or non-standard subsets of $]0^-,1^+[$. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership.

The structure of the paper is as follows: the next section is a preliminary discussion; the third section describes the formulation of linear programing problem using the proposed model; the fourth section presents some illustrative examples to put on view how the approach can be applied; the last section summarizes the conclusions and gives an outlook for future research.

2 Some Preliminaries

2.1 Neutrosophic Set [2]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function T (x), an indeterminacy-membership function (x) and a falsity-membership function (x). $\mathrm{T}(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x){:}X \to]0^-,1^+[$, $\mathrm{I}_A(x){:}X \to]0-,1+[$ and $F_A(x){:}X \to]0^-,1^+[$. There is no restriction on the sum of $\mathrm{T}(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \mathrm{T}_A(x) \leq \sup I_A(x) \leq F_A(x) \leq 3^+$.

2.2 Single Valued Neutrosophic Sets (SVNS) [7,8]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$A = \{\langle x, \mathrm{T}_A(x), I_A(x)\ , F_A(x)\rangle : x \in X\}$, where $\mathrm{T}_A(x){:}X \to [0,1]$, $I_A(x){:}X \to [0,1]$ and $F_A(x){:}X \to [0,1]$ with $0 \leq \mathrm{T}_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $\mathrm{T}(x)$, $I_A(x)$and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, a SVN number is denoted by A= (a, b, c), where a, b, c ∈ [0, 1] and a+b+c≤3.

2.3 Complement [3]

The complement of a single valued neutrosophic set A is denoted by $_C$ (A) and is defined by

$T_c(A)(x) = F(A)(x)$,

$I_c(A)(x) = 1 - I(A)(x)$,

$F_c(A)(x) = T(A)(x)$, for all x in X.

2.4 Union [3]

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A∪B, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$T(C)(x) = max\ (T(A)(x), T(B)(x))$,

$I(C)(x) = max\ (I(A)(x), I(B)(x))$,

$F(C)(x) = min(F(A)(x), F(B)(x))$ for all x in X.

2.5 Intersection [3]

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A∩B, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$T(C)(x) = min\ (T(A)(x) ,T(B)(x))$,

$I(C)(x) = min\ ((A)(x) ,I(B)(x))$, $F(C)(x) = max(F(A)(x) ,F(B)(x))$ for *all* x *in* X

3 Neutrosophic Linear Programming Problem

Linear programming problem with neutrosophic coefficients (NLPP) is defined as the following:

Maximize Z= $\sum_{j=1}^{n} c_j x_j$

Subject to

$\sum_{j=1}^{n} \tilde{a}_{ij}^{n} x_j \leq b_i \quad 1 \leq i \leq m$ (1)

$x_j \geq 0, \qquad 1 \leq j \leq n$

where a_{ij}^{n} *is* a neutrosophic number.

The single valued neutrosophic number (a_{ij}^{n}) is donated by A=(a,b,c) where a,b,c ∈ [0,1] And a,b,c ≤ 3

The truth- membership function of neutrosophic number a_{ij}^{n} is defined as:

$$T\, a_{ij}^{n}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_2-x}{a_{3-a_2}} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \quad (2)$$

The indeterminacy- membership function of neutrosophic number $a_{ij}^{n}$is defined as:

$$I\, a_{ij}^{n}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & b_1 \leq x \leq b_2 \\ \frac{b_2-x}{b_{3-b_2}} & b_2 \leq x \leq b_3 \\ 0 & otherwise \end{cases} \quad (3)$$

And its falsity- membership function of neutrosophic number $a_{ij}^{\sim n}$ is defined as:

$$F\, a_{ij}^{n}(x) = \begin{cases} \frac{x-C_1}{C_2-C_1} & C_1 \leq x \leq C_2 \\ \frac{C_2-x}{C_{3-C_2}} & C_2 \leq x \leq C_3 \\ 1 & otherwise \end{cases} \quad (4)$$

Then we find the upper and lower bounds of the objective function for truth-membership, indeterminacy and falsity membership as follows:

$z_U^T = \max\{z(x_i^*\ )\}$ and z_l^T =min$\{z(x_i^*\ )\}$ where 1$\leq i \leq k$

$z_{L=}^F z_L^T$ And $z_{u=}^F z_u^T - R(z_u^T - z_L^T)$

$z_{U=}^I z_U^I\ and\ z_{l=}^I z_{l=}^I - S(z_u^T - z_L^T)$

where R, S are predetermined real number in (0, 1).

The truth membership, indeterminacy membership, falsity membership of objective function are as follows:

$$T_O^{(Z)} = \begin{cases} 1 & if\ \ z \geq z_u^T \\ \frac{z-z_L^T}{z_u^T-z_L^T} & if\ \ z_L^T \leq z \leq z_u^T \\ 0 & if\ z\ < z_L^T \end{cases} \quad (5)$$

$$I_O^{(Z)} = \begin{cases} 1 & if\ z \geq z_u^T \\ \frac{z-z_L^I}{z_u^I-z_L^I} & if\ \ z_L^T \leq z \leq z_u^T \\ 0 & if\ z\ < z_L^T \end{cases} \quad (6)$$

$$F_O^{(Z)} = \begin{cases} 1 & if\ z \geq z_u^T \\ \frac{z_u^F - Z}{z_u^F - z_L^F} & if\ \ z_L^T \leq z \leq z_u^T \\ 0 & if\ z\ < z_L^T \end{cases} \quad (7)$$

The neutrosophic set of the i^{th} constraint c_i is defined as:

$$T_{c_i}^{(x)} = \begin{cases} 1 & if\quad b_i \geq \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \\ \frac{b_i - \sum_{j=1}^{n} a_{ij\,x_j}}{\sum_{j=1}^{n} d_{ij\,x_j}} & if\quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \\ 0 & if\ \ b_i\ < \sum_{j=1}^{n} a_{ij\,x_j} \end{cases} \quad (8)$$

$$I_{c_i}^{(x)} = \begin{cases} 0\ \ if\quad b_i < \sum_{j=1}^{n} a_{ij\,x_j} \\ \frac{b_i - \sum_{j=1}^{n} d_{ij\,x_j}}{\sum_{j=1}^{n} a_{ij\,x_j}}\ If\quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \\ 0\ if\ b_i \geq \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \end{cases} \quad (9)$$

$$F_{c_i}^{(x)} = \begin{cases} 1 & if\quad b_i < \sum_{j=1}^{n} a_{ij\,x_j} \\ 1 - T_{c_i}^{(x)} & if\quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \\ 0 & if\ b_i \geq \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j \end{cases} \quad (10)$$

4 Neutrosophic Optimization Model

In our neutrosophic model we want to maximize the degree of acceptance and minimize the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Neutrosophic optimization model can be defined as:

$$\begin{aligned} & maxT_{(x)} \\ & minF_{(x)} \\ & minI_{(x)} \\ & \textit{Subject to} \\ & T_{(X)} \geq F_{(x)}\ , \\ & T_{(X)} \geq I_{(x)}\ , \\ & 0 \leq T_{(X)} + I_{(x)} + F_{(x)} \leq 3, \\ & T_{(X)},\quad I_{(X)}\ ,\quad F_{(X)} \geq 0,\quad x \geq 0\ , \end{aligned} \quad (11)$$

where $T_{(x)}, F_{(x)}, I_{(x)}$ denote the degree of acceptance, rejection, and indeterminacy of x respectively.

The above problem is equivalent to the following:

$max\ \alpha,\ min\ \beta\ , min\ \theta$

Subject to

$$\alpha \leq \mathrm{T}\ (x),$$
$$\beta \leq \mathrm{F}\ (x),$$
$$\theta \leq \mathrm{I}\ (x),$$
$$\alpha \geq \beta,$$
$$\alpha \geq \theta,$$
$$0 \leq \alpha + \beta + \theta \leq 3, \qquad (12)$$
$$x \geq 0,$$

where α denotes the minimal acceptable degree, β denotes the maximal degree of rejection and θ denotes the maximal degree of indeterminacy.

The neutrosophic optimization model can be changed into the following optimization model:

$max(\alpha - \beta - \theta)$

Subject to

$$\alpha \leq \mathrm{T}\ (x), \qquad (13)$$
$$\beta \geq \mathrm{F}\ (x),$$
$$\theta \geq \mathrm{I}\ (x),$$
$$\alpha \geq \beta,$$
$$\alpha \geq \theta,$$
$$0 \leq \alpha + \beta + \theta \leq 3,$$
$$\alpha, \beta, \theta \geq 0\ ,$$
$$x \geq 0\ .$$

The previous model can be written as:

min (1- α) $\beta\ \theta$

Subject to

$$\alpha \leq \mathrm{T}\,(x)$$
$$\beta \geq \mathrm{F}\,(x)$$
$$\theta \geq \mathrm{I}\,(x)$$
$$\alpha \geq \beta$$
$$\alpha \geq \theta$$
$$0 \leq \alpha + \beta + \theta \leq 3 \tag{14}$$
$$x \geq 0.$$

5 The Algorithm for Solving Neutrosophic Linear Programming Problem (NLPP)

Step 1. solve the objective function subject to the constraints.

Step 2. create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3. declare goals and tolerance.

Step 4. construct membership functions.

Step 5. set α, β, θ in the interval $]^{-}0, 1^{+}[$ for each neutrosophic number.

Step 6. find the upper and lower bound of objective function as we illustrated previously in section 3.

Step 7. construct neutrosophic optimization model as in equation (13).

6 Numerical Examples

To measure the efficiency of our proposed model, we solved many numerical examples.

6.1. Illustrative Example #1

Beaver Creek Pottery Company is a small crafts operation run by a Native American tribal council. The company employs skilled artisans to produce clay bowls and mugs with authentic Native American designs and colours. The two primary resources used by the company are special pottery clay and skilled labour. Given these limited resources, the company desires to know how many bowls and mugs to produce each day in order to maximize profit. The two products have the following resource requirements for production and profit per item produced presented in Table 1:

Table 1. Resource requirements of two products

product	Resource Requirements		
	Labour(Hr./Unit)	Clay (Lb./Unit)	Profit($/Unit)
Bowl	$\tilde{1}$	$\tilde{4}$	$\widetilde{40}$
Mug	$\tilde{2}$	$\tilde{3}$	$\widetilde{50}$

There are around 40 hours of labour and around 120 pounds of clay available each day for production. We will formulate this problem as a neutrosophic linear programming model as follows:

max $\widetilde{40}x_1 + \widetilde{50}x_2$

S.t.

$\tilde{1}x_1 + \tilde{2}x_2 \leq \widetilde{40}$ $\quad \tilde{4}x_1 + \tilde{3}x_2 \leq \widetilde{120}$

$$x_1, x_2 \geq 0 \tag{15}$$

where

C_1=$\widetilde{40} = \{(30,\ 40,\ 50), (0.7,\ 0.4,\ 0.3)\}$;

C=$\widetilde{50} = \{(40,\ 50,\ 60), (0.6,\ 0.5,\ 0.2)\}$;

a_{11}=$\tilde{1} = \{(0.5,\ 1,\ 3), (0.6,\ 0.4,\ 0.1)\}$;

a_{12}=$\tilde{2} = \{(0,\ 2,\ 6), (0.6,\ 0.4,\ 0.1)\}$;

a_{21}=$\tilde{4} = \{(1,\ 4,\ 12), (0.4,\ 0.3,\ 0.2)\}$;

a_{22}=$\tilde{3} = \{(1, 3,\ 10), (0.7,\ 0.4,\ 0.3)\}$;

b_1=$\widetilde{40} = \{(20,\ 40,\ 60), (0.4,\ 0.3,\ 0.5)\}$;

b_2=$\widetilde{120} = \{(100,\ 120,\ 140), (0.7,\ 0.4,\ 0.3)\}$;

The equivalent crisp formulation is:

max $15x_1 + 18x_2$

S.t

$x_1 + x_2 \leq 12$

$3x_1 + 2x_2 \leq 45$

$x_1, x_2 \geq 0$

The optimal solution is $x_1 = 0$; $x_2 = 12$; with optimal objective value = 216$.

6.2. Illustrative Example #2

$$
\begin{aligned}
& max\ \tilde{5}x_1 + \tilde{3}x_2 \\
& s.t. \\
& \tilde{4}x_1 + \tilde{3}x_2 \le \tilde{12} \\
& \tilde{1}x_1 + \tilde{3}x_2 \le \tilde{6} \\
& x_1, x_2 \ge 0
\end{aligned}
\tag{16}
$$

where

$c_1=\tilde{5}=$ {(4, 5, 6), (0.5, 0.8, 0.3)};

$c_2=\tilde{3}=$ {(2.5, 3, 3.2), (0.6, 0.4, 0)};

$a_{11}=\tilde{4}=$ {(3.5, 4, 4.1), (0.75, 0.5, 0.25)};

$a_{12}=\tilde{3}=$ {(2.5, 3, 3.2), (0.2, 0.8, 0.4)};

$a_{21}=\tilde{1}=$ {(0, 1, 2), (0.15, 0.5, 0)};

$a_{22}=\tilde{3}=$ {(2.8, 3, 3.2), (0.75, 0.5, 0.25)};

$b_1=\tilde{12}=$ {(11, 12, 13), (0.2, 0.6, 0.5)};

$b_2=\tilde{6}=$ {(5.5, 6, 7.5), (0.8, 0.6, 0.4)}.

The equivalent crisp formulation is:

max 1.3125x_1 +0.0158x_2

S.t

2.5375x_1+0.54375x_2≤2.475

0.3093x_1+1.125$x_2 \le 2.1375$

x_1, x_2≥0

The optimal solution is $x_1 = 1$; $x_2 = 0$; with optimal objective value 1 $.

6.3. Illustrative Example #3

$$\begin{aligned} & max\ \tilde{25}x_1 + \tilde{48}x_2 \\ & s.t. \\ & 15x_1 + 30x_2 \le 45000 \\ & 24x_1 + 6x_2 \le 24000 \\ & 21x_1 + 14x_2 \le 28000 \\ & x_1, x_2 \ge 0 \end{aligned} \tag{17}$$

where

$c_1 = \tilde{25}$ = {(19, 25, 33), (0.8, 0.1, 0.4)};

$c_2 = \tilde{48}$ = {(44, 48, 54), (0.75, 0.25, 0)}.

The corresponding crisp linear programs given as follows:

max $11.069x_1 + 22.8125x_2$

$s.t$

$15x_1 + 30x_2 \le 45000$

$24x_1 + 6x_2 \le 24000$

$x_1, x_2 \ge 0$

The optimal solution is $x_1 = 0$; $x_2 = 1500$; with optimal objective value 34219 $

6.4. Illustrative Example #4

$$\begin{aligned} & max\ 25x_1 + 48x_2 \\ & s.t. \\ & \tilde{15}x_1 + \tilde{30}x_2 \le 45\tilde{0}00 \\ & \tilde{24}x_1 + \tilde{6}x_2 \le 24\tilde{0}00 \\ & \tilde{21}x_1 + \tilde{14}x_2 \le 28\tilde{0}00 \\ & x_1, x_2 \ge 0 \end{aligned} \tag{18}$$

where

$a_{11}= \tilde{15} = \{(14, 15, 17), (0.75, 0.5, 0.25)\}$;

$a_{12}= \tilde{30} = \{(25, 30, 34), (0.25, 0.7, 0.4)\}$;

$a_{21}= \tilde{24} = \{(21, 24, 26), (0.4, 0.6, 0)\}$;

$a_{22}= \tilde{6} = \{(4, 6, 8), (0.75, 0.5, 0.25)\}$;

$a_{31}= \tilde{21} = \{(17, 21, 22), (1, 0.25, 0)\}$;

$a_{32}= \tilde{14} = \{(12, 14, 19), (0.6, 0.4, 0)\}$;

$b_1= \tilde{45000} = \{(44980, 45000, 45030), (0.3, 0.4, 0.8)$;

$b2= \tilde{24000} = \{(23980, 24000, 24050), (0.4, 0.25, 0.5)\}$;

$b_3= \tilde{28000} = \{(27990, 28000, 28030), (0.9, 0.2, 0)\}$.

The associated crisp linear programs model will be:

$max\ 25x_1 + 48x_2$

$s.t$

$5.75x_1 + 6.397x_2 \leq 9282$

$10.312x_1 + 6.187x_2 \leq 14178.37$

$x_1, x_2 \geq 0$

The optimal solution is $x_1 = 0$; $x_2 = 1451$; with optimal objective value 69648$

6.5. Illustrative Example#5

$$\begin{aligned}
&\max 7x_1 + 5x_2 \\
&s.t. \\
&\tilde{1}x_1 + \tilde{2}x_2 \leq 6 \\
&\tilde{4}x_1 + \tilde{3}x_2 \leq 12 \\
&x_1, x_2 \geq 0
\end{aligned} \tag{19}$$

where

$a_{11}=\tilde{1}$ = {(0.5, 1, 2), (0.2, 0.6, 0.3)};

$a_{12}=\tilde{2}$ = {(2.5, 3, 3.2), (0.6, 0.4, 0.1)};

$a_{21}=\tilde{4}$ = {(3.5, 4, 4.1), (0.5, 0.25, 0.25)};

$a_{22}=\tilde{3}$ = {(2.5, 3, 3.2), (0.75, 0.25, 0)};

The associated crisp linear programs model will be:

$max\ 7x_1 + 5x_2$

S. t

$0.284x_1 + 1.142x_2 \leq 6$

$1.45x_1 + 1.36x_2 \leq 12$

$x_1, x_2 \geq 0$

The optimal solution is $x_1 = 4$; $x_2 = 4$; with optimal objective value 48$.

The result of our NLP model in this example is better than the results obtained by intuitionistic fuzzy set [4].

7 Conclusions and Future Work

Neutrosophic sets and fuzzy sets are two hot research topics. In this paper, we propose linear programming model based on neutrosophic environment, simultaneously considering the degrees of acceptance, indeterminacy, and rejection of objectives, by proposed model for solving neutrosophic linear programming problems (NlPP). In the proposed model, we maximize the degrees of acceptance and minimize indeterminacy and rejection of objectives. NlPP was transformed into a crisp programming model using truth membership, indeterminacy membership, and falsity membership functions. We also give numerical examples to show the efficiency of the proposed method. As far as future directions are concerned, these will include studying the duality theory of linear programming problems based on Neutrosophic.

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