Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers

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Abstract— In this work, a neutrosophic network method is proposed for finding the shortest path length with single valued trapezoidal neutrosophic number. The proposed algorithm gives the shortest path length using score function from source node to destination node. Here the weights of the edges are considered to be single valued trapezoidal neutrosophic number. Finally, a numerical example is used to illustrate the efficiency of the proposed approach.

Keywords— Single valued trapezoidal neutrosophic number; Score function; Network; Shortest path problem.

I. INTRODUCTION

In 1998, the concept of the neutrosophic set (NS for short) and neutrosophic logic were introduced by Smarandache in [1, 2] in order to efficiently handle the indeterminate and inconsistent information in real world. Neutrosophic set is a generalization of the theory of fuzzy set [3], intuitionistic fuzzy sets [4], interval-valued fuzzy sets [5] and interval-valued intuitionistic fuzzy sets [6]. The concept of the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval ]0, 1[

Recently, based on the neutrosophic set theory, Subas [9] presented the concept of triangular and trapezoidal neutrosophic numbers and applied to multiple-attribute decision making problems. Then Biswas et al [10] presented a special case of trapezoidal neutrosophic numbers and applied to multiple-attribute decision making problems by introducing the cosine similarity measure. Deli and Subas [11] presented the single valued trapezoidal neutrosophic numbers (SVN-numbers) as a generalization of the intuitionistic trapezoidal fuzzy numbers and proposed a methodology for solving multiple-attribute decision making problems with SVN-numbers. In addition, Thamaraiselvi and Santhi [12] introduced a mathematical representation of a transportation problems in neutrosophic environment based on single valued trapezoidal neutrosophic numbers and also provided the solution method.

The shortest path problem (SPP) is one of the most fundamental and well-known combinatorial problems that appear in various fields of science and engineering, e.g. road networks application, transportation, routing in communication channels and scheduling problems. The main objective of the shortest path problem is to find a path with minimum length between any pair of vertices. The edge (arc) length of the network may represent the real life quantities such as, time, cost, etc. In a classical shortest path problem, the distances of the edge between different nodes of a network are assumed to be certain. Numerous algorithms have been developed with the weights on edges on network being fuzzy numbers, intuitionistic fuzzy numbers, vague numbers [13-16].
Recently, Broumi et al. [17-22] presented the concept of neutrosophic graphs, interval valued neutrosophic graphs and bipolar single valued neutrosophic graphs. To this day, only a few papers dealing with shortest path problem in neutrosophic environment. The paper proposed by Broumi et al. [23] is one of the first on this subject. The authors proposed an algorithm for solving neutrosophic shortest path problem based on score function. The same authors [24] proposed another algorithm for solving shortest path problem in a bipolar neutrosophic environment. Also, in [25] they proposed the shortest path algorithm in a network with its edge lengths as interval valued neutrosophic numbers.

The goal of this work is to propose an approach for solving shortest path problem in a network where edge weights are characterized by a single valued trapezoidal neutrosophic numbers. In order to do, the paper is organized as follows: In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets and single valued trapezoidal neutrosophic sets. In Section 3, we propose some modified operations of single valued trapezoidal neutrosophic numbers. In section 4, a network terminology is presented. In section 5, we propose an algorithm for finding the shortest path and shortest distance in single valued trapezoidal neutrosophic graph. In section 6, we illustrate a practical example which is solved by the proposed algorithm. Finally, some concluding remarks are presented in section 7.

II. PRELIMINARIES

In this section, some basic concepts and definitions on neutrosophic sets, single valued neutrosophic sets and single valued trapezoidal neutrosophic sets are reviewed from the literature.

**Definition 2.1 [1].** Let \( X \) be a space of points (objects) with generic elements in \( X \) denoted by \( x \); then the neutrosophic set \( A \) (NS) is an object having the form \( A = \{< x: () \}, x \in X\}, \) where the functions \( T, I, F: X \rightarrow [0, 1] \) define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element \( x \in X \) to the set \( A \) with the condition:

\[
0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3.
\]

The functions \( T_a(x), I_a(x) \) and \( F_a(x) \) are real standard or nonstandard subsets of \([0, 1] \). Since it is difficult to apply NSs to practical problems, Wang et al. [7] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

**Definition 2.2 [7].** Let \( X \) be a space of points (objects) with generic elements in \( X \) denoted by \( x \). A single valued neutrosophic set \( A \) (SVNS) is characterized by truth-membership function \( T_a(x) \), an indeterminacy-membership function \( I_a(x) \) and a falsity-membership function \( F_a(x) \). For each point \( x \) in \( X \) \( T_a(x), I_a(x), F_a(x) \in [0, 1] \). A SVNS can be written as:

\[
A = \{< x: T_a(x), I_a(x), F_a(x) >, x \in X \}
\]

**Definition 2.3 [11].** A single valued trapezoidal neutrosophic number (SVTN-number) \( \tilde{a} = (a_1, b_1, c_1, d_1) \in \mathbb{R} \) is a special neutrosophic set on the real number set \( \mathbb{R} \) whose truth membership, indeterminacy-membership, and a falsity-membership are given as follows:

\[
T_a(x) = \begin{cases} 
\frac{(x-a)}{(b-a)} & \text{if } a \leq x \leq b \\
\frac{(b-a)}{(d-c)} & \text{if } c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}
\]

\[
I_a(x) = \begin{cases} 
\frac{(b-x+I_a(x-a))}{(b-a)} & \text{if } a \leq x \leq b \\
\frac{(d-c)}{d-c} & \text{if } c \leq x \leq d \\
1 & \text{otherwise}
\end{cases}
\]

\[
F_a(x) = \begin{cases} 
\frac{(b-x+F_a(x-a))}{(b-a)} & \text{if } a \leq x \leq b \\
\frac{(d-c)}{d-c} & \text{if } c \leq x \leq d \\
1 & \text{otherwise}
\end{cases}
\]

Where \( 0 \leq T_a \leq 1; 0 \leq I_a \leq 1; 0 \leq F_a \leq 1 \) and \( 0 \leq T_a + I_a + F_a \leq 3; \) \( a_1, b_1, c_1, d_1 \in \mathbb{R} \)

**Definition 2.3 [11].** Let \( \tilde{A} = (a_1, b_1, c_1, d_1) \) and \( \tilde{B} = (b_2, c_2, d_2) \) be two single valued trapezoidal neutrosophic numbers. Then, the operations for SVTN-numbers are defined as follows:

(i) \( \tilde{A} + \tilde{B} = (a_1 + b_1, b_2, c_2, d_2) \)

(ii) \( \tilde{A} \cdot \tilde{B} = (\min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)) \)

(iii) \( \tilde{A} \triangledown \tilde{B} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4) \cdot (T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)) \)

A convenient method for comparing of single valued trapezoidal neutrosophic number is by use of score function.

**Definition 2.4 [11].** Let \( \tilde{A} = (a_1, b_1, c_1, d_1) \) be a single valued trapezoidal neutrosophic number. Then, the score function \( s(\tilde{A}) \) and accuracy function \( a(\tilde{A}) \) of a SVTN-numbers are defined as follows:
Definition 2.5 [11]. Let \( \tilde{A}_1 \) and \( \tilde{A}_2 \) be two SVTN-numbers the ranking of \( \tilde{A}_1 \) and \( \tilde{A}_2 \) by score function is defined as follows:

(i) If \( s(\tilde{A}_1) < s(\tilde{A}_2) \) then \( \tilde{A}_1 \prec \tilde{A}_2 \)

(ii) If \( s(\tilde{A}_1) = s(\tilde{A}_2) \) and if

1. \( a(\tilde{A}_1) < a(\tilde{A}_2) \) then \( \tilde{A}_1 \prec \tilde{A}_2 \)
2. \( a(\tilde{A}_1) > a(\tilde{A}_2) \) then \( \tilde{A}_1 \succ \tilde{A}_2 \)
3. \( a(\tilde{A}_1) = a(\tilde{A}_2) \) then \( \tilde{A}_1 \equiv \tilde{A}_2 \)

III. ARITHMETIC OPERATIONS BETWEEN TWO SV-TRAPEZOIDAL NEUTROSOPHIC NUMBERS

In this subsection, we make a slight modification of the operations for SVTNNs as defined below:

\begin{align*}
\text{(i) } & a(\tilde{A}_1) = \left( \frac{1}{12} \left[ a_1 + a_2 + a_3 + a_4 \right] \times \left[ 2 + T_1 - I_1 - F_1 \right] \right) \\
\text{(ii) } & a(\tilde{A}_2) = \left( \frac{1}{12} \left[ a_1 + a_2 + a_3 + a_4 \right] \times \left[ 2 + T_2 - I_2 - F_2 \right] \right)
\end{align*}

Step 5 Let the destination node (node n) be labeled as \( [\tilde{d}_n, l] \), then the single valued trapezoidal neutrosophic shortest distance between source node and destination node is \( \tilde{d}_n \).

Step 6 Since destination node is labeled as \( [\tilde{d}_n, l] \), so, to find the single valued trapezoidal neutrosophic shortest path between source node and destination node, check the label of node \( l \). Let it be \( [\tilde{d}_l, p] \), now check the label of node \( p \) and so on. Repeat the same procedure until node 1 is obtained.

IV. NETWORK TERMINOLOGY

Consider a directed network \( G = (V, E) \) consisting of a finite set of nodes \( V = \{1, 2, ..., n\} \) and a set of \( m \) directed edges \( E \subseteq V \times V \). Each edge is denoted by an ordered pair \((i, j)\) where \( i, j \in V \) and \( i \neq j \). In this network, we specify two nodes, denoted by \( s \) and \( t \), which are the source node and the destination node, respectively. We define a path as a sequence \( P = [i, l_1, j] \) of alternating nodes and edges. The existence of at least one path \( P \) in \( G \) is assumed for every \( i \in V \). The shortest path from \( i \) to \( j \) is denoted as \( d(P) \) is defined as

\[ d(P) = \sum_{(i,j) \in P} d_{ij} \]  

Remark: A node \( i \) is said to be predecessor node of node \( j \) if

(i) Node \( i \) is directly connected to node \( j \).
(ii) The direction of path connecting node \( i \) and \( j \) from \( i \) to \( j \).

V. SINGLE VALUED TRAPEZOIDAL NEUTROSOPHIC PATH PROBLEM

In this section, motivated by the work of Kumar [14], an algorithm is proposed to find the path of minimum distance between the source node (i) and the destination node (j) in a single valued trapezoidal neutrosophic graph.

The main steps of the algorithm are as follows:

Step 1 Assume \( \tilde{d}_i = (0, 0, 0, 0; 0, 1, 1) \) and label the source node (say node 1) as \( \tilde{d}_1 = (0, 0, 0, 0; 0, 1, 1) \). The label indicating that the node has no predecessor.

Step 2 Find \( \tilde{d}_j = \min \{ \tilde{d}_i \cap \tilde{d}_j ; i=2,3,...,n \} \).

Step 3 If minimum occurs corresponding to unique value of \( i \), then label node \( j \) as \( \tilde{d}_j = \min \{ \tilde{d}_i \} \). If minimum occurs corresponding to more than one values of \( i \), then it represents that there are more than one single valued trapezoidal neutrosophic path between source node and node \( j \) but single valued trapezoidal neutrosophic distance along path is \( \tilde{d}_j \), so choose any value of \( i \).

Step 4 Let the destination node (node \( n \)) be labeled as \( [\tilde{d}_n, l] \), then the single valued trapezoidal neutrosophic shortest distance between source node and destination node is \( \tilde{d}_n \).

Step 5 Since destination node is labeled as \( [\tilde{d}_n, l] \), so, to find the single valued trapezoidal neutrosophic shortest path between source node and destination node, check the label of node \( l \). Let it be \( [\tilde{d}_l, p] \), now check the label of node \( p \) and so on. Repeat the same procedure until node 1 is obtained.

Step 6 Now the single valued trapezoidal neutrosophic shortest path can be obtained by combining all the nodes obtained by the step 5.

Remark 5.1 Let \( \tilde{A}_j ; i = 1, 2, ..., n \) be a set of single valued trapezoidal neutrosophic numbers, if \( S(\tilde{A}_j) < S(\tilde{A}_i) \), for all \( i \), the single valued trapezoidal neutrosophic number is the minimum of \( \tilde{A}_j \).
In this network each edge have been assigned to single valued trapezoidal neutrosophic number as follows:

<table>
<thead>
<tr>
<th>Edges</th>
<th>single valued trapezoidal Neutrosophic distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>$&lt; (1, 2, 3, 4); 0.4, 0.6, 0.7&gt;$</td>
</tr>
<tr>
<td>1-3</td>
<td>$&lt; (2, 5, 7, 8); 0.2, 0.3, 0.4&gt;$</td>
</tr>
<tr>
<td>2-3</td>
<td>$&lt; (3, 7, 8, 9); 0.1, 0.4, 0.6&gt;$</td>
</tr>
<tr>
<td>2-5</td>
<td>$&lt; (1, 5, 7, 9); 0.7, 0.6, 0.8&gt;$</td>
</tr>
<tr>
<td>3-4</td>
<td>$&lt; (2, 4, 8, 9); 0.5, 0.3, 0.1&gt;$</td>
</tr>
<tr>
<td>4-6</td>
<td>$&lt; (7, 8, 9, 10); 0.3, 0.2, 0.6&gt;$</td>
</tr>
<tr>
<td>5-6</td>
<td>$&lt; (2, 4, 5, 7); 0.6, 0.5, 0.3&gt;$</td>
</tr>
</tbody>
</table>

Table 1. Weights of the graphs

The calculations for this problem are as follows:

Since node 6 is the destination node, so $n=6$.

Assume $d_i = (0, 0, 0, 0); 0, 1, 1 >$ and label the source node (say node 1) as $< (0, 0, 0, 0); 0, 1, 1 , 1 >$, the value of $d_j; j=2, 3, 4, 5, 6$ can be obtained as follows:

**Iteration 1** Since only node 1 is the predecessor node of node 2, so putting $i=1$ and $j=2$ in step 2 of the proposed algorithm, the value of $d_2$ is

$d_2 = \min \{ \tilde{d}_2 \oplus \tilde{d}_{12} \} = \min \{ (0, 0, 0, 0); 0, 1, 1 > \oplus (1, 2, 3, 4); 0.4, 0.6, 0.7 > \}$

Since minimum occurs corresponding to $i=1$, so label node 2 as $< (1, 2, 3, 4); 0.4, 0.6, 0.7 >$.

**Iteration 2** The predecessor node of node 3 are node 1 and node 2, so putting $i=1, 2$ and $j=3$ in step 2 of the proposed algorithm, the value of $d_3$ is

$d_3 = \min \{ \tilde{d}_3 \oplus \tilde{d}_{13} \oplus \tilde{d}_{23} \} = \min \{ (0, 0, 0, 0); 0, 1, 1 > \oplus (2, 5, 7, 8); 0.2, 0.3, 0.4 > \oplus (1, 2, 3, 4); 0.4, 0.6, 0.7 > \oplus (3, 7, 8, 9); 0.1, 0.4, 0.6 > \}$

$S(\{ (2, 5, 7, 8); 0.2, 0.3, 0.4 > \}$

$= \frac{1}{12} [a_1 + a_2 + a_3 + a_4] \times [2 + T_1 - I_1 - F_1] = 2.75$

$= S(\{ (4, 9, 11, 13); 0.46, 0.24, 0.42 > ) = 5.55$

Since $S(\{ (2, 5, 7, 8); 0.2, 0.3, 0.4 > ) < S(\{ (4, 9, 11, 13); 0.46, 0.24, 0.42 > )$

So minimum $\{ (2, 5, 7, 8); 0.2, 0.3, 0.4 >, (4, 9, 11, 13); 0.46, 0.24, 0.42 > \}$

$= (2, 5, 7, 8); 0.2, 0.3, 0.4 >$

Since minimum occurs corresponding to $i=1$, so label node 3 as $\{ (2, 5, 7, 8); 0.2, 0.3, 0.4 >, 1 \}$

**Iteration 3** The predecessor node of node 4 is node 3, so putting $i=3$ and $j=4$ in step 2 of the proposed algorithm, the value of $d_4$ is

$d_4 = \min \{ \tilde{d}_4 \oplus \tilde{d}_{34} \} = \min \{ (2, 5, 7, 8); 0.2, 0.3, 0.4 > \oplus (2, 4, 8, 9); 0.5, 0.3, 0.1 > \}$

$= (4, 9, 15, 17); 0.6, 0.09, 0.04 >$

Since minimum occurs corresponding to $i=5$, so label node 4 as $\{ (2, 5, 7, 8); 0.2, 0.3, 0.4 >, 3 \}$

**Iteration 4** The predecessor node of node 5 are node 4 and node 3, so putting $i=2$ and $j=5$ in step 2 of the proposed algorithm, the value of $d_5$ is

$d_5 = \min \{ \tilde{d}_5 \oplus \tilde{d}_{45} \oplus \tilde{d}_{54} \} = \min \{ (1, 2, 3, 4); 0.4, 0.6, 0.7 > \oplus (1, 5, 7, 9); 0.7, 0.6, 0.8 > \oplus (2, 5, 7, 8); 0.2, 0.3, 0.4 > \oplus (3, 4, 5, 10); 0.3, 0.4, 0.7 > \}$

$= (4, 9, 15, 17); 0.6, 0.09, 0.04 >$

Since minimum occurs corresponding to $i=5$, so label node 5 as $\{ (4, 9, 15, 17); 0.6, 0.09, 0.04 >, 2 \}$

**Iteration 5** The predecessor node of node 6 are node 4 and node 5, so putting $i=4$ and $j=6$ in step 2 of the proposed algorithm, the value of $d_6$ is

$d_6 = \min \{ \tilde{d}_6 \oplus \tilde{d}_{46} \oplus \tilde{d}_{56} \} = \min \{ (4, 9, 15, 17); 0.6, 0.09, 0.04 > \oplus (7, 8, 9, 10); 0.3, 0.2, 0.6 > \oplus (2, 7, 10, 13); 0.82, 0.36, 0.56 > \oplus (2, 4, 5, 7); 0.6, 0.5, 0.3 > \}$

$= (4, 9, 15, 17); 0.93, 0.18, 0.17 >$

$= S(\{ (11, 17, 24, 27); 0.72, 0.018, 0.024 >, (4, 11, 15, 20); 0.93, 0.18, 0.17 > \}$

Since minimum occurs corresponding to $i=5$, so label node 6 as $\{ (4, 11, 15, 20); 0.93, 0.18, 0.17 >, 5 \}$

Since node 6 is the destination node of the given network, so the single valued trapezoidal neutrosophic shortest distance...
the performance of the proposed methodology for the shortest path. Next, we will research the application of this algorithm.

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REFERENCES


between node 1 and node 6 is <(4, 11, 15, 20); 0.93, 0.18, 0.17>.

Now the single valued trapezoidal neutrosophic shortest path between node 1 and node 6 can be obtained by using the following procedure:

Since node 6 is labeled by [<(4, 11, 15, 20); 0.93, 0.18, 0.17>, 5], which represents that we are coming from node 5. Node 5 is labeled by [<(2, 7, 10, 13); 0.82, 0.36, 0.56>, 2], which represents that we are coming from node 2. Node 2 is labeled by [<(1, 2, 3, 4); 0.4, 0.6, 0.7>, 1], which represents that we are coming from node 1. Now the single valued trapezoidal neutrosophic shortest path between node 1 and node 6 is obtained by joining all the obtained nodes. Hence the single valued trapezoidal neutrosophic shortest path is 1 → 2 → 5 → 6.

The single valued trapezoidal neutrosophic shortest distance and the single valued trapezoidal neutrosophic shortest path of all nodes from node 1 is shown in the table 2 and the labeling of each node is shown in figure 2.

![Diagram](image_url)

**Table 2. Tabular representation of different single valued trapezoidal neutrosophic shortest paths**

<table>
<thead>
<tr>
<th>Node No. (j)</th>
<th>(d_i)</th>
<th>Single valued trapezoidal Neutrosophic shortest path between jth and 1st node</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>&lt;(1, 2, 3, 4); 0.4, 0.6, 0.7&gt;</td>
<td>1 → 2</td>
</tr>
<tr>
<td>3</td>
<td>&lt;(2, 5, 7, 8); 0.2, 0.3, 0.4&gt;</td>
<td>1 → 3</td>
</tr>
<tr>
<td>4</td>
<td>&lt;(4, 9, 15, 17); 0.6, 0.09, 0.04&gt;</td>
<td>1 → 3 → 4</td>
</tr>
<tr>
<td>5</td>
<td>&lt;(2, 7, 10, 13); 0.82, 0.36, 0.56&gt;</td>
<td>1 → 2 → 5</td>
</tr>
<tr>
<td>6</td>
<td>&lt;(4, 11, 15, 20); 0.93, 0.18, 0.17&gt;</td>
<td>1 → 2 → 5 → 6</td>
</tr>
</tbody>
</table>

**FIG 2. Network with single valued trapezoidal neutrosophic shortest distance of each node from node 1**

VI. CONCLUSION

In this paper, we have developed an algorithm for solving shortest path problem on a network with single valued trapezoidal neutrosophic edge lengths. The process of ranking the path is very useful to make decisions in choosing the best of all possible path alternatives. Numerical example via six node network showed the performance of the proposed