J. Appl. Math. Comput.
DOI 10.1007/s12190-017-1098-z

ORIGINAL RESEARCH

Accessible single-valued neutrosophic graphs

M. Hamidi[1] · **A. Borumand Saeid**[2]

Received: 31 December 2016

Abstract This paper derived single-valued neutrosophic graphs from single-valued neutrosophic hypergraphs via strong equivalence relation. We show that any weak single-valued neutrosophic graph is a derived single-valued neutrosophic graph and any linear weak single-valued neutrosophic tree is an extendable linear single-valued neutrosophic tree.

Keywords Single-valued neutrosophic (graphs) hypergraphs · Strong equivalence relation · Extendable single-valued neutrosophic

Mathematics Subject Classification 03E72 · 05C72 · 05C78 · 05C99

1 Introduction

Neutrosophy, as a newly-born science is a branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. It can be defined as the incidence of the application of a law, an axiom, an idea, a conceptual accredited construction on an unclear, indeterminate phenomenon, contradictory to the purpose of making it intelligible. Neutrosophic sets and systems are a tool for publications on advanced studies in neutrosophy, neutrosophic set, neu-

✉ A. Borumand Saeid
arsham@uk.ac.ir

M. Hamidi
m.hamidi@pnu.ac.ir

[1] Department of Mathematics, Faculty of Mathematics, Payame Noor University, Tehran, Iran

[2] Department of Pure Mathematics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran

Published online: 27 March 2017

trosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

Neutrosophic set and neutrosophic logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Most of the problems in engineering, medical science, economics, environments etc. have various uncertainties. In 1995, Smarandache talked for the first time about neutrosophy and in 1999 and 2005 [11,14] he initiated the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. Alkhazaleh et al. generalized the concept of fuzzy soft set to neutrosophic soft set and they gave some applications of this concept in decision making and medical diagnosis [4].

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in their relationships or in both, normally that we need to design a fuzzy graph model. The extension of fuzzy graph theory [12,16] have been developed by several researchers including intuitionistic fuzzy graphs [1,13] considered the vertex sets and edge sets as intuitionistic fuzzy sets.

Smarandache [15] have defined four main categories of neutrosophic graphs, two of which are based on literal indeterminacy (I), which are called; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and have gained popularity among the researchers due to their applications via real world problems [7,17]. The other two graphs are based on (t, i, f) components and are called; The (t, i, f)-Edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al. [5] introduced a third neutrosophic graph model. This model allowed the attachment of truth-membership (t), indeterminacy-membership (i) and falsity-membership degrees (f) both to vertices and edges, and investigated some of their properties.

Fuzzy hypergraph was introduced by the Kaufmann [10]. Lee-kwang et al. generalized the concept of fuzzy hypergraph and redefined it to be useful for fuzzy partition of a system. Akram and Dudek [2] investigated some properties of intuitionistic fuzzy hypergraph and gave applications of intuitionistic fuzzy hypergraph.

Akram et al. [3] are defined the concepts of single-valued neutrosophic hypergraph, line graph of single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph and transversal single-valued neutrosophic hypergraph.

Regarding these points, the aim of this paper is to generalize the notion of single-valued neutrosophic graphs by considering the notion of strong equivalence relation and to define the concept of extendable single-valued neutrosophic graphs. It is a normal question about the relationships between extendable single-valued neutrosophic graphs and extendable single-valued neutrosophic hypergraphs. From here comes the main motivation for this and in this regard, we have considered the quotient of single-valued neutrosophic hypergraphs via equivalence relations. Also, we want to establish the relationship between (α, β, γ)-level single-valued neutrosophic graphs and (α, β, γ)-level single-valued neutrosophic hypergraphs. Moreover, by using strong equivalence relation, we have defined a well-defined operation on single-valued neutrosophic hypergraphs that the quotient of any single-valued neutrosophic hypergraphs via this relation is a single-valued neutrosophic graph.

We use single-valued neutrosophic hypergraphs to represent of the complex systems as networks, social, biological, ecological and technological systems where the use of complex networks gives very limited to information about the structure of the system. By introducing the concept of the complex hyper-network, the use of complex hypernetworks appears to be a necessary for exploring these systems and representation their relationships. We have introduced several valuable measures as truth-membership, indeterminacy and falsity-membership values for studying complex hyper-networks, such as node and hypergraph centralities as well as clustering coefficients for both the hyper-networks and the networks.

2 Preliminaries

In this section, we recall some definitions and results that are indispensable to our research paper.

Definition 2.1 [6] Let $G = \{x_1, x_2, \ldots, x_n\}$ be a finite set. A hypergraph on G is a family $H = (G, \{E_i\}_{i=1}^{m})$ of subsets of G such that

(i) for all $1 \leq i \leq m$, $E_i \neq \emptyset$;

(ii) $\bigcup_{i=1}^{m} E_i = G$.

A *simple hypergraph* (*Sperner family*) is a hypergraph $H = (G, \{E_i\}_{i=1}^{m})$ such that

(iii) $E_i \subset E_j \Longrightarrow i = j$.

The elements $x_1, x_2, \ldots, x_n$ of G are called *vertices*, and the sets $E_1, E_2, \ldots, E_m$ are the *edges* (hyperedges) of the hypergraph. For any $1 \leq k \leq m$ if $|E_k| \geq 2$, then E_k is represented by a solid line surrounding its vertices, if $|E_k| = 1$ by a cycle on the element (loop). If for all $1 \leq k \leq m|E_k| = 2$, the hypergraph becomes an ordinary (undirected) graph.

Definition 2.2 [8] Let $(G, \{E_x\}_{x \in G})$ be a hypergraph, where for any $x \in G$, E_x is one of hyperedges such that $x \in E_x$. Then a binary relation ρ on G is defined as follows: for every integer $n \geq 1$, ρ_n is defined as follows:

$$x \rho_n y \Longleftrightarrow |E_x^m| = |E_y^m|, \text{ where } |E_x^m| = \min\{|E_t|;\ x \in E_t\} \text{ or } |E_x^m| \leq |E_x|$$

and $n = \min\{deg(x), deg(y)\}$.

Obviously the relation $\rho = \bigcup_{n \geq 1} \rho_n$ is an equivalence relation on G. We denote the set of all equivalence classes of ρ by G/ρ. Hence $G/\rho = \{\rho(x) \mid x \in G\}$.

Theorem 2.3 *[8] Let $(G, \{E_x\}_{x \in G})$ be a hypergraph. Then there exists an operation $*$ on G/ρ such that $(G/\rho, *)$ is a graph.*

Definition 2.4 [8] Let $H = (G, \{E_x\}_{x \in G})$ be a hypergraph. Then H is called a complete hypergraph, if for any $x, y \in G$ there exits a hyperedge E such that $\{x, y\} \subseteq E$ and a complete hypergraph with n elements is shown by K_n^*. Let $H = (G, \{E_i\}_{i=1}^{n+1})$ be a complete hypergraph.

- (i) H is called a joint complete hypergraph, if for any $1 \leq i \leq n$, $|E_i| = i$, $E_i \subseteq E_{i+1}$ and $|E_{n+1}| = n$;
- (ii) H is called a discrete complete hypergraph, if for any $1 \leq i \neq j \leq n$, $|E_i| = |E_j|$, $E_i \cap E_j = \emptyset$ and $|E_{n+1}| = n$;

Definition 2.5 [18] Let X be a set. A single-valued neutrosophic set A in X (SVN-SA) is a function $A : X \longrightarrow [0, 1] \times [0, 1] \times [0, 1]$ with the form $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ where the functions T_A, I_A, F_A define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Moreover, $Supp(A) = \{x \mid T_A(x) \neq 0, I_A(x) \neq 0, F_A(x) \neq 0\}$ is a crisp set.

Definition 2.6 [5] A single-valued neutrosophic graph (SVN-G) is defined to be a form $G = (V, E, A, B)$ where

- (i) $V = \{v_1, v_2, \ldots, v_n\}$, $T_A, I_A, F_A : V \longrightarrow [0, 1]$ denote the degree of membership, degree of indeterminacy and non-membership of the element $v_i \in V$; respectively, and for every $1 \leq i \leq n$, we have $0 \leq T_A(v_i)+I_A(v_i)+F_A(v_i) \leq 3$.
- (ii) $E \subseteq V \times V$, $T_B, I_B, F_B : E \longrightarrow [0, 1]$ are called degree of the truth-membership, the indeterminacy-membership and the falsity-membership of the edge $(v_i, v_j) \in E$ respectively, such that for any $1 \leq i, j \leq n$, we have $T_B(v_i, v_j) \leq \min\{T_A(v_i), T_A(v_j)\}$, $I_B(v_i, v_j) \geq \max\{I_A(v_i), I_A(v_j)\}$, $F_B(v_i, v_j) \geq \max\{F_A(v_i), F_A(v_j)\}$ and $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$. Also A is called the single-valued neutrosophic vertex set of V and B is called the single-valued neutrosophic edge set of E.

Definition 2.7 [3]

- (i) A single-valued neutrosophic hypergraph (SVN-HG) is defined to be a pair $H = (V, \{E_i\}_{i=1}^m)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of vertices and $\{E_i = \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}\}_{i=1}^m$ is a finite family of non-trivial neutrosophic subsets of the vertex V such that $V = \bigcup_{i=1}^m supp(E_i)$. Also $\{E_i\}_{i=1}^m$ is called the family of single-valued neutrosophic hyperedges of H and V is the crisp vertex set of H.
- (ii) Let $1 \leq \alpha, \beta, \gamma \leq 1$, then $A^{(\alpha,\beta,\gamma)} = \{x \in X \mid T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}$ is called (α, β, γ)-level subset of A.

3 (Weak) single-valued neutrosophic hypergraphs (graphs) (SVN-HG)

In this section, we introduce concept of weak single-valued neutrosophic graph and via equivalence relations, construct quotient single-valued neutrosophic hypergraphs.

Moreover, for any $(\alpha, \beta, \gamma) \in [0, 1]^3$, we investigate on (α, β, γ)-level hypergraphs and show that any finite set can be a (α, β, γ)-level (partitioned) hypergraph.

Let $H = (G, \{E_i\}_{i=1}^n)$ be a hypergraph, $1 \leq i, j \leq n$ and $k \in \mathbb{N}$. Then H is called a partitioned hypergraph, if $\mathcal{P} = \{E_1, E_2, \ldots, E_n\}$ is a partition of G. We will denote the set of partitioned hypergraphs with $|\mathcal{P}| = k$ on G that $|E_i| = |E_j|$, by $\mathcal{P}_h^{(k)}(H)$ and the set of all partitioned hypergraphs on H, by $\mathcal{P}_h(H)$.

Proposition 3.1 *Let G be a finite set and R be an equivalence relation on H. Then the following statements hold:*

(i) $H = \big(G, \{R(x)\}_{x \in G}\big)$ *is a (partitioned) hypergraph;*
(ii) $H = \big(G, E_{x,y} = \{R(x) \cup R(y)\}_{x,y \in G}\big)$ *is a complete (hyper)graph.*

Proof Since R is an equivalence relation on H, we get that $\mathcal{P} = \{E_x = R(x)\}_{x \in H}$ is a partition of H and so $H = \big(G, \{R(x)\}_{x \in G}\big)$ is a (partitioned) hypergraph. Moreover, $\bigcup_{x \in G} R(x) = G$ implies that $H = \big(G, E_{x,y} = \{R(x) \cup R(y)\}_{x,y \in G}\big)$ is a complete (hyper)graph, especially whence for any $x \in G$, $|R(x)| = 1$, then $H = \big(G, E_{x,y} = \{R(x) \cup R(y)\}_{x,y \in G}\big)$ is a complete graph. □

Corollary 3.2 *The following statements are hold.*

(i) Let G be a set, $|G| = n$ and $\mathcal{Q} = \{G/R \mid R$ is an equivalence relation on $G\}$. Then $|\mathcal{Q}| = \sum_{k=1}^{n} \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n$.
(ii) Any finite set can be a (partitioned) single-valued neutrosophic hypergraph.
(iii) Any finite set can be a complete single-valued neutrosophic hypergraph.

Proof (i) [9]. (ii), (iii) Let $G = \{a_1, a_2, \ldots, a_n\}$. By Proposition 3.1, there exists $r \leq n$ such that $H = (G, \{E_i = R(x_i)\}_{i=1}^r)$ is a partitioned hypergraph. For any $1 \leq i \leq r$, consider $E_i = \{(x_i, i/(10^n), (i+1)/(10^n), (i+2)/(10^n))\}$. Since for any $n \in \mathbb{N}$, $3n + 3 \leq 10^n$, we obtain $H = (G, \{E_i = R(x_i)\}_{i=1}^r)$ which it is a partitioned single-valued neutrosophic hypergraph. □

Definition 3.3 Let $G = (V, E, A, B)$ be a single-valued neutrosophic graph. Then $G = (V, E, A, B)$ is called a weak single-valued neutrosophic graph, if $supp(A) = V$ and for any $v_i, v_j \in V$ have $T_B(v_i, v_j) = \min\{T_A(v_i), T_A(v_j)\}$, $I_B(v_i, v_j) = \max\{I_A(v_i), I_A(v_j)\}$ and $F_B(v_i, v_j) = \max\{F_A(v_i), F_A(v_j)\}$.

Example 3.4 Let $V = \{a_1, a_2, \ldots, a_n\}$. Consider the complete graph K_n and define $A : V \to [0, 1]$ by $T_A(a_i) = 1/i$, $I_A(a_i) = 1/(i+1)$, $F_A(a_i) = 1/(i+2)$ and $B : V \times V \to [0, 1]$ by $T_B(a_i, a_j) = T_A(a_i) \times T_A(a_j)$, $I_B(a_i, a_j) = F_B(a_i, a_j) = I_A(a_i) + T_A(a_j)$. It is clear that (V, A, B) is a complete single-valued neutrosophic graph and $supp(A) = V$.

Corollary 3.5 *Any finite set can be a complete weak single-valued neutrosophic graph.*

Proof It is obtained from Corollary 3.2. □

Lemma 3.6 *Let X be a finite set and $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ be a single-valued neutrosophic set in X. If R is an equivalence relation on X, then $A/R = \{(R(x), T_{R(A)}(R(x)), I_{R(A)}(R(x)), F_{R(A)}(R(x)) \mid x \in X\}$ is a single-valued neutrosophic set, where $T_{R(A)}(R(x)) = \bigwedge_{t\ R\ x} T_A(t)$, $I_{R(A)}(R(x)) = \bigvee_{t\ R\ x} I_A(t)$ and $F_{R(A)}(R(x)) = \bigvee_{t\ R\ x} F_A(t)$.*

Proof Let $X = \{x_1, x_2, \ldots, x_n\}$ and $\mathcal{P} = \{R(x_1), R(x_2), \ldots, R(x_k)\}$ be a partition of X, where $k \leq n$. Since for any $x_i \in X$, $T_A(x_i) \leq 1$, $I_A(x_i) \leq 1$ and $F_A(x_i) \leq 1$, we get that $\bigwedge_{t\ R\ x_i} T_A(t) \leq 1$, $\bigvee_{t\ R\ x_i} I_A(t) \leq 1$ and $\bigvee_{t\ R\ x_i} F_A(t) \leq 1$. Hence for any $1 \leq i \leq k$, $0 \leq \bigwedge_{t\ R\ x_i} T_A(t) + \bigvee_{t\ R\ x_i} I_A(t) + \bigvee_{t\ R\ x_i} F_A(t) \leq 3$ and so $R(A) = \{(R(x_i), \bigwedge_{t\ R\ x_i} T_A(t), \bigvee_{t\ R\ x_i} I_A(t), \bigvee_{t\ R\ x_i} F_A(t))\}_{i=1}^{k}$ is a single-valued neutrosophic set in X/R. □

Theorem 3.7 *Let $\overline{V} = \{v_1, v_2, \ldots, v_n\}$ and $H = (\overline{V}, \{v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}_{i=1}^{m})$ be a single-valued neutrosophic hypergraph. If R is an equivalence relation on H, then $H/R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{j=1}^{n}\big)$ is a partitioned single-valued neutrosophic hypergraph.*

Proof By Lemma 3.6, $\{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{j=1}^{n}$ is a finite family of single-valued neutrosophic subsets of $\overline{V}/R$. Since $\overline{V} = \bigcup_{i=1}^{m} supp(E_i)$, we get that $\bigcup_{i=1}^{m} supp(R(E_i)) = R(\bigcup_{i=1}^{m} supp(E_i)) = R(\overline{V})$. It follows that $H/R = (R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{j=1}^{n})$ is a single-valued neutrosophic hypergraph. Since R is an equivalence relation on $\overline{V}$, for any $x \neq y \in \overline{V}$ we get that $R(x) \cap R(y) = \emptyset$ and so it is a partitioned single-valued neutrosophic hypergraph. □

Example 3.8 Consider a joint complete single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_i\}_{i=1}^{n})$, where $\overline{V} = \{a_1, a_2, \ldots, a_n\}$ and for any $1 \leq i \leq n$, $E_i = \{(a_i, i/10^n, (i+1)/10^n), (i+2)/10^n)\}$. Clearly $R = \{(a_i, a_i), (a_r, a_s) \mid r + s = n + 1, 1 \leq i \leq n\}$ is an equivalence relation on $\overline{V}$ and so we obtain $\overline{V}/R = \{R(a_1), R(a_2), R(a_3), \ldots, R(a_{(n/2)-1}), R(a_{n/2})\}$. It follows that

$$H/R = \Bigg(\Big\{R(a_1), R(a_2), R(a_3), \ldots, R(a_{(n/2)-1}), R(a_{n/2})\Big\},$$
$$\times\Big\{\big(R(a_i), i/10^n, (i+1)/10^n, (i+2)/10^n\big), \big(R(a_{n-i+1}),$$
$$\times(n-i+1)/10^n, (n-i+2)/10^n, (n-i+3)/10^n\big)\Big\}_{i=1}^{n/2}\Bigg).$$

Computation shows that H/R is a partitioned single-valued neutrosophic hypergraph.

Corollary 3.9 *Let* $\overline{V} = \{v_1, v_2, \ldots, v_n\}$ *and* $H = (\overline{V}, \{v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}_{i=1}^{m})$ *be a single-valued neutrosophic hypergraph. If* R *is an equivalence relation on* H*, then* $H \upharpoonright R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{i=1}^{m}\big)$ *is a single-valued neutrosophic hypergraph, where*

$$T_{R(A)}(R(x)) = \bigwedge_{x\ R\ t \in E_i} T_A(t),\ I_{R(A)}(R(x)) = \bigvee_{x\ R\ t \in E_i} I_A(t)$$

and $F_{R(A)}(R(x)) = \bigvee_{x\ R\ t \in E_i} F_A(t)$.

Example 3.10 Let $H = (\{a, b, c, d\}, E_1, E_2)$ be a single-valued neutrosophic hypergraph in Fig. 1, and $R = \{(x, x), (a, c), (c, a) \mid x \in \{a, b, c, d\}\}$.

Clearly $H \upharpoonright R$ is obtained in Fig. 2a and H/R is obtained in Fig. 2b.

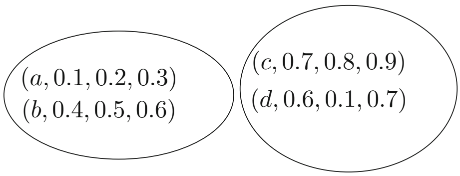
Corollary 3.11 *Let* $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ *be a single-valued neutrosophic hypergraph and* R *be an equivalence relation on* H*. Then*

(i) if $H/R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{j=1}^{t}\big)$ *and* $H \upharpoonright R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{i=1}^{s}\big)$*, then* $m \leq s < t$*;*

(ii) if $R = \bigcup_{i=1}^{m} E_i \times E_i$*, then* $H/R = H \upharpoonright R$*.*

Definition 3.12 Let $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ be a single-valued neutrosophic hypergraph, $0 \leq \alpha, \beta, \gamma \leq 1$ and $E_i^{(\alpha,\beta,\gamma)} = \{x \in \overline{V} \mid T_{E_i}(x) \geq \alpha, I_{E_i}(x) \geq \beta, F_{E_i}(x) \leq$

Fig. 1 Partitioned SVN-HG



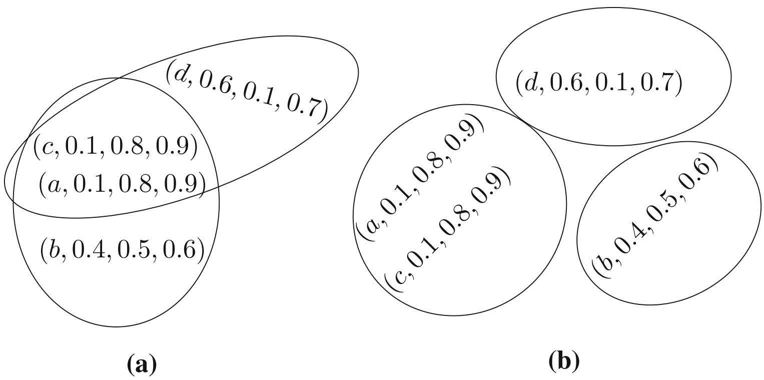


Fig. 2 SVN-HG 2*a* and 2*b*. **a** SVN-HG $H \upharpoonright R$ **b** SVN-HG H/R

$\gamma\}$, where $1 \leq i \leq m$. Then $H^{(\alpha,\beta,\gamma)} = (\overline{V}^{(\alpha,\beta,\gamma)} = \bigcup_{i=1}^{m} E_i^{(\alpha,\beta,\gamma)}, E^{(\alpha,\beta,\gamma)} = \{E_i^{(\alpha,\beta,\gamma)}\}_{i=1}^{m})$ is called an (a strong) (α, β, γ)-level hypergraph if (for any $1 \leq i \leq m$ we have, $E_i^{(\alpha,\beta,\gamma)} \neq \emptyset$) $E^{(\alpha,\beta,\gamma)} \neq \emptyset$.

Proposition 3.13 *Let $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ be a single-valued neutrosophic hypergraph and $0 \leq \alpha, \beta, \gamma \leq 1$. If $H^{(\alpha,\beta,\gamma)} = (V^{(\alpha,\beta,\gamma)}, E^{(\alpha,\beta,\gamma)})$ is a (α, β, γ)-level hypergraph, then it is a single-valued neutrosophic hypergraph.*

Proof Since $H^{(\alpha,\beta,\gamma)}$ is a (α, β, γ)-level hypergraph, for any $1 \leq i \leq m$ we have, $E_i^{(\alpha,\beta,\gamma)} \neq \emptyset$, we obtain $r \leq m$ such that for any $1 \leq i \leq r$, $E_i^{(\alpha,\beta,\gamma)} \neq \emptyset$. Now consider $\overline{V}^{(\alpha,\beta,\gamma)} = \bigcup_{i=1}^{r} E_i^{(\alpha,\beta,\gamma)}$, where for any $1 \leq i \leq r$,

$$E_i^{(\alpha,\beta,\gamma)} = \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}_{i=1}^{r}.$$

Hence $H^{(\alpha,\beta,\gamma)} = (V^{(\alpha,\beta,\gamma)}, \{E_i^{(\alpha,\beta,\gamma)}\}_{i=1}^{r})$ is a single-valued neutrosophic hypergraph. □

Theorem 3.14 *Let $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ be a single-valued neutrosophic hypergraph and $0 \leq \alpha, \beta, \gamma \leq 1$. If R is an equivalence relation on H and $H^{(\alpha,\beta,\gamma)} = (V^{(\alpha,\beta,\gamma)}, E^{(\alpha,\beta,\gamma)})$ is a (α, β, γ)-level hypergraph, then*

- *(i) $H^{(\alpha,\beta,\gamma)}/R = (R(V^{(\alpha,\beta,\gamma)}), R(E^{(\alpha,\beta,\gamma)}))$ is a single-valued neutrosophic hypergraph and is a (α, β, γ)-level hypergraph.*
- *(ii) $H^{(\alpha,\beta,\gamma)} \Rsh R = (R(V^{(\alpha,\beta,\gamma)}), R(E^{(\alpha,\beta,\gamma)}))$ is a single-valued neutrosophic hypergraph and is a (α, β, γ)-level hypergraph.*
- *(iii) $(H/R)^{(\alpha,\beta,\gamma)} = (R(V)^{(\alpha,\beta,\gamma)}, R(E)^{(\alpha,\beta,\gamma)})$ is a single-valued neutrosophic hypergraph.*

Proof $(i), (ii)$ Let $\overline{V} = \{x_1, x_2, \ldots, x_n\}$. Since $E^{(\alpha,\beta,\gamma)}) \neq \emptyset$, then there exists $1 \leq i \leq m$, so that $E_i^{(\alpha,\beta,\gamma)} \neq \emptyset$. Let $x_i \in E_i^{(\alpha,\beta,\gamma)}$, then $R(x_i) \in R(V^{(\alpha,\beta,\gamma)})$ and $R(E_i^{(\alpha,\beta,\gamma)}) \neq \emptyset$ and is a hyperedge. Since R is an equivalence relation and $\bigcup_{i=1}^{n} E_i^{(\alpha,\beta,\gamma)} = V^{(\alpha,\beta,\gamma)}$, we get that $H^{(\alpha,\beta,\gamma)}/R = (R(V^{(\alpha,\beta,\gamma)}), R(E^{(\alpha,\beta,\gamma)}))$ is a hypergraph. □

Example 3.15 Consider the single-valued neutrosophic hypergraph in Fig. 1 and equivalence relation R in Example 3.10. Let $\alpha = 0.1$, $\beta = 0.2$ and $\gamma = 0.8$, then we obtain that $E_1^{(\alpha,\beta,\gamma)} = \{a, b\}$ and $E_2^{(\alpha,\beta,\gamma)} = \{d\}$ and so we have the hypergraph in Fig. 3. Since $\overline{V}^{(\alpha,\beta,\gamma)} = \{a, b, d\}$, we get that $\overline{V}^{(\alpha,\beta,\gamma)}/R = \{\{a, c\}, \{b\}, \{d\}\}$ and so we have the hypergraph in Fig. 4b. Now, consider $\overline{V}/R = \{\{a, c\}, \{b\}, \{d\}\}$, since $(R(E_1))^{(\alpha,\beta,\gamma)} = \{\}$, $(R(E_2))^{(\alpha,\beta,\gamma)} = \{\}$ and $(R(E_3))^{(\alpha,\beta,\gamma)} = \{b\}$, we get the hypergraph $(H/R)^{(\alpha,\beta,\gamma)} = ((\overline{V}/R)^{(\alpha,\beta,\gamma)}, (E/R)^{(\alpha,\beta,\gamma)})$ in Fig. 4a.

Example 3.15, shows that necessarily, $(H/R)^{(\alpha,\beta,\gamma)}$ is not an (α, β, γ)-level hypergraph.

Theorem 3.16 *Let $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ be a single-valued neutrosophic hypergraph. Then there exists $0 \leq \alpha, \beta, \gamma \leq 1$ such that $H^{(\alpha,\beta,\gamma)} = H$ and so $H^{(\alpha,\beta,\gamma)}/R = H/R$.*

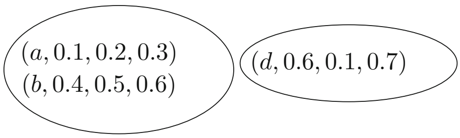


Fig. 3 (α, β, γ)-level hypergraph

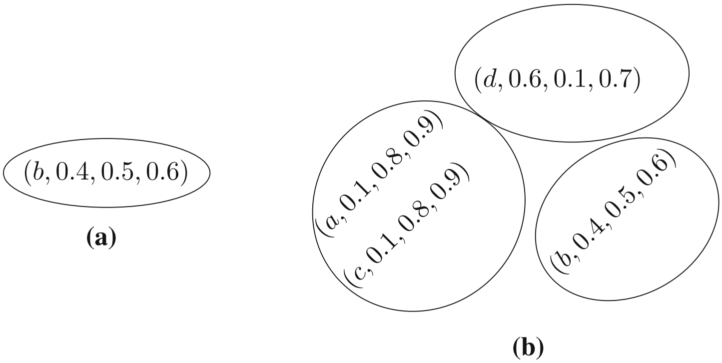


Fig. 4 SVN-HG 4a and 4b. **a** SVN-HG $(H/R)^{(\alpha,\beta,\gamma)}$ **b** SVN-HG $H^{(\alpha,\beta,\gamma)}/R$

Proof Let $\alpha = \bigwedge_{i=1}^{m} T_{E_i}$, $\beta = \bigwedge_{i=1}^{m} I_{E_i}$ and $\gamma = \bigvee_{i=1}^{m} F_{E_i}$. Then for any $1 \leq i \leq m$ and any $x \in E_i$ we get $T_{E_i}(x) \geq (\bigwedge_{i=1}^{m} T_{E_i})(x) = \alpha$. In a similar way we obtain that $I_{E_i}(x) \geq \beta$ and $F_{E_i}(x) \leq \gamma$. Since for any $1 \leq i \leq m$, $E_i^{(\alpha,\beta,\gamma)} = E_i$, we get that $E^{(\alpha,\beta,\gamma)} \neq \emptyset$, $\overline{V}^{(\alpha,\beta,\gamma)} = \bigcup E^{(\alpha,\beta,\gamma)} = \overline{V}$ and so $(\bigcup_{i=1}^{m} E_i^{(\alpha,\beta,\gamma)}, \{E_i^{(\alpha,\beta,\gamma)}\}_{i=1}^{m})$ $= H$.

Corollary 3.17 *Let* $H = (\overline{V}, \{E_i\}_{i=1}^{m})$ *be a single-valued neutrosophic hypergraph. Then there exists* $0 \leq \alpha, \beta, \gamma \leq 1$ *such that* $(H/R)^{(\alpha,\beta,\gamma)} = H^{(\alpha,\beta,\gamma)}/R$.

Example 3.18 Consider the single-valued neutrosophic hypergraph in Fig. 1 and equivalence relation R in Example 3.10. Let $\alpha = 0.1$, $\beta = 0.1$ and $\gamma = 0.9$, then we obtain that $E_1^{(\alpha,\beta,\gamma)} = \{a, b\}$ and $E_2^{(\alpha,\beta,\gamma)} = \{c, d\}$ and so we have the hypergraph in Fig. 1.

Corollary 3.19 *The following statements are hold:*

(i) Any finite set can be a (α, β, γ)*-level (partitioned) hypergraph.*
(ii) Any finite set can be a (α, β, γ)*-level complete (hyper)graph.*

4 Extendable single-valued neutrosophic graphs

In this section, we will define concept of derivable and extendable single-valued neutrosophic graphs and will show that any weak single-valued neutrosophic graph is a derived single-valued neutrosophic graph.

Definition 4.1 (*i*) Let $G = (V, E, A, B)$ be a single-valued neutrosophic graph and $H = (\overline{V}, \{\overline{E_x}\}_{x\in\overline{V}})$ be a single-valued neutrosophic hypergraph. We say that the single-valued neutrosophic graph G is derived from the single-valued neutrosophic hypergraph H if G is isomorphic to a nontrivial quotient of H.($G \cong H/\rho$)

(*ii*) A single-valued neutrosophic graph $G = (V, E, A, B)$ with underlying set V is called an *extendable* single-valued neutrosophic graph, if there exists a single-valued neutrosophic hypergraph $H = (\overline{V}, \{\overline{E_x}\}_{x\in\overline{V}})$ and $n \in \mathbb{N}$ such that $|(V, A, B)| = |(\overline{V}, \{\overline{E_x}\}_{x\in\overline{V}})| - n$, and graph G is derived from hypergraph H. If $V = \overline{V}$ we will say that it is an extended single-valued neutrosophic graph.

Theorem 4.2 *Let $H = (\overline{V}, \{E_i\}_{i=1}^m)$ be a single-valued neutrosophic hypergraph. Then there exists an operation "$*$" on H/ρ such that $(H/\rho, *)$ is a single-valued neutrosophic graph.*

Proof By Theorem 3.7, $H/R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{j=1}^n\big)$ is a partitioned single-valued neutrosophic hypergraph, where

$$T_{R(E_i)}(R(x)) = \bigwedge_{x\ R\ t\in X} T_{E_i}(t),\ I_{R(E_i)}(R(x)) = \bigvee_{x\ R\ t\in X} I_{E_i}(t) \text{ and}$$
$$F_{R(E_i)}(R(x)) = \bigvee_{x\ R\ t\in X} F_{E_i}(t).$$

For any $\rho(x) = \rho((x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)))$ and $\rho(y) = \rho((y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\rho$, define an operation "$*$" on H/ρ by

$$\rho(x) * \rho(x) = \begin{cases} \widehat{\rho(x), \rho(y)} & \text{if } E_x \cap E_y \neq \emptyset, \\ \widehat{\emptyset} & \text{otherwise,} \end{cases}$$

where for any $x, y \in G$, $(\widehat{\rho(x), \rho(y)})$ is represented as an ordinary (simple) edge and $\widehat{\emptyset} = \widehat{\rho(x)}$ means that there is not edge. It is easy to see that

$$H/\rho = \big(\rho(\overline{V}), \{\rho(v_j), T_{\rho(E_i)}(\rho(v_j)), I_{\rho(E_i)}(\rho(v_j)), F_{\rho(E_i)}(\rho(v_j))\}_{j=1}^n, *\big)$$

is a graph. Now, define $\overline{T}_{\rho(E_i)}, \overline{I}_{\rho(E_i)}, \overline{F}_{\rho(E_i)} : \rho(\overline{V}) \times \rho(\overline{V}) \longrightarrow [0, 1]$ by $\overline{T}_{\rho(E_i)}(\rho(x), \rho(y)) = \bigwedge_{a\rho x,\ b\rho y}(T_{\rho(E_i)}(a) \wedge T_{\rho(E_i)}(b)), \overline{I}_{\rho(E_i)}(\rho(x), \rho(y)) = \bigvee_{a\rho x,\ b\rho y}(I_{\rho(E_i)}(a) \vee I_{\rho(E_i)}(b))$ and $\overline{F}_{\rho(E_i)}(\rho(x), \rho(y)) = \bigvee_{a\rho x,\ b\rho y}(F_{\rho(E_i)}(a) \vee F_{\rho(E_i)}(b))$. It is clear to see that $\overline{T}_{\rho(E_i)}(\rho(x), \rho(y)) \leq (T_{\rho(E_i)}(\rho(x)) \wedge T_{\rho(E_i)}(\rho(y)))$, $\overline{I}_{\rho(E_i)}(\rho(x), \rho(y)) \geq (I_{\rho(E_i)}(\rho(x)) \vee I_{\rho(E_i)}(\rho(y)))$ and $\overline{F}_{\rho(E_i)}(\rho(x), \rho(y)) \geq (F_{\rho(E_i)}(\rho(x)) \vee F_{\rho(E_i)}(\rho(y)))$. Hence $H/\rho = \big(\rho(\overline{V}), \{\rho(v_j), T_{\rho(E_i)}(\rho(v_j)), I_{\rho(E_i)}(\rho(v_j)), F_{\rho(E_i)}(\rho(v_j))\}_{j=1}^n, *\big)$ is a single-valued neutrosophic graph. □

Example 4.3 Let $H = (\{a, b, c, d, e, f, g\}, \{E_1, E_2, E_3\})$ be a single-valued neutrosophic hypergraph in Fig. 5.

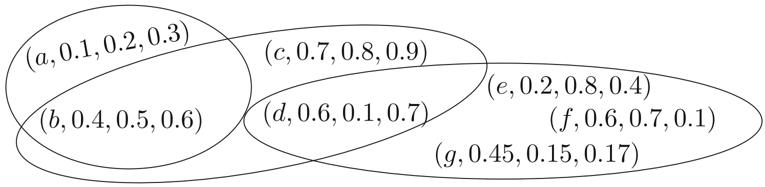


Fig. 5 Partitioned SVN-HG

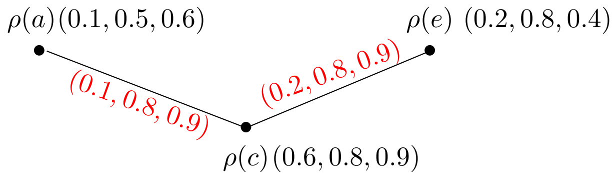


Fig. 6 SVN-G (H/ρ)

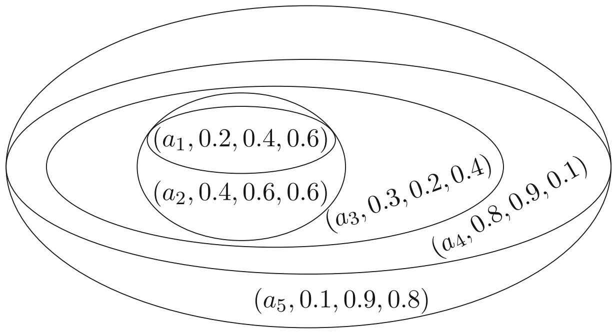


Fig. 7 Joint complete SVN-HG

We have $E_a^m = E_b^m = \{a, b\}$, $E_c^m = E_d^m = \{b, c, d\}$, and $E_e^m = E_f^m = E_g^m = \{d, e, f, g\}$, thus we obtain $\rho(a) = \rho(b) = \{a, b\}$, $\rho(c) = \rho(d) = \{c, d\}$ and $\rho(e) = \rho(f) = \rho(g) = \{e, f, g\}$. It follows that $H/\rho = (\rho(\overline{V}), \rho(\{E_1, E_2, E_3\}), *)$ is the single-valued neutrosophic graph in Fig. 6.

Example 4.4 Let $\overline{V} = \{a_1, a_2, a_3, a_4, a_5\}$. Then consider the joint complete single-valued neutrosophic hypergraph $H = (\overline{V}, E_1, E_2, E_3, E_4, E_5)$ in Fig. 7. We have $E_{a_1}^m = \{a_1\}$, $E_{a_2}^m = \{a_1, a_2\}$, $E_{a_3}^m = \{a_1, a_2, a_3\}$, $E_{a_4}^m = \{a_1, a_2, a_3, a_4\}$ and $E_{a_5}^m = \{a_1, a_2, a_3, a_4, a_5\}$, thus we obtain $\rho(a_1) = \{a_1\}$, $\rho(a_2) = \{a_2\}$, $\rho(a_3) = \{a_3\}$, $\rho(a_4) = \{a_4\}$ and $\rho(a_5) = \{a_5\}$.

It follows that $H/\rho = (\rho(\overline{V}), \{(\rho(a_i), T_{\rho(E_i)}(\rho(a_i)), I_{\rho(E_i)}(\rho(a_i)), F_{\rho(E_i)}(\rho(a_i)))\}_{i=1}^{5}$ where is obtained in Fig. 8.

Theorem 4.5 *Any single-valued neutrosophic graph is a derived single-valued neutrosophic graph if and only if it is a weak single-valued neutrosophic graph.*

Proof Let $G = (V, E, A, B, *')$ be a weak single-valued neutrosophic graph, in such a way that $V = \{a_1, a_2, \ldots, a_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$, where $m \leq n$. Suppose

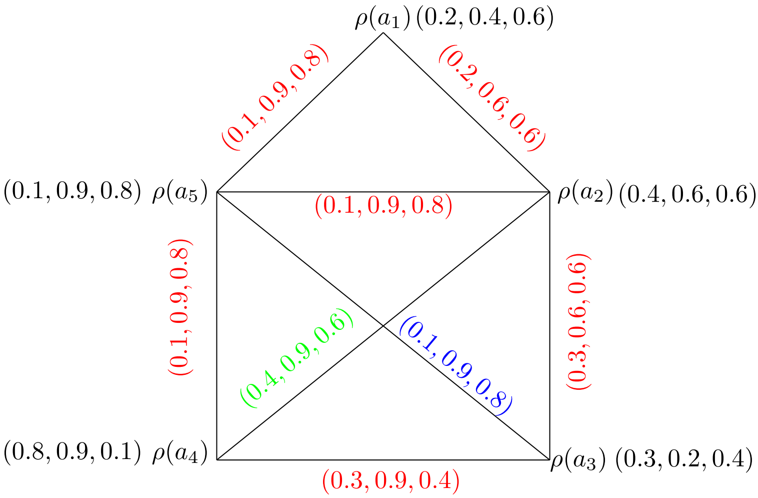


Fig. 8 Derived cycle SVN-G K_4

that for any $1 \leq i \leq m$, $e_i = \{a_i, a_{i'}\} = a_i *' a_{i'}$. Define a single-valued neutrosophic hypergraph $\overline{G} = (\overline{V}, \{\overline{E_i}\}_{i=1}^n)$ as follows:

$$\overline{E_i} = \{(a_i, T_A(a_i), I_A(a_i), F_A(a_i))\} \cup A_i$$

such that for any $1 \leq k \leq n$, we have $|A_k| = k$, $A_k = \{(x, T_A(x), I_A(x), F_A(x)) \mid T_A(x) \neq 0, I_A(x) \neq 0, F_A(x) \neq 0\}$ and for any $1 \leq i, i' \leq n$, $|\overline{E_i}| < |\overline{E_{i+1}}|$ and $\overline{E_i} \cap \overline{E_{i'}} \neq \emptyset$. It is easy to see that $\overline{V} = \bigcup_{i=1}^n A_i \cup V$ and since $G = (V, E, A, B, *')$ is a weak single-valued neutrosophic graph, we get that $H = (\overline{V}, \{\overline{E_i}\}_{i=1}^n)$ is a single-valued neutrosophic hypergraph. Clearly for any $1 \leq i \leq n$, $\rho(a_i) = \overline{E_i}$ and since $\overline{E_i} \cap \overline{E_{i'}} \neq \emptyset$, we get that $H/\rho = \{\rho(a_i) \mid 1 \leq i \leq n\}$ and so obtain

$$\rho(a_i) * \rho(a_j) = \begin{cases} \widehat{\rho(a_i), \rho(a_{i'})} & \text{if } j = i', \\ \widehat{\emptyset} & \text{if } j = i. \end{cases}$$

Now define a map $\varphi : (H/\rho, *) \longrightarrow G = (V, E, A, B, *')$ by $\varphi(\rho(a_i, T_A(a_i), I_A(a_i), F_A(a_i))) = (a_i, T_A(a_i), I_A(a_i), F_A(a_i))$ and $\varphi((\widehat{\rho(a_i), \rho(a_{i'})})) = e_i$. Let $x, y \in \overline{V}$. If $\rho(x) = \rho(y)$, then $|\overline{E_x}| = |\overline{E_y}|$ and so $\overline{E_x} = \overline{E_y}$. Thus $\varphi(\rho(x)) = \varphi(\rho(y))$. Since for any $1 \leq i, i' \leq n$,

$$\varphi(\rho(a_i) * \rho(a_{i'})) = \varphi((\widehat{\rho(a_i), \rho(a_{i'})})) = e_i = a_i *' a_{i'} = \varphi(\rho(a_i)) *' \varphi(\rho(a_{i'})),$$

in other words, if $\rho(a_i)$ and $\rho(a_i')$ in G/ρ are adjacent, then $\varphi(\rho(a_i))$ and $\varphi(\rho(a_i'))$ in G are adjacent. So φ is a homomorphism. It is easy to see that φ is bijection and so is an isomorphism. It follows that by Theorem 4.2, any weak single-valued neutrosophic graph is a derived single-valued neutrosophic graph. By definition of single-valued neutrosophic hypergraph, the converse of theorem is obtained immediately. □

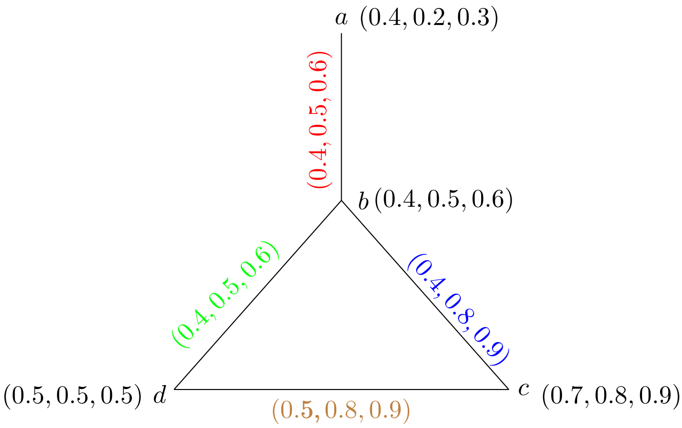


Fig. 9 Extendable SVN-G *G*

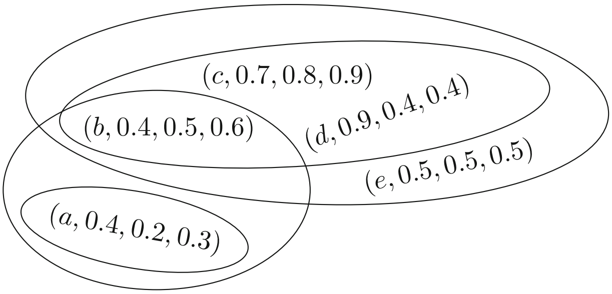


Fig. 10 SVN-HG *H*

Corollary 4.6 *Let* $0 \leq \alpha, \beta, \gamma \leq 1$. *Then any weak single-valued neutrosophic graph can be a derived* (α, β, γ)*-level graph.*

Example 4.7 Consider the single-valued neutrosophic graph (G, A, B) in Fig. 9. Now, consider the single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^4)$, where $\overline{V} = \{a, b, c, d, e\}$, $E_1 = \{(a, 0.4, 0.2, 0.3)\}$, $E_2 = \{(a, 0.4, 0.2, 0.3), (b, 0.4, 0.5, 0.6)\}$, $E_3 = \{(b, 0.4, 0.5, 0.6), (c, 0.7, 0.8, 0.9), (d, 0.4, 0.4, 0.4)\}$ and

$$E_4 = \{(b, 0.4, 0.5, 0.6), (c, 0.7, 0.8, 0.9), (d, 0.4, 0.4, 0.4), (e, 0.5, 0.5, 0.5)\}.$$

Then $(\overline{V}, E_1, E_2, E_3, E_4)$ is a single-valued neutrosophic hypergraph in Fig. 10.

Since $E_a^m = \{(a, 0.4, 0.2, 0.3)\}$, $E_b^m = \{(a, 0.4, 0.2, 0.3), (b, 0.4, 0.5, 0.6)\}$,
$E_c^m = \{(b, 0.4, 0.5, 0.6), (c, 0.7, 0.8, 0.9), (d, 0.9, 0.4, 0.4)\}$and
$E_e^m = \{(b, 0.4, 0.5, 0.6), (c, 0.7, 0.8, 0.9), (d, 0.9, 0.4, 0.4), (e, 0.5, 0.5, 0.5)\}$,

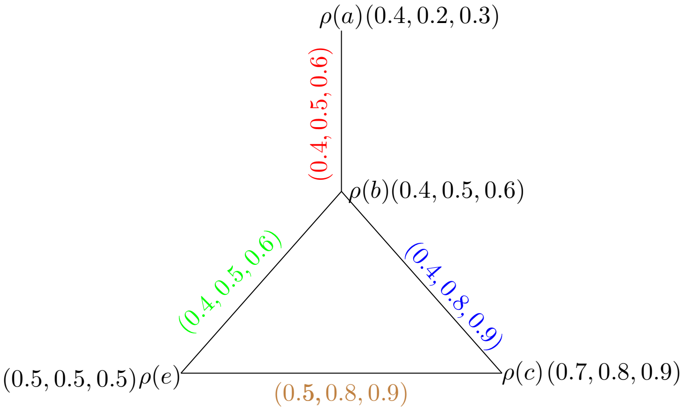


Fig. 11 Derivable SVN-G H/ρ

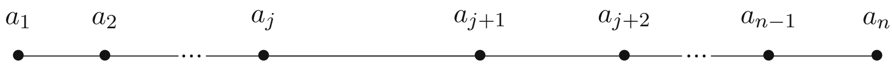


Fig. 12 Linear tree T^l_m

we get that

$$\rho((a,0.4,0.2,0.3)) = \{(a,0.4,0.2,0.3)\}, \rho((b,0.4,0.5,0.6)) = \{(b,0.4,0.5,0.6)\},$$
$$\times\, \rho((c,0.7,0.8,0.9)) = \{(c,0.7,0.8,0.9),(d,0.9,0.4,0.4)\} \text{ and}$$
$$\times\, \rho((e,0.5,0.5,0.5)) = \{(e,0.5,0.5,0.5)\}.$$

So we obtained the single-valued neutrosophic graph in Fig. 11.

Let $V = \{a_1, a_2, \ldots, a_m\}$. Then we denote the linear tree on V in Fig. 12 and denote it by T^l_m.

Theorem 4.8 *Let $m \in \mathbb{N}$. Then linear weak single-valued neutrosophic tree (T^l_m, A, B) is an extendable linear single-valued neutrosophic tree.*

Proof Let $T^l_m = (V, E)$ be a linear single-valued neutrosophic tree where $V = \{a_1, a_2, \ldots, a_m\}$. Now, consider a single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^n)$, where for any $1 \leq i \leq m, |E_i| = i, (a_i, T_A(a_i), I_A(a_i), F_A(a_i)) \in E_i, E_i \cap E_{i+1} = \emptyset, |E_{i+1}| - |E_i| = 1$ and for any $1 \leq j \neq i, i+1 \leq m, E_j \cap E_i = \emptyset$. It is easy to see that $|\overline{V}| > |V|$ and $E^m_{a_i} = \{(a_i, T_A(a_i), I_A(a_i), F_A(a_i)), (x, T_A(x), I_A(x), F_A(x)) \mid x \in \overline{V}\}$ which in $|E^m_{a_i}| = i$. It follows that for any $1 \leq i \leq m, \rho((a_i, T_A(a_i), I_A(a_i), F_A(a_i))) = \{(a_i, T_A(a_i), I_A(a_i), F_A(a_i))\}$ and

$$\rho(a_i) * \rho(a_j) = \begin{cases} \widehat{\rho(a_i), \rho(a_{i'})} & \text{if } i+1 = i', \\ \widehat{\emptyset} & \text{if } i+1 \neq i'. \end{cases}$$

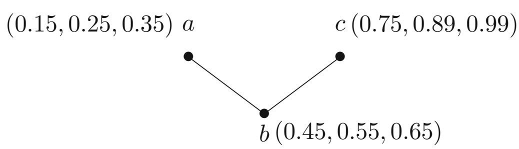


Fig. 13 Linear tree T_3^l

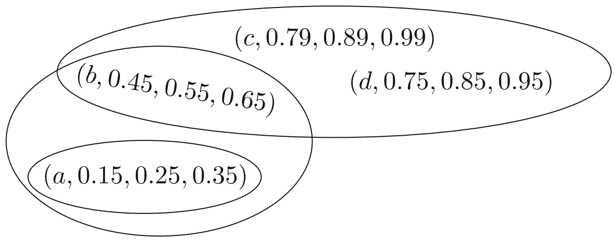


Fig. 14 SVN-HG H

Since $T_m^l = (V, E)$ is a linear single-valued neutrosophic tree, for any $1 \le i, j \le m$ we get that

$$\begin{aligned} T_B(\rho((a_i), \rho((a_i)) &= T_B(\{a_i\}, \{a_j\}) = \{T_B(a_i, a_j)\} \le \{T_A(a_i) \wedge T_A(a_j)\} \\ &= \{T_A(a_i)\} \wedge \{T_A(a_j)\} = T_B(\rho((a_i)) \wedge T_B(\rho((a_j)). \end{aligned}$$

In a similar way can see that $I_B(\rho((a_i), \rho((a_i)) \ge I_B(\rho((a_i)) \vee I_B(\rho((a_j))$ and $F_B(\rho((a_i), \rho((a_i)) \ge F_B(\rho((a_i)) \vee F_B(\rho((a_j))$. Since $(\overline{V}, \{E_j\}_{j=1}^n)/\rho \cong (T_m^l, A, B)$, $(\overline{V}, \{E_j\}_{j=1}^n)/\rho$ is a linear single-valued neutrosophic tree and $|\overline{V}| > |V|$ we get that (T_m^l, A, B) is an extendable linear single-valued neutrosophic tree. □

Example 4.9 Consider the linear single-valued neutrosophic tree (T_3^l, A, B) in Fig. 13.

Now, consider a single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^4)$, where $\overline{V} = \{a, b, c, d, e\}$, $E_1 = \{(a, 0.15, 0.25, 0.35)\}$, $E_2 = \{(a, 0.15, 0.25, 0.35), (b, 0.45, 0.55, 0.65)\}$, $E_3 = \{(b, 0.45, 0.55, 0.65), (c, 0.79, 0.89, 0.99), (d, 0.75, 0.85, 0.95)\}$.

Then $(\overline{V}, E_1, E_2, E_3)$ is the single-valued neutrosophic hypergraph in Fig. 14. Since $E_a^m = \{(a, 0.15, 0.25, 0.35)\}$, $E_b^m = \{(a, 0.15, 0.25, 0.35), (b, 0.45, 0.55, 0.65)\}$ and $E_d^m = \{(b, 0.45, 0.55, 0.65), (c, 0.79, 0.89, 0.99), (d, 0.75, 0.85, 0.95)\}$, we get that $\rho((a, 0.15, 0.25, 0.35)) = \{(a, 0.15, 0.25, 0.35)\}$, $\rho((b, 0.45, 0.55, 0.65)) = \{(b, 0.45, 0.55, 0.65)\}$ and $\rho((d, 0.75, 0.89, 0.99)) = \{(c, 0.75, 0.89, 0.99), (d, 0.75, 0.89, 0.99)\}$. So we obtain the single-valued neutrosophic graph in Fig. 15.

It is clear that $(\overline{V}, \{E_j\}_{j=1}^4)/\rho \cong (T_3^l, A, B)$.

Corollary 4.10 *Linear weak single-valued neutrosophic tree* (T_m^l, A, B) *is an extended if and only if* $m = 1$ *or* $m = 2$.

Theorem 4.11 *Let* $n \in \mathbb{N}$. *Then complete weak single-valued neutrosophic graph* (K_n, A, B) *is an extended complete single-valued neutrosophic graph.*

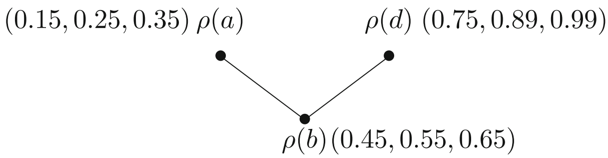


Fig. 15 Derived linear SVN-T T_3^l

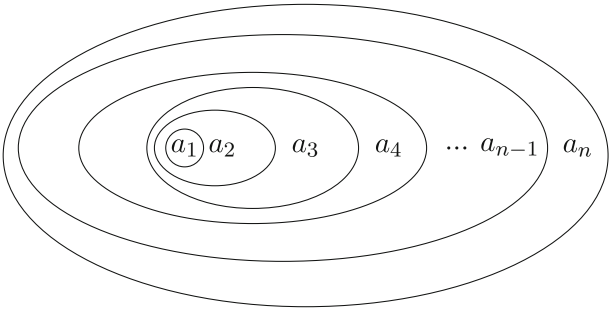


Fig. 16 Joint complete SVN-HG K_n^*

Proof Let $G = \{a_1, a_2, \ldots, a_n\}$ and (K_n, A, B) be a complete single-valued neutrosophic graph. Now, consider the single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^n)$, where $\overline{V} = \{a_1, a_2, \ldots, a_n\}$ and for any $1 \leq i \leq n, E_i = \{(a_1, T_A(a_1), I_A(a_1), F_A(a_1)), \ldots, (a_i, T_A(a_i), I_A(a_i), F_A(a_i))\}$ in Fig. 16.

Since for any $1 \leq i \leq n$, $E_i^m = \{(a_1, T_A(a_1), I_A(a_1), F_A(a_1)), \ldots, (a_i, T_A(a_i), I_A(a_i), F_A(a_i))\}$, we get that for any $1 \leq i \leq n$, $\rho((a_i, T_A(a_i), I_A(a_i), F_A(a_i))) = \{(a_i, T_A(a_i), I_A(a_i), F_A(a_i))\}$ and so for any $1 \leq i \neq j \leq n$, $\rho(a_i)*\rho(a_j) = \rho(\widehat{a_i), \rho(a}_j)$. Since (K_n, A, B) is a complete single-valued neutrosophic graph, for any $1 \leq i, j \leq n$ we get

$$
\begin{aligned}
F_B(\rho((a_i), \rho((a_i)) &= F_B(\{a_i\}, \{a_j\}) = \{F_B(a_i, a_j)\} \geq \{F_A(a_i) \vee F_A(a_j)\} \\
&= \{F_A(a_i)\} \vee \{F_A(a_j)\} = F_B(\rho((a_i)) \vee F_B(\rho((a_j)).
\end{aligned}
$$

In a similar way can see that $T_B(\rho((a_i), \rho((a_i)) \leq T_B(\rho((a_i)) \wedge T_B(\rho((a_j))$ and $I_B(\rho((a_i), \rho((a_i)) \geq I_B(\rho((a_i)) \vee I_B(\rho((a_j))$. Hence $H = (\overline{V}, \{E_j\}_{j=1}^n) \cong (K_n, A, B)$ and since $\overline{V} = n$, have (K_n, A, B) is an extended complete single-valued neutrosophic graph. □

Example 4.12 Consider the complete single-valued neutrosophic graph (K_4, A, B) in Fig. 17. Now, for $G = \{a, b, c, d\}$consider single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^4)$, where

$$E_1 = \{(a, 0.6, 0.8, 0.9)\}, E_2 = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6)\},$$
$$E_3 = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6), (c, 0.5, 0.6, 0.7)\} \text{ and}$$

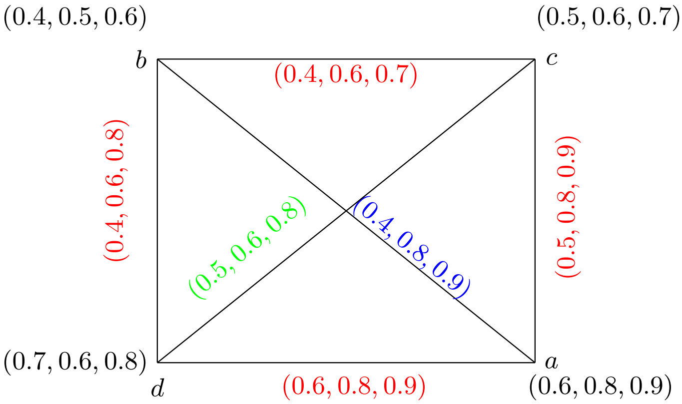
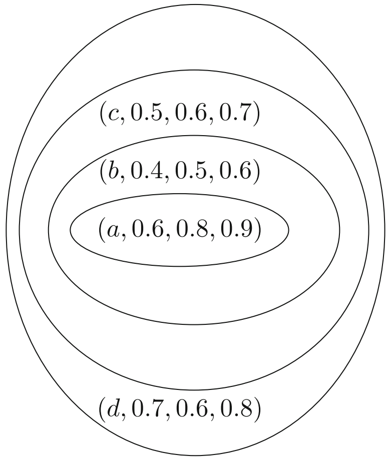


Fig. 17 Cycle SVN-G K_4

Fig. 18 SVN-HG H



$$E_4 = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6), (c, 0.5, 0.6, 0.7), (d, 0.7, 0.6, 0.8)\}.$$

Then (G, E_1, E_2, E_3, E_4) is a hypergraph in Fig. 18.

$$\begin{aligned}&\text{Since } E_a^m = \{(a, 0.6, 0.8, 0.9)\},\ E_b^m = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6)\},\\ &\quad E_c^m = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6), (c, 0.5, 0.6, 0.7)\} \text{ and}\\ &\quad E_d^m = \{(a, 0.6, 0.8, 0.9), (b, 0.4, 0.5, 0.6), (c, 0.5, 0.6, 0.7), (d, 0.7, 0.6, 0.8)\},\\ &\text{we get that} \rho((a, 0.6, 0.8, 0.9)) = \{(a, 0.6, 0.8, 0.9)\},\ \rho((b, 0.4, 0.5, 0.6))\\ &\quad = \{(b, 0.4, 0.5, 0.6)\},\ \rho((c, 0.5, 0.6, 0.7)) = \{(c, 0.5, 0.6, 0.7)\} \text{ and}\\ &\qquad \times \rho((d, 0.7, 0.6, 0.8)) = \{(d, 0.7, 0.6, 0.8)\}.\end{aligned}$$

So we obtain the single-valued neutrosophic graph in Fig. 19. It is clear that $(\overline{V}, \{E_j\}_{j=1}^4)/\rho \cong (K_4, A, B)$.

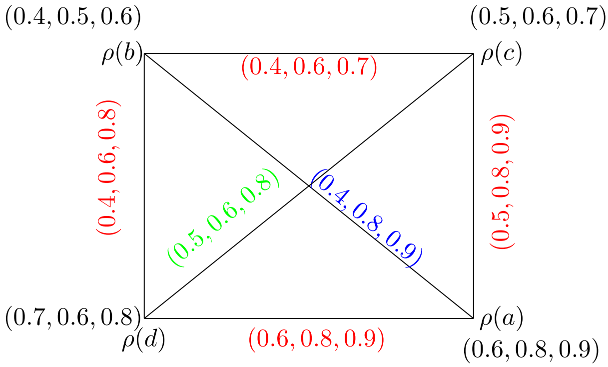


Fig. 19 Derivable complete K_4

Corollary 4.13 *Let (G, A, B) be a weak single-valued neutrosophic graph. (G, A, B) is an extended single-valued neutrosophic graph if and only if is a (G, A, B) be a complete weak single-valued neutrosophic graph.*

Theorem 4.14 *Let $m, n \in \mathbb{N}$. Then complete weak single-valued neutrosophic bigraph $(K_{m,n}, A, B)$ is an extendable complete single-valued neutrosophic bigraph.*

Proof Let $V = V \cup V' = \{a_1, a_2, \ldots, a_m, a'_1, a'_2, \ldots, a'_n\}$ and $(K_{m,n}, V \cup V', A, B)$ be a complete single-valued neutrosophic bigraph. Consider hypergraph $H = (\overline{V}, \{E_i\}_{i=1}^m \cup \{E'_j\}_{j=1}^n)$, where $\overline{V}$ is set of all vertices of hyperedges of H, for any $1 \leq i \leq m, 1 \leq j \leq n, a_i \in E_i, a'_j \in E'_j, |E_{i+1}| = |E_i| + 1, |E'_1| = |E_m| + 1, |E'_{j+1}| = |E'_j| + 1, \bigcap_{i=1}^m E_i = \emptyset, \bigcap_{j=1}^n E'_j = \emptyset$ and for any $1 \leq i \leq m$, $1 \leq j \leq n$, $E_i \cap E'_j \neq \emptyset$. By definition for any $1 \leq i \leq m$, $1 \leq j \leq n$, $E_i^m = E_i$, $E'^m_j = E'_j$, $\rho(a_i) = E_i$ and $\rho(a'_j) = E'_j$. Hence for any $1 \leq i \leq m$ and $1 \leq j \leq n$ we obtain

$$\rho(x) * \rho(y) = \begin{cases} \widehat{\rho(x), \rho(y)} & \text{if } \{x, y\} = \{a_i, a'_j\}, \\ \widehat{\emptyset} & \text{otherwise.} \end{cases}$$

Thus $H = (\overline{V}, \{E_i\}_{i=1}^m \cup \{E'_j\}_{j=1}^n) \cong K_{m,n}$.

Let $\overline{V} = \{(a_i, T_A(a_i), I_A(a_i), F_A(a_i))\}_{i=1}^m \bigcup \{(b_r, T_A(b_r), I_A(b_r), F_A(b_r))\}_{r=1}^l \bigcup \{(a'_j, T_A(a'_j), I_A(a'_j), F_A(a'_j))\}_{j=1}^n \bigcup \{(b'_s, T_A(b'_s), I_A(b'_s), F_A(b'_s))\}_{s=1}^p$, such that for any $1 \leq i \leq m, 1 \leq r \leq l, 1 \leq j \leq n, 1 \leq s \leq p, T_A(a_i) \leq T_A(b_r), I_A(a_i) \geq I_A(b_r), F_A(a_i) \geq F_A(b_r), T_A(a'_j) \leq T_A(b'_s), I_A(a'_j) \geq I_A(b'_s)$ and $F_A(a'_j) \geq F_A(b'_s)$. It is easy to see that for any $1 \leq i \leq m, 1 \leq j \leq n$ we get that,

$$\begin{aligned} &T_{\rho(E_i)}(\rho(a_i)) = T_{E_i}(a_i), I_{\rho(E_i)}(\rho(a_i)) = I_{E_i}(a_i), \\ &\quad F_{\rho(E_i)}(\rho(a_i)) = F_{E_i}(a_i), T_{\rho(E'_j)}(\rho(a'_j)) = T_{E'_j}(a_j), \\ &\quad I_{\rho(E'_j)}(\rho(a'_j)) = I_{E'_j}(a'_j) \text{ and} \end{aligned}$$

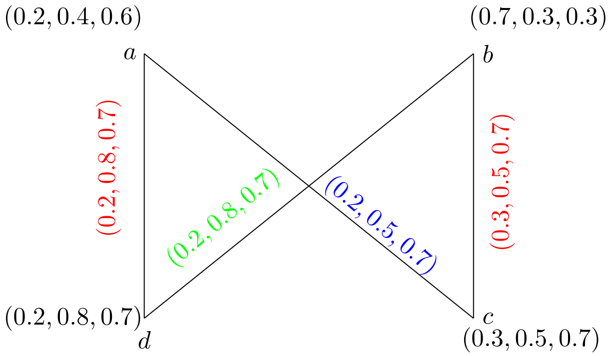


Fig. 20 Cycle SVN-G K_4

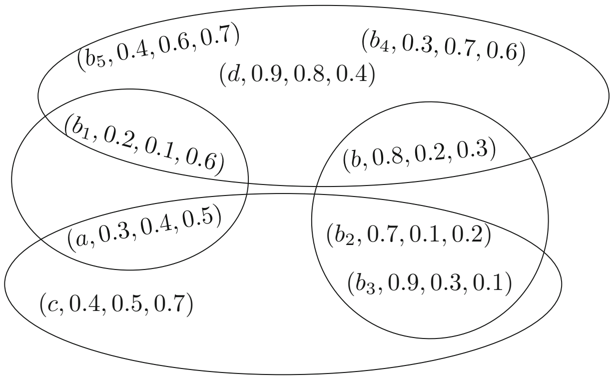


Fig. 21 SVN-HG H

$$F_{\rho(E'_j)}(\rho(a'_j)) = F_{E'_j}(a'_j).$$

Moreover, $\overline{T}_{\rho(E_i)}(\rho(a_i), \rho(a'_j)) \leq T_{E_i}(a_i) \wedge T_{E'_j}(a'_j), \overline{I}_{\rho(E_i)}(\rho(a_i), \rho(a'_j)) \geq I_{E_i}(a_i) \wedge I_{E'_j}(a'_j)$ and $\overline{F}_{\rho(E_i)}(\rho(a_i), \rho(a'_j)) \geq F_{E_i}(a_i) \wedge F_{E'_j}(a'_j)$. It follows that H/ρ is a complete single-valued neutrosophic bigraph. □

Example 4.15 Consider the complete single-valued neutrosophic bigraph $(K_{2,2}, A, B)$ in Fig. 20. Now, consider single-valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_{j=1}^4)$ in Fig. 21. By the following computations,

$$E_a^m = \{(a, 0.3, 0.4, 0.5), (b_1, 0.2, 0.1, 0.6)\},$$
$$E_b^m = \{(b, 0.8, 0.2, 0.3), (b_2, 0.7, 0.1, 0.2), (b_3, 0.9, 0.3, 0.1)\},$$
$$E_c^m = \{(c, 0.4, 0.5, 0.7), (a, 0.3, 0.4, 0.5), (b_2, 0.7, 0.1, 0.2), (b_3, 0.9, 0.3, 0.1)\} \text{ and}$$
$$E_d^m = \{(d, 0.9, 0.8, 0.4), (b_5, 0.4, 0.6, 0.7), (b_4, 0.3, 0.7, 0.6), (b_1, 0.2, 0.1, 0.6), \times(b, 0.8, 0.2, 0.3)\},$$

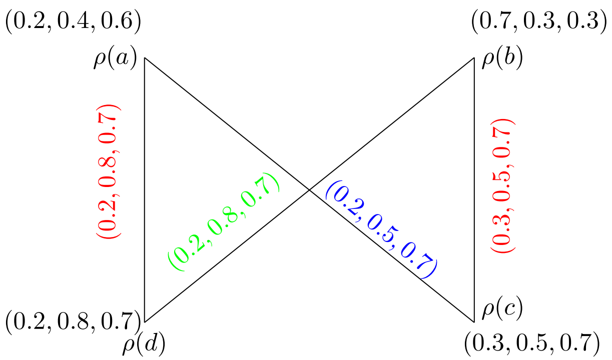


Fig. 22 Derivable complete SVN-G $K_{2,2}$

we get that,

$$\begin{aligned}\rho((a,0.2,0.4,0.6)) &= \{(a,0.3,0.4,0.5),(b_1,0.2,0.1,0.6)\},\\ \rho((b,0.7,0.3,0.3)) &= \{(b,0.8,0.2,0.3),(b_2,0.7,0.1,0.2),(b_3,0.9,0.3,0.1)\},\\ \rho((c,0.3,0.5,0.7)) &= \{(c,0.4,0.5,0.7),(a,0.3,0.4,0.5),(b_2,0.7,0.1,0.2),\\ &\quad\times(b_3,0.9,0.3,0.1)\}\text{and in a similar way,}\\ \rho((d,0.2,0.8,0.7)) &= \{(d,0.9,0.8,0.4),(b_5,0.4,0.6,0.7),(b_4,0.3,0.7,0.6),\\ &\quad\times(b_1,0.2,0.1,0.6),(b,0.8,0.2,0.3)\}.\end{aligned}$$

So we obtain the single-valued neutrosophic graph in Fig. 22. It is clear that $(\overline{V},\{E_j\}_{j=1}^{4})/\rho \cong (K_{2,2},A,B)$.

Corollary 4.16 *Let $(K_{m,n},A,B)$ be a complete weak single-valued neutrosophic bigraph. Then $(K_{m,n},A,B)$ is an extended complete weak single-valued neutrosophic bigraph if and only if $m=n=1$.*

5 An applications of accessible single-valued neutrosophic (hyper)graphs in complex networks

In this section, we describe some applications of accessible single-valued neutrosophic graphs.

The study of complex networks play a main role in the important area of multidisciplinary research involving physics, chemistry, biology, social sciences, and information sciences. These systems are commonly represented by means of simple or directed graphs that consist of sets of nodes representing the objects under investigation, e.g., people or groups of people, molecular entities, computers, etc., joined together in pairs by links if the corresponding nodes are related by some kind of relationship. These networks include the internet, the world wide web, social networks, information networks, neural networks, food webs, and protein–protein interaction networks. In some cases the use of simple or directed graphs to represent complex networks does not provide a complete description of the real-world systems under investigation. For instance, in a collaboration network represented as a simple graph, we only know whether scientists have collaborated or not, but we can not know whether three or more authors linked together in the network were coauthors of the same paper or not. A possible solution to this problem is to represent the collaboration network as a bipartite graph in which a disjoint set of nodes represents papers and another disjoint set represents authors. However, in this case the homogeneity in the definition of nodes is lost, because we have certain nodes that represent papers and others that represent authors. In the study of connectivity, clustering and other topological properties, this distinction between two classes of nodes with completely different interpretations may lead to artifacts in the data.

A natural way of representing these systems is to use the hypergraphs. In the hypergraphs, hyper-edges can relate groups of more than two nodes. Thus, we can represent the collaboration network as a hypergraph in which nodes represent authors and hyper-edges represent the groups of authors that have published papers together. Despite the fact that complex weighted networks have been covered in some detail in the physical literature, there are no reports on the use of hypergraphs to represent complex systems. Consequently, we will formally introduce the hypergraph concept as a generalization for representing complex networks and will call them complex hyper-networks. The hypergraph concept includes, as particular cases, a wide variety of other mathematical structures that are appropriate for the study of complex networks. Since still these representations are unsuccessful to deal with all the competitions of the world, for that purpose SVN-HG are introduced. Now, we discuss applications of SVN-HG to study the competition along with algorithms. The SVN-G have many utilizations in different areas, where by using the especial equivalence relations, we connect SVN-G and SVN-HG. We will first show some examples of complex systems for which hypergraph representation is necessary.

Example 5.1 In social networks nodes represent people or groups of people, normally called actors, that are connected by pairs according to some pattern of contact or interactions between them. Such patterns can be of friendship, collaboration, business relationships, etc. There are some cases in which hypergraph representations of the social network are indispensable. Let $X = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a society and $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ be names of its people. These people create some groups as $E_1 = \{a_1, a_2, a_3\}$, $E_2 = \{a_4, a_3\}$ and $E_1 = \{a_4, a_5, a_6, a_7\}$. Let, the degree of contribution in the business relationships of a_1 is $10/100$, degree of indeterminacy of contribution is $0/100$ and degree of false-contribution is $15/100$, i.e. the truth-membership, indeterminacy-membership and falsity-membership values of the vertex human is $(0.1, 0, 0.15)$. The likeness, indeterminacy and dislikeness of contribution in the business relationships this society is shown in the Table 1.

Consider the social complex network $H = (\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}, \{E_1, E_2, E_3\})$ in Fig. 23. Since

$$E^m_{a_1} = E^m_{a_2} = \{(a_1, 0.1, 0.05, 0.15), (a_2, 0.2, 0.21, 0.25), (a_3, 0.3, 0.05, 0.35)\},$$
$$E^m_{a_3} = E^m_{a_4} = \{(a_3, 0.3, 0.05, 0.35), (a_4, 0.4, 0.42, 0.45)\} \text{ and}$$

Table 1 Likeness, indeterminacy and dislikness of a social network

People	Truth-membership	Indeterminacy-membership	Falsity-membership
a_1	0.1	0.05	0.15
a_2	0.2	0.21	0.25
a_3	0.3	0.05	0.35
a_4	0.4	0.42	0.45
a_5	0.5	0.05	0.55
a_6	0.6	0.62	0.65
a_7	0.7	0.72	0.75

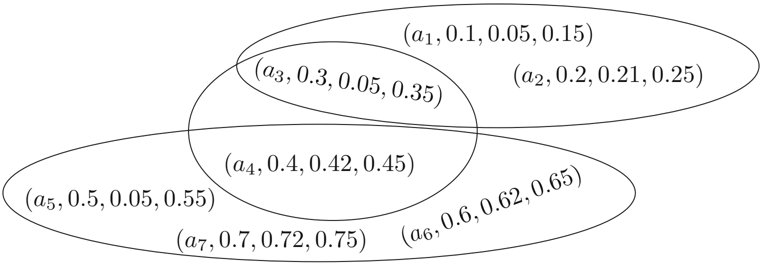


Fig. 23 Social complex network

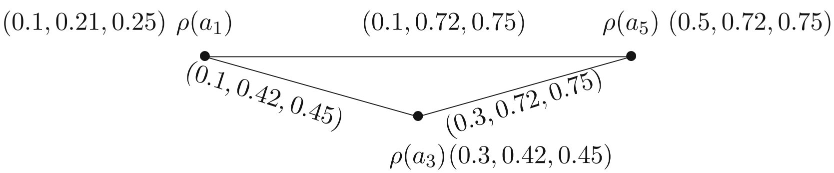


Fig. 24 Social network

$$E^m_{a_5} = E^m_{a_6} = E^m_{a_7} = \{(a_4, 0.4, 0.42, 0.45), (a_5, 0.5, 0.05, 0.55), (a_6, 0.6, 0.62, 0.65), \times(a_7, 0.7, 0.72, 0.75)\},$$

we get that

$$\rho((a_1, 0.1, 0.05, 0.15)) = \{(a_1, 0.1, 0.21, 0.35), (a_2, 0.1, 0.21, 0.35)\},$$
$$\rho(a_3, 0.3, 0.05, 0.35)) = \{(a_3, 0.3, 0.42, 0.45), (a_4, 0.3, 0.42, 0.45)\} \text{ and}$$
$$\rho((a_5, 0.4, 0.72, 0.75)) = \{(a_5, 0.4, 0.72, 0.75), (a_6, 0.4, 0.72, 0.75), \times (a_7, 0.4, 0.72, 0.75)\}.$$

So we obtain the single-valued neutrosophic graph in Fig. 24. By Fig. 24, for society X, we have 3 representatives $\rho(a_1)$, $\rho(a_2)$ and $\rho(a_3)$ where the likeness, indeterminacy and dislikeness of contribution in the business relationships of group of this society is shown in the Table 2.

Table 2 Likeness, indeterminacy and dislikness of SVN-G

Group representative	Group representative	Truth	Indeterminacy	Falsity
$\rho(a_1)$	$\rho(a_5)$	0.1	0.72	0.75
$\rho(a_1)$	$\rho(a_3)$	0.1	0.42	0.45
$\rho(a_3)$	$\rho(a_5)$	0.3	0.72	0.75

Table 3 Likeness, indeterminacy and dislikness of social networks

People	Truth-membership	Indeterminacy-membership	Falsity-membership
a	0.6	0.5	0.4
a'	0.1	0.2	0.3
b	0.2	0.3	0.4
b'	0.3	0.2	0.1
c	0.8	0.9	0.05
d	0.9	0.8	0.7
d'	0.5	0.6	0.7
e	0.4	0.5	0.6
f	0.7	0.8	0.9
g	0.99	0.05	0.9

Example 5.2 Trophic relations in ecological systems are normally represented through the use of food webs, which are oriented graphs (digraphs) whose nodes represent species and links represent trophic relations between species. Another way of representing food webs is by means of competition graphs, which have the same set of nodes as the food web but in which two nodes are connected if, and only if, the corresponding species compete for the same prey in the food web. In the competition graph we can only know if two linked species have some common prey, but we can not know the composition of the whole group of species that compete for common prey. In order to solve this problem a competition hypergraph has been proposed in which nodes represent species in the food web and hyper-edges represent groups of species that compete for common prey. It has been shown that in many cases competition hyper-networks yield a more detailed description of the predation relations among the species in the food web than competition graphs. Let $X = \{a, a', b, b', c, d, d', e, f, g\}$ be a ecological system and $a, a', b, b', c, d, d', e, f, g$ be species. These species create some groups species as $E_1 = \{a, a'\}$, $E_2 = \{a, b', b\}$, $E_3 = \{b, c, d', d\}$ and $E_4 = \{d, g, f, e, a'\}$. Let, the degree of contribution in the business relationships of a is $60/100$, degree of indeterminacy of contribution is $50/100$ and degree of false-contribution is $40/100$, i.e. the truth-membership, indeterminacy-membership and falsity-membership values of the vertex human is $(0.6, 0.5, 0.4)$. The likeness, indeterminacy and dislikeness of contribution in the business relationships this society is shown in the Table 3.

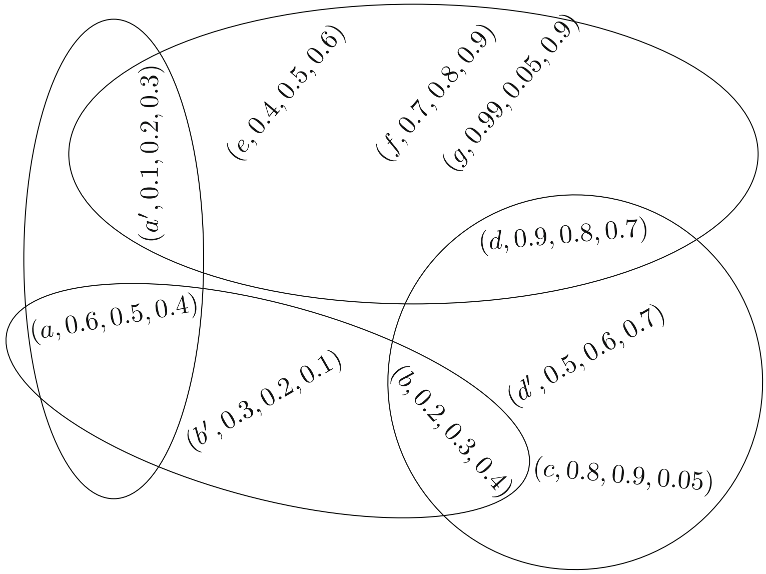


Fig. 25 Food competition hyper-network

Consider the food competition hyper-network is illustrated in Fig. 25. Since

$$\begin{aligned} E_a^m &= E_{a'}^m = \{(a, 0.6, 0.5, 0.4), (a', 0.1, 0.2, 0.3)\},\\ E_b^m &= E_{b'}^m = \{(a, 0.6, 0.5, 0.4), (b, 0.2, 0.3, 0.4), (b', 0.3, 0.2, 0.1)\} \text{ and}\\ E_c^m &= E_d^m = E_{d'}^m = \{(b, 0.2, 0.3, 0.4), (c, 0.8, 0.9, 0.05), (d, 0.9, 0.8, 0.7),\\ &\qquad \times (d', 0.5, 0.6, 0.7)\}, \text{ and}\\ E_e^m &= E_f^m = E_g^m = \{(d, 0.9, 0.8, 0.7), (e, 0.4, 0.5, 0.6), (f, 0.7, 0.8, 0.9),\\ &\qquad \times (g, 0.99, 0.05, 0.9), (a', 0.1, 0.2, 0.3)\}, \end{aligned}$$

we get that

$$\begin{aligned} \rho((a, 0.1, 0.5, 0.4)) &= \{(a, 0.1, 0.5, 0.4), (a', 0.1, 0.5, 0.4)\},\\ \rho(b, 0.2, 0.3, 0.4)) &= \{(b, 0.2, 0.3, 0.4), (b', 0.2, 0.3, 0.4)\}\\ \rho((c, 0.5, 0.9, 0.7)) &= \{(c, 0.5, 0.9, 0.7), (d, 0.5, 0.9, 0.7), (d', 0.5, 0.9, 0.7)\} \text{ and}\\ \rho((e, 0.4, 0.8, 0.9)) &= \{(e, 0.4, 0.8, 0.9), (f, 0.4, 0.8, 0.9), (g, 0.4, 0.8, 0.9)\}. \end{aligned}$$

So we obtain the single-valued neutrosophic graph in Fig. 26. By Fig. 26, for society X, we have 4 representatives $\rho(a)$, $\rho(b)$, $\rho(c)$ and $\rho(e)$ where the likeness, indeterminacy and dislikeness of trophic relations between species of group of this society is shown in the Table 4.

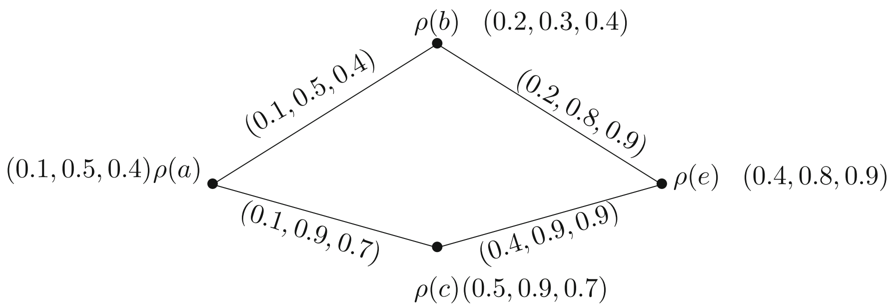


Fig. 26 Food web

Table 4 Likeness, indeterminacy and dislikness of SVN-G

Group representative	Group representative	Truth	Indeterminacy	Falsity
$\rho(a)$	$\rho(b)$	0.1	0.5	0.4
$\rho(a)$	$\rho(c)$	0.1	0.9	0.7
$\rho(b)$	$\rho(e)$	0.2	0.8	0.9
$\rho(c)$	$\rho(e)$	0.4	0.9	0.9

6 Conclusion

The current paper has considered the notion of single-valued neutrosophic hypergraph, single-valued neutrosophic graph and by introducing weak single-valued neutrosophic graph, we have established a relation between them. Also:

(i) Any weak single-valued neutrosophic graph is a derived single-valued neutrosophic graph.
(ii) Every linear weak single-valued neutrosophic tree (T_m^l, A, B) is an extendable linear single-valued neutrosophic tree.
(iii) All complete weak single-valued neutrosophic graphs (K_n, A, B) are extended complete single-valued neutrosophic graphs.
(iv) Any complete weak single-valued neutrosophic bigraph $(K_{m,n}, A, B)$ is an extendable complete single-valued neutrosophic bigraph.
(v) The concept of intuitionistic neutrosophic sets provides an additional possibility to represent imprecise, uncertainty, inconsistent and incomplete information which exists in real situations. In this research paper, we have described the concept of single-valued neutrosophic graphs. We have also presented applications of single-valued neutrosophic hypergraphs and single-valued neutrosophic graphs in food webs and social networks.

We hope that these results are helpful for further studies in graph theory. In our future studies, we hope to obtain more results regarding graphs, hypergraphs and their applications.

Compliance with ethical standards

Conflict of interest Authors declare that they have no conflict of interest.

Human and animal rights This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Akram, M., Davvaz, B.: Strong intuitionistic fuzzy graphs. Filomat **26**(1), 177–196 (2012)
2. Akram, M., Dudek, W.A.: Intuitionistic fuzzy hypergraphs with applications. Inf. Sci. **218**, 182–193 (2013)
3. Akram, M., Shahzadi, S., Saeid, A.B.: Single–valued neutrosophic hypergraph, TWMS J. App. Eng. Math. (to appear)
4. Alkhazaleh, S., Salleh, A.R., Hassan, N.: Neutrosophic soft set. Adv. Decis. Sci. (2011)
5. Broumi, S., Talea, M., Bakkali, A., Samarandache, F.: Single valued neutrosophic graph. J. New Theory **10**, 86–101 (2016)
6. Berge, C.: Graphs and Hypergraphs. North-Holland Publishing Company, London (1973)
7. Devadoss, A.V., Rajkumar, A., Praveena, N.J.P.: A study on miracles through holy bible using neutrosophic cognitive maps (NCMS), Int. J. Comput. Appl. **69**(3), 22–27 (2013)
8. Hamidi, M., Saeid, A.B.: Creating and computing graphs from hypergraphs, (submitted)
9. Hamidi, M., Saeid, A.B.: On derivable trees. (submitted)
10. Kauffmann, A.: Introduction a la theorie des sous–ensembles flous. Masson, Paris (1977)
11. Maji, P.K.: Neutrosophic soft set. Ann. Fuzzy Math. Inform. **5**, 157–168 (2013)
12. Gani, A.Nagoor, Latha, S.R.: On irregular fuzzy graphs. Appl. Math. Sci. **6**(11), 517–523 (2012)
13. Gani, A.N., Begum, S.S.: Degree, order and size in intuitionistic fuzzy graphs. Int. J. Algorithms Comput Math. **3**(3), 11–16 (2010)
14. Smarandache, F.: Neutrosophic set—a generalisation of the intuitionistic fuzzy sets. Int. J. Pure Appl. Math. **24**, 287–297 (2005)
15. Smarandache, F.: Refined literal indeterminacy and the multiplication law of sub-indeterminacies. Neutrosophic Sets Syst. **9**, 58–63 (2015)
16. Smarandache, F.: Types of neutrosophic graphs and neutrosophic algebraic structures together with their applications in technology. Seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania (06 June 2015)
17. Kandasamy, W.B.V, Ilanthenral, K., Smarandache, F.: Neutrosophic graphs: a new dimension to graph theory, Europo Nova, Belgium. (2015)
18. Wang, H., Smarandache, F., Zhang, Y., Sunderraman, R.: Single valued neutrosophic sets. Multisspace Multistruct. **4**, 410–413 (2010)