Abstract—The Critical Path Method (CPM) is one of several related techniques for planning and managing of complicated projects in real world applications. In many situations, the data obtained for decision makers are only approximate, which gives rise of neutrosophic critical path problem. In this paper, the proposed method has been made to find the critical path in network diagram, whose activity time uncertain. The vague parameters in the network are represented by triangular neutrosophic numbers, instead of crisp numbers. At the end of paper, two illustrative examples are provided to validate the proposed approach.

Keywords—Neutrosophic Sets, Project Management, CPM, Score and Accuracy Functions.

I. INTRODUCTION

Project management is concerned with selecting, planning, execution and control of projects in order to meet or exceed stakeholders’ need or expectation from project. Two techniques of project management, namely Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) were developed in 1950s. [1] The successful implementation of CPM and Program Evaluation and Review Technique (PERT) where stakeholders’ need or expectation from project. Two techniques execution and control of projects in order to meet or exceed.

Steps involved in CPM include [2]:
1. Develop Work Breakdown Structure of a project, estimate the resources needed and establish precedence relationship among activities.
2. Translate the activities into network.
3. Carry out network computation and prepare schedule of the activities.

In CPM the main problem is wrongly calculated activity durations, of large projects that have many activities. The planned value of activity duration time may change under certain circumstances and may not be presented in a precise manner due to the error of the measuring technique or instruments etc. It has been obvious that neutrosophic set theory is more appropriate to model uncertainty that is associated with parameters such as activity duration time and resource availability in CPM.

This paper is organized as follows:
In section 2, the basic concepts neutrosophic sets are briefly reviewed. In section 3, the mathematiclal model of neutrosophic CPM and the proposed algorithm is presented. In section 4, two numerical examples are illustrated. Finally section 5 concludes the paper with future work.

II. PRELIMINARIES

In this section, the basic definitions involving neutrosophic set, single valued neutrosophic sets, triangular neutrosophic numbers and operations on triangular neutrosophic numbers are outlined.

Definition 3. [3, 5-7] Let $X$ be a space of points (objects) and $x\in X$. A neutrosophic set $A$ in $X$ is defined by a truth-membership function $T_{A}(x)$, an indeterminacy-membership function $I_{A}(x)$ and a falsity-membership function $F_{A}(x)$, $T_{A}(x)$, $I_{A}(x)$ and $F_{A}(x)$ are real standard or real nonstandard subsets of $[0, 1]$. That is $T_{A}(x): X \rightarrow [0, 1]$, $I_{A}(x): X \rightarrow [0, 1]$ and $F_{A}(x): X \rightarrow [0, 1]$. There is no restriction on the sum of $T_{A}(x)$, $I_{A}(x)$ and $F_{A}(x)$, so $0 \leq \sup T_{A}(x)+ \sup I_{A}(x)+ \sup F_{A}(x) \leq 3$.

Definition 4. [3, 7] Let $X$ be a universe of discourse. A single valued neutrosophic set $A$ over $X$ is an object having the form $A=\{x. T_{A}(x), I_{A}(x), F_{A}(x): x \in X\}$, where $T_{A}(x): X \rightarrow [0, 1]$, $I_{A}(x): X \rightarrow [0, 1]$ and $F_{A}(x): X \rightarrow [0, 1]$ with $0 \leq T_{A}(x)+ I_{A}(x)+ F_{A}(x) \leq 3$ for all $x \in X$. The intervals $T_{A}(x)$, $I_{A}(x)$ and $F_{A}(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$ to $A$, respectively. For convenience, a SVN number is denoted by $A=\{a, b, c\}$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

Definition 5. [4, 5] Let $\alpha_{\bar{A}}, \beta_{\bar{A}}, \beta_{\bar{A}} \in [0, 1]$ and $a_{1}, a_{2}, a_{3} \in R$ such that $a_{1} \leq a_{2} \leq a_{3}$ . Then a single valued triangular neutrosophic number, $\bar{A}=(a_{1}, a_{2}, a_{3})$ is a special neutrosophic set on the real line set $R$, whose truth-membership,
indeterminacy-membership, and falsity-membership functions are given as follows:

\[ T_a(x) = \begin{cases} 
\alpha_a \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\
\alpha_a & \text{if } x = a_2 \\
\alpha_a \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 < x \leq a_3 
\end{cases} \]

(1)

\[ F_a(x) = \begin{cases} 
\beta_a \frac{x - a_2 + \beta_a(x - a_1)}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\
\beta_a & \text{if } x = a_2 \\
\beta_a \frac{x - a_2 + \beta_a(a_3 - x)}{a_3 - a_2} & \text{if } a_2 < x \leq a_3 
\end{cases} \]

(2)

(3)

Where \( \alpha_a, \theta_a, \) and \( \beta_a \) denote the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued triangular neutrosophic number \( \tilde{a} = \langle (a_1, a_2, a_3); \alpha_a, \theta_a, \beta_a \rangle \) may express an ill-defined quantity about \( a \), which is approximately equal to \( a \).

**Definition 4.** [4] Let \( \tilde{a} = \langle (a_1, a_2, a_3); \alpha_a, \theta_a, \beta_a \rangle \) and \( \tilde{b} = \langle (b_1, b_2, b_3); \alpha_b, \theta_b, \beta_b \rangle \) be two single valued triangular neutrosophic numbers and \( \gamma \neq 0 \) be any real number. Then,

\[ \tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle \]

\[ \tilde{a} - \tilde{b} = \langle (a_1 - b_3 - a_2 - b_2, a_3 - b_3); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle \]

\[ \tilde{a} \tilde{b} = \begin{cases} 
\langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle & \text{if } (a_3 > 0, b_3 > 0) \\
\langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle & \text{if } (a_3 < 0, b_3 > 0) \\
\langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle & \text{if } (a_3 > 0, b_3 < 0) \\
\langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_a \land \alpha_b, \theta_a \lor \theta_b, \beta_a \lor \beta_b \rangle & \text{if } (a_3 < 0, b_3 < 0)
\end{cases} \]

The CPM in neutrosophic environment takes the following form:

A network \( N = (E, A, \tilde{T}) \), being a project model, is given. \( E \) is a set of events (nodes) and \( A \subset E \times E \) is a set of activities. \( \tilde{T} \) is a triangular neutrosophic number and stand for activity duration.

To obtain crisp model of neutrosophic CPM we should use the following equations:

\[ S(\tilde{a}) = \frac{1}{16} [a_1 + b_1 + c_1] \times (2 + \alpha_a - \theta_a - \beta_a) \]

(4)

And

\[ A(\tilde{a}) = \frac{1}{16} [a_1 + b_1 + c_1] \times (2 + \alpha_a + \theta_a + \beta_a) \]

(5)

Is called the score and accuracy degrees of \( \tilde{a} \), respectively. The neutrosophic CPM model can be represented by a crisp model using truth membership, indeterminacy membership, and falsity membership functions and the score and accuracy degrees of \( \tilde{a} \), using equations (1), (2), (3) and (4), (5) respectively.

Then the CPM with crisp activity times becomes:

A network \( N = (E, A, T) \), being a project model, is given. \( E \) is set of events (nodes) and \( A \subset E \times E \) is a set of activities. The set \( E = \{1, 2, ..., n\} \) is labeled in such a way that the following
Slack for ith event = it can be delayed without affecting the project completion time. Slack or Float is cushion available on event/activity by which Critical path is the longest path in the network. At critical path, $T_f = \text{minimum } (T_j^f + T_{ij})$, calculate all $T_f^i$ for ith event, select maximum value.

$T_i^l = \text{minimum } (T_j^l - T_{ij})$, calculate all $T_i^l$ for ith event, select minimum value.

$T_i^s$ = Start= Latest occurrence time of successor event $j$.

$T_i^f$ = Finish= Latest occurrence time of an activity $ij$.

$T_i^o$ = Start= Earliest occurrence time of an activity $ij$.

Slack for ith event = $T_i^l - T_i^o$, for events on critical path, slack is zero.

From the previous steps we can conclude the proposed algorithm as follows:

1. To deal with uncertain, inconsistent and incomplete information about activity time, we considered activity time of CPM technique as triangular neutrosophic number.
2. Calculate membership functions of each triangular neutrosophic number, using equations 1, 2 and 3.
3. Obtain crisp model of neutrosophic CPM using equations (4) and (5) as we illustrated previously.
5. Determine floats and critical path, which is the longest path in network.
6. Determine expected project completion time.

**IV. ILLUSTRATIVE EXAMPLES**

To explain the proposed approach in a better way, we solved two numerical examples and steps of solution are determined clearly.

**A. NUMERICAL EXAMPLE 1**

An application deals with the realization of a road connection between two famous cities in Egypt namely Cairo and Zagazig. Linguistics terms such as "approximately between" and "around" can be properly represented by approximate reasoning of neutrosophic set theory. Here triangular neutrosophic numbers are used to describe the duration of each task of project. As a real time application of this model, the following example is considered. The project manager wishes to construct a possible route from Cairo (s) to Zagazig (d). Given a road map of Egypt on which the times taken between each pair of successive intersection are marked, to determine the critical path from source vertex (s) to the destination vertex (d). Activities and their neutrosophic durations are presented in table 1.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Neutrosophic Activity Time (days)</th>
<th>Immediate predecessors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>About 2 days (1,2,3;0.8,0.5,0.3)</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>About 3 days (2,3,8;0.6,0.3,0.5)</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>About 3 days (1,3,10;0.9,0.7,0.6)</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>About 2 days (1,2,6;0.5,0.6,0.4)</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>About 5 days (2,5,11;0.8,0.6,0.7)</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>About 4 days (1,4,8;0.4,0.6,0.8)</td>
<td>C</td>
</tr>
<tr>
<td>G</td>
<td>About 5 days (3,5,20;0.8,0.3,0.2)</td>
<td>C</td>
</tr>
<tr>
<td>H</td>
<td>About 6 days (4,6,10;0.8,0.5,0.3)</td>
<td>D</td>
</tr>
<tr>
<td>I</td>
<td>About 7 days (5,7,15;0.3,0.5,0.4)</td>
<td>F,E</td>
</tr>
<tr>
<td>J</td>
<td>About 5 days (3,5,7;0.8,0.5,0.7)</td>
<td>H,G</td>
</tr>
</tbody>
</table>

**Step 1:** Neutrosophic model of project take the following form: $N= (E, A, \tilde{T})$, where $E$ is asset of events (nodes) and $A \subseteq E \times E$ is a set of activities. $\tilde{T}$ is a triangular neutrosophic number and stand for activity time.

**Step 2:** Obtaining crisp model of problem by using equations (4) and (5). Activities and their crisp durations are presented in table 2.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Activity Time (days)</th>
<th>Immediate predecessors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>G</td>
<td>4</td>
<td>C</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
<td>F,E</td>
</tr>
<tr>
<td>J</td>
<td>1</td>
<td>H,G</td>
</tr>
</tbody>
</table>
Step 3: Draw network diagram of CPM.
Network diagram of CPM using Microsoft Project 2010 presented in Fig.1.

Fig. 1. Network of activities with critical path

Step 4: Determine critical path, which is the longest path in the network.
From Fig.1, we find that the critical path is A-C-G-J and is denoted by red line.

Step 5: Calculate project completion time.
The expected project completion time = $t_A + t_C + t_G + t_J = 8$ days.

B. NUMERICAL EXAMPLE 2

Let us consider neutrosophic CPM and try to obtain crisp model from it. Since you are given the following data for a project.

TABLE 3. INPUT DATA FOR NEUTROSOPHIC CPM.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Neutrosophic Activity Time(days)</th>
<th>Immediate Predecessors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>8</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>6</td>
<td>C</td>
</tr>
<tr>
<td>F</td>
<td>10</td>
<td>D,E</td>
</tr>
</tbody>
</table>

Time in the previous table considered as a triangular neutrosophic numbers.

Let, $\hat{2} = ((0,2,4); 0.8,0.6,0.4)$, $\hat{8} = ((4,8,15); 0.2,0.3,0.5)$,
$\hat{4} = ((1,4,12); 0.2,0.5,0.6)$, $\hat{6} = ((2,6,18); 0.5,0.4,0.9)$,
$\hat{5} = ((1,5,10); 0.8,0.2,0.4)$, $\hat{10} = ((2,10,22); 0.7,0.2,0.5)$.

To obtain crisp values of each triangular neutrosophic number, we should calculate score function of each neutrosophic number using equation (4).
The expected time of each activity are presented in table 4.

TABLE 4. INPUT DATA FOR CRISP CPM.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Immediate Predecessors</th>
<th>Activity Time(days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>D,E</td>
<td>4</td>
</tr>
</tbody>
</table>

After obtaining crisp values of activity time we can solve the critical path method easily, and determine critical path efficiently.
To draw network of activities with critical path we used Microsoft project program.

Fig. 2. Network of activities with critical path

From Fig.2, we find that the critical path is A-C-E-F and is denoted by red line.
The expected project completion time = $t_A + t_C + t_E + t_F = 9$ days.

V. CONCLUSION

Neutrosophic set is a generalization of classical set, fuzzy set and intuitionistic fuzzy set because it not only considers the truth-membership and falsity-membership but also an indeterminacy function which is very obvious in real life situations. In this paper, we have considered activity time of CPM as triangular neutrosophic numbers and we used score function to obtain crisp values of activity time. In future, the research will be extended to deal with different project management techniques.

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REFERENCES


