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Ratio-Product Type Exponential Estimator for Estimating Finite Population Mean Using Information on Auxiliary Attribute

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Abstract

In practice, the information regarding the population proportion possessing certain attribute is easily available, see Jhajj et.al. (2006). For estimating the population mean $\overline{Y}$ of the study variable y, following Bahl and Tuteja (1991), a ratio-product type exponential estimator has been proposed by using the known information of population proportion possessing an attribute (highly correlated with y) in simple random sampling. The expressions for the bias and the mean-squared error (MSE) of the estimator and its minimum value have been obtained. The proposed estimator has an improvement over mean per unit estimator, ratio and product type exponential estimators as well as Naik and Gupta (1996) estimators. The results have also been extended to the case of two phase sampling. The results obtained have been illustrated numerically by taking some empirical populations considered in the literature.

Keywords: Proportion, bias, mean-squared error, two phase sampling.

1. Introduction

In survey sampling, the use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with the auxiliary variable x. but in several practical situations, instead of existence of auxiliary variables there exists some auxiliary attributes, which are highly correlated with study variable y, such as

(i) Amount of milk produced and a particular breed of cow. (ii) Yield of wheat crop and a particular variety of wheat etc. (see Shabbir and Gupta (2006)).

In such situations, taking the advantage of point biserial correlation between the study variable and the auxiliary attribute, the estimators of parameters of interest can be constructed by using prior knowledge of the parameters of auxiliary attribute.

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N. let y_i and ϕ_i denote the observations on variable y and ϕ respectively for the ith unit ($i = 1,2,..., N$). We note that ϕ_i = 1, if ith unit of population possesses attribute ϕ and ϕ_i = 0, otherwise. Let $A = \sum_{i=1}^{N} \phi_i$ and $a = \sum_{i=1}^{n} \phi_i$ denote the total number of units in the population and sample respectively possessing attribute ϕ. Let $P = \frac{A}{N}$ and $p = \frac{a}{n}$ denote the proportion of units in the population and sample respectively possessing attribute ϕ.

In order to have an estimate of the population mean $\overline{Y}$ of the study variable y, assuming the knowledge of the population proportion P, Naik and Gupta (1996) defined ratio and product estimators of population when the prior information of population proportion of units, possessing the same attribute is available. Naik and Gupta (1996) proposed following estimators:

$$t_1 = \bar{y}\left(\frac{P}{p}\right) \tag{1.1}$$

$$t_2 = \bar{y}\left(\frac{p}{P}\right) \tag{1.2}$$

The MSE of t_1 and t_2 up to the first order of approximation are

$$\mathrm{MSE}(t_1) = f_1 \bar{Y}^2 \left[C_y^2 + C_p^2 (1 - 2K_p)\right] \tag{1.3}$$

$$\mathrm{MSE}(t_2) = f_1 \bar{Y}^2 \left[C_y^2 + C_p^2 (1 + 2K_p)\right] \tag{1.4}$$

where $C_y^2 = \dfrac{S_y^2}{\bar{Y}^2}, C_p^2 = \dfrac{S_\phi^2}{P^2}, f_1 = \dfrac{1}{n} - \dfrac{1}{N}, K_p = \rho_{pb} \dfrac{C_y}{C_p}, S_y^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2,$

$S_\phi^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}(\phi_i - P)^2$, $S_{y\phi} = \dfrac{1}{N-1}\left(\sum_{i=1}^{N} y_i \phi_i - NP\bar{Y}\right)$ and

$\rho_{pb} = \dfrac{S_{y\phi}}{S_y S\phi}$ is the point biserial correlation coefficient.

Following Bahl and Tuteja (1991), we propose the following ratio and product exponential estimators

$$t_3 = \bar{y}\exp\left(\frac{P - p}{P + p}\right) \tag{1.5}$$

$$t_4 = \bar{y}\exp\left(\frac{p - P}{p + P}\right) \tag{1.6}$$

2. Bias and MSE of t_3 and t_4

To obtain the bias and MSE of t_3 to the first degree of approximation, we define

$e_y = \dfrac{(\bar{y} - \bar{Y})}{\bar{Y}}, e_\phi = \dfrac{(p - P)}{P}$, therefore E ($e_i$) = 0. $i = (y, \phi)$,

$E(e_y^2) = f_1 C_y^2$, $E(e_\phi^2) = f_1 C_p^2, E(e_y e_\phi) = f_1 \rho_{pb} C_y C_p$.

Expressing (1.5) in terms of e's, we have

$$t_3 = \overline{Y}(1+e_y)\exp\left[\frac{P-P(1+e_\phi)}{P+P(1+e_\phi)}\right]$$

$$= \overline{Y}(1+e_y)\exp\left[\frac{-e_\phi}{(2+e_\phi)}\right] \qquad (2.1)$$

Expanding the right hand side of (2.1) and retaining terms up to second powers of e's, we have

$$t_3 = \overline{Y}\left[1+e_y-\frac{e_\phi}{2}+\frac{e_\phi^2}{8}-\frac{e_y e_\phi}{2}\right] \qquad (2.2)$$

Taking expectations of both sides of (2.2) and then subtracting $\overline{Y}$ from both sides, we get the bias of the estimator t_3 up to the first order of approximation, as

$$B(t_3) = f_1\overline{Y}\frac{C_p^2}{2}\left(\frac{1}{4}-K_p\right) \qquad (2.3)$$

From (2.2), we have

$$\left(t_3-\overline{Y}\right) \cong \overline{Y}\left[e_y-\frac{e_\phi}{2}\right] \qquad (2.4)$$

Squaring both sides of (2.4) and then taking expectations we get MSE of the estimator t_3, up to the first order of approximation as

$$MSE(t_3) = f_1\overline{Y}^2\left[C_y^2+C_p^2(\frac{1}{4}-K_p)\right] \qquad (2.5)$$

To obtain the bias and MSE of t_4 to the first degree of approximation, we express (1.6) in terms of e's

$$t_4 = \bar{Y}(1+e_y)\exp\left[\frac{P(1+e_\phi)-P}{P(1+e_\phi)+P}\right] \tag{2.6}$$

and following the above procedure, we get the bias and MSE of t_4 as follows

$$B(t_4) = f_1\bar{Y}\frac{C_p^2}{2}\left(\frac{1}{4}+K_p\right) \tag{2.7}$$

$$MSE(t_4) = f_1\bar{Y}^2\left[C_y^2 + C_p^2(\frac{1}{4}+K_p)\right] \tag{2.8}$$

3. Proposed class of estimators

It has been theoretically established that, in general, the linear regression estimator is more efficient than the ratio (product) estimator except when the regression line of y on x passes through the neighborhood of the origin, in which case the efficiencies of these estimators are almost equal. Also in many practical situations the regression line does not pass through the neighborhood of the origin. In these situations, the ratio estimator does not perform as good as the linear regression estimator. The ratio estimator does not perform well as the linear regression estimator does.

Following Singh and Espejo (2003), we propose following class of ratio-product type exponential estimators:

$$t_5 = \bar{y}\left[\alpha\exp\left(\frac{P-p}{P+p}\right)+(1-\alpha)\exp\left(\frac{p-P}{p+P}\right)\right] \tag{3.1}$$

where α is a real constant to be determined such that the MSE of t_5 is minimum.

For α=1, t_5 reduces to the estimator $t_3 = \bar{y}\exp\left(\frac{P-p}{P+p}\right)$ and for α = 0, it reduces to

$t_4 = \bar{y}\exp\left(\frac{p-P}{p+P}\right)$.

Bias and MSE of t_5:

Expressing (3.1) in terms of e's, we have

$$t_5 = \overline{Y}\left(1+e_y\right)\left[\alpha \exp\left\{\frac{P-P\left(1+e_\phi\right)}{P+P\left(1+e_\phi\right)}\right\}+(1-\alpha)\exp\left\{\frac{P(1+e_\phi)-P}{P(1+e_\phi)+P}\right\}\right]$$

$$= \overline{Y}\left(1+e_y\right)\left[\alpha \exp\left\{\frac{-e_\phi}{2}\right\}+(1-\alpha)\exp\left\{\frac{e_\phi}{2}\right\}\right] \tag{3.2}$$

Expanding the right hand side of (3.2) and retaining terms up to second powers of e's, we have

$$t_5 = \overline{Y}\left[1+e_y+\frac{e_\phi}{2}-\alpha e_\phi+\frac{e_\phi^2}{8}+e_y e_\phi-\alpha e_y e_\phi\right] \tag{3.3}$$

Taking expectations of both sides of (3.3) and then subtracting $\overline{Y}$ from both sides, we get the bias of the estimator t_5 up to the first order of approximation, as

$$B(t_5) = f_1\overline{Y}\left[\frac{C_p^2}{8}+\rho_{pb}C_yC_p\left(\frac{1}{2}-\alpha\right)\right] \tag{3.4}$$

From (3.3), we have

$$\left(t_5-\overline{Y}\right) \cong \overline{Y}\left[e_y+e_\phi\left(\frac{1}{2}-\alpha\right)\right] \tag{3.5}$$

Squaring both sides of (3.5) and then taking expectations we get MSE of the estimator t_5, up to the first order of approximation as

$$MSE(t_5) = f_1\overline{Y}^2\left[C_y^2+C_p^2\left(\frac{1}{4}+\alpha^2-\alpha\right)+2\rho_{pb}C_yC_p\left(\frac{1}{2}-\alpha\right)\right] \tag{3.6}$$

Minimization of (3.6) with respect to α yields optimum value of as

$$\alpha = \frac{2K_p+1}{2} = \alpha_0 \text{ (Say)} \tag{3.7}$$

Substitution of (3.7) in (3.1) yields the optimum estimator for t_5 as $(t_5)_{opt}$ (say)with minimum MSE as

$$\min.MSE(t_5) = f_1 \bar{Y}^2 C_y^2 \left(1 - \rho_{pb}^2\right) = M(t_5)_{opt} \tag{3.8}$$

which is same as that of traditional linear regression estimator.

4. Efficiency comparisons

In this section, the conditions for which the proposed estimator t_5 is better than $\bar{y}$, t_1, t_2, t_3, and t_4 have been obtained. The variance of $\bar{y}$ is given by

$$var(\bar{y}) = f_1 \bar{Y}^2 C_y^2 \tag{4.1}$$

To compare the efficiency of the proposed estimator t_5 with the existing estimator, from (4.1) and (1.3), (1.4), (2.5), (2.8) and (3.8), we have

$$var(\bar{y}) - M(t_5)_0 = \rho_{pb}^2 \geq 0. \tag{4.2}$$

$$MSE(t_1) - M(t_5)_0 = \left(C_p - \rho_{pb} C_y\right)^2 \geq 0. \tag{4.3}$$

$$MSE(t_2) - M(t_5)_0 = \left(C_p + \rho_{pb} C_y\right)^2 \geq 0. \tag{4.4}$$

$$MSE(t_3) - M(t_5)_0 = \left(\frac{C_p^2}{2} - \rho_{pb} C_y\right)^2 \geq 0. \tag{4.5}$$

$$MSE(t_4) - M(t_5)_0 = \left(\frac{C_p^2}{2} + \rho_{pb} C_y\right)^2 \geq 0. \tag{4.6}$$

Using (4.2)-(4.6), we conclude that the proposed estimator t_5 outperforms $\bar{y}$, t_1, t_2, t_3, and t_4.

5. Empirical study

We now compare the performance of various estimators considered here using the following data sets:

Population 1. [Source: Sukhatme and Sukhatme (1970), p. 256]

y = number of villages in the circles and

ϕ = A circle consisting more than five villages.

N = 89, $\bar{Y}$ =3.360, P = 0.1236, ρ_{pb} = 0.766, C_y = 0.60400, C_p = 2.19012.

Population 2. [Source: Mukhopadhyaya, (2000), p. 44]

Y= Household size and

ϕ = A household that availed an agricultural loan from a bank.

N = 25, $\bar{Y}$ =9.44, P = 0.400, ρ_{pb} = -0.387, C_y = 0.17028, C_p = 1.27478.

The percent relative efficiency (PRE's) of the estimators $\bar{y}$, t_1-t_4 and $(t_5)_{opt}$ with respect to unusual unbiased estimator $\bar{y}$ have been computed and compiled in table 5.1.

Table 5.1: PRE of various estimators with respect to $\bar{y}$.

Estimator	**PRE's (., $\bar{y}$)**	
	Population	
	I	**II**
$\bar{y}$	100	100
t_1	11.63	1.59
t_2	5.07	1.94
t_3	66.24	5.57
t_4	14.15	8.24
$(t_5)_0$	241.98	117.61

Table 5.1 shows that the proposed estimator t_5 under optimum condition performs better than the usual sample mean $\bar{y}$, Naik and Gupta (1996) estimators (t_1 and t_2) and the ratio and product type exponential estimators (t_3 and t_4).

6. Double sampling

In some practical situations when P is not known a priori, the technique of two-phase sampling is used. Let p' denote the proportion of units possessing attribute ϕ in the first phase sample of size n'; p denote the proportion of units possessing attribute ϕ in the second phase sample of size n < n' and $\bar{y}$ denote the mean of the study variable y in the second phase sample.

When P is not known, two-phase ratio and product type exponential estimator are given by

$$t_6 = \bar{y}\exp\left(\frac{p'-p}{p'+p}\right) \tag{6.1}$$

$$t_7 = \bar{y}\exp\left(\frac{p-p'}{p+p'}\right) \tag{6.2}$$

To obtain the bias and MSE of t_6 and t_7, we write

$$\bar{y} = \bar{Y}(1+e_y),\ p = P(1+e_\phi),\ p' = P(1+e'_\phi)$$

such that

$$E(e_y) = E(e_\phi) = E(e'_\phi) = 0.$$

and

$$E(e_y^2) = f_1C_y^2,\ E(e_\phi^2) = f_1C_p^2,\ E(e'_\phi)^2 = f_2C_p^2,\ E(e_\phi e'_\phi) = f_2\rho_{pb}C_yC_p.$$

where $f_2 = \frac{1}{n'} - \frac{1}{N}$.

Expressing (6.1) in terms of e's, we have

$$t_6 = \bar{Y}(1+e_y)\exp\left[\frac{P(1+e'_\phi) - P(1+e_\phi)}{P(1+e'_\phi) + P(1+e_\phi)}\right]$$

$$= \bar{Y}(1+e_y)\exp\left[\frac{e'_\phi - e_\phi}{2}\right] \tag{6.3}$$

Expanding the right hand side of (6.3) and retaining terms up to second powers of e's, we have

$$t_6 = \bar{Y}\left[1 + e_y + \frac{e'_\phi}{2} - \frac{e_\phi}{2} + \frac{e'^2_\phi}{8} + \frac{e^2_\phi}{8} - \frac{e_\phi e'_\phi}{4} + \frac{e_y e'_\phi}{2} - \frac{e_y e_\phi}{2}\right] \tag{6.4}$$

Taking expectations of both sides of (6.4) and then subtracting $\bar{Y}$ from both sides, we get the bias of the estimator t_6 up to the first order of approximation, as

$$B(t_6) = f_3 \bar{Y} \frac{C_p^2}{4}\left(1 - 2K_p\right) \tag{6.5}$$

From (6.4), we have

$$\left(t_6 - \bar{Y}\right) \cong \bar{Y}\left[e_y + \frac{(e'_\phi - e_\phi)}{2}\right] \tag{6.6}$$

Squaring both sides of (6.6) and then taking expectations we get MSE of the estimator t_6, up to the first order of approximation as

$$MSE(t_6) = \bar{Y}^2\left[f_1 C_y^2 + f_3 \frac{C_p^2}{4}\left(1 - 4K_p\right)\right] \tag{6.7}$$

To obtain the bias and MSE of t_7 to the first degree of approximation, we express (6.2) in terms of e's as

$$t_7 = \bar{Y}(1+e_y)\exp\left[\frac{P(1+e_\phi) - P(1+e'_\phi)}{P(1+e_\phi) + P(1+e'_\phi)}\right]$$

$$= \bar{Y}(1+e_y)\exp\left[\frac{e_\phi - e'_\phi}{2}\right] \tag{6.8}$$

Expanding the right hand side of (6.8) and retaining terms up to second powers of e's, we have

$$t_7 = \bar{Y}\left[1 + e_y + \frac{e_\phi}{2} - \frac{e'_\phi}{2} + \frac{e'^2_\phi}{8} + \frac{e^2_\phi}{8} - \frac{e_\phi e'_\phi}{4} + \frac{e_y e_\phi}{2} - \frac{e_y e'_\phi}{2}\right] \tag{6.9}$$

Taking expectations of both sides of (6.9) and then subtracting $\bar{Y}$ from both sides, we get the bias of the estimator t_7 up to the first order of approximation, as

$$B(t_7) = f_3 \bar{Y} \frac{C_p^2}{4}\left(1 + 2K_p\right) \tag{6.10}$$

From (6.9), we have

$$\left(t_7 - \bar{Y}\right) \cong \bar{Y}\left[e_y + \frac{(e_\phi - e'_\phi)}{2}\right] \tag{6.11}$$

Squaring both sides of (6.11) and then taking expectations we get MSE of the estimator t_7, up to the first order of approximation as

$$MSE(t_7) = \bar{Y}^2\left[f_1 C_y^2 + f_3 \frac{C_p^2}{4}\left(1 + 4K_p\right)\right] \tag{6.12}$$

7. Proposed class of estimators in double sampling

We propose the following class of estimators in double sampling

$$t_8 = \bar{y}\left[\alpha_1 \exp\left(\frac{p' - p}{p' + p}\right) + (1 - \alpha_1)\exp\left(\frac{p - p'}{p + p'}\right)\right] \tag{7.1}$$

where α_1 is a real constant to be determined such that the MSE of t_8 is minimum.

For $\alpha_1 = 1$, t_8 reduces to the estimator $t_6 = \bar{y}\exp\left(\frac{p'-p}{p'+p}\right)$ and for $\alpha_1 = 0$, it reduces to $t_7 = \bar{y}\exp\left(\frac{p-p'}{p+p'}\right)$.

Bias and MSE of t_8:

Expressing (7.1) in terms of e's, we have

$$t_8 = \bar{Y}(1+e_y)\left[\alpha_1 \exp\left\{\frac{P(1+e'_\phi)-P(1+e\phi)}{P(1+e'_\phi)+P(1+e\phi)}\right\}+(1-\alpha_1)\exp\left\{\frac{P(1+e_\phi)-P(1+e'_\phi)}{P(1+e_\phi)+P(1+e'_\phi)}\right\}\right]$$

$$= \bar{Y}(1+e_y)\left[\alpha_1 \exp\left\{\frac{e'_\phi - e_\phi}{2}\right\}+(1-\alpha_1)\exp\left\{\frac{e_\phi - e'_\phi}{2}\right\}\right] \qquad (7.2)$$

Expanding the right hand side of (7.2) and retaining terms up to second powers of e's, we have

$$t_8 = \bar{Y}[1+e_y+\frac{e_\phi}{2}-\frac{e'_\phi}{2}-\alpha_1 e_\phi+\alpha_1 e'_\phi+\frac{e_\phi^2}{8}+\frac{e'^{\,2}_\phi}{8}+\frac{e_y e_\phi}{2}-\frac{e_y e'_\phi}{2}$$

$$-\frac{e_\phi e'_\phi}{4}+\alpha_1 e_y e'_\phi-\alpha_1 e_y e_\phi] \qquad (7.3)$$

Taking expectations of both sides of (7.3) and then subtracting $\bar{Y}$ from both sides, we get the bias of the estimator t_8 up to the first order of approximation, as

$$B(t_8) = f_3\bar{Y}\frac{C_p^2}{8}\left[1-8K_p\left(\alpha_1-\frac{1}{2}\right)\right] \qquad (7.4)$$

From (7.3), we have

$$(t_8-\bar{Y}) \cong \bar{Y}\left[e_y-\left(\alpha_1-\frac{1}{2}\right)e_\phi+\left(\alpha_1-\frac{1}{2}\right)e'_\phi\right] \qquad (7.5)$$

Squaring both sides of (7.5) and then taking expectations we get MSE of the estimator t_8, up to the first order of approximation as

$$MSE(t_8) = \bar{Y}^2\left[f_1C_y^2 + f_3C_p^2\left(\alpha_1 - \frac{1}{2}\right)\left\{\left(\alpha_1 - \frac{1}{2}\right) - 2K_p\right\}\right] \quad (7.6)$$

Minimization of (7.6) with respect to α_1 yields optimum value of as

$$\alpha_1 = \frac{2K_p + 1}{2} = \alpha_{10}\,(\text{Say}) \quad (7.7)$$

Substitution of (7.7) in (7.1) yields the optimum estimator for t_8 as $(t_8)_{opt}$ (say) with minimum MSE as

$$\min.MSE(t_8) = \bar{Y}^2C_y^2\left(f_1 - f_3\rho_{pb}^2\right) = M(t_8)_o, \text{ (say)} \quad (7.8)$$

which is same as that of traditional linear regression estimator.

8. Efficiency comparisons

The MSE of usual two-phase ratio and product estimator is given by

$$MSE(t_9) = \bar{Y}^2\left[f_1C_y^2 + f_3C_p^2(1 - 2K_p)\right] \quad (8.1)$$

$$MSE(t_{10}) = \bar{Y}^2\left[f_1C_y^2 + f_3C_p^2(1 + 2K_p)\right] \quad (8.2)$$

From (4.1), (6.7), (6.12), (8.1), (8.2) and (7.8) we have

$$var(\bar{y}) - M(t_8)_0 = f_3\rho_{pb}^2 \geq 0. \quad (8.3)$$

$$MSE(t_6) - M(t_8)_0 = f_3\left(\frac{C_p}{2} - \rho_{pb}C_y\right)^2 \geq 0. \quad (8.4)$$

$$MSE(t_7) - M(t_8)_0 = f_3\left(\frac{C_p}{2} + \rho_{pb}C_y\right)^2 \geq 0. \quad (8.5)$$

$$MSE(t_9) - M(t_8)_0 = f_3\left(C_p - \rho_{pb}C_y\right)^2 \geq 0. \quad (8.6)$$

$$MSE(t_{10}) - M(t_8)_0 = f_3\left(C_p + \rho_{pb}C_y\right)^2 \geq 0\,. \tag{8.7}$$

From (8.3)-(8.7), we conclude that our proposed estimator t_8 is better than $\bar{y}$, t_6, t_7, t_9, and t_{10}.

9. Empirical study

The various results obtained in the previous section are now examined with the help of following data:

Population 1. [Source: Sukhatme and Sukhatme(1970), p. 256]

$N = 89$, $n' = 45$, $n = 23$, $\bar{y} = 1322$, $p = 0.1304$, $p' = 0.1333$, $\rho_{pb} = 0.408$, $C_y = 0.69144$, $C_p = 2.7005$.

Population 2. [Source: Mukhopadhyaya(2000), p. 44]

$N = 25$, $n' = 13$, $n = 7$, $\bar{y} = 7.143$, $p = 0.294$, $p' = 0.308$, $\rho_{pb} = -0.314$, $C_y = 0.36442$, $Cp = 1.34701$.

Table 9.1: PRE of various estimators (double sampling) with respect to $\bar{y}$.

Estimator	**PRE's (., $\bar{y}$)**	
	Population	
	I	**II**
$\bar{y}$	100	100
t_6	40.59	25.42
t_7	21.90	40.89
t_9	11.16	8.89
t_{10}	7.60	12.09
$(t_8)_0$	112.32	106.74

Table 9.1 shows that the proposed estimator t_8 under optimum condition performs better than the usual sample mean $\bar{y}$, t_6, $t_{7,}$ t_9, and t_{10}.

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