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The Alexandrov-Urysohn Compactness On Single Valued Neutrosophic S^* Centered Systems

Abstract

In this paper we present the notion of the single valued neutrosophic S^* maximal compact extension in single valued neutrosophic S^* centered system. Moreover, the concept of single valued neutrosophic S^* absolute is applied to establish the Alexandrov -Urysohn compactness criterion. Some of the basic properties are characterized.

Keywords

Single valued neutrosophic S^* centered system, single valued neutrosophic $S^*\theta-$ homeomorphism, single valued neutrosophic $S^*\theta-$ continuous functions.

1. Introduction

Florentin Smarandache [9] combined the non- standard analysis with a tri component logic/set, probability theory with philosophy and proposed the term neutrosophic which means knowledge of neutral thoughts. This neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic set. In 1998, Florentin Smarandache [6] defined the single valued neutrosophic set involving the concept of standard analysis. Stone [10, 11] applied the apparatus of Boolean rings to investigate spaces more general than completely regular ones, related to some extent to the function-theoretic approach. Using these methods tone [10, 11] obtained a number of important results on Hausdorff spaces and in fact introduced the important topological construction that was later called the absolute. The first proof of Alexandrov-Urysohn compactness criterion without any axiom of countability was given by Stone [10, 11].Cech extension in topological spaces and Alexandrov-Urysohn compactness criterion were constructed by Iliadis and Fomin[7].

In this paper, the concept of absolute in single valued neutrosophic S^* structure space and the single valued neutrosophic S^* maximal compact extension β(S) (single valued neutrosophic S^* cech extension) of an arbitrary single valued neutrosophic S^* completely regular space is introduced. Further, the Alexandrov -Urysohn compactness criterion on single valued neutrosophic S^* structure has been studied.

2. Preliminaries

Definition 2.1: [6]

Let *X* be a space of points (objects), with a generic element in *X* denoted by *x*. A single valued neutrosophic set (SVNS) *A* in *X* is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A.

For each point *x* in *X*, $T_A(x), I_A(x), F_A(x) \in [0,1]$. When *X* is continuous, a SVNS *A* can be written as *A*, $\int_X \langle T_A(x), I_A(x), F_A(x)\rangle / x, x \in X$.

When *X* is discrete, a SVNS *A* can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i)\rangle / x_i, x_i \in X$$

Definition 2.2: [3]

Let X be a non- empty set and S a collection of all single valued neutrosophic sets of X.A single valued neutrosophic S^* structure on S is a collection S^* of subsets of S having the following properties

1. φ and S are in S^*.
2. The union of the elements of any sub-collection of S^* is in S^*.
3. The intersection of the elements of any finite sub-collection of S^* is in S^*.

The collection S together with the structure S^* is called single valued neutrosophic S^* structure space. The members of S^* are called single valued neutrosophic S^* open sets. The complement of single valued neutrosophic S^* open set is said to be a single valued neutrosophic S^* closed set.

Example 2.3: [3]

Let $X = \{a,b\}$, $S = \left\{ \frac{a}{\langle 0.8,0.3,0.5\rangle}, \frac{b}{\langle 0.7,0.4,0.6\rangle} \right\}$, $S^* = \{S, \phi, S_1, S_2, S_3, S_4\}$ where,

$S_1 = \left\{ \frac{a}{\langle 0.6,0.1,0.7\rangle}, \frac{b}{\langle 0.5,0.2,0.8\rangle} \right\},\ S_2 = \left\{ \frac{a}{\langle 0.4,0.2,0.6\rangle}, \frac{b}{\langle 0.5,0.3,0.9\rangle} \right\}$,

$S_3 = \left\{ \frac{a}{\langle 0.4,0.1,0.7\rangle}, \frac{b}{\langle 0.5,0.2,0.9\rangle} \right\}, S_4 = \left\{ \frac{a}{\langle 0.6,0.2,0.6\rangle}, \frac{b}{\langle 0.5,0.3,0.8\rangle} \right\}.$

Here (S, S^*) is a structure space.

Definition 2.4: [3]

Let A be a member of S. A single valued neutrosophic S* open set U in (S, S^*) is said to be a single valued neutrosophic S* open neighborhood of A if $A \in G \subset U$ for some single valued neutrosophic S* open set G in (S, S^*).

Example 2.5: [3]

Let $X = \{a,b\}$, $S = \left\{ \frac{a}{\langle 0.8,0.3,0.5\rangle}, \frac{b}{\langle 0.7,0.4,0.6\rangle} \right\}$, $S^* = \{S, \phi, S_1, S_2, S_3, S_4\}$ where,

$S_1 = \left\{ \frac{a}{\langle 0.6,0.1,0.7\rangle}, \frac{b}{\langle 0.5,0.2,0.8\rangle} \right\},\ S_2 = \left\{ \frac{a}{\langle 0.4,0.2,0.6\rangle}, \frac{b}{\langle 0.5,0.3,0.9\rangle} \right\}$,

$S_3 = \left\{ \frac{a}{\langle 0.4,0.1,0.7\rangle}, \frac{b}{\langle 0.5,0.2,0.9\rangle} \right\}, S_4 = \left\{ \frac{a}{\langle 0.6,0.2,0.6\rangle}, \frac{b}{\langle 0.5,0.3,0.8\rangle} \right\}.$

Let $A = \left\{ \frac{a}{\langle 0.4,0.1,0.8\rangle}, \frac{b}{\langle 0.3,0.1,0.9\rangle} \right\}.$

Here, $A \in S_1 \subset S_4$. S_4 is the single valued neutrosophic S* open neighbourhood of A.

Definition 2.6: [3]

Let (S, S^*) be a single valued neutrosophic S* structure space and $A = \langle x, T_A, I_A, F_A \rangle$ be a single valued neutrosophic set in X. Then the single valued neutrosophic S* closure of A (briefly SV N S*cl(A)) and single valued neutrosophic S* interior of A (briefly SVN S*int(A)) are respectively defined by

SVN S*cl(A) $= \bigcap$ {K: K is a single valued neutrosophic S* closed sets in S and A $\subseteq$ K}

SVN S*int(A) = $\bigcup$ {G: G is a single valued neutrosophic S* open sets in S and G $\subseteq$ A}.

Example 2.7: [3]

Let $X=\{a,b\}$, $S=\left\{\frac{a}{\langle 0.8,0.3,0.5\rangle},\frac{b}{\langle 0.7,0.4,0.6\rangle}\right\}$, $S^{*}=\{S,\phi,S_1,S_2,S_3,S_4\}$ where,

$S_1=\left\{\frac{a}{\langle 0.6,0.1,0.7\rangle},\frac{b}{\langle 0.5,0.2,0.8\rangle}\right\}$, $S_2=\left\{\frac{a}{\langle 0.4,0.2,0.6\rangle},\frac{b}{\langle 0.5,0.3,0.9\rangle}\right\}$,

$S_3=\left\{\frac{a}{\langle 0.4,0.1,0.7\rangle},\frac{b}{\langle 0.5,0.2,0.9\rangle}\right\}$, $S_4=\left\{\frac{a}{\langle 0.6,0.2,0.6\rangle},\frac{b}{\langle 0.5,0.3,0.8\rangle}\right\}$.

$S_1^c=\left\{\frac{a}{\langle 0.7,0.9,0.6\rangle},\frac{b}{\langle 0.8,0.8,0.5\rangle}\right\}$, $S_2^c=\left\{\frac{a}{\langle 0.6,0.8,0.4\rangle},\frac{b}{\langle 0.9,0.7,0.5\rangle}\right\}$,

$S_3^c=\left\{\frac{a}{\langle 0.7,0.9,0.4\rangle},\frac{b}{\langle 0.9,0.8,0.5\rangle}\right\}$, $S_4^c=\left\{\frac{a}{\langle 0.6,0.8,0.6\rangle},\frac{b}{\langle 0.8,0.7,0.5\rangle}\right\}$.

Let $A=\left\{\frac{a}{\langle 0.5,0.3,0.6\rangle},\frac{b}{\langle 0.7,0.4,0.9\rangle}\right\}$. Then $SVN\,S^*\mathrm{int}(A)=\{S_3\}$.

$SVN\,S^*cl(A)=\{S_4^c\}$.

Definition 2.8: [3]

The ordered pair (S, S^*) is called a single valued neutrosophic S^* Hausdorff space if for each pair A_1, A_2 of disjoint members of S, there exist disjoint single valued neutrosophic S^* open sets U_1 and U_2 such that $A_1 \subseteq U_1$ and $A_2 \subseteq U_2$.

Example 2.9: [3]

Let $X=\{a,b\}$, $S=\left\{\frac{a}{\langle 1,1,0\rangle},\frac{b}{\langle 1,1,0\rangle}\right\}$, $S^{*}=\{S,\phi,S_1,S_2,S_3\}$ where,

$S_1=\left\{\frac{a}{\langle 0.5,0,1\rangle},\frac{b}{\langle 0,0.3,0.4\rangle}\right\}$, $S_2=\left\{\frac{a}{\langle 0.5,0.2,0.5\rangle},\frac{b}{\langle 0.7,0.3,0.4\rangle}\right\}$, $S_3=\left\{\frac{a}{\langle 0,0.2,0.5\rangle},\frac{b}{\langle 0.7,0,1\rangle}\right\}$.

Let $A_1=\left\{\frac{a}{\langle 0.3,0,1\rangle},\frac{b}{\langle 0,0.1,1\rangle}\right\}$, $A_2=\left\{\frac{a}{\langle 0,0.1,0.6\rangle},\frac{b}{\langle 0.5,0,1\rangle}\right\}$.

Here A_1 and A_2 are disjoint members of S and S_1, S_2 are disjoint single valued neutrosophic S* open sets such that $A_1 \subseteq S_1$ *and* $A_2 \subseteq S_2$.

Hence the ordered pair (S, S^*) is a single valued neutrosophic S* Hausdorff space.

Definition 2.10: [3]

Let (S_1, S_1^*) and (S_2, S_2^*) be any two single valued neutrosophic S* structure spaces and let $f:(S_1, S_1^*) \to (S_2, S_2^*)$ be a function. Then f is said to be single valued neutrosophic S* continuous iff the pre image of each single valued neutrosophic S_2^* open set in (S_2, S_2^*) is a single valued neutrosophic S_1^* open set in (S_1, S_1^*).

Definition 2.11: [3]

Let (S_1, S_1^*) and (S_2, S_2^*) be any two single valued neutrosophic S* structure spaces and let $f:(S_1, S_1^*) \to (S_2, S_2^*)$ be a bijective function. If both the functions f and the inverse function $f^{-1}:(S_2, S_2^*) \to (S_1, S_1^*)$ are single valued neutrosophic S* continuous then f is called single valued neutrosophic S* homeomorphism.

Definition 2.12: [4]

Let f be a function from a single valued neutrosophic S* structure space (S_1, S_1^*) into a single valued neutrosophic S* structure space (S_2, S_2^*) with $f(A_1) = f(A_2)$ where $A_1 \in (S_1, S_1^*)$ and $A_2 \in (S_2, S_2^*)$. Then f is called a single valued neutrosophic S*θ continuous at A_1 if for every neighbourhood O_{A_2} of A_2, there exists a neighbourhood O_{A_1} of A_1 such that $f(SVNS*cl(O_{A_1})) \subset SVNS*cl(O_{A_2})$. The function is called single valued neutrosophic S*$\theta-$ continuous if it is single valued neutrosophic S*$\theta-$ continuous at every member of S_1.

Definition 2.13: [3]

A function is called a single valued neutrosophic S*θ− homeomorphism if it is single valued neutrosophic S* one to one and single valued neutrosophic S*θ− continuous in both directions.

Definition 2.14: [3]

Let (S, S*) be a single valued neutrosophic S* Hausdorff space. A system $p = \{U_\alpha : \alpha = 1, 2, 3, ...n\}$ of single valued neutrosophic S* open sets is called a single valued neutrosophic S* centered system if any finite collection of the sets of the system has a non- empty intersection .

Example 2.15: [3]

Let $X = \{a,b\}$, $S = \left\{\frac{a}{\langle 1,1,0\rangle}, \frac{b}{\langle 1,1,0\rangle}\right\}$, $S^* = \{S, \phi, S_1, S_2, S_3\}$ where,

$S_1 = \left\{ \frac{a}{\langle 0.5,0,1 \rangle}, \frac{b}{\langle 0,0.3,0.4 \rangle} \right\}$, $S_2 = \left\{ \frac{a}{\langle 0.5,0.2,0.5 \rangle}, \frac{b}{\langle 0.7,0.3,0.4 \rangle} \right\}$, $S_3 = \left\{ \frac{a}{\langle 0,0.2,0.5 \rangle}, \frac{b}{\langle 0.7,0,1 \rangle} \right\}$.

Let $A_1 = \left\{ \frac{a}{\langle 0.3,0,1 \rangle}, \frac{b}{\langle 0,0.1,1 \rangle} \right\}$, $A_2 = \left\{ \frac{a}{\langle 0,0.1,0.6 \rangle}, \frac{b}{\langle 0.5,0,1 \rangle} \right\}$ .Let us consider the system $p_1 = \{S_\alpha, \alpha = 1,2,3\}$. p_1 is a single valued neutrosophic S* centered system since S_1, S_2 has a non -empty intersection.

Let $p_2 = \{S_\alpha : \alpha = 1,2\}$ is also a single valued neutrosophic S* centered system.

Here p_1 is a maximal single valued neutrosophic S* centered system.

Definition 2.16: [3]

The single valued neutrosophic S^* centered system p is called a maximal single valued neutrosophic S^* centered system or a single valued neutrosophic S^* end if it cannot be included in any larger single valued neutrosophic S^* centered system of single valued neutrosophic S^* open sets.

Definition 2.17: [3]

A subset A of a single valued neutrosophic S^* structure space (S, S^*) is said to be an everywhere single valued neutrosophic S^* dense subset in (S, S^*) if $SVNS^*cl(A) = S$.

Definition 2.18: [3]

A subset of a single valued neutrosophic S^* structure space (S, S^*) is said to be a nowhere single valued neutrosophic S^* dense subset in (S, S^*) if $X \setminus A^c$ is everywhere single valued neutrosophic S^* dense subset.

3. Single valued neutrosophic Cech extension

Definition 3.1:

A single valued neutrosophic S^* centered system $p = \{U_\alpha\}$ of single valued neutrosophic S^* open sets of S is called a single valued neutrosophic S^* completely regular system if for any $U_\alpha \in p$ there exists a $V_\alpha \in p$ and a single valued neutrosophic S^* continuous function f on S such that $f(A) = 1$ for $A \in S \setminus U_\alpha$, $f(A) = 0$ for $A \in V_\alpha$ and $0 \leq f(A) \leq 1$ for any $A \in S$. In this case V_α. is a single valued neutrosophic S^* completely regularly contained in U_α.

Definition 3.2:

A single valued neutrosophic S^* completely regular system is called a single valued neutrosophic S^* completely regular end if it is not contained in any larger single valued neutrosophic S^* completely regular system.

Definition 3.3:

Let (S_1, S_1^*) and (S_2, S_2^*) be any two single valued neutrosophic S^* structure spaces. A function $f:(S_1, S_1^*) \to (S_2, S_2^*)$ of a single valued neutrosophic S^* structure space (S_1, S_1^*) onto a single valued neutrosophic S^* structure space (S_2, S_2^*) is a quotient function (or natural function) if, whenever V is a single valued neutrosophic S_2^* open set in (S_2, S_2^*), $f^{-1}(V)$ is a single valued neutrosophic S_1^* open set in (S_1, S_1^*) and conversely.

Note 3.4:

The maximal single valued neutrosophic S^* centered systems of single valued neutrosophic S^* open sets (single valued neutrosophic S^* ends) regarded as elements of the single valued neutrosophic S^* space $\theta(S)$, fall into two classes: those single valued neutrosophic S^* ends each of which contains all the single valued neutrosophic S^* open neighbourhoods of one (obviously only one) member of S, and the single valued neutrosophic S^* ends not containing such systems of single valued neutrosophic S^* open neighbourhoods. The single valued neutrosophic S^* ends of the first type can be regarded as representing the members of the original single valued neutrosophic S^* space S and those of the second type as corresponding to holes in S.

Definition 3.5:

The collection of all single valued neutrosophic S^* ends of the first type in $\theta(S)$ is a single valued neutrosophic S^* completely regular space and it is also called the single valued neutrosophic S^* absolute of S which is denoted by *w(S)*.

In *w(S)*, each member $V \in S$ is represented by single valued neutrosophic S^* ends containing all single valued neutrosophic S^* open neighbourhoods of S. It is obvious that $w(S) = \bigcup_{V \in S} B(V)$ where $B(V)$ are the single valued neutrosophic S^* ends p of S that contain all the single valued neutrosophic S^* open neighbourhoods of V .The subset *w(S)* is mapped in a natural way onto S .If $p \in w(S)$, then by definition $\pi_S(p) = V$, where V is the member whose single valued neutrosophic S^* open neighbourhoods all belong to p and π_S is the natural function of $w(S)$ onto S.

Lemma 3.6:

A single valued neutrosophic S^* centered system $\{U_\alpha\}$ of all single valued neutrosophic S^* open neighbourhoods of a member A in a single valued neutrosophic S^* completely regular space S is a single valued neutrosophic S^* completely regular end.

Proof:

Here, $\{U_\alpha\}$ is a single valued neutrosophic S^* completely regular centered system .The Lemma will be proved if it is possible to show that $\{U_\alpha\}$ is not contained in any other single valued neutrosophic S^* completely regular system. As a contrary, suppose that $\{V_\alpha\}$ is a single valued neutrosophic S^* centered completely regular system containing $\{U_\alpha\}$ with $V_{\alpha_1} \notin \{U_\alpha\}$. Since V_{α_1} meets every single valued neutrosophic S^* open neighbourhood of A , $A \in SVNS^*cl(V_{\alpha_1}) \setminus V_{\alpha_1}$ .Let V_{α_2} be an element of $\{V_\alpha\}$ such that $SVNS^*cl(V_{\alpha_2})$ $\subseteq V_{\alpha_1}$.But then $A \in SVNS^*cl(V_{\alpha_2})$.It follows that V_{α_2} does not meet any of the single valued neutrosophic S^* open neighbourhoods of A , so $\{V_\alpha\}$ cannot be a single valued neutrosophic S^* centered system containing $\{U_\alpha\}$.

Now we construct a single valued neutrosophic S^* structure space which is denoted by $\alpha'(S)$ .Itsmembers are all single valued neutrosophic S^* completely regular ends of S, and its single valued neutrosophic S^* topology is defined as follows: Choose an arbitrary single valued neutrosophic S^* open set U in S and the collection O_U of all single valued neutrosophic S^* centered completely regular ends of S that contain U as a member is to be a single valued neutrosophic S^* open neighbourhood of each of them.

Lemma 3.7:

A single valued neutrosophic S^* completely regular end $p = \{U_\alpha\}$ of a single valued neutrosophic S^* structure space S has the following properties:

1. If $U\beta \supseteq U_\alpha \in p$, then $U\beta \in p$.
2. The intersection of any finite number of members of p belongs to p.

Proof:

Assertion (1) is obvious.

(2) Let $U_{\alpha 1}, U_{\alpha 2}, ... U_{\alpha n} \in p, U'_{\alpha 1}, U'_{\alpha 2}, ... U'_{\alpha n} \in p$ and $f_1, f_2, ... f_n$ be functions such that $f_i(A) = 0$ on $SVNS * cl(U'_{\alpha i}), f_i(A) = 1$ on $S \setminus U_{\alpha i}$. Then the function $f_1(A) + f_2(A) + ... + f_n(A)$ is zero on $SVNS * cl(U'_{\alpha 1}) \cap ..... \cap SVNS * cl(U'_{\alpha n})$ and a fortiori on $SVNS * cl((U'_{\alpha 1}) \cap ... \cap (U'_{\alpha n}))$.

Since $S \setminus (\cap U_{\alpha i}) = \bigcup (S \setminus U_{\alpha i})$, then $f_1(A) + ... + f_n(A) \geq 1$ at each member of $S \setminus (\cap U_{\alpha i})$. Putting $f(A) = 1$ whenever $f_1(A) + ... + f_n(A) \geq 1$ and $f(A) = f_1(A) + ... + f_n(A)$. When this sum is less than 1, it may be obtained a function $f(A)$ such that $0 \leq f(A) \leq 1, f(A) = 0$ on $\cap SVNS * cl(U'_{\alpha n})$ and $f(A) = 1$ on $S \setminus (\cap U_{\alpha})$. Hence the single valued neutrosophic S^* system p must contain $\cap U_{\alpha i}$ and $\cap U'_{\alpha i}$, otherwise it would not be maximal single valued neutrosophic S^* completely regular system.

Corollary 3.8:

$O_U \cap O_V = O_{U \cap V}$. For if $p \in O_U \cap O_V$, then $U \in p$ and $V \in p$. By Lemma 3.7, $U \cap V \in p$, that is, $p \in O_{U \cap V}$. Therefore, $O_U \cap O_V \subseteq O_{U \cap V}$. If $p \in O_{U \cap V}$, then $U \cap V \in p$ and by the same Lemma $U \in p$ and $V \in p$. Therefore, $p \in O_U$ and $p \in O_V$. That is, $p \in O_U \cap O_V$. Hence, $O_{U \cap V} \subseteq O_U \cap O_V$. Thus, $O_U \cap O_V = O_{U \cap V}$.

Lemma 3.9:

A single valued neutrosophic S^* structure space $\alpha'(S)$ is a single valued neutrosophic S^* Hausdorff extension of S.

Proof:

The single valued neutrosophic S^* structure space $\alpha'(S)$ is a single valued neutrosophic S^* Hausdorff space. Let p and q be any two disjoint members of $\alpha'(S)$. Then it is easy to find $U \in p$ and $V \in p$ such that $U \cap V = \varphi$, for otherwise the single valued neutrosophic S^* centered system consisting of all the members of p and all the members of q would be a single valued neutrosophic S^* centered completely regular system containing p and q, which is impossible. O_U and O_V, associated with this U and V, are disjoint single valued neutrosophic S^* open neighbourhoods of p and q in $\alpha'(S)$.

It shall be identified that the member $A \in S$ with the single valued neutrosophic S* end p_A = {U$_\alpha$} consisting of all the single valued neutrosophic S* open neighbourhood of A in S. Then $O_U \cap S = U$, which shows that S is single valued neutrosophic S^* topologically embedded in $\alpha'(S)$, and since it is easy to see that S is everywhere single valued neutrosophic S* dense in $\alpha'(S)$. Therefore, $\alpha'(S)$ is a single valued neutrosophic S*extension of S. This proves the Lemma.

Note 3.10:

In a single valued neutrosophic S* completely regular space, the single valued neutrosophic S*canonical neighbourhood forms a base.

Lemma 3.11:

A single valued neutrosophic S* structure space $\alpha'(S)$can be continuously mapped onto every single valued neutrosophic S* compact extension of S in such a way that the members of S remain fixed.

Proof:

Let $b(S)$ be any single valued neutrosophic S* compact extension of S. Each member A $\in$ b(S) determines the single valued neutrosophic S* centered system p_A = {U$_\alpha$} consisting of all single valued neutrosophic S* open neighbourhoods of A in $b(S)$ .By Lemma 3.6 , this is a single valued neutrosophic S* completely regular system and a single valued neutrosophic S* maximal .The single valued neutrosophic S* centered system q_A = {V$_\alpha$ = U$_\alpha$ $\cap$ S} is a single valued neutrosophic S* completely regular system .If d = {H$_\alpha$} $\in$ $\alpha'(S)$ contains a single valued neutrosophic S* centered system q_A, then we define φ on $\alpha'(S)$ as φ(d) = q_A .Since $b(S)$ is a single valued neutrosophic S* Hausdorff extension , d can contain only one such single valued neutrosophic S* centered system q_A .Hence the function φ is well-defined .Since an arbitrary single valued neutrosophic S* completely regular system can be extended to a single valued neutrosophic S* completely regular end , φ is onto. It is easy to see that if $A \in S$, then $\varphi(A) = A$.φ is defined on the whole of $\alpha'(S)$.For if d = {H$_\alpha$} $\in \alpha'(S)$, then $\bigcap_\alpha \left(SVNS^* cl(H_\alpha)\right)^{b(S)} \neq \phi$ (because b(S) is a single valued neutrosophic S*compact space).Let $A \in \bigcap_\alpha \left(SVNS^* cl(H_\alpha)\right)^{b(S)}$.Then the single valued neutrosophic S^*centered system $d \cup q_A$ consisting of all members $H_\alpha \in d$ and all members $U_\alpha \in q_A$ is a single valued neutrosophic S^*completely regular and since d is a single valued neutrosophic S^* maximal completely regular system , $d \cup q_A = d$, that is , $q_A \subseteq d$, so that $\varphi(d) = A$. Let $A \in b(S)$ and U_α be any single valued neutrosophic S^* open neighbourhood of A in $b(S)$. Assuming U_α is a canonical single valued neutrosophic S^* open neighbourhood. Put $V_\alpha = U_\alpha \cap S$. Let $d \in \alpha'(S)$ and $\varphi(d)$=A. Then O_{V_α} is a single valued neutrosophic S^* open neighbourhood of d in $\alpha'(S)$.To show that $\varphi(O_{V_\alpha}) \subseteq SVNS^*cl(U_\alpha)$.For this, it is clear that $V_\alpha \in q'_A$ if and only if $A' \in U_\alpha$. Now , if $d' \in O_{V_\alpha}$, then $V_\alpha \in d'$.If $\varphi(d') = A' \notin SVNS^*cl(U_\alpha)$ then some single valued neutrosophic S^* open neighbourhood of A' which does not contained in U_α, but then $V_\alpha \notin q_{A'}$, so that $V_\alpha \notin d'$, that is $d' \notin O_{V_\alpha}$, This contradicts our assumption .Since b(S) is single valued neutrosophic S^* regular space ,φ is a single valued neutrosophic S^* continuous and the Lemma is proved .The single valued neutrosophic set O_U , where U is a canonical single valued neutrosophic S^* open set of S forms a base in $\alpha'(S)$.

Lemma 3.12:

The single valued neutrosophic S^* structure space $\alpha'(S)$ is a single valued neutrosophic S^* completely regular space.

Proof:

Let $p=\{U_\alpha\} \in \alpha'(S)$ and let U_2 be any single valued neutrosophic S^* completely regular contained in the canonical single valued neutrosophic S^* open set U_1.Assume that $SVNS^*cl(O_{U_2}) \not\subset (\alpha'(S) \backslash O_{U_1})$.Then there is a member $q = \{SVNS^*cl(V_\alpha)\}$ such that $q \in SVNS^*cl(O_{U_2}) \cap (\alpha'(S) \backslash O_{U_1})$.The relation $q \in SVNS^*cl(O_{U_2})$ means that every $V_\alpha \in q$ meets U_2 and the relation $q \in \alpha'(S) \backslash O_{U_1}$, equivalent to $q \notin O_{U_1}$ means that every V_α meets $S \backslash U_1$.Since U_1 is a canonical single valued neutrosophic S^* open set , it follows that V_α meets $S \backslash SVNS^*cl(U_1)$.

If V_1 and V_2 are single valued neutrosophic S^* open sets such that V_1 is single valued neutrosophic S^* completely regularly contained in $S \backslash SVNS^*cl(V_2)$ and $V = V_1 \cap V_2 \in q$, then either $V_1 \in q$ or $V_2 \in q$. Now , let $f(B)$ be a function that is zero on SV N S^*cl(U_2) and 1 on $S \backslash U_1$.Such a function exists , since U_2 is single valued neutrosophic S^* completely regularly contained in U_1 .Also let $0 < a < b < 1$ and let $\Gamma(a, b)$ be the single valued neutrosophic S^* open set $\{B : a < f(B) < b\}$.By assumption , every $V_\alpha \in q$ has non- empty intersection with $\Gamma(a, b)$.For otherwise , V_α splits into two single valued neutrosophic S^* open sets $V_{\alpha 1}$ and $V_{\alpha 2}$ such that $V_{\alpha 1}$ is single valued neutrosophic S^* completely regularly contained in S\SV N S^*cl($V_{\alpha 2}$) and $V_{\alpha 2} \cap U_2 = \varphi$, $V_{\alpha 1} \cap$ (S\SVN S^*cl(U_1)) $= \varphi$. The last equation contradicts the fact that q $\in$ SV N S^*cl (O_{U_2}) $\cap (\alpha'(S) \backslash O_{U_1})$.

Consider the single valued neutrosophic S^* open sets $\Gamma(a, b)$ where $0 < a < a_0 < b_0 < b < 1$ and a_0 and b_0 are fixed .They form a single valued neutrosophic S^* completely regular system which must be contained in q .But $\Gamma(a, b) \subset U_1$ that is , $\Gamma(a, b) \cap \Gamma(S \backslash SVN\ S^*cl(U_1)) = \varphi$ and hence , $q \notin (\alpha'(S) \backslash\ O_{U_1})$. This contradiction shows that SV N S^*cl (O_{U_2}) $\subseteq O_{U_1}$, from which it follows that $\alpha'(S)$ is a single valued neutrosophic S^* regular space . To prove that it is a single valued neutrosophic S^* completely regular space. Let $\Gamma_t, 0 \leq t \leq 1$, denote the set of all $B \in$ S for which $f(B) < t$. It has shown that if $t_1 < t_2$, then $SVNS^*cl(O_{\Gamma_{t_1}}) \subseteq O_{\Gamma_{t_2}}$ .Hence it follows that O_{U_2} is single valued neutrosophic S^* completely regularly contained in O_{U_1} .

Lemma 3.13:

The single valued neutrosophic S^* structure space $\alpha'(S)$ is a single valued neutrosophic S^* compact.

Proof:

If H is a single valued neutrosophic S^* open set of $\alpha'(S)$, then there exists a single valued neutrosophic S^* open set $U(H)$ of S such that $H \subseteq O_{U(H)} \subseteq SVNS^*cl(H)$ then $U(H) = \bigcup_\alpha U_\alpha$.If H is single valued neutrosophic S^* completely regularly embedded in G , then $O_{U(H)}$ is clearly single valued neutrosophic S^* completely regularly embedded in $O_{U(G)}$.Suppose that $\alpha'(S)$ is not a single valued neutrosophic S^* compact space. Then by

Tychonoff's theorem, there exists a single valued neutrosophic S* completely regular space $\alpha'(S) \cup \xi$, containing $\alpha'(S)$ as an everywhere single valued neutrosophic S* dense set. Let H_α be the set of all single valued neutrosophic S* open sets of $\alpha'(S)$ for which $H_\alpha \cup \xi$ is a single valued neutrosophic S* open neighbourhood of ξ in $\alpha'(S) \cup \xi$. Then, $\{H_\alpha\}$ is a single valued neutrosophic S* completely regular system in $\alpha'(S)$. Hence $O_{U(H_\alpha)}$ is also a single valued neutrosophic S* completely regular space. Thus, $\cap O_{U(H_\alpha)} = \varphi$. Since $\alpha'(S)$ is a single valued neutrosophic S* centered system $p = \{\cup (H_\alpha)\}$ of single valued neutrosophic S* completely regular too. But $p \in O_{U(H_\alpha)}$ for every $\alpha \in \Lambda$, that is $\bigcap_\alpha O_{U(H_\alpha)} = \varphi$. This contradiction proves the lemma.

Proposition 3.14:

For any single valued neutrosophic S* completely regular space *S*, the single valued neutrosophic S* structure space $\alpha'(S)$ coincides with the Cech extension $\beta(S)$ upto a single valued neutrosophic S* homeomorphism leaving the members of *S* fixed.

Proof:

The proof follows immediately from Lemma 3.11 and Lemma 3.13 and the uniqueness of a maximal single valued neutrosophic S* compact extension.

4. The Alexandrov - Urysohn compactness

In this section, the concept of single valued neutrosophic S* absolute is applied to establish the Alexandrov - Urysohn compactness.

Property 4.1:

If $F_1 \subset F_2 \subset F_3 \subset .... \subset F_n = S$ with F_1 non -empty, then $\bigcap_{i=1}^{n} \widetilde{F}_i \neq \phi$ (in particular, if *F* is non-empty, so is $\widetilde{F}$).

Proof:

Let $B \in F_1$ and let $q' = \{G'\}$ be a single valued neutrosophic S* end of F_1 containing a single valued neutrosophic S* centered system of single valued neutrosophic S* open sets G', in F, such that $B \in SVNS^*int(SVNS^*cl(G'))$. It may be assumed that it has been constructed systems $q^i = \{G^i\}$ of F_i such that q^i contains all the single valued neutrosophic S^* open sets $G^i \subseteq F_i$ for which B $\in$ SVNS**int*(SVNS*cl(G^i)) and all single valued neutrosophic S^* open sets whose intersection with F_{i-1} is some G^{i-1}.

Now construct q^{i+1}. By definition, q^{i+1}, consists of all sets $G^{i+1} \subseteq F_{i+1}$ for which B $\in$ SVNS**int*(SVNS**cl*(G^{i+1})) and of all single valued neutrosophic S^* open sets whose intersection with F_i is some G^i. It is easy to show that q^{i+1} is a single valued neutrosophic S^* centered system. Thus for each i construct a single valued neutrosophic S^* centered system q^i. Let $p = \{H\}$ denote the single valued neutrosophic S^* end of S containing q^n. To show that $p \in \bigcap_{i=1}^{n} \widetilde{F}_i$. It follows from the construction of p that if $H \cap F_i \in q^i$ for some i and some single valued neutrosophic S^* open set H in S, then $H \in p$. To show that $p \in \widetilde{F}_i$. Let H be a single valued neutrosophic S^* open set of S such that B $\in$ SVNS*int(SVNS*cl(H $\cap$ F$_i$)). Then $H \cap F_i \in q^i$ and hence $H \in p$, that is, $p \in \widetilde{F}_i$. Hence the proof.

Property 4.2:

If F is a single valued neutrosophic S^*H− closed, then $\widetilde{F}$ is single valued neutrosophic S^*compact (and hence single valued neutrosophic S^* closed in θ(S)).

Proof:

Let $\{H_\alpha\}$ be any single valued neutrosophic S^* covering of $\widetilde{F}$ by single valued neutrosophic S^* open sets in $\widetilde{F}$. They may be extended to single valued neutrosophic S^*open in *w(S)*. It may assume that each of that each of the extended sets has the form O_U, where U is a single valued neutrosophic S^* open set in *S*. Otherwise $\{H_\alpha\}$ may be replaced by a finer covering for which this condition holds. So it may be assumed that $\{H_\alpha\}$ is a single valued neutrosophic S^* covering of F by sets single valued neutrosophic S^* open in *w(S)* of the form O_{U_α}, where U_α is single valued neutrosophic S^* open in *S*. Let B $\in$ F. Let H_β^B denote the union of a finite number of single valued neutrosophic S^* open sets H_α covering the single valued neutrosophic S^* compact set $\pi_s^{-1}(B)$. It is clear that H_β^B has the form $O_{U_\beta^B}$, where U_β^B is a single valued neutrosophic S^* open set in *S* and is maximal among the single valued neutrosophic S^* open sets H for which $O_H = O_{U_\beta^B}$. From the above, it follows that the single valued neutrosophic S^*centered system $\{SVNS^* \operatorname{int}(U_{\beta}^{B} \cap F)\}$is a single valued neutrosophic S^* covering of F .Since F is single valued neutrosophic S^*H- closed, choose a finite number of members of this single valued neutrosophic S^* covering such that $\bigcup_{i=1}^{n} SVNS^* cl(SVNS^* \operatorname{int}(SVNS^* cl(U_{\beta i}^{B} \cap F))) = F$,where the closure is taken in F in both cases. To show that $\bigcup_{i=1}^{n} O_{U_{\beta i}^{B}} \supseteq \widetilde{F}$ .Since the union $\bigcup_{i=1}^{n} O_{U_{\beta i}^{B}} = U$ has the property that $B \in SVNS^* \operatorname{int}(SVNS^* cl(F \cap U))$for any B,then an arbitrary single valued neutrosophic S^* end $p \in \widetilde{F}$ contains U and hence belongs to some $O_{U_{\beta}^{B}}$. Thus, for only those $H\alpha$ that make up $O_{U_{\beta i}^{B}}$ and take their intersections with $\widetilde{F}$, the required finite covering is obtained.

Definition 4.3:

A single valued neutrosophic S^* Hausdorff space S is a single valued neutrosophic S^* compact space if and only if every (not necessarily countable) well-ordered decreasing sequence of non-empty single valued neutrosophic S^* closed sets has a non-empty intersection.

Theorem 4.4: (Alexandrov - Urysohn compactness)

A single valued neutrosophic S^* Hausdorff space S is a single valued neutrosophic S^* compact space if and only if each of its single valued neutrosophic S^* closed subset is single valued neutrosophic S^*H- closed.

Proof:

Necessity:

The necessity of this condition follows from Property 4.2. Since in a single valued neutrosophic S^* compact space every single valued neutrosophic S^* closed subset is a single valued neutrosophic S^* compact space and hence single valued neutrosophic S^*H- closed.

Sufficiency:

Let S be a single valued neutrosophic S^* Hausdorff space, *w(S)* be its single valued neutrosophic S^* absolute and π_S be a single valued neutrosophic S^* natural function of *w(S)* onto S. Also let F be any single valued neutrosophic S^* subset of S. It can be associated it with a certain single valued neutrosophic S^* subset $\widetilde{F}$ of *w(S)*, defined by saying that the member $p \in \pi_s^{-1}(B), B \in S$, belongs to $\widetilde{F}$ if $p \in O_U$ for every U satisfying the condition $B \in SVNS^*\operatorname{int}(SVNS^*cl(U \cap F))$.By the construction of $\widetilde{F}$ it is contained in the complete single valued neutrosophic S^* inverse image $\pi_s^{-1}(F)$ of F in $w(S)$. $\widetilde{F}$ is called as the reduced inverse image of F in $w(S)$. The proof of the Alexandrov - Urysohn compactness in single valued neutrosophic S^* topology is based on the properties discussed above. For suppose that the conditions of the theorem are satisfied and that $\{F_\alpha\}$ is a well-ordered decreasing system of single valued neutrosophic S^* closed sets of S. Then by Property 4.1, the set $\widetilde{F}_\alpha$ form a single valued neutrosophic S^* centered system in $w(S)$. Also, since all the $F_\alpha^{'}$s are single valued neutrosophic S^* compact space (Property 4.2), hence, $\bigcap_\alpha \widetilde{F}_\alpha \neq \phi$. Let $C \in \widetilde{F}$. Then $\pi_S(C) \in F_\alpha$ for every α, that is, $\bigcap_\alpha F_\alpha = \varphi$, as required.

Property 4.5:

Any well-ordered sequence of decreasing single valued neutrosophic S^*H- closed sets in a single valued neutrosophic S^* Hausdorff space has a non-empty intersection.

Proof:

From the proof of Property 4.1 it is easy to see that $\pi_S(\widetilde{F}) = F$. However, in general $\widetilde{F}$ does not coincide with $\pi_s^{-1}(F)$. Also, in the proof of Theorem 4.4, it cannot be taken $\pi_s^{-1}(F)$ instead of $\widetilde{F}$, since the complete inverse image of a single valued neutrosophic S^* H- closed set need not be single valued neutrosophic S^* compact . In fact , let S be a single valued neutrosophic S^* Hausdorff space and F a single valued neutrosophic S^*H- closed subset such that there is a member $A \in S \backslash F$ for which there does not exist disjoint single valued neutrosophic S^* open neighbourhoods of A and F. Note that in a single valued neutrosophic S^* Hausdorff space two disjoint single valued neutrosophic S^* compact sets have disjoint single valued neutrosophic S^* open neighbourhoods. If $\pi_s^{-1}(F)$ were single valued neutrosophic S^* compact, then the single valued neutrosophic S^* compact sets $\pi_s^{-1}(F)$ and $\pi_s^{-1}(A)$ would have disjoint single valued neutrosophic S^* open neighbourhoods in $w(S)$, say U and V. Then it follows from the proof of the theorem that $SVNS^*int(SVNS^*cl(\pi_S(U))$ and $SVNS^*int(SVNS^*cl(\pi_S(V)))$ would be disjoint single valued neutrosophic S^* open neighbourhoods of F and A in S, which contradicts our assumption.

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