A novel trapezoidal intuitionistic fuzzy information axiom approach: An application to multicriteria landfill site selection

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Abstract

Information axiom is used to determine the best option among the potential alternatives. In case of uncertain parameters, information axiom has been extended to its ordinary fuzzy versions in the literature. In this paper, we developed a trapezoidal intuitionistic fuzzy information axiom model in order to handle the hesitancy of experts. The developed model aggregates the judgments of more than one expert and proposes a new defuzzification method for trapezoidal intuitionistic fuzzy sets. Our model was applied to a multicriteria landfill site selection problem and a sensitivity analysis was conducted to check the robustness of the given decisions.

1. Introduction

A landfill site is defined as a site for the disposal of waste materials by using various methods. Landfill sites are required to dispose of waste materials that it is impossible or hard to be reused or recycled. Although governments spend time and money for developing new technologies in order to reduce the amount of waste materials, there is still being huge quantity of waste materials coming from both household and commercial sector. Therefore, determining the best site for landfill facility is a critical issue in urban planning process since it includes vital impact on the economy, ecology, and the environmental health of the region. The main purpose in the landfill site selection process is to determine the best location that minimizes hazards to the environment and public health (Uyan, 2014). The determining of an optimum location for landfill among potential alternative locations is a multiple criteria decision-making problem including both quantitative and qualitative criteria. Since the measurements of qualitative criteria include imprecise or vague, the conventional approaches to facility location problem tend to be less effective in dealing with these impreciseness or vagueness (Kahraman et al., 2003).

Ordinary fuzzy set theory proposed by Zadeh (1965) has been used in order to cope with vagueness or uncertainty in human thinking process. However, it is hard to define an exact membership function by using ordinary fuzzy sets when there is some hesitancy of experts. Therefore, the membership and non-membership functions of a fuzzy set are simultaneously defined in an intuitionistic fuzzy set (IFS). IFS requires sum of degrees of membership and non-membership values of
any element to be at most equal to 1. IFSs are the excellent extension of fuzzy sets that can consider the experts’ hesitancy and vagueness together. IFSs have been later extended to Pythagorean fuzzy sets in order to provide a larger range for membership degrees (Yager, 2013). Intuitionistic and Pythagorean fuzzy judgments assigned by different experts can be aggregated by various aggregation functions (Garg, 2016, 2017a, b; Garg and Arora, 2017; Garg et al., 2017).

The contribution of the proposed multicriteria information axiom method is the captured uncertainty and hesitancy in decision-making process through intuitionistic fuzzy sets. The incorporation of experts’ hesitancies into the decision model is the main research area of the recently developed fuzzy set extensions such as intuitionistic fuzzy sets, type-2 fuzzy sets, hesitant fuzzy sets, Pythagorean fuzzy sets, etc. Information axiom measures the information content which indicates the best design with the highest probability of success. It compares the design range and system range and selects the best alternative as the one having the most overlapped ranges. The proposed method is different from the other approaches since decision-making is based on the comparisons of these ranges.

In the literature, there are several ordinary FIA works that try to capture the vagueness and impreciseness in the definition of system and design requirements. However, these works cannot address the hesitancy of experts. The motivation of our study is to develop intuitionistic trapezoidal FIA in order to consider this hesitancy. It is the first time that FIA is extended to intuitionistic trapezoidal fuzzy sets in this paper. For this purpose, new definitions and formulas for FIA will be presented. In addition, an original area based defuzzification method has been proposed to the literature. In the scope of this paper, landfill site selection problem will be solved by using the proposed trapezoidal intuitionistic fuzzy information axiom.

The rest of this paper is organized as follows. Section 2 presents a literature review for multicriteria landfill site selection. Section 3 gives the evaluation criteria for landfill site selection problem. Section 4 presents the basics of fuzzy information axiom method. Section 5 provides intuitionistic sets and Section 6 proposes intuitionistic fuzzy information axiom. Section 7 presents the solution of a landfill site selection problem and a sensitivity analysis. Finally, the concluding remarks are given in Section 8.

### 2. Literature review: Multicriteria landfill site selection

The waste management process is a vital problem for urban planning and it includes various steps which are collection, transport, processing, recycling or disposing, and monitoring. The effectiveness of this process is based on the selection of the most suitable landfill site. An inappropriate decision on landfill site selection has negative impacts on environmental, economical, and social factors (Moghaddas and Namaghi, 2011; Nazari et al. 2012). The solution of this problem is hard, complex, and time-consuming process because of the evaluation of various conflicting criteria. Therefore, during the last decade, the problem has attracted a lot of research works and academic studies. The list of the most common used criteria in the literature is given in Table 1. According to the table, the most preferred criteria are Distance to surface water resources (DSW), Distance to roads (DR), Distance to residential areas (DRA), Distanceland use (LU), and Slope of the land (SL), respectively. The usage percentages of the evaluation criteria are presented in Fig. 1.

Table 2 presents the methods used for the evaluation of landfill site selection problem. In the literature, different models and methods are used in order to determine appropriate site for landfill. An integrated method combining analytic hierarchy process (AHP) and geographic information system (GIS) is the most preferred method in the literature. Additionally, some other site selection methods such as genetic algorithms (Lee et al., 2010), regression based methods (Bilginol et al., 2015), spatial multi-criteria analysis (van Haaren and Fthenakis, 2011), agent based models (Sirikijpanichkul et al., 2007), and fuzzy based models (Nazari et al., 2012) can be used for landfill site selection. Fig. 2 presents the usage percentages of evaluation methods in the literature.

### 3. Evaluation criteria for landfill site selection

In the literature, there are lots of criteria that have been considered for the solution problem. In the scope of this study, the criteria affecting the selection of a landfill site are classified into three main groups. These groups are Environmental factors, Social factors and Economical factors. Definition of these criteria and their design ranges are as follows:
Table 1
Evaluation criteria for landfill site selection.

<table>
<thead>
<tr>
<th>Environmental factors</th>
<th>Social factors</th>
<th>Economic factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSW</td>
<td>DRA</td>
<td>SL</td>
</tr>
<tr>
<td>GWD</td>
<td>DIA</td>
<td>DR</td>
</tr>
<tr>
<td>PS</td>
<td>WD</td>
<td>DI</td>
</tr>
<tr>
<td>LU</td>
<td>DTA</td>
<td>LP</td>
</tr>
<tr>
<td>L</td>
<td>PDA</td>
<td>IC</td>
</tr>
<tr>
<td>H</td>
<td>DRW</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>DA</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>DIA</td>
<td></td>
</tr>
<tr>
<td>DF</td>
<td>DPA</td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>DRW</td>
<td></td>
</tr>
<tr>
<td>DFA</td>
<td>DRA</td>
<td></td>
</tr>
</tbody>
</table>

- Distance to surface water resources (DSW), Ground water depth (GWD), Permeability of the soil (PS), Land use (LU), Lithology (L), Hydrology (H), Geology (G), Topography (T), Distance to fault (DF), Landslide (LS), Distance to flood plain area (DFA), Distance to residential areas (DRA), Distance to industrial areas (DIA), Wind directions (WD), Distance to historical/tourist areas (DTA), Distance to protected area (DPA), Distance to railway (DRW), Distance to airports (DA), Slope of the land (SL), Distance to roads (DR), Distance to infrastructure (power lines, water pipe lines) (DI), Land prices (LP), Initial costs (Setup Costs) (IC)
### Tools and Methods used for landfill site selection problem.

<table>
<thead>
<tr>
<th>Tools/Methods</th>
<th>GIS</th>
<th>Other techniques</th>
<th>Fuzzy sets</th>
<th>Application location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahrani et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td>✔</td>
<td>Shabestar, Iran</td>
</tr>
<tr>
<td>Djokanovic et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td>✔</td>
<td>Pančevo, Serbia</td>
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<tr>
<td>Rahmat et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td>✔</td>
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<tr>
<td>Torabi-Kaveh et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td>✔</td>
<td>Iranshahr, Iran</td>
</tr>
<tr>
<td>El Magziri et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td>✔</td>
<td>Mohammadia, Morocco</td>
</tr>
<tr>
<td>Chubuk et al., 2016</td>
<td>✔</td>
<td>AHP</td>
<td></td>
<td>Al-Hillah Qadhaa</td>
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<tr>
<td>Hanine et al., 2016</td>
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<td>AHP</td>
<td></td>
<td>Casablanca, Morocco</td>
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<tr>
<td>Eskandari et al., 2015</td>
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<td>SAW</td>
<td></td>
<td>Marvdasht, Iran</td>
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<tr>
<td>El Baba et al., 2015</td>
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<td>AHP</td>
<td></td>
<td>Gaza Strip</td>
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<tr>
<td>Arkoc, 2014</td>
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<td></td>
<td></td>
<td>Corinth</td>
</tr>
<tr>
<td>Uyan, 2014</td>
<td>✔</td>
<td>AHP</td>
<td></td>
<td>Konya</td>
</tr>
<tr>
<td>Shahabi et al., 2014</td>
<td>✔</td>
<td>Weighted linear combination Boolean logic</td>
<td></td>
<td>Saqqez</td>
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<td>Kumar and Mohammad, 2013</td>
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<td>AHP</td>
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<td>Isalou et al., 2013</td>
<td></td>
<td></td>
<td></td>
<td>Mahshahr</td>
</tr>
<tr>
<td>Alavi et al., 2013</td>
<td></td>
<td>Delphi</td>
<td></td>
<td>Polog Region, Macedonia</td>
</tr>
<tr>
<td>Donevska et al., 2012</td>
<td>✔</td>
<td>AHP</td>
<td></td>
<td>Serbian</td>
</tr>
<tr>
<td>Vasiljević et al., 2012</td>
<td></td>
<td>AHP</td>
<td></td>
<td>Polog Region, Macedonia</td>
</tr>
<tr>
<td>Gorsevski et al., 2012</td>
<td>✔</td>
<td>OWA</td>
<td></td>
<td>Trabzon</td>
</tr>
<tr>
<td>Vildirim, 2012</td>
<td>✔</td>
<td>AHP</td>
<td></td>
<td>Trabzon</td>
</tr>
<tr>
<td>Moghadass and Namaghi, 2011</td>
<td></td>
<td></td>
<td></td>
<td>Khorasan Razavi</td>
</tr>
<tr>
<td>Sener et al., 2010</td>
<td></td>
<td>AHP</td>
<td></td>
<td>Beysehir</td>
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<td>Nas et al., 2010</td>
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<td>AHP</td>
<td></td>
<td>Konya</td>
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<td>Ersoy and Bulut, 2009</td>
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<td>AHP</td>
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<td>Trabzon</td>
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<td>Guiqin et al., 2009</td>
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<td>AHP</td>
<td></td>
<td>Beijing</td>
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<td>Ramjeawon and Beerachee, 2008</td>
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<td>AHP</td>
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<td>Mauritius</td>
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<td>Sener et al., 2010</td>
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<td>SAW</td>
<td></td>
<td>Ankara</td>
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<tr>
<td>Molo et al., 2006</td>
<td>✔</td>
<td>AHP</td>
<td></td>
<td>Cacheoim de Itapemirim</td>
</tr>
</tbody>
</table>

### Environmental factors

The first group factors are defined as Distance to surface water resources (DSW), Ground water depth (GWD), Permeability of the overlaying layer (POL), Land cover (LNC), Land use (LU), and Natural disaster potential (NDP). These factors are benefit criteria such that the biggest score represents the best or most suitable value.

#### 3.1. Distance to surface water resources (DSW)

Since landfills create noxious gases and leachate, a landfill should be located far away from surface water resources. Table 3 presents the distances between landfills and surface waters. The minimum value means the worst case such that landfill site cannot be located to an area closer than minimum value. Maximum value means the best case that the site takes the best performance score.

#### 3.1.2. Ground water depth (GWD)

This criterion represents that a landfill site should be located in an area where ground water is sufficiently deep so that the ground-water quality is not affected (Kumar and Hassan, 2013). The distances to the groundwater used in the literature are given in Table 4.

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**Fig. 2.** Usage percentages of evaluation methods in the literature.
(Arkoc, 2014). However, some researchers (Yal and Akgun, 2013, Afzali et al. 2014) considered the land use factor into two categories: suitable and unsuitable. The areas such as moors, forests, irrigated fields, gardens and pastures are considered as unsuitable whereas the dry fields and the abandoned land areas are assumed to be suitable.

3.1.5. Land use

Land use represents how people use a landscape. Alavi et al. (2013) divided this factor into residential, agricultural, industrial, and unused lands whereas Gorsevski et al. (2012) classified it as water, forest, barren, and agriculture. Isalou et al. (2013) preferred agriculture and garden criteria as land use factors.

3.1.6. Natural disaster potential (NDP)

This criterion represents potential of disaster such as distance to fault (DF), distance to flood plain area (DFA), landslide (LS) or erosion potential. Among these disasters, the most considered one in the literature is distance to fault factor. The magnitudes of distances used in the literature are given in Table 5. Yal and Akgun (2013) divided distance to fault factor into the following categories as high, moderate, and low or no landslide.

Table 5

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Min</th>
<th>Max</th>
<th># of category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Afzali et al., 2014</td>
<td>m</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yal and Akgun, 2013</td>
<td>m</td>
<td>100</td>
<td>5000</td>
<td>5</td>
</tr>
<tr>
<td>Gorsevski et al., 2012</td>
<td>m</td>
<td>500</td>
<td>1000</td>
<td>3</td>
</tr>
<tr>
<td>Ersoy and Bulut, 2009</td>
<td>m</td>
<td>60</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

3.2. Social factors

In this section, social factors consist of Distance to residential areas (DRA), Distance to industrial areas (DIA), Wind directions (WD), Distance to protected area (DPA), and Distance to transportation centers (Railways, Airports, Stations) (DTC).

3.2.1. Distance to residential areas (DRA)

Placing the landfill site close to a residential area has a negative effect since it adversely affects public health, land value, and future developments of cities (Nas et al., 2010). The distances used for this criterion in the literature are given in Table 6.

Table 6

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Min</th>
<th>Max</th>
<th># of category</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>Uyan, 2014</td>
<td>m</td>
<td>1000</td>
<td>4000</td>
<td>5</td>
</tr>
<tr>
<td>Yal and Akgun, 2013</td>
<td>m</td>
<td>500</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>Isalou et al., 2013</td>
<td>m</td>
<td>300</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>Alavi et al., 2013</td>
<td>m</td>
<td>2000</td>
<td>15000</td>
<td>5</td>
</tr>
<tr>
<td>Vasiljević et al., 2012</td>
<td>m</td>
<td>500</td>
<td>2500</td>
<td>5</td>
</tr>
<tr>
<td>Nas et al., 2010</td>
<td>m</td>
<td>1000</td>
<td>10000</td>
<td>11</td>
</tr>
<tr>
<td>Ersoy and Bulut, 2009</td>
<td>m</td>
<td>1000</td>
<td>10000</td>
<td>4</td>
</tr>
</tbody>
</table>

3.2.2. Wind directions (WD)

This criterion avoids site selection downwind otherwise there would be a possibility to transfer wastes toward the community (Isalou et al., 2013).

3.2.3. Distance to historical/tourist areas (DTA)

A landfill site should not be placed on a site close to historic/cultural sites since prevailing wind directions may negatively affect tourism activities. A buffer zone for this criterion is defined between 1000 and 4000 m with five categories by Uyan (2014) while it is up to 2000 m in Ramjeawon and Beerachee (2008).

3.2.5. Distance to transportation centers (Railways, Airports, Stations) (DTC)

Landfill sites should be selected far from transportation centers. For instance, landfill sites close to airports may cause bird hazards (Alavi et al., 2013). Distances to transportation centers in the literature are given in Table 8.

3.3. Economical factors

Economical factors are divided into two categories as Setup Factors and Operational Factors. Setup factors consists of initial costs (IC), slope of the land (SL), and land prices (LP) while operational factors include distance to roads (DR), distance to infrastructure (DI), accessibility, distance to waste production centers (WPC), and landfill capacity.

3.3.1. Setup factors

Initial costs (IC): This criterion includes the assessment of the number of landowners involved in land acquisition (Ramjeawon and Beerachee, 2008).

Slope of the land (SL): Since the slope of the land may lead to higher excavation costs, it is considered as a cost factor by researchers. In other words, steep slopes increase setup costs. Land slopes used in the literature are given in Table 9.

Land prices (LP): Land price is another cost factor that may lead to higher setup costs. Wang et al. (2009) consider this criterion in five categories in terms of suitability of the land for landfill site.

3.3.2. Operational factors

Distance to roads (DR): Proximity to roads makes the sites less attractive because of additional costs. Therefore, the suitability ranking decreases as you move away from the main roads (Yal and Akgun, 2014;
Uyan, 2014). Table 10 presents the distances to main roads in the literature.

**Distance to infrastructure (DI):** This criterion includes structures such as power lines, water pipe lines, telecommunications, etc. Availability of this structure near landfill site is a requirement (Ramjeawon and Beerachee, 2008; Vasiljević et al., 2012). However, Uyan (2014) considers it as a cost criterion. In his study, landfill site is selected with a certain distance from infrastructure systems in order to prevent damage to systems. Sener et al. (2006) consider this criterion by dividing it into two categories: suitable and unsuitable areas.

**Accessibility:** This criterion presents traffic movements on the access roads leading to the site (Ramjeawon and Beerachee, 2008). Heavy traffic increases the transportation costs.

**Distance to waste production centers (WPC):** This criterion is an important factor in terms of operational costs since landfill sites near to the waste production centers lead to decreasing transportation costs (Wang et al., 2009). Wang et al. (2009) define distances in five categories in a range between 500 and 2000 m.

**Landfill capacity:** The site should serve for at least 15 years (Ramjeawon and Beerachee, 2008) and it should provide opportunities for expansion.

### 4. Information Axiom

Information Axiom (IA) is the second axiom of Axiomatic Design (AD) methodology which is proposed by Suh (1990, 2001). The method consists of two axioms. The first one is known as Independence Axiom. Based on these axioms, the objective of the AD is to compose a new design and/or to improve the existing design in a systematic basis to design processes by providing a thinking process to create a new design and/or to improve the existing design (Suh, 2005). The first axiom Independence Axiom requires maintaining independence of functional requirements (FRs) which are related to main characteristics of a design. The second axiom requires maintaining minimum information content of a design. The relation between independence axiom and information axiom is that information axiom is used to select the best alternative having the minimum information content among the alternatives which satisfy independence axiom (Suh, 1990). The information axiom is symbolized by the information content (I) that is related to the probability of satisfying the design goals representing by FRs. Eq. (1) is used to the information content of a design (Suh, 1990, 2001)

\[ I_i = \log_2 \frac{1}{p_i} \]  

(1)

where \( p_i \) is the probability of success for a given FR with DP. The probability of success \( p_i \) is calculated by the intersection of a system range and a design range defined by designers. The intersection area between system range and design range \( (dr) \) is named common area \( (A_{dr}) \) or common range. Fig. 3 shows how to calculate the probability of a design goal for a single FR. Design range gives the upper and lower values of the design goal and the system probability density function (system pdf) indicates the distribution of the system. Then, the \( p_i \) is calculated as follows;

\[ p_i = \int_{dr} p_i (FR_i) \, dFR_i \]  

(2)

where \( p_i (FR_i) \) is the density function.

If there are more than one FR, information content is calculated as follows (Suh, 1990);

\[ I_{system} = \log_2 \left( \prod_{i=1}^{m} p_i \right) \]  

(3)

\[ I_{system} = \log_2 \left( \sum_{i=1}^{m} \log_2 \left( \frac{1}{p_i} \right) \right) \]  

(4)

\[ I_{system} = -\sum_{i=1}^{m} \log_2 p_i = \sum_{i=1}^{m} \log_2 \left( \frac{1}{p_i} \right) \]  

(5)

IA has been extended under fuzzy environment by Kulak and Kahraman (2005a, b) and Kahraman and Cebi (2009) proposed new definitions and formulations for the method in order to extent the applications of the method under fuzziness. These definitions and formulations have been used for the solution of various types of axiomatic design problems under fuzzy environment. They used fuzzy numbers to depict the design goal and properties of the alternatives. Eq. (6) is used to calculate the information content under fuzzy environment. Fig. 4 illustrates the information content calculation procedure with triangular fuzzy numbers. Both system and design ranges consist of triangular fuzzy numbers. So, information content is calculated by Eq. (7) for the given design in Fig. 4.

\[ I = \log_2 \left( \frac{Area \, of \, System \, Range}{Common \, Area} \right) \]  

(6)

\[ I = \log_2 \left( \frac{\gamma - \alpha}{\mu_1 (\gamma - \beta)} \right) \]  

(7)

In Fig. 5, both system and design ranges are given with trapezoidal fuzzy numbers. So, the information content value for the given figure is calculated by Eq. (8).

\[ I = \log_2 \left( \frac{\delta - \alpha + \gamma - \tau}{\delta - \beta + \gamma - \zeta} \right) \]  

(8)
In the general case of \( m \) FRs, the total information content for the entire system is calculated by Eq. (9).

\[
I_i = \sum_{j=1}^{m} I_j \tag{9}
\]

where \( i \) refers to \( i \)th alternative.

5. Intuitionistic fuzzy sets

Intuitionistic fuzzy sets have been developed by Atanassov (1986). They have been widely used in the literature (Chang et al., 2008; Yager, 2009; Cevik Onar et al., 2015; Kahraman et al., 2015a, b, 2016a, b). This extension of fuzzy sets (Atanassov, 1986, 1999) considers the membership value as well as the non-membership value to describe any \( x \) in \( X \) such that the sum of membership and non-membership is at most equal to 1.

**Definition 1.** Let \( X \neq \emptyset \) be a given set. An intuitionistic fuzzy set in \( X \) is an object \( A \) given by

\[
\tilde{A} = \left\{ \left( x, \mu_A(x), \nu_A(x) \right) : x \in X \right\},
\]

where \( \mu_A : X \rightarrow [0, 1] \) and \( \nu_A : X \rightarrow [0, 1] \) satisfy the condition

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1,
\]

for every \( x \in X \). Hesitancy is equal to “\( 1 - (\mu_A(x) + \nu_A(x)) \)”. 

**Definition 2.** Trapezoidal Intuitionistic Fuzzy Numbers (TrIFNs): A TrIFN \( \tilde{A} \) is an intuitionistic fuzzy subset in \( \mathbb{R} \) with membership function and non-membership function as follows

\[
\mu_A(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\
1, & \text{for } a_2 \leq x \leq a_3 \\
\frac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
\nu_A(x) = \begin{cases} 
\frac{a_2 - x}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\
0, & \text{for } a_2 \leq x \leq a_3 \\
\frac{x - a_1}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4 \\
1, & \text{otherwise}
\end{cases}
\]

where \( a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a'_4 \), \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) and TrIFN is denoted by \( \tilde{A}_{TIFN} = \left( a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4 \right) \) (see Fig. 6).

\[
\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)
\]

**Definition 3.** Let \( \tilde{A} \) and \( \tilde{B} \) be two Atanassov’s IFSs in the set \( X \). The intersection of \( \tilde{A} \) and \( \tilde{B} \) is defined as in Eq. (14) (Bora and Neog, 2012). It is illustrated in Fig. 7.

\[
\tilde{A} \cap \tilde{B} = \left\{ x, \min \left( \mu_A(x), \mu_B(x) \right), \max \left( \nu_A(x), \nu_B(x) \right) \right\} : x \in X \right\}.
\]

Let \( \tilde{A}_{TIFN} = \left( a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4 \right) \) and \( \tilde{B}_{TIFN} = \left( b_1, b_2, b_3, b_4, b'_1, b'_2, b'_3, b'_4 \right) \). Assume that the magnitudes of these parameters are as in Fig. 7. In this figure, \( \mu_{\tilde{A}}(x) \) and \( \mu_{\tilde{B}}(x) \) represent the membership...
functions of the fuzzy sets $\tilde{A}$ and $\tilde{B}$, respectively whereas $\nu_{\tilde{A}}(x_i)$ and $\nu_{\tilde{B}}(x_i)$ are the non-membership functions of $\tilde{A}$ and $\tilde{B}$, respectively. The intersection, $\tilde{A}_{TIFN} \cap \tilde{B}_{TIFN}$ is a Trapezoidal Intuitionistic Fuzzy Number (TrIFN) denoted by $C_{TIFN}$ = $\left( c_1, c_2, c_3, c_4, e_1, e_2, e_3, e_4 \right)$. This intersection operation is illustrated in Fig. 7. The red colored line represents the intersection of membership functions of $\tilde{A}$ and $\tilde{B}$, which is denoted by $C_{TIFN}$.

5.1. Aggregation operators for TrIFNs

Aggregation of preferences is an important problem in decision making problems. Hesitancy of different experts can be considered by aggregating intuitionistic fuzzy information. Zhang and Liu (2010) proposed an aggregation method for trapezoidal intuitionistic fuzzy numbers as follows;

Suppose $I_i = \left( [a_i^T, a_i^{M1}, a_i^{M2}, a_i^U], [b_i^T, b_i^{M1}, b_i^{M2}, b_i^U] \right)$ ($i = 1, 2, ..., n$) is a set of TrIFNs, then the result is a TrIFN aggregated by Eq. (15), and

$$f_m(I_1, I_2, ..., I_n) = \left\{ \begin{array}{l}
1 - \prod_{i=1}^{n} (1 - \mu_{a_i}^{w_i}), 1 - \prod_{i=1}^{n} (1 - \mu_{b_i}^{w_i}), \\
1 - \prod_{i=1}^{n} (1 - \mu_{a_i}^{w_i} - \mu_{b_i}^{w_i}), 1 - \prod_{i=1}^{n} (1 - \mu_{a_i}^{w_i} - \mu_{b_i}^{w_i})
\end{array} \right\}. $$

(15)

where, $w = \left( w_1, w_2, ..., w_n \right)^T$ is the weight vector of $I_i$ ($i = 1, 2, ..., n$), $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$.

Eq. (15) is transformed to Eq. (16) given in Box I, since the widespread of non-membership function must be larger than that of membership function and the midpoints of these functions must be equal to each other. These two conditions provide that the sum of membership and non-membership degrees for any $x$ is at most equal to 1.

5.2. A new proposal for the defuzzification of TrIFNs

A TrIFN can be represented by taking the inverse of a non-membership value as in Fig. 8 (Kahraman et al., 2017) (Kahraman et al., 2017). The defuzzification of a TrIFN can be calculated as in Eq. (17):

$$\text{Defuzzified } \tilde{A} = \frac{(a_3 + a_4 - a_1 - a_2) + (a_1 + a_4 - a_1 - a_2)}{4}. $$

(17)

6. Proposed intuitionistic fuzzy information axiom

In this paper, new definitions for intuitionistic fuzzy information axiom have been proposed to the literature. In Fig. 9, the intuitionistic fuzzy common area of system range ($\tilde{S}R$) and design range ($\tilde{D}R$) is illustrated. The defuzzified system range (DSR) is obtained by using Eq. (18):

$$\text{DSR} = \frac{(a'_1 + a'_4 - a'_1 - a'_2) + (a'_1 + a'_4 - a'_1 - a'_2)}{4}. $$

(18)

The common area can be obtained by the average of the common areas covered by membership functions and complement of non-membership functions of both system range ($\tilde{S}R$) and design range ($\tilde{D}R$). This is the average of gray and blue diagonal areas. The defuzzified common area can be obtained by Eq. (19).

$$\text{DCA} = \frac{\text{Area}(\tilde{S}R \cap \tilde{D}R) + \text{Area}(\tilde{NS}R \cap \tilde{N}D\tilde{R})}{2}. $$

(19)

When the common area is a TrIFN as in Fig. 10, Eq. (20) calculates the defuzzified common area (DCA) by considering the heights of the membership and non-membership functions in the defuzzification process.

$$\text{DCA} = \frac{\mu_1 \times (\mu'_1 + \mu'_4 - \mu'_1 - \mu'_2) + \mu_4 \times (\mu'_1 + \mu'_4 - \mu'_1 - \mu'_2)}{4} $$

(20)

The common area may also be a triangular IFN as in Fig. 10.

The defuzzified value (DCA) of triangular intuitionistic fuzzy common areas of system range ($\tilde{S}R$) and design range ($\tilde{D}R$) can be calculated by using Eq. (21).

$$\text{DCA} = \frac{\mu_1 \times (\mu'_1 + \mu'_4 - \mu'_1 - \mu'_2) + \mu_4 \times (\mu'_1 - \mu'_1)}{4} $$

(21)

Let us assume that there is a decision matrix which consists of $m$ alternatives and $n$ criteria. To obtain the total information content $I_i^T$ of any design, Eq. (22) is used.

$$I_i^T = \sum_{j=1}^{n} I_{ij}, j = 1, 2, ..., m \text{ and } i = 1, 2, ..., n. $$

(22)

If the criteria in the problem have different importance weights, the total weighted information content $I_i^{wT}$ is given by Eq. (23).

$$I_i^{wT} = \sum_{j=1}^{n} I_{ij} w_j, j = 1, 2, ..., n. $$

(23)

The performance of the proposed IFIA model is examined through the sample calculations given in Table 11. Table 11 shows the calculations of eight alternative system ranges for a given intuitionistic fuzzy design range (0.6, 0.7, 0.8, 0.9; 0.5, 0.7, 0.8, 1). In order to find the information content of each alternative, DSR value is first calculated. For Alternative 1, the DSR is calculated using Eq. (18):

$$(0.6 + 0.8 - 0.5 - 0.3) + (0.6 + 0.7 - 0.5 - 0.4) = 0.5. $$

In order to calculate the DCA value, the intersection of design range and alternative system range should be determined. The intersection
IA and ordinary fuzzy IA models. When the size of the information content becomes smaller, which is wholly compatible with the classical model, the common area between $\Delta R$ and $\bar{D}R$ gets larger, the information content becomes smaller, which is wholly compatible with the classical IA and ordinary fuzzy IA models. The common area between $\Delta R$ and $\bar{D}R$ gets larger, the information content becomes smaller, which is wholly compatible with the classical IA and ordinary fuzzy IA models.

\[ f_\omega = \left( \begin{array}{c} \max \left( 1 - \prod_{i=1}^{n} (1 - a_i^{(M)})^w, \prod_{i=1}^{n} (b_i^M)^w \right) \times \left( 1 - \prod_{i=1}^{n} (1 - a_i^{(L)})^w \right) \\
\min \left( 1 - \prod_{i=1}^{n} (1 - a_i^{(M)})^w, \prod_{i=1}^{n} (b_i^L)^w \right) \times \left( 1 - \prod_{i=1}^{n} (1 - a_i^{(L)})^w \right) \\
\sqrt{\prod_{i=1}^{n} (1 - a_i^{(M)})^w, \prod_{i=1}^{n} (b_i^M)^w} \end{array} \right) \]

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $I_i (i = 1, 2, \ldots, n), \omega_i \in [0, 1]$. $\sum_{i=1}^{n} \omega_i = 1$.

![Box I.](https://example.com/box1.png)

Fig. 11. Waste management facilities of Istanbul Metropolitan Municipality (URL1).

Table 11 Performance of the proposed model.

<table>
<thead>
<tr>
<th>Design range: (0.6, 0.7, 0.8, 0.9, 0.5, 0.7, 0.8, 1) Alternative system ranges</th>
<th>DSR</th>
<th>DCA</th>
<th>DSR/DCA</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.4, 0.5, 0.6, 0.7, 0.3, 0.5, 0.6, 0.8)</td>
<td>0.500</td>
<td>0.069</td>
<td>7.273</td>
<td>2.862</td>
</tr>
<tr>
<td>(0.45, 0.55, 0.65, 0.75, 0.35, 0.55, 0.65, 0.85)</td>
<td>0.500</td>
<td>0.105</td>
<td>4.776</td>
<td>2.256</td>
</tr>
<tr>
<td>(0.5, 0.6, 0.7, 0.8, 0.4, 0.6, 0.7, 0.9)</td>
<td>0.500</td>
<td>0.150</td>
<td>3.333</td>
<td>1.737</td>
</tr>
<tr>
<td>(0.55, 0.65, 0.75, 0.85, 0.45, 0.65, 0.75, 0.95)</td>
<td>0.500</td>
<td>0.200</td>
<td>2.500</td>
<td>1.322</td>
</tr>
<tr>
<td>(0.6, 0.7, 0.8, 0.9, 0.5, 0.7, 0.8, 1)</td>
<td>0.500</td>
<td>0.250</td>
<td>2.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(0.65, 0.75, 0.85, 0.95, 0.55, 0.75, 0.85, 1)</td>
<td>0.475</td>
<td>0.200</td>
<td>2.375</td>
<td>1.248</td>
</tr>
<tr>
<td>(0.7, 0.8, 0.9, 1, 0.6, 0.8, 0.9, 1)</td>
<td>0.450</td>
<td>0.150</td>
<td>3.000</td>
<td>1.585</td>
</tr>
<tr>
<td>(0.75, 0.85, 0.95, 1.0, 0.65, 0.85, 0.95, 1)</td>
<td>0.400</td>
<td>0.105</td>
<td>3.821</td>
<td>1.934</td>
</tr>
<tr>
<td>(0.8, 0.9, 1, 0.7, 0.9, 1, 1)</td>
<td>0.350</td>
<td>0.069</td>
<td>5.091</td>
<td>2.348</td>
</tr>
</tbody>
</table>

7. Application: Landfill site selection in Istanbul

7.1. Problem definition

Istanbul is the most crowded city in Turkey with the population of almost 15 million and it is the cultural, economic, and historic center of the country. Furthermore, it is the largest city in Europe and it is a kind of bridge connecting Asia and Europe. The geographic coordinates of Istanbul, Turkey are latitude 41°00′49″ N and longitude 28°56′58″ E with 39 m elevation above sea level. The city is administered by the Istanbul Metropolitan Municipality. In the city, solid wastes are
collected by the district municipalities every day and wastes are brought to the solid waste transfer stations located at various points of Istanbul. Then, these solid wastes are transported to sanitary landfills. Fig. 11 presents the solid waste facilities of the city.

An average of 16,500 tons of solid waste are disposed per day. For this purpose, Istanbul Metropolitan Municipality has three sanitary landfills including Kemerburgaz-Odayeri, Silivri-Seymen and Şile-Kömürçüoda. The capacities of these sites are 2.33×10^6 m^3, 2.64×10^6 m^3, and 2.27×10^6 m^3, respectively. In the near future, the capacities of these sanitary landfills will be fulfilled and new landfills are required to be determined. Hence, the main objective of this study is to determine a new sanitary landfill for Istanbul. The possible alternatives have been determined as Gümüşpinar (A1), Dağyenice (A2), and Cifultan (A3) since they have the suitable large areas for landfill. The locations of these alternatives are illustrated in Fig. 12.

7.2. Problem solution

The criteria used in the evaluation are given in Table 12. The criteria are divided into four categories; “Environmental factors”, “Social factors”, “Setup factors”, and “Operational factors”. Environmental factors involves “Distances to surface water resources”, “Land use”, and “Natural disaster potential”. “Residential areas”, “Wind direction”, and “Protected areas” are included under social factors. Setup factors consist of “Initial cost” and “Land price” while “Distance to roads”, “Distance to recycling plants”, “Landfill capacity” and “Distance to waste transfer stations” are taken into consideration under operational factors. The criterion “Distances to surface water resources” is obtained by measuring Euclidean distance of the location alternatives to surface water resources (Fig. 13). The landfill site should be far away from water resource to prevent surface water pollution. The criterion “Land use” involves land uses of the location alternatives such as agricultural areas, residential areas, tourism areas, industrial areas etc. Under this criterion, the current potential of the location are evaluated. Unused lands are considered as the most suitable while residential and tourism lands are considered as unsuitable. The Euclidean distance to fault line is used for the criterion “Natural disaster potential”. The landfill site should be far away from fault line to prevent increases on natural disaster. The criterion “Residential areas” presents the effects of the locations to residential areas. The landfill site should be far away from residential areas to prevent negative effects of landfills on residential population such as dust, noise, odor etc. The criterion “Wind direction” involves the wind effects of the location alternatives on the residential areas since winds carry dust and odor of the site to residential area. In general, the dominant wind for Istanbul is the northeast and the second dominant wind direction is southwest (URL2). The relationship among location of the landfills, residential area and wind direction are considered under this criterion. The criterion “protected areas” provides the distance to natural, ecological and/or cultural values. The criterion “Initial costs” includes design and construction costs such as new road construction, infrastructure costs, improvements to existing facilities etc. The criterion “Land price” presents the cost of purchasing facility site. The criterion “Distance to roads” presents Euclidean distance to highway. Since distances to main roads increase operation costs it is an important criterion. Hence, farther distance is considered as the worst grade. The distance to recycling plants given in Fig. 13 is considered by the criterion “Distance to recycling plants”. The recycling plants are presented in Fig. 11. When the criterion distance to roads is considered, the shorter distances to recycling plants are desired. The criterion “Landfill capacity” includes current capacity of the site and potential expansion of the site. The distance to the waste transfer stations which are the processing sites for the temporary deposition is considered under the criterion “Distance to waste transfer stations”. As the other distances, this criterion also affects the operation costs. The locations of solid waste transfer stations are given in Fig. 11. There are eight waste transfer stations; Silivri, Şile, Halkali, Aydını, Halkalı Yenibosna, Kavçukbakalköy, Baruthane, and Hekimbaş. The distances of any potential alternative to the waste transfer station are obtained by the fuzzy arithmetic mean of the shortest distances of the two transfer locations.

The data types are presented in Table 12. They are composed of trapezoidal fuzzy numbers for tangible criteria and linguistic subjective judgments for intangible criteria. Three experts assigned their linguistic evaluations for the intangible criteria.

The criteria that involve distances are considered as either tangible or intangible depending on the following rule. If there is only one area (such as residential area or surface water) under consideration, we...
define its distance to the landfill location as a fuzzy number (Tangible). If there is more than one area with various population densities, the distances cannot be easily measured with respect to the landfill location. Under such circumstances, the appropriateness of the distance to the residential areas is defined by a linguistic term.

The three experts mentioned in the study represent the point of views of their organizations. The expert 1 represents the point of view of the specialists from the local government. The expert 2 represents the point of view of academicians studying on the subject. Finally, the expert 3 represents the point of view of the non-profit organizations.

The collected data are given in Tables 13 and 14, respectively.

As an illustrative calculation, let us show the information content of A1 for FR11 criterion. The design range is \((2, 5, 23, 25; 1, 5, 23, 26)\) while the system range is \((20, 21, 23, 24; 20, 21, 23, 24)\). The common area is obtained as \((20, 21, 23, 24; 20, 21, 23, 24)\) which is the same as the system range. As a result, the ratio of the common area to the system area becomes 1, and the information content is \(\log(1) = 0\).

In a similar way, the information contents of the alternatives are calculated for each criterion. In order to find the overall information content of an alternative, the weighted sum of the criteria information contents are summed up assuming that the weights of the criteria are equal. For example, total information content of Alternative 1 is calculated as:

\[
\text{Total } I = 0.083 \times (0 + 0.306 + 0 + \cdots + 5.466) = 1.020.
\]
Additionally, we consulted an environmental expert for his evaluation. Calculations indicate that unless the criterion weight for DR44 is not negatively, A3 is insensitively the best alternative. A1 becomes the best DR43. Even an increase in the weight of DR41 affects all alternatives DR22. As the weight of DR13 increases, a2 gets the superiority over A1. This is also true for DR21 and weights of DR11 and DR12 increase, A3 is always the selected alternative.

Consider the evaluations for C31 by the three experts: (0.5, 0.6, 0.7, 0.8; 0.4, 0.6, 0.7, 0.9), (0.7, 0.8, 0.9, 1; 0.7, 0.8, 0.9, 1), and (0.3, 0.4, 0.5, 0.6; 0.2, 0.4, 0.5, 0.7). Let the aggregated value be \(A_{31} = \left( u_{1}^{T}, u_{1}^{M}, a_{1}^{M}, a_{1}^{U}, b_{1}^{U}, b_{1}^{M}, b_{1}^{L}, b_{1}^{L} \right) \), then the following results are obtained:

\[
\begin{align*}
    a_{1}^{M} &= \max \left( \left( 1 - 0.5 \right)^{33} \times \left( 1 - 0.7 \right)^{33} \times \left( 1 - 0.3 \right)^{33} \times \left( 1 - 0.8 \right)^{33} \right) \times \left( 0.4 \times 33 \times 0.7 \times 33 \times 0.2 \right) = 0.528 \\
    b_{1}^{L} &= \min \left( \left( 1 - 0.5^{33} \right) \times \left( 1 - 0.7^{33} \right) \times \left( 1 - 0.3^{33} \right) \times \left( 1 - 0.8^{33} \right) \right) \times \left( 0.4^{33} \times 0.7^{33} \times 0.2^{33} \right) = 0.383 \\
    a_{1}^{M} &= \sqrt{1 - \left( 1 - 0.6^{33} \right) \times \left( 1 - 0.8^{33} \right) \times \left( 1 - 0.4^{33} \right) \times \left( 0.6^{33} \times 0.8^{33} \times 0.4^{33} \right) \times 0.606} \\
    a_{1}^{M} &= \sqrt{\left( 1 - 0.6^{33} \right) \times \left( 1 - 0.8^{33} \right) \times \left( 1 - 0.4^{33} \right) \times \left( 0.6^{33} \times 0.8^{33} \times 0.4^{33} \right) \times 0.606} \\
    a_{1}^{U} &= \min \left( \left( 1 - 0.8^{33} \right) \times \left( 1 - 0.6^{33} \right) \times \left( 0.8^{33} \times 0.6^{33} \times 0.4^{33} \right) \times 0.716 \\
    b_{1}^{U} &= \max \left( \left( 1 - 0.5^{33} \right) \times \left( 1 - 0.7^{33} \right) \times \left( 1 - 0.3^{33} \right) \times \left( 1 - 0.8^{33} \right) \right) \times \left( 0.4^{33} \times 0.7^{33} \times 0.2^{33} \right) = 1
\end{align*}
\]

7.3. Sensitivity analysis

Sensitivity analysis is conducted to reveal the robustness of the given decisions. The weights of the criteria are changed and the rankings of alternatives are observed. The results of sensitivity analysis are given in Fig. 14.

From the sensitivity graphs in Fig. 14, we conclude that as the weights of DR11 and DR12 increase, A3 is always the selected alternative with an increasing superiority. This is also true for DR21 and DR22. As the weight of DR13 increases, a2 gets the superiority over A1. Increases in the weight of DR23 affect A3 negatively and cause A2 to be selected. Similar to DR13, important DRs for A2 are DR31, DR32, and DR43. Even an increase in the weight of DR41 affects all alternatives negatively, A3 is insensitively the best alternative. A1 becomes the best alternative when the weight of DR42 is larger than 0.25 or the weight of DR44 is larger than 0.20.

All the calculations above are based on equal criteria weights. These calculations indicate that unless the criterion weight for DR44 is not larger than about 0.15, Alternative A1 is always the selected alternative. Additionally, we consulted an environmental expert for his evaluation on the importance degrees of the considered criteria. Based on the environmental expert’s evaluation, the Information Contents in Table 19 have been obtained. These results indicate that the ranking is still the same as the previous analyses.

7.4. Comparative analysis and discussion

In this sub-section we compare our study with Kahraman and Cebi (2009)’s study to show the advantages. In Kahraman and Cebi (2009)’s study, ordinary fuzzy sets have been used to capture the vagueness in the design parameters, ignoring the possible hesitancies of experts about the values of design parameters. The present study handles both the membership and nonmembership degrees assigned by the experts that the sum of these degrees does not necessarily have to be equal to 1.0. The comparison with ordinary FIA presents the difference of the present work with Kahraman and Cebi (2009)’s study.

Table 20 presents the results obtained from ordinary fuzzy information axiom approach. Based on these results, A1 is selected as the best alternative and the rank of the alternatives are same in the both approaches. However, there are differences among the numerical values obtained from ordinary FIA and trapezoidal intuitionistic FIA approaches.

The comparative analysis by using equal weights for FRs shows that the information contents of 16 evaluations are the same in both

<table>
<thead>
<tr>
<th>C21</th>
<th>C22</th>
<th>C23</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>A1</td>
<td>(0.5, 0.6, 0.7, 0.8; 0.4, 0.6, 0.7, 0.9)</td>
</tr>
<tr>
<td>C22</td>
<td>A2</td>
<td>(0.7, 0.8, 0.9, 1; 0.7, 0.8, 0.9, 1)</td>
</tr>
<tr>
<td>C23</td>
<td>A3</td>
<td>(0.3, 0.4, 0.5, 0.6, 0.2, 0.4, 0.5, 0.7)</td>
</tr>
</tbody>
</table>

Table 16: TriFNs for intangible criteria.
Table 17
Design ranges and the aggregated preferences for the alternatives.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Evaluations</th>
<th>Evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR11</td>
<td>FR (2.5,23,25;1,5,23,26)</td>
<td>CR31 (0.6,0.75,0.95,1;0.55,0.75,0.95,1)</td>
</tr>
<tr>
<td>A1</td>
<td>(20,21,23,24;20,21,23,24)</td>
<td>(0.528,0.606,0.716,0.857;0.383,0.606,0.716,0.857)</td>
</tr>
<tr>
<td>A2</td>
<td>(4.5,10,11,4.5,10,11)</td>
<td>(0.32,0.392,0.499,0.657;0.159,0.392,0.499,0.657)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.05,2.3,0.5,2.3)</td>
<td>(0.75,0.8,0.9,0.8,0.9,0.9)</td>
</tr>
<tr>
<td>CR12</td>
<td>FR (0.6,0.73,0.9,1;0.55,0.73,0.9,1)</td>
<td>CR32 (0.4,0.49,0.54,0.56;0.383,0.49,0.54,0.56)</td>
</tr>
<tr>
<td>A1</td>
<td>(0.578,0.671,0.776,0.932;0.482,0.671,0.776,0.932)</td>
<td>(40,45,55,60;40,45,55,60)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.644,0.737,0.842,0.965;0.581,0.737,0.842,0.965)</td>
<td>(100,110,130,140;100,110,130,140)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.374,0.467,0.569,0.683;0.252,0.467,0.569,0.683)</td>
<td>(1000,1500,1150,1200;1000,1500,1150,1200)</td>
</tr>
<tr>
<td>CR13</td>
<td>FR (42.49,55.65;41.49,55.66)</td>
<td>CR41 (0.5,5.2,3,0.5,2,4)</td>
</tr>
<tr>
<td>A1</td>
<td>(48,50,52,54;48,50,52,54)</td>
<td>(0.5,1.2,0,0.5,1,2)</td>
</tr>
<tr>
<td>A2</td>
<td>(36.38,42.44;36.38,42.44)</td>
<td>(1,3,4,6,1,3,4)</td>
</tr>
<tr>
<td>A3</td>
<td>(41.43,47.49,41.43,47.49)</td>
<td>(1,2.4,5,1,2.4,5)</td>
</tr>
<tr>
<td>CR21</td>
<td>FR (0.45,0.65,0.85,1;0.4,0.65,0.85,1)</td>
<td>CR42 (0.2,0.4,0.5,0.6;0.2,0.4,0.5,0.6)</td>
</tr>
<tr>
<td>A1</td>
<td>(0.3,0.4,0.5,0.6;0.2,0.4,0.5,0.6)</td>
<td>(66,68,72,74;66,68,72,74)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.578,0.671,0.776,0.932;0.482,0.671,0.776,0.932)</td>
<td>(46,48,52,54,46,48,52,54)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.172,0.262,0.362,0.467;0.079,0.262,0.362,0.467)</td>
<td>(21,23,27,29,21,23,27,29)</td>
</tr>
<tr>
<td>CR22</td>
<td>FR (0.45,0.65,1,1;0.4,0.65,1,1)</td>
<td>CR43 (0.5,0.65,0.9,1;0.5,0.65,0.9,1)</td>
</tr>
<tr>
<td>A1</td>
<td>(0.578,0.671,0.776,0.932;0.482,0.671,0.776,0.932)</td>
<td>(0.578,0.671,0.776,0.932;0.482,0.671,0.776,0.932)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.441,0.533,0.635,0.748;0.317,0.533,0.635,0.748)</td>
<td>(0.239,0.328,0.431,0.524,0.239,0.328,0.431,0.524)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.172,0.262,0.362,0.467;0.079,0.262,0.362,0.467)</td>
<td>(0.644,0.737,0.842,0.965,0.644,0.737,0.842,0.965)</td>
</tr>
<tr>
<td>CR23</td>
<td>FR (0.5,0.75,0.95,1;0.45,0.75,0.95,1)</td>
<td>CR44 (0.13,37,48,13,33,48)</td>
</tr>
<tr>
<td>A1</td>
<td>(0.5,0.6,0.7,0.8;0.4,0.6,0.7,0.8)</td>
<td>(46,48,52,54,46,48,52,54)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.441,0.533,0.635,0.748;0.317,0.533,0.635,0.748)</td>
<td>(41,43,47,49,41,43,47,49)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.7,0.8,0.9,1;0.7,0.8,0.9,1)</td>
<td>(31,33,37,39,31,33,37,39)</td>
</tr>
</tbody>
</table>

Fig. 14. Sensitivity analysis.
Fig. 14. (continued)

Table 18
Information contents.

<table>
<thead>
<tr>
<th>FRs</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR11</td>
<td>0.000</td>
<td>0.000</td>
<td>3.100</td>
</tr>
<tr>
<td>FR12</td>
<td>0.306</td>
<td>0.000</td>
<td>2.790</td>
</tr>
<tr>
<td>FR13</td>
<td>0.000</td>
<td>4.158</td>
<td>1.019</td>
</tr>
<tr>
<td>FR21</td>
<td>1.862</td>
<td>0.000</td>
<td>4.060</td>
</tr>
<tr>
<td>FR22</td>
<td>0.000</td>
<td>0.552</td>
<td>4.060</td>
</tr>
<tr>
<td>FR23</td>
<td>0.595</td>
<td>1.154</td>
<td>0.000</td>
</tr>
<tr>
<td>FR31</td>
<td>0.853</td>
<td>4.417</td>
<td>0.000</td>
</tr>
<tr>
<td>FR32</td>
<td>0.000</td>
<td>0.000</td>
<td>3.855</td>
</tr>
<tr>
<td>FR41</td>
<td>0.000</td>
<td>0.415</td>
<td>1.263</td>
</tr>
<tr>
<td>FR42</td>
<td>3.158</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>FR43</td>
<td>0.000</td>
<td>3.568</td>
<td>0.000</td>
</tr>
<tr>
<td>FR44</td>
<td>5.466</td>
<td>1.851</td>
<td>0.000</td>
</tr>
<tr>
<td>Total</td>
<td>1.020</td>
<td>1.343</td>
<td>1.679</td>
</tr>
</tbody>
</table>

16 approaches. However, 16 evaluations show a decrease while 4 evaluations indicate an increase. In two functional requirements (FR12 and FR22) information contents exhibit both an increase and a decrease. These changes are caused by the consideration of hesitancies of experts. Even these changes in our case have not affected the rankings, it may cause a significant change in the rankings in another case study. Besides, the consideration of different weights for FRs may put forward

Table 19
Weighted information content based on environmental expert’s evaluation.

<table>
<thead>
<tr>
<th>FRs</th>
<th>Criteria weights</th>
<th>Normalized weights</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR11</td>
<td>100</td>
<td>0.109</td>
<td>0</td>
<td>0</td>
<td>3.1</td>
</tr>
<tr>
<td>FR12</td>
<td>70</td>
<td>0.076</td>
<td>0.306</td>
<td>0</td>
<td>2.79</td>
</tr>
<tr>
<td>FR13</td>
<td>90</td>
<td>0.098</td>
<td>0</td>
<td>4.158</td>
<td>1.019</td>
</tr>
<tr>
<td>FR21</td>
<td>80</td>
<td>0.087</td>
<td>1.862</td>
<td>0</td>
<td>4.06</td>
</tr>
<tr>
<td>FR22</td>
<td>70</td>
<td>0.076</td>
<td>0</td>
<td>0.552</td>
<td>4.06</td>
</tr>
<tr>
<td>FR23</td>
<td>60</td>
<td>0.065</td>
<td>0.595</td>
<td>1.154</td>
<td>0</td>
</tr>
<tr>
<td>FR31</td>
<td>80</td>
<td>0.087</td>
<td>0.853</td>
<td>4.417</td>
<td>0</td>
</tr>
<tr>
<td>FR32</td>
<td>90</td>
<td>0.098</td>
<td>0</td>
<td>0</td>
<td>3.855</td>
</tr>
<tr>
<td>FR41</td>
<td>60</td>
<td>0.065</td>
<td>0</td>
<td>0.415</td>
<td>1.263</td>
</tr>
<tr>
<td>FR42</td>
<td>70</td>
<td>0.076</td>
<td>3.158</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FR43</td>
<td>80</td>
<td>0.087</td>
<td>0</td>
<td>3.568</td>
<td>0</td>
</tr>
<tr>
<td>FR44</td>
<td>70</td>
<td>0.076</td>
<td>5.466</td>
<td>1.851</td>
<td>0</td>
</tr>
<tr>
<td>Weighted Total</td>
<td>0.954</td>
<td>1.387</td>
<td>1.771</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the superiority of the proposed intuitionistic fuzzy FIA. It can be concluded that the trapezoidal intuitionistic fuzzy information approach may present drastic results when the hesitancy on the evaluation is larger.

8. Conclusion

Multicriteria information axiom is a very useful tool for the evaluation of alternatives when the data of design requirements and functional
requirements can be obtained. Vague and subjective evaluations for DRs and FRs can be captured by using fuzzy sets. The proposed trapezoidal intuitionistic fuzzy information axiom could handle these type of assessments including the hesitancy of the experts and successfully evaluated the landfill alternatives. The proposed model utilizes the common areas of trapezoidal intuitionistic fuzzy numbers for DRs and FRs. A careful calculation for common areas is needed in the proposed model since you may face various shapes of intersection of DRs and FRs. The proposed model also includes new defuzzification methods for intuitionistic fuzzy sets and a modified aggregation method.

A real landfill site selection problem has been solved by using 4 main criteria, 12 sub-criteria, and 3 alternatives. Sensitivity analyses showed that the obtained rankings are sensitive to the weights of the criteria. This requires a careful evaluation process in the implementation of the proposed method.

For further research, instead of intuitionistic fuzzy sets, we suggest other extensions of fuzzy sets such as type-2 fuzzy sets, hesitant fuzzy sets, Pythagorean fuzzy sets, or neutrosophic sets to be used in this work. The obtained results can be compared with our results.

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