A Multi-attribute VIKOR Decision-making Method Based on Hesitant Neutrosophic Sets

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Abstract
By combining neutrosophic sets and hesitant fuzzy sets, the definition of hesitant neutrosophic sets is presented. Based on the Euclidean distance and the cosine similarity formula, the similarity formula of neutrosophic sets is constructed. Then its similarity formula is also provided. According to the similarity formula of hesitant neutrosophic sets, a new multi-attribute VIKOR decision-making method for hesitant neutrosophic information is developed. At last an illustrative example is given to verify its practicality and effectiveness.

Key words: Hesitant Neutrosophic Set, VIKOR Method, Similarity Measure

1. INTRODUCTION
In the real world, the information on attributes of the object is often imprecise or uncertain. Due to the shortage of human understanding of uncertainty problems, the attribute values in the decision-making problems are usually incomplete. In order to solve this problem, in 1965, Zadeh(1969), put forward the concept of fuzzy set to a certain extent, made up the shortcoming of the classical set theory. However, because of the complexity of the human mind and personal qualities, decision makers have different opinions for the same problems, and are often difficult to reach an agreement. Fuzzy sets are unable to describe this phenomenon. In order to deal with the complicated situation, Torra(2010) proposed hesitant fuzzy sets which were the expansion form of fuzzy sets. The hesitant fuzzy set allowed one element belonging to a collection of membership could be several possible values, which could reflect the different preference. The hesitant fuzzy set are applied into multi-attribute decision-making (MADM). Xu and Xia(2011) gave the concepts of distance, correlation and similarity between the two hesitation fuzzy sets, and then discussed the corresponding relationship between them. Yager RR (1986) defined the correlation coefficient between the two hesitation fuzzy sets, which could be used to measure the relationship between them. With the complex evolution of processing problems, there are many forms of hesitant fuzzy sets and other mathematical processing tools, which make the description of real-world objects more powerful. Based on the sets, Atanassov (1989) proposed the intuitionistic fuzzy sets (IFS) which considered a membership function $T_a(x)$ and a non-membership function $F_a(x)$. However, IFSs could only deal with incomplete information, but could not do the indeterminate information and inconsistent information.

In 1999, Smarandache developed the neutrosophic set (NS) from a philosophical perspective which composed of three parts $T_a(x), I_a(x), F_a(x)$. In neutrosophic sets, the uncertainty can be quantitative and clear, where uncertainty degree and non-truth degree are independent of each other. And this assumption is very important in information fusion. Broumi and Smarandache put forward a new method calculating the distance between neutrosophic set based on Hausdorff distance, at the same time constructed similarity measurement method by the distance of induction. Under the environment of neutrosophic set, Wang (2010) put forward a single-valued neutrosophic set which the three function satisfied $T_a(x), I_a(x), F_a(x) \in [0,1]$ and $0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3$; Ye gave that the cosine similarity degree is a special case of coefficient in a single-valued neutrosophic set. Based on those, Wang proposed the concept of the interval neutrosophic sets. Ye defined the measures between them and constructed a new method of MADM.

Inspired by articles, YANG yong-wei combined the two concepts and proposed the definition of hesitant neutrosophic sets which make them more capable of dealing with uncertain information. At the same time, the similarity function of neutrosophic sets is given by the Euclidean distance and the cosine similarity formula.

In real decision-making environment, sometime MADM has some problems: First, the existing decision-making methods tend to focus on the absolute value of attributes, ignoring the psychological effect of the relative value of attribute on decision maker; Second, in the existing decision-making methods, the attribute index values are often regarded as a set of isolated vector values, ignoring the synergistic effect of the index values on
the overall scheme. Therefore, in the real decision-making, the attribute value should be evaluated according to the reference value, and the scheme is modified simultaneously, in order to prevent the negative effects of individual indicators of poor performance from being neutralized or overshadowed by other indicators and not being objectively reflected. Some schemes with extremely unbalanced performance may be selected as excellent schemes without fully reflecting the synergy between the attributes, resulting in evaluation or decision-making errors.

In order to make up for the shortcomings of the above multi-attribute group decision-making methods and improve the scientific and reliability of the decision-making results, this paper introduces the collaborative thinking into the multi-attribute group decision-making model and proposes a new multi-attribute VIKOR decision-making method based on hesitant neutrosophic sets.

2. THE DEFINITIONS AND THE MULTI-ATTRIBUTE VIKOR METHOD

2.1 The definitions of hesitant neutrosophic sets

The first we give the definitions about the hesitant neutrosophic sets. From philosophical point of view, Smarandache originally presented the concept of a neutrosophic set \( A \) in a universal set \( X \).

\[ \text{Definition 1} \text{ Let } X \text{ be a universe of discourse, for any } x \in X, \text{ the set } A = \{ (x(T_a(x), I_a(x), F_a(x)) \mid x \in X \} \text{ is called a neutrosophic set, which is characterized independently by a truth-membership function } T_a(x), \text{ an indeterminacy-membership function } I_a(x) \text{ and a falsity-membership function } F_a(x). \text{ The functions } T_a(x), I_a(x) \text{ and } F_a(x) \text{ in } X \text{ are real standard or nonstandard subsets } [0,1], \text{ such that } T_a(x) : X \rightarrow [0,1], \]

\[ I_a(x) : X \rightarrow [0,1] \text{ and } F_a(x) : X \rightarrow [0,1]. \text{ Then, the sum of } T_a(x), I_a(x) \text{ and } F_a(x) \text{ satisfies the condition } 0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3. \]

But it is very difficult to apply the neutrosophic set to science and engineering fields. Wang et al. introduced the concept of a single-valued neutrosophic set as a subclass of the neutrosophic set and gave the following definition.

\[ \text{Definition 2} \text{ Let } X \text{ be a universe of discourse, then a single-valued neutrosophic set is defined as } \]

\[ A = \{ x(T_a(x), I_a(x), F_a(x)) \mid x \in X \}, \text{ where } T_a(x) \rightarrow [0,1], \quad I_a(x) \rightarrow [0,1], \quad F_a(x) \rightarrow [0,1] \text{ with } 0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3 \text{ for every } x \in X \text{. The value denote the truth characterized individually by a truth-membership function } T_a(x), \text{ an indeterminacy-membership function } I_a(x) \text{ and a falsity-membership function } F_a(x). \]

Let A and B be two neutrosophic sets on a universe of discourse \( X = \{ x_1, x_2, \ldots, x_n \} \), the Euclidean distance

\[ d(A, B) = \left\{ \frac{1}{3n} \sum_{i=1}^{n} \{ (T_a(x_i) - T_b(x_i))^2 + (I_a(x_i) - I_b(x_i))^2 + (F_a(x_i) - F_b(x_i))^2 \}^{1/2} \right\} (1) \]

However, sometimes it is very difficult to give the membership of an element, not because we have errors, or because we have some values for the probability distribution, but because we have many possible values. For this situation, Torra introduced a set of hesitation set.

\[ \text{Definition 3} \text{ Let } X \text{ be a universe of discourse, then a hesitation fuzzy set (HFS) is defined as } \]

\[ A = \{ (x, h_a(x)) \mid x \in X \}, \text{ where } h_a(x) \rightarrow [0,1]. \text{ For confidence, Xu and Xia call } h_a(x) \text{ be hesitation fuzzy element (HFE).} \]

In practical decision-making problems, due to the insufficiency of the effective information, it is difficult for decision makers to express their views with exact values, so YANG Yong-wei introduced the definition of hesitation hesitant neutrosophic set.

\[ \text{Definition 4} \text{ Let } X \text{ be a universe of discourse, a hesitant neutrosophic set(HNS) is defined as } \]

\[ A = \{ (x, h_a(x)) \mid x \in X \}, \text{ where } h_a(x) = \{ (h_a^T(x), h_a^I(x), h_a^F(x)) \mid x \in X \}, \text{ and } h_a^T(x) : X \rightarrow [0,1], h_a^I(x) : X \rightarrow [0,1], h_a^F(x) : X \rightarrow [0,1] \text{ are the truth-membership function, indeterminacy-membership function and falsity-membership function respectively satisfying } 0 \leq h_a^T(x) + h_a^I(x) + h_a^F(x) \leq 3. \]

\[ \text{Definition 5} \text{ Let } X \text{ be a universe of discourse, } A = \{ (x, h_a(x)) \mid x \in X \} \text{ be a hesitant neutrosophic set, where hesitantneutrosophic element } h_a(x) = \{ h_{ak}(x), h_{ak}^T(x), h_{ak}^I(x), h_{ak}^F(x) \mid k = 1,2, \ldots, l \} \text{. We define the average value } \overline{h}_a(x) \text{ as the following:} \]

\[ \overline{h}_a(x) = (\overline{h}_a^T(x), \overline{h}_a^I(x), \overline{h}_a^F(x)) \]

where \( \overline{h}_a^T(x) = \frac{1}{l} \sum_{k=1}^{l} h_{ak}^T(x), \overline{h}_a^I(x) = \frac{1}{l} \sum_{k=1}^{l} h_{ak}^I(x), \overline{h}_a^F(x) = \frac{1}{l} \sum_{k=1}^{l} h_{ak}^F(x). \)
And the variance \( \delta_{A}(x_i) \) of hesitant neutrosophic value \( h_A(x_i) \) is
\[
\delta_{A}(x_i) = (\delta_{A}^{T}(x_i), \delta_{A}^{H}(x_i), \delta_{A}^{F}(x_i))
\]
where
\[
\delta_{A}^{T}(x_i) = \frac{1}{l} \sum_{i=1}^{l} (h_{1}(x_i) - \bar{h}_{1}(x_i))^2, \quad \delta_{A}^{H}(x_i) = \frac{1}{l} \sum_{i=1}^{l} (h_{2}(x_i) - \bar{h}_{2}(x_i))^2, \quad \delta_{A}^{F}(x_i) = \frac{1}{l} \sum_{i=1}^{l} (h_{3}(x_i) - \bar{h}_{3}(x_i))^2.
\]

Based on the Euclidean distance of neutrosophic set, we give the Euclidean distance of two hesitant neutrosophic elements:
\[
d(h_1, h_2) = \sqrt{\frac{1}{3l} \sum_{i=1}^{l} \left[ (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 \right].}
\]

where \( (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 = \max\{l_1, l_2\} \), \( h_{1}(x_i) \) is the \( k \)th element in \( h_1 \), and the elements are arranged in descending order, satisfying \( (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 \geq (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 \). So \( \tau : (1,2,\cdots,l) \rightarrow (1,2,\cdots,l) \) is regarded as permutation.

### 2.2 The similarity of hesitant neutrosophic sets

Wang proposed the similarity of hesitant neutrosophic sets based on the cosine similarity.

**Definition 6** Let \( X \) be a universe of discourse, the functions \( S : NS(X) \times NS(X) \rightarrow [0,1] \) is called the similarity function on \( NS(X) \), if it satisfies:

\[ S(A, B) = \begin{cases} 
0 & \text{if } A \subseteq C, \text{ and } B \subseteq C, \text{ then } S(A, C) \leq \min\{S(A, B), S(B, C)\} \\
S(A, B) = 1 & \text{if only if } A = B; \\
S(A, B) = \frac{1}{1 + d(A, B)}.
\]

For hesitant neutrosophic information, cosine similarity measurement formula focuses on the component where the truth-membership function, indeterminacy-membership function and falsity-membership function play a major role, while ignoring the influence of attribute information as a whole in decision-making. So we give distance-induced similarity using Hausdorff similarity.

**Theorem 1** Suppose \( A, B \) be two hesitant neutrosophic sets on a universe of discourse \( X = \{x_1, x_2, \cdots, x_n\} \), then the Euclidean distance of them is

\( d(A, B) = \frac{1}{n} \sum_{i=1}^{n} \cos^2 \left[ \frac{\pi}{2} (I_{1}(x_i) - I_{2}(x_i) \cup I_{1}(x_i) - I_{2}(x_i) \cup F_1(x_i) - F_2(x_i)) \right]. \)

(1) cosine similarity:
\[
\rho(A, B) = S(A, B) = \frac{1}{n} \sum_{i=1}^{n} \cos^2 \left[ \frac{\pi}{2} (I_{1}(x_i) - I_{2}(x_i) \cup I_{1}(x_i) - I_{2}(x_i) \cup F_1(x_i) - F_2(x_i)) \right].
\]

(2) distance-induced similarity: \( S(A, B) = 1 - d(A, B) \).

(3) Hausdorff similarity: \( S(A, B) = \frac{1}{1 + d(A, B)} \).

For hesitant neutrosophic information, cosine similarity measurement formula focuses on the component where the truth-membership function, indeterminacy-membership function and falsity-membership function play a major role, while ignoring the influence of attribute information as a whole in decision-making. So we give distance-induced similarity using Hausdorff similarity.

**Theorem 2** Suppose \( A, B \) be two hesitant neutrosophic sets on a universe of discourse \( X = \{x_1, x_2, \cdots, x_n\} \), \( d(A, B) \) and \( \rho(A, B) \) are the Euclidean distance and cosine similarity, so the similarity formula of \( A, B \) is
\[
S(A, B) = \frac{\rho(A, B)}{\rho(A, B) + d(A, B)}.
\]

\[ \sqrt{\left( \frac{1}{3l} \sum_{i=1}^{l} \left[ (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 \right] \right) + \frac{1}{3l} \sum_{i=1}^{l} \left[ (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 + (h_{1}(x_i) - h_{2}(x_i))^2 \right].} \]

Where \( h_{1,i} = (h_{1,i}^{T}(x), h_{1,i}^{H}(x), h_{1,i}^{F}(x)) \) and \( h_{2,i} = (h_{2,i}^{T}(x), h_{2,i}^{H}(x), h_{2,i}^{F}(x)) \) are two hesitant neutrosophic numbers on a universe of discourse \( X = \{x_1, x_2, \cdots, x_n\} \), then the similarity of \( h_1 \) and \( h_2 \) is
\[
S(h_1, h_2) = S(h_{1,i}, h_{2,i}), (i = 1, 2, 3, 4)
\]

Where \( h_{1,i} = (h_{1,i}^{T}(x), h_{1,i}^{F}(x), h_{1,i}^{R}(x)) \) and \( h_{2,i} = (h_{2,i}^{T}(x), h_{2,i}^{R}(x), h_{2,i}^{F}(x)) \), \( l = \max \left\{ l_1, l_2 \right\} \), \( \tau : (1, 2, \cdots, l) \rightarrow (1, 2, \cdots, l) \) is a permutation.

Now the similarity of hesitant neutrosophic set is defined as:

**Definition 7** Suppose \( h_1 = \{h_{1,i} = (h_{1,i}^{T}(x), h_{1,i}^{H}(x), h_{1,i}^{F}(x)) \} \) and \( h_2 = \{h_{2,i} = (h_{2,i}^{T}(x), h_{2,i}^{H}(x), h_{2,i}^{F}(x)) \} \) are two hesitant neutrosophic numbers on a universe of discourse \( X = \{x_1, x_2, \cdots, x_n\} \), then the similarity of \( h_1 \) and \( h_2 \) is
\[
S(h_1, h_2) = S(h_{1,i}, h_{2,i}), (i = 1, 2, 3, 4)
\]

Where \( h_{1,i} = (h_{1,i}^{T}(x), h_{1,i}^{F}(x), h_{1,i}^{R}(x)) \) and \( h_{2,i} = (h_{2,i}^{T}(x), h_{2,i}^{R}(x), h_{2,i}^{F}(x)) \), \( l = \max \left\{ l_1, l_2 \right\} \), \( \tau : (1, 2, \cdots, l) \rightarrow (1, 2, \cdots, l) \) is a permutation.

Now the similarity of hesitant neutrosophic set is defined as:

**Definition 8** Let \( X = \{x_1, x_2, \cdots, x_n\} \) be a universe of discourse, and \( A, B \) be two hesitant neutrosophic sets, then \( S(A, B) \) is called as the similarity of \( A, B \), if it satisfies:

\( 0 \leq S(A, B) \leq 1 \).
(S2) \( \overline{S}(A,B) = 1 \) if and only if \( A = B \);  
(S3) \( \overline{S}(A,B) = \overline{S}(B,A) \)

In the process of multiple attribute decision making, the importance of attributes is often different, so we need to assign different weights. Therefore, the weights of elements should be considered, and we give under the weights of similarity of the hesitantneutrosophic set.

**Theorem 3** Suppose \( X \) be a universe of discourse, \( A = \{(x_i, h(x_i)) | i = 1, 2, \cdots, n \} \) and \( B = \{(x_j, h(x_j)) | i = 1, 2, \cdots, n \} \) are the hesitantneutrosophic sets on \( X \), \( \omega = (\omega_1, \omega_2, \cdots, \omega_n) \) is a weight, where \( \omega_i \in [0,1], \sum \omega_i = 1 \), then the formula \( \overline{S}(A,B) = \sum \omega_i \overline{S}(h(x_i), h(x_j)) \) is the similarity measure satisfying the definition 8.

3. THE MULTI-ATTRIBUTE VIKOR METHOD BASED ON HESITANTNEUTROSOPHIC SETS

The VIKOR method is put forward by Opricovic in 1998, which is a kind of multiple attribute decision making method based on ideal point solution. And its basic idea is first to determine the ideal solution and negative ideal solution, then according to the value of each alternative, the schemes are prefered for the proximity to the ideal solution. Although the VIKOR method is similar to classic TOPSIS method, Opricovic by comparing the two methods, points out that the optimal solution by the TOPSIS is not necessarily the most close to the ideal point solution. The VIKOR method is a kind of compromise ranking method based on an ideal, which can achieve the optimal ranking of finite decision-making schemes by maximizing group utility and minimizing individual regret.

In this paper, we extend the traditional method to hesitantneutrosophic environment, and propose a new VIKOR method combing the grey correlation degree.

Let \( Y = (Y_1, Y_2, \cdots, Y_n) \) be a discrete set of alternatives, \( G = (G_1, G_2, \cdots, G_n) \) be a set of attributes, \( \omega = (\omega_1, \omega_2, \cdots, \omega_n) \) be the weight of attributes, where \( \omega_i \in [0,1], \sum \omega_i = 1 \). Then decision makers \( D = (D_1, D_2, \cdots, D_n) \) provide his decision matrix \( H^k = (h_{ij}^k)_{n \times n} \), where \( h_{ij} \) for alternative \( Y_i \) under the attribute \( G_j \). Suppose that the matrix \( H^k = (h_{ij}^k)_{n \times n} \) is hesitantneutrosophic decision matrix, where \( h_{ij} \) is expressed by the hesitantneutrosophic fuzzy number.

In MADM process, we construct a new VIKOR method based on synergy degree.

Step 1 Add average value \( \overline{h}_{ij} \) in every hesitantneutrosophic fuzzy number, and the length is \( l_{ij} = \max l_{ij} \).

Step 2 If the attribute types are the same type, the decision matrixes \( H^k = (h_{ij}^k)_{n \times n} \) do not need to be standardized; If the attribute types are different, they are normalized. The \( H^k = (h_{ij}^k)_{n \times n} \) is translated into \( D^k = (d_{ij}^k)_{n \times n} \),

\[
d_{ij}^k = \begin{cases} h_{ij} & \text{benefit index} \\ k & \text{cost index} \end{cases}
\]

Step 3 In the multi-attribute decision-making, ideal solutions are often used to determine the optimal solution decision set. Although ideal solutions may not exist in the real world, it provides a practical theoretical framework for the evaluation of alternatives. Determine the positive ideal solution and the negative ideal solution:

\( Y^+ = (d_1^*, d_2^*, \cdots, d_n^*) \), where \( d_i^* = (\max_{j}(d_{ij}^{(i)}), \min_{j}(d_{ij}^{(i)}), \min_{j}(d_{ij}^{(i)}) \), \( Y^- = (d_1^-, d_2^-, \cdots, d_n^-) \), where \( d_i^- = (\min_{j}(d_{ij}^{(i)}), \max_{j}(d_{ij}^{(i)}), \max_{j}(d_{ij}^{(i)}) \).

Step 4 Calculate the group utility value \( V_i \) and individual regret value \( R_i \) based on similarity measure:

\[
V_i = \sum_{j} \omega_j \frac{d_{ij}^+ - d_{ij}^-}{d_{ij}^* - d_{ij}^-} = \sum_{j} \omega_j \left( \frac{\overline{S}(d_{ij}^* - d_{ij}^-)}{\overline{S}(d_{ij}^* - d_{ij}^-)} \right)
\]

\[
R_i = \max_{j} \omega_j \frac{d_{ij}^+ - d_{ij}^-}{d_{ij}^* - d_{ij}^-} = \max_{j} \omega_j \left( \frac{\overline{S}(d_{ij}^* - d_{ij}^-)}{\overline{S}(d_{ij}^* - d_{ij}^-)} \right)
\]

Step 5 Calculate the compromise evaluation value using the group utility value \( V_i \) and individual regret value:

\[
Q_i = \lambda \left( \frac{V_i - V^+}{V^- - V^+} \right) + (1 - \lambda) \left( \frac{R_i - R^+}{R^- - R^+} \right)
\]
where $V^+ = \min V_i$, $V^- = \max V_i$, $R^+ = \min R_i$, $R^- = \max R_i$, $\lambda \in [0,1]$ is the maximum group utility weight, if $\lambda = 0.5$, then it indicates that decisions are made in a balanced compromise. In VIKOR, we get $\lambda = 0.5$.

Step 6 Rank the order of alternatives according to the $Q_i, V_i, R_i$, and get three orders $Q_1, V_1, R_1$.

Step 7 Determine the compromise solution.

4. NUMERICAL EXAMPLE

Suppose a software company wants to recruit a system administrator. After preliminary screening, three candidates $Y_1, Y_2, Y_3$ need to be finalized. Two experts give their decision-making matrix according to $G_1$ emotional stability, $G_2$ verbal communication skills, $G_3$ working experience and $G_4$ confidence, where the values of the evaluation are hesitant neutrosophic fuzzy numbers. Suppose the weight is $\omega = (0.30, 0.2, 0.15, 0.35)$.

Table 1. Decision making matrix of the first expert

<table>
<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>{(0.5,0.3,0.7),{(0.9,0.6,0.4)}</td>
<td>{(0.8,0.3,0.5)}</td>
<td>{(0.7,0.2,0.5),{(0.2,0.6,0.5)}</td>
</tr>
<tr>
<td>$G_2$</td>
<td>{(0.4,0.7,0.6),{(0.4,0.6,0.9)}</td>
<td>{(0.6,0.7,0.3)}</td>
<td>{(0.8,0.4,0.7),{(0.9,0.3,0.4),{(0.7,0.5,0.6)}</td>
</tr>
<tr>
<td>$G_3$</td>
<td>{(0.7,0.2,0.8)}</td>
<td>{(0.7,0.5,0.4)}</td>
<td>{(0.8,0.5,0.3)}</td>
</tr>
</tbody>
</table>

Table 2. Decision making matrix of the second expert

<table>
<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>{(0.5,0.2,0.6),{(0.9,0.5,0.4)}</td>
<td>{(0.7,0.2,0.5)}</td>
<td>{(0.8,0.1,0.2)}</td>
</tr>
<tr>
<td>$G_2$</td>
<td>{(0.4,0.7,0.6),{(0.4,0.6,0.8)}</td>
<td>{(0.7,0.6,0.3)}</td>
<td>{(0.8,0.3,0.4),{(0.7,0.3,0.5)}</td>
</tr>
<tr>
<td>$G_3$</td>
<td>{(0.6,0.2,0.7)}</td>
<td>{0.6,0.5,0.4)}</td>
<td>{(0.8,0.4,0.3)}</td>
</tr>
</tbody>
</table>

Add the average value $d^0_i$ in the hesitant neutrosophic fuzzy numbers $d^i_j$. Since all attributes are benefit indicators, it is not necessary to standardize the decision matrix. Then according to the steps, we calculate Group utility value, individual regret value and compromise evaluation value (see Table 3).

Table 3. Group utility value, individual regret value and compromise evaluation value

<table>
<thead>
<tr>
<th></th>
<th>$Q_i$</th>
<th>$V_i$</th>
<th>$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>0.402</td>
<td>0.117</td>
<td>0.099</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>0.409</td>
<td>0.113</td>
<td>0.500</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>0.406</td>
<td>0.134</td>
<td>0.767</td>
</tr>
</tbody>
</table>

So we can obtain the rank order $Y_1 \succ Y_2 \succ Y_3$ according to the magnitude of the compromise evaluation value. The company will choose the candidate $Y_1$. The result satisfies the acceptable stability criteria and acceptable dominance criteria.

5. CONCLUSIONS

The multi-attribute VIKOR decision-making method of hesitant neutrosophic sets is proposed in this paper. By combining neutrosophic sets and hesitant fuzzy sets, the definition of hesitant neutrosophic sets is presented. Based on the Euclidean distance and the cosine similarity formula, the similarity formula of neutrosophic sets is constructed. Then its similarity formula is also provided. According to the similarity formula of hesitant neutrosophic sets, a new multi-attribute VIKOR decision-making method for hesitant neutrosophic information is developed. At last an illustrative example is given to verify its practicality and effectiveness.

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REFERENCES


