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AN EXTENDED GREY RELATIONAL ANALYSIS BASED INTERVAL NEUTROSOPHIC MULTI ATTRIBUTE DECISION MAKING FOR WEAVER SELECTION

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Abstract – The paper proposes a multi-attribute decision making method based on extended grey relation analysis under interval neutrosophic environment. The interval neutrosophic set is an important decision making apparatus that can handle imprecise, indeterminate, inconsistence information. The rating of the alternatives with respect to certain attribute considered by the expert is characterized by linguistic variables that can be represented by interval neutrosophic sets. In the selection process, the attributes are identified from the experts' opinion. The weight of each attribute is completely unknown and maximizing deviation method is employed in order to determine them. Then, an extended grey relational analysis technique is developed to find the ranking order of all alternatives. Finally, an illustrative numerical example for weaver selection in Khadi Institution is provided to show the effectiveness and applicability of the developed approach.

***Keywords* –** *Multi-attribute decision making, linguistic variable, interval neutrosophic set, grey relational analysis, weaver selection.*

1 Introduction

Zadeh [25] coined the term 'degree of membership' and defined the concept of fuzzy set in order to deal with uncertainty. Atanassov [1] incorporated the degree of non-membership in the concept of fuzzy set as an independent component and defined the concept of intuitionistic fuzzy set. Smarandache [13,14] grounded the term 'degree of indeterminacy as an independent component and defined the concept of neutrosophic set from the philosophical point of view to deal with incomplete, indeterminate and inconsistent

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information which exist in real world decision making problems. In neutrosophic set, truth membership, indeterminacy membership, falsity membership functions are independent and they are real standard or non-standard subsets of$]0^-, 1^+[$. However, NS is difficult to apply in practical decision making situation. To employ the concept of neutrosophic set in practical fields, Wang et al. [19] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS) since single value is an instance of set value. In SVNS, the truth membership, indeterminacy membership, falsity membership functions are subsets of [0, 1]. SVNS is identified as a useful tool for practical scientific and engineering applications. However, decision information may be provided with intervals rather than real numbers due to lack of knowledge of the decision maker. Therefore, Wang et al. [18] defined set-theoretic operators on interval neutrosophic set (INS) which is more flexible and practical than SVNS. INS is much easier to handle incomplete, indeterminate and inconsistent information.

Multi-attribute decision making (MADM) is one of the fastest developing areas during past few decades and it has been employed to solve different practical problems such as economic evaluation, planning and design, investment, transportation, marketing, operations research, management science, etc. The objective of MADM is to select the most desirable alternative from a set of alternatives with respect to multiple and often conflicting attributes. During last five years many methodologies [2-7, 10-12, 15-17, 20-24] have been proposed for MADM under neutrosophic environment. Ye [24] studied MADM method by using correlation coefficient of SVNSs. Ye and Zhang [23] proposed similarity measures between SVNSs based on maximum and minimum operators and developed a MADM method based on weighted similarity measures of SVNSs under single valued neutrosophic assessments. Liu and Wang [11] proposed a single valued neutrosophic normalized weighted Bonferroni mean operator for solving multi-attribute group decision making (MAGDM) problem. Broumi and Smarandache [6] proposed a neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation operator and a neutrosophic trapezoid linguistic weighted geometric aggregation operator for MADM problems with single valued neutrosophic assessment. Biswas et al. [5] extended the concept of technique of order preference by similarity to ideal solution (TOPSIS) for solving MADM problems with SVNS information.

Chi and Liu [7] established an extended TOPSIS method for MADM problems where the attribute weights are unknown and attribute values are expressed in terms of INSs. Ye [21] discussed distance-based similarity measures for solving MADM problems with completely unknown weights for decision makers and attributes under single valued neutrosophic environment. Ye [22] proposed an interval neutrosophic linguistic weighted arithmetic average operator and an interval neutrosophic linguistic weighted geometric average operator and developed a method for solving MADM problems with interval neutrosophic linguistic information. Şahin and Karabacak [12] developed a simple inclusion measure for solving MADM problem under interval neutrosophioc environment.

Deng [8] originally developed grey relational analysis (GRA) in order to solve uncertainty problems under discrete data and incomplete information. In the field of neutrosophy, Biswas et al. [3] applied the concept of GRA to formulate an approach for solving MADM problem with SVNS information where the information about attribute weights are fully unknown to the DM. Biswas et al. [2] also studied neutrosophic MADM with unknown weight information using modified GRA.

The main objective of this paper is to extend the concept of GRA to develop a new approach for solving MADM problems under INS information. The attributes are obtained in terms of linguistic variables which can be transformed into INSs. Here, the weights of the attributes are completely unknown and maximizing deviation method [20] is applied in order to determine the unknown attribute weights. Then, virtual positive ideal solution (PIS) and negative ideal solution (NIS) [7] are identified by selecting the best values for each attribute from all alternatives. Finally, neutrosophic grey relational coefficient of each alternative is calculated in order to rank the alternatives.

The remaining part of the paper is constructed as follows: Section 2 presents preliminaries of neutrosophic set and also provides transformation rule between linguistic variables and INSs. Section 3 is devoted to develop an extended GRA method for solving MADM problems. In Section 4, an illustrative example is solved in order to demonstrate the effectiveness of the proposed approach. Finally, the last Section concludes the paper.

2 Preliminaries of Neutrosophic Sets

Neutrosophy [13] is a new branch of philosophy grounded by Smarandache. Neutrosophy is the origin of the concept of neutrosophic set.

2.1 Some Basic Definitions

Definition 2.1. [13] Let *X* be a space of objects with generic element in *X* represented by *x*. Then a NS is defined by

$$\mathrm{A} = \{x, \langle \mathrm{T_A}(x), \mathrm{I_A}(x), \mathrm{F_A}(x) \rangle \mid x \in \mathrm{X}\}$$

Where, $\mathrm{T_A}(x): X \to\ ]0^-, 1^+[$; $\mathrm{I_A}(x): X \to\ ]0^-, 1^+[$; $\mathrm{F_A}(x): X \to\ ]0^-, 1^+[$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, repectively. It is to be noted that $0^- \le \mathrm{T_A}(x) + \mathrm{I_A}(x) + \mathrm{F_A}(x) \le 3^+$.

Definition 2.2. [19] Let *X* be a universal space of objects with generic element in *X* represented by *x*. Then a SVNS $N \subset X$ is characterized by a truth-membership function $\mathrm{T_N}(x)$, an indeterminacy-membership function $\mathrm{I_N}(x)$, and a falsity-membership function $\mathrm{F_N}(x)$ with $\mathrm{T_N}(x), \mathrm{I_N}(x), \mathrm{F_N}(x) \in [0, 1]$ for each point $x \in X$. Here, it is to be noted that for a SVNS we have, $0 \le \sup \mathrm{T_N}(x) + \sup \mathrm{I_N}(x) + \sup \mathrm{F_N}(x) \le 3$.

Definition 2.3. [18] Consider *X* be a universal space of points with generic element in *X* represented by *x*. Then an INS is defined as follows

$$P = \{x, \langle \mathrm{T_P}(x), \mathrm{I_P}(x), \mathrm{F_P}(x) \rangle \mid x \in X\}$$

Here, $\mathrm{T_P}(x), \mathrm{I_P}(x), \mathrm{F_P}(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively with $\mathrm{T_P}(x), \mathrm{I_P}(x), \mathrm{F_P}(x) \subseteq [0, 1]$ for each point $x \in X$ and $0 \le \sup(\mathrm{T_P}(x)) + \sup(\mathrm{I_P}(x)) + \sup(\mathrm{F_P}(x)) \le 3$. For convenience, we can write $x = ([\mathrm{T^L}, \mathrm{T^U}], [\mathrm{I^L}, \mathrm{I^U}], [\mathrm{F^L}, \mathrm{F^U}])$ and *x* is called interval neutrosophic value (INV).

2.2 The Operational Rules of INS

Consider a = $([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U])$ and b = $([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U])$ be two INVs, then the operational definitions [7] are presented as given as follows:

(1) The complement of a is $\bar{a} = ([F_1^L, F_1^U], [1-I_1^U, 1-I_1^L],[T_1^L, T_1^U])$
(2) $a+b = ([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U],[F_1^L F_2^L, F_1^U F_2^U])$
(3) $a.b = ([T_1^L T_2^L, T_1^U T_2^U], [I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U],[F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U])$
(4) $ma = ([1 - (1 - T_1^L)^m, 1 - (1 - T_1^U)^m], [(I_1^L)^m, (I_1^U)^m], [(F_1^L)^m, (F_1^U)^m]), m > 0$
(5) $a^m = ([(T_1^L)^m, (T_1^U)^m], 1 - (1 - I_1^L)^m, 1 - (1 - I_1^U)^m, [1 - (1 - F_1^L)^m, 1 - (1 - F_1^U)^m]), m > 0$

Definition [7]. Let *a* = $([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U])$ and b = $([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U])$ be two INVs, then the Hamming distance between *a* and *b* is presented as follows.

$$r_H(a, b) = 1/6(|T_1^L - T_2^L| + |T_1^U - T_2^U| + |I_1^L - I_2^L| + |I_1^U - I_2^U| + |F_1^L - F_2^L| + |F_1^U - F_2^U|)$$

Definition [7]. Consider a = $([T_1^L, T_1^U], [I_1^L, I_1^U],[F_1^L, F_1^U])$ and b = $([T_2^L, T_2^U], [I_2^L, I_2^U],[F_2^L, F_2^U])$ be two INVs, then the Euclidean distance between a and b is defined as given below.

$$r_E(a, b) = \sqrt{1/6\left((T_1^L - T_2^L)^2 + (T_1^U - T_2^U)^2 + (I_1^L - I_2^L)^2 + (I_1^U - I_2^U)^2 + (F_1^L - F_2^L)^2 + (F_1^U - F_2^U)^2\right)}$$

Definition [7]. Let *A* = $([T_i^L, T_i^U], [I_i^L, I_i^U],[F_i^L, F_i^U])$, (i = 1, 2, ..., m) and B = $([\hat{T}_i^L, \hat{T}_i^U], [\hat{I}_i^L, \hat{I}_i^U],[\hat{F}_i^L, \hat{F}_i^U])$,(i = 1, 2, ..., m) be two INSs, then the Hamming distance between *A* and *B* is presented as follows.

$$r_H(A, B) = 1/6m \sum_{i=1}^{m} (|T_i^L - \hat{T}_i^L| + |T_i^U - \hat{T}_i^U| + |I_i^L - \hat{I}_i^L| + |I_i^U - \hat{I}_i^U| + |F_i^L - \hat{F}_i^L| + |F_i^U - \hat{F}_i^U|).$$

Definition [7]. Let *A* = $([T_i^L, T_i^U], [I_i^L, I_i^U],[F_i^L, F_i^U])$, (i = 1, 2, ..., m) and *B* = $([\hat{T}_i^L, \hat{T}_i^U], [\hat{I}_i^L, \hat{I}_i^U],[\hat{F}_i^L, \hat{F}_i^U])$,(i = 1, 2, ..., m) be two INSs, then the Euclidean distance between *A* and *B* is presented as given below.

$$r_E(A,B) = \sqrt{1/6m \sum_{i=1}^{m} \left((T_i^L - \hat{T}_i^L)^2 + (T_i^U - \hat{T}_i^U)^2 + (I_i^L - \hat{I}_i^L)^2 + (I_i^U - \hat{I}_i^U)^2 + (F_i^L - \hat{F}_i^L)^2 + (F_i^U - \hat{F}_i^U)^2\right)}$$

2.3 Transformation between Linguistic Variables and INS

A linguistic variable is a variable whose values are expressed in either words or sentences in a natural language. The rating of the alternatives with respect to certain qualitative attribute can be presented in terms of linguistic variable such as extreme good, very good, good, and medium good, etc. Linguistic variables can be transformed into INSs (see Table 1).

Table 1. Transformation between the linguistic variables and INVs

Linguistic variables	INVs
Extreme good (EG)	([0.95, 1], [0.05, 0. 1], [0, 0.1])
Very good (VG)	([0.75, 0.95], [0.1, 0.15], [0.1, 0.2])
Good (G)	([0.6, 0.75], [0.1, 0.2], [0.2, 0.25])
Medium Good (MG)	([0.5, 0.6], [0.2, 0.25], [0.25, 0.35])
Medium (M)	([0.4, 0.5], [0.2, 0.3], [0.35, 0.45])
Medium low (ML)	([0.3, 0.4], [0.15, 0.25], [0.45, 0.5])
Low (L)	([0.2, 0.3], [0.1, 0.2], [0.5, 0.65])
Very low (VL)	([0.05, 0.2], [0.1, 0.15], [0.65, 0.8])
Extreme low (EL)	([0, 0.05], [0.05, 0. 1], [0.8, 0.95])

3 An Extended GRA Method for Solving MADM Problems based on INS

Consider a MADM problem with p alternatives and q attributes. Let G = $\{g_1, g_2, \ldots, g_p\}$, $(p \geq 2)$ denotes the set of alternatives and H $=\{h_1, h_2, \ldots, h_q\}$, $(q \geq 2)$ represents the set of attributes. Also let W = $\{w_1, w_2, \ldots, w_q\}$ be the weighting vector of the attributes with $\sum_{j=1}^{q} w_j = 1$. w_j (> 0), (j = 1, 2, …, q) reflects the relative importance of the attributes and we assume that w_j, (j = 1, 2, …, q) is completely unknown in the decision making process. The attributes are obtained in linguistic variables, which can be expressed by INSs. In the following steps, we describe the extended GRA method under INSs for ranking the alternatives.

Step 1. Construction of decision matrix

Let the rating of alternative g_i, (i = 1, 2, …, p) with respect to the attribute h_j, (j = 1, 2, …, q) is obtained in terms linguistic variable that can be expressed in terms of INVs by using the Table 1. Then construct the decision matrix $C = [c_{ij}]_{p \times q}$ as follows:

$$C = [c_{ij}]_{p \times q} = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1q} \\ c_{21} & c_{22} & \ldots & c_{2q} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ c_{p1} & c_{p2} & \ldots & c_{pq} \end{bmatrix} \quad (1)$$

where $c_{ij}=([T_{ij}^L, T_{ij}^U],[I_{ij}^L, I_{ij}^U],[F_{ij}^L, F_{ij}^U]),(i=1,2,...,p;j=1,2,...,q)$

Step 2. Standardization of decision matrix

To eliminate the influence of different physical dimensions to decision results, we standardize the decision matrix due to Chi and Liu [7]. Suppose the standardized decision matrix $S = [s_{ij}]_{p \times q}$ is presented as follows.

$$S = [s_{ij}]_{p\times q} = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1q} \\ s_{21} & s_{22} & \dots & s_{2q} \\ . & . & \dots & . \\ . & . & \dots & . \\ s_{p1} & s_{p2} & \dots & s_{pq} \end{bmatrix} \tag{2}$$

where $s_{ij} = ([\tilde{T}^L_{ij}, \tilde{T}^U_{ij}], [\tilde{I}^L_{ij}, \tilde{I}^U_{ij}], [\tilde{F}^L_{ij}, \tilde{F}^U_{ij}])$, (i = 1, 2, ..., p; j = 1, 2, ..., q). Here, it is to be noted that

$$c_{ij} = s_{ij}, \text{ if j is benefit type of attribute}$$

$$c_{ij} = \bar{s}_{ij}, \text{ if j is cost type of attribute,}$$

where $\bar{s}_{ij}$ is the complement of s_{ij}.

Step 3. Determination of the attribute weights

The weights of the attributes are not always known to the DM in the decision making situation. Also, the weights are not equal in general. Since we assume that the weights of the attributes are completely unknown, we apply maximizing deviation method of Yang [20] in order to determine the unknown attribute weights. The method is based on the concept that if the attribute values of all alternatives for a specified attribute have a small deviations, then small weight is provided for this attribute. If the attribute values of all alternatives for a particular attribute have greater deviations, we can offer greater weight for this attribute. However, if the attribute values of all alternatives for a given attribute are equal then the weight of such attribute may be taken as 0.

The deviation values of alternatives G_i to all other alternatives with respect to attribute H_j can be described as $L_{ij}(w_j) = \sum_{t=1}^{p} r(c_{ij}, c_{tj})\, w_j$, then

$$L_j(w_j) = \sum_{i=1}^{p} L_{ij}(w_j) = \sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

denotes total deviations of all alternatives to the other alternatives for the attribute H_j.

$$L(w_j) = \sum_{j=1}^{q} L_j(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

denotes the deviation of all attributes for all alternatives to the other alternatives. Then the optimization model is presented as follows:

$$\text{Maximize } L(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj}) w_j$$

$$\text{Subject to } \sum_{j=1}^{q} w_j^2 = 1,\ w_j \ge 0,\ j = 1, 2, \dots, q. \tag{3}$$

We can obtain attribute weight [20] as follows:

$$w_j = \frac{\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}{\sqrt{\sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r^2(c_{ij}, c_{tj})}}, \ j = 1, 2, \ldots, q. \quad (4)$$

Then, the normalized attribute weight based on the above model is obtained as given below.

$$w_j = \frac{\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}{\sum_{j=1}^{q}\sum_{i=1}^{p}\sum_{t=1}^{p} r(c_{ij}, c_{tj})}, \ j = 1, 2, \ldots, q. \quad (5)$$

Step 4. Determination of the weighted decision matrix

The weighted decision matrix is constructed as follows:

$$Z = [z_{ij}]_{p\times q} = \begin{bmatrix} w_1 s_{11} & w_2 s_{12} & \cdots & w_q s_{1q} \\ w_1 s_{21} & w_2 s_{22} & \cdots & w_q s_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ w_1 s_{p1} & w_2 s_{p2} & \cdots & w_q s_{pq} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1q} \\ z_{21} & z_{22} & \cdots & z_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ z_{p1} & z_{p2} & \cdots & z_{pq} \end{bmatrix},$$

where $z_{ij} = ([\breve{T}_{ij}^L, \breve{T}_{ij}^U], [\breve{I}_{ij}^L, \breve{I}_{ij}^U], [\breve{F}_{ij}^L, \breve{F}_{ij}^U])$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Step 5. Determination of interval PIS and NIS

Chi and Liu [7] defined the interval PIS (p_j^+) and interval NIS (n_j^+) for INS as given below.

$$p_j^+ = ([1, 1], [0, 0], [0, 0]), \ j = 1, 2, \ldots, q \quad (7)$$

$$n_j^- = ([0, 0], [1, 1], [1, 1]), \ j = 1, 2, \ldots, q \quad (8)$$

The virtual interval PIS and interval NIS can also be recognized by determining the best and worst values respectively for each attribute from all alternatives as shown below.

$$p_j^+ = ([\operatorname{Max}_i \breve{T}_{ij}^L, \operatorname{Max}_i \breve{T}_{ij}^U], [\operatorname{Min}_i \breve{I}_{ij}^L, \operatorname{Min}_i \breve{I}_{ij}^U], [\operatorname{Min}_i \breve{F}_{ij}^L, \operatorname{Min}_i \breve{F}_{ij}^U]) \quad (9)$$

$$n_j^+ = ([\operatorname{Min}_i \breve{T}_{ij}^L, \operatorname{Min}_i \breve{T}_{ij}^U], [\operatorname{Max}_i \breve{I}_{ij}^L, \operatorname{Max}_i \breve{I}_{ij}^U], [\operatorname{Max}_i \breve{F}_{ij}^L, \operatorname{Max}_i \breve{F}_{ij}^U]). \quad (10)$$

Step 6. Determination of neutrosophic grey relational coefficient of each alternative from PIS and NIS

The grey relational coefficient of each alternative from PIS is obtained from the following formula:

$$\zeta_{ij}^{+} = \frac{\underset{i}{Min}\underset{j}{Min}\Delta_{ij}^{+} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{+}}{\Delta_{ij}^{+} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{+}} \tag{11}$$

where $\Delta_{ij}^{+} = r(z_{ij}, p_j^{+})$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Also, the grey relational coefficient of each alternative from NIS is obtained from the formula given below:

$$\zeta_{ij}^{-} = \frac{\underset{i}{Min}\underset{j}{Min}\Delta_{ij}^{-} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{-}}{\Delta_{ij}^{-} + \sigma \underset{i}{Max}\underset{j}{Max}\Delta_{ij}^{-}} \tag{12}$$

where $\Delta_{ij}^{-} = r(z_{ij}, p_j^{-})$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Here, $\sigma \in [0, 1]$ denotes the the environmental coefficient and it is a free parameter. σ is used to adjust the difference of the relational coefficient. Generally, we set $\sigma = 0.5$ in the decision making circumstances.

Step 7. Determination of degree of neutrosophic grey relational coefficient

The degree of neutrosophic grey relational coefficient of each alternative from PIS and NIS are calculated respectively by using the follwing formula:

$$\zeta_i^{+} = \sum_{j=1}^{q} \zeta_{ij}^{+}\ ;\ \zeta_i^{-} = \sum_{j=1}^{q} \zeta_{ij}^{-}, \text{ for } (i = 1, 2, ..., p). \tag{13}$$

Step 8. Determination neutrosophic relative relational degree

The neutrosophic relative relational degree is obtained from the the following equation

$$R_i = \frac{\zeta_j^{+}}{\zeta_j^{+} + \zeta_j^{-}},\ i = 1, 2, ..., p. \tag{14}$$

Step 9. Ranking the alternatives

Rank the alternatives g_i based on the relative relational degree. The highest value of R_i reflects the most important alternative.

4 An Illustrative Example

A Khadi Institution desires to recruit two most competent weavers g_1, g_2, g_3 from a list of three weavers. In order to identify the key attributes of weaver selection, we interviewed Khadi domain experts of Chak, a Gram Panchayet area of Murshidabad, West Bengal, India. After analyzing the data the seven most important attributes for weaver selection are identified as: skill (h_1), previous experience (h_2), honesty (h_3), physical fitness (h_4), locality of the weaver (h_5), personality (h_6), economic condition of the weaver (h_7) [9]. Here, the seven attributes are benefit type attributes and the weights of the attributes are completely unknown. The Khadi Institution then hire a Khadi expert in order to select the most suitable weaver based on the seven attributes. Generally, the Khadi expert is hired from the locality who knows the weavers strength and weakness very well. The Khadi expert provides linguistic variables to represent the rating of the weavers with respect to the above attributes as shown in the Table 2. Then our objective is to choose the most appropriate weaver based on the proposed approach. In the following steps, we present the proposed approach for weaver selection.

Step 1: We convert the linguistic decision matrix as shown in Table 2 into INVs decision matrix by using Table 1. The decision matrix is constructed as in Table 3.

Step 2: We use Euclidean distance defined in Eq. 5 to obtain r (c_{ij}, c_{tj}), i = t = 1, 2, …, p; j = 1, 2, …, q and we determine the weights of the attributes by using Eq. 7 as follows:

$w_1 = w_2 = 0.096$, $w_3 = w_4 = 0.176$, $w_5 = 0.096$, $w_6 = 0.151$, $w_7 = 0.207$ such that $\sum_{j=1}^{7} w_j = 1$, $w_j \geq 0$, j = 1, 2, …, 7.

Step 3: We determine z_{ij}, i = 1, 2, 3; j = 1, 2, …, 7 by using Eq. 6. The weighted decision matrix is provided in Table 4.

Step 4: The virtual interval PIS (p_j^+) and virtual interval NIS (n_j^+), j = 1, 2, …, 7 are identified as given below.

p_1^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_2^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_3^+ = ([0.216, 0.41], [0.667, 0.716], [0.667, 0.753]);
p_4^+ = ([0.216, 0.41], [0.667, 0.716], [0.667, 0.753]);
p_5^+ = ([0.125, 0.25], [0.802, 0.833], [0.802, 0.857]);
p_6^+ = ([0.129, 0.189], [0.706, 0.784], [0.784, 0.811]);
p_7^+ = ([0.173, 0.249], [0.621, 0.717], [0.717, 0.75]).

n_1^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_2^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_3^+ = ([0.115, 0.149], [0.753 0.783], [0. 783, 0.831]);
n_4^+ = ([0.115, 0.149], [0.753 0.783], [0. 783, 0.831]);
n_5^+ = ([0.084, 0.125], [0.802, 0.857], [0.857, 0.875]);
n_6^+ = ([0.074, 0.099], [0.784, 0.838], [0.853, 0.886]);
n_7^+ = ([0.071, 0.1], [0.717, 0.75], [0.848, 0.866]).

Step 5: The interval neutrosophic grey relational coefficient of each alternative from virtual PIS and NIS are calculated respectively by using the Eq.9 and Eq. 10 as follows:

$$\zeta_{ij}^{+} = \begin{bmatrix} 0.000 & 0.000 & 0.135 & 0.135 & 0.059 & 0.000 & 0.065 \\ 0.059 & 0.059 & 0.000 & 0.053 & 0.059 & 0.029 & 0.019 \\ 0.000 & 0.059 & 0.053 & 0.000 & 0.000 & 0.071 & 0.114 \end{bmatrix}$$

$$\zeta_{ij}^{-} = \begin{bmatrix} 1.000 & 1.000 & 0.333 & 0.333 & 0.534 & 1.000 & 0.509 \\ 0.534 & 0.534 & 1.000 & 0.560 & 0.534 & 0.699 & 0.780 \\ 1.000 & 0.534 & 0.560 & 1.000 & 1.000 & 0.487 & 0.372 \end{bmatrix}$$

Step 6: The degree or grade of interval neutrosophic grey relational coefficient of each alternative from PIS and NIS are obtained respectively by using the Eq. 11and Eq. 12 as given below.

$\zeta_1^{+} = 0.394$, $\zeta_2^{+} = 0.278$, $\zeta_3^{+} = 0.297$; $\zeta_1^{-} = 4.709$, $\zeta_2^{-} = 4.641$, $\zeta_3^{-} = 4.953$

Step 7: The interval neutrosophic relative relational degree is obtained as follows:

$$R_1 = = 0.077209,\ R_2 = = 0.056516,\ R_3 = = 0.056571$$

Finally, we rank the order of all alternatives according to the decending order of R_i as:

$$R_1 > R_3 > R_2$$

So, g_1, g_3 are the most suitable weavers for Khadi Institution.

5. Conclusion

In this paper, we have developed an alternative method for MADM problems with unknown weight information under interval neutrosophic environment. The attributes with respect to certain alternative are represented by linguistic variables rather than numerical values and the linguistic variables are expressed in terms of interval valued neutrosophic set. The unknown weights of the attributes are obtained by using maximizing deviation method. Then modified GRA method is proposed to solve the MADM problems. Finally, an illustrative numerical example for weaver selection in Khadi Institution is demonstrated to show the applicability of the proposed method. The authors hope that the proposed method can be effective for solving practical decision making problems such as pattern recognition, databases, medical diagnosis, decision making, etc.

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