Some Types of Neutrosophic Crisp Sets and Neutrosophic Crisp Relations

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Abstract**—**The purpose of this paper is to introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

Index Terms**—**Neutrosophic set, neutrosophic crisp sets, neutrosophic crisp relations, generalized neutrosophic set, Intuitionistic neutrosophic Set.

I. Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [16, 17, 18], and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 19] such as a neutrosophic set theory. In this paper we introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set. Section 3 presents new types of neutrosophic crisp sets and studied some of their basic properties. Section 4 presents the concept of neutrosophic crisp relations . Finally we concludes the paper.

II. Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16 , 17, 18], and Salama et al. [4,5]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is nonstandard unit interval.

Definition 2.1 *[9, 13, 15]*

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp event in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$, Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:

i)Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii)Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv)Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

iii) Type3: $X_N = \langle X, X, \phi \rangle$,

iv) Type4: $X_N = \langle X, X, X \rangle$,

Definition 2.2 *[9, 13, 15]*

Let $A = \langle A_1, A_2, A_3 \rangle$ a NCE or UNCE on X , then the complement of the set A (A^c, for short) maybe defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

One can define several relations and operations between NCS as follows:

Definition 2.3 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$)

($A \subseteq B$) may be defined as two types:

1)Type1:

$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or

2)Type2:

$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$

Definition 2.4 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ are NCSS Then

1) $A \cap B$ may be defined as two types:

i) Type1:

$A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or

ii) Type2:

$A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2) $A \cup B$ may be defined as two types:

i) Type 1: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

or

ii)Type 2: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

Proposition 2.1 *[9, 13, 15]*

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets in X, then

1) $\cap A_j$ may be defined two types as :

i)Type1: $\cap A_j = \langle \cap Aj_1, \cap A_{j2}, \cup A_{j3} \rangle$,or

ii)Type2: $\cap A_j = \langle \cap Aj_1, \cup A_{j2}, \cup A_{j3} \rangle$.

2) $\cup A_j$ may be defined two types as :

i)Type1: $\cup A_j = \langle \cup Aj_1, \cap A_{j2}, \cap A_{j3} \rangle$ or

ii)Type2: $\cup A_j = \langle \cup Aj_1, \cup A_{j2}, \cap A_{j3} \rangle$.

III. New Types of Neutrosophic Crisp Sets

We shall now consider some possible definitions for some types of neutrosophic crisp sets

Definition 3.1

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$ where A_1, A_2 and A_3 are subsets of X .

Definition 3.2

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) (Neutrosophic Crisp Set with Type 1) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$. (NCS-Type1 for short).

2) (Neutrosophic Crisp Set with Type 2) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type2 for short).

3) (Neutrosophic Crisp Set with Type 3) If satisfying

$A_1 \cap A_2 \cap A_3 = \phi$ and

$A_1 \cup A_2 \cup A_3 = X$. (NCS-Type3 for short).

Definition 3.3

1) (Neutrosophic Set [7]): Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x \ , \nu_A(x) \le 1^+$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

2) (Generalized Neutrosophic Set [8]**)**: Let X be a non-empty fixed set. A generalized neutrosophic (GNS for short) set A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x \ , \nu_A(x) \le 1^+$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

3) (Intuitionistic Neutrosophic Set [16]**)**. Let X be a non-empty fixed set. An intuitionistic neutrosophic set A (INS for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \le 0.5$, $\mu_A(x) \wedge \nu_A(x) \le 0.5$, $\sigma_A(x) \wedge \nu_A(x) \le 0.5$, and ${}^-0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+$.

A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously on NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS ⟶ Generalized NS ⟶ NS

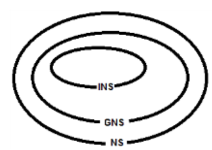

Fig.1: Represents the relation between types of NS

Corollary 3.1

Let X non-empty fixed set and $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be INS on X

Then:

1) Type1- A^c of INS be a GNS.
2) Type2- A^c of INS be a INS.
3) Type3- A^c of INS be a GNS.

Proof

Since A INS then $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$, and $\mu_A(x) \wedge \sigma_A(x) \le 0.5, \nu_A(x) \wedge \mu_A(x) \le 0.5$ $\nu_A(x) \wedge \sigma_A(x) \le 0.5$ Implies $\mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x) \le 0.5$ then is not to be Type1- A^c INS. On other hand the Type 2- A^c, $A^c = \langle \nu_A(x), \sigma_A(x), \mu_A(x) \rangle$ be INS and Type 3- A^c, $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ and $\sigma^c{}_A(x) \le 0.5$ implies to $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ GNS and not to be INS

Example 3.1

Let $X = \{a, b, c\}$, and A, B, C are neutrosophic sets on X,

$A = \langle 0.7, 0.9, 0.8) \setminus a, (0.6, 0.7, 0.6) \setminus b, (0.9, 0.7, 0.8 \setminus c \rangle$,

$B = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$

$C = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.8, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$

By the Definition 3.3 no.3

$\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \geq 0.5$, A be not GNS and INS,

$B = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$

not INS, where $\sigma_A(b) = 0.4 < 0.5$. Since $\mu_B(x) \wedge \sigma_B(x) \wedge \nu_B(x) \leq 0.5$ then B is a GNS but not INS.

$A^c = \langle 0.3, 0.1, 0.2) \setminus a, (0.4, 0.3, 0.4) \setminus b, (0.1, 0.3, 0.2 \setminus c \rangle$

be a GNS, but not INS.

$B^c = \langle 0.3, 0.1, 0.5) \setminus a, (0.4, 0.6, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c \rangle$

be a GNS, but not INS, C be INS and GNS,

$C^c = \langle 0.3, 0.1, 0.5) \setminus a, (0.4, 0.2, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c \rangle$

be a GNS but not INS.

Definition 3.4

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$,

Salama et al in [6,13] constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:

i) Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii) Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv) Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

v) Type3: $X_N = \langle X, X, \phi \rangle$,

vi) Type4: $X_N = \langle X, X, X \rangle$,

Definition 3.5

A NCS-Type1 ϕ_{N_1}, X_{N_1} in X as follows:

1) ϕ_{N1} may be defined as three types:

i) Type1: $\phi_{N_1} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N_1} = \langle \phi, X, \phi \rangle$, or

iii) Type3: $\phi_N = \langle \phi, \phi, \phi \rangle$.

2) X_{N1} may be defined as one type

Type1: $X_{N_1} = \langle X, \phi, \phi \rangle$.

Definition 3.6

A NCS-Type2, ϕ_{N_2}, X_{N2} in X as follows:

1) ϕ_{N2} may be defined as two types:

i) Type1: $\phi_{N_2} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N_2} = \langle \phi, X, \phi \rangle$

2) X_{N2} may be defined as one type

Type1: $X_{N_2} = \langle X, \phi, \phi \rangle$

Definition 3.7

a NCS-Type 3, ϕ_{N3}, X_{N3} in X as follows:

1) ϕ_{N3} may be defined as three types:

i) Type1: $\phi_{N3} = \langle \phi, \phi, X \rangle$, or

ii) Type2: $\phi_{N3} = \langle \phi, X, \phi \rangle$, or

iii) Type3: $\phi_{N3} = \langle \phi, X, X \rangle$.

2) X_{N3} may be defined as three types

i)Type1: $X_{N3} = \langle X, \phi, \phi \rangle$,

ii)Type2: $X_{N3} = \langle X, X, \phi \rangle$,

iii)Type3: $X_{N3} = \langle X, \phi, X \rangle$,

Corollary 3.1

In general

1-Every NCS-Type 1, 2, 3 are NCS.
2-Every NCS-Type 1 not to be NCS-Type2, 3.
3-Every NCS-Type 2 not to be NCS-Type1, 3.
4-Every NCS-Type 3 not to be NCS-Type2, 1, 2.
5-Every crisp set be NCS.

The following Venn diagram represents the relation between NCSs

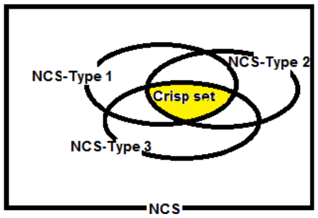

Fig. 2: Venn diagram represents the relation between NCSs

Example 3.2

Let A, B, C, D are NCSs on $X = \{a,b,c,d,e,f\}$, the following types of neutrosophic crisp sets

i) $A = \langle \{a\},\{b\},\{c\} \rangle$ be a NCS-Type 1, but not NCS-Type 2 and Type 3

ii) $B = \langle \{a,b\},\{c,d\},\{f,e\} \rangle$ be a NCS-Type 1, 2, 3

iii) $C = \langle \{a,b,c,d\},\{e\},\{a,b,f\} \rangle$ be a NCS-Type 3 but not NCS-Type 1, 2.

iv) $D = \langle \{a,b,c,d\},\{a,b,c\},\{a,b,d,f\} \rangle$ be a NCS but not NCS-Type 1, 2, 3.

The complement for A, B, C, D may be equals

The complement of A

i)Type 1: $A^c = \langle \{b,c,d,e,f\},\{a,c,d,e,f\},\{a,b,d,e,f\} \rangle$ be a NCS but not NCS—Type1, 2,3

ii)Type 2: $A^c = \langle \{c\},\{b\},\{a\} \rangle$ be a NCS-Type 3 but not NCS—Type1, 2

iii)Type 3: $A^c = \langle \{c\},\{a,c,d,e,f\},\{a\} \rangle$ be a NCS-Type 1 but not NCS—Type 2, 3.

The complement of B may be equals

i)Type 1: $B^c = \langle \{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii) Type 2: $B^c = \langle \{e,f\},\{c,d\},\{a,b\} \rangle$ be NCS-Type 1, 2, 3.

iii)Type 3: $B^c = \langle \{e,f\},\{a,b,e,f\},\{a,b\} \rangle$ be NCS-Type 3, but not NCS-Type 1, 2.

The complement of C may be equals

i)Type 1: $C^c = \langle \{e,f\},\{a,b,c,d,f\},\{c,d,e\} \rangle$.

ii)Type 2: $C^c = \langle \{a,b,f\},\{e\},\{a,b,c,d\} \rangle$,

iii)Type 3: $C^c = \langle \{a,b,f\},\{a,b,c,d\},\{a,b,c,d\} \rangle$,

The complement of D may be equals

i)Type 1: $D^c = \langle \{e,f\},\{d,e,f\},\{c,e\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii)Type 2: $D^c = \langle \{a,b,d,f\},\{a,b,c\},\{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

iii)Type 3: $D^c = \langle \{a,b,d,f\},\{d,e,f\},\{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

Definition 3.8

Let X be a non-empty set, $A = \langle A_1, A_2, A_3 \rangle$

1) If A be a NCS-Type1 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement Type1: $A^c = \langle A_3, A_2, A_1 \rangle$.

2) If A be a NCS-Type 2 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement $A^c = \langle A_3, A_2, A_1 \rangle$.

3)If A be NCS-Type3 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

Example 3.3

Let $X = \{a,b,c,d,e,f\}$, $A = \langle \{a,b,c,d\},\{e\},\{f\} \rangle$ be a NCS-Type 2, $B = \langle \{a,b,c\},\{\phi\},\{d,e\} \rangle$ be a NCS-Type1., $C = \langle \{a,b\},\{c,d\},\{e,f\} \rangle$ NCS-Type 3, then the complement $A = \langle \{a,b,c,d\},\{e\},\{f\} \rangle$, $A^c = \langle \{f\},\{e\},\{a,b,c,d\} \rangle$ NCS-Type 2, the complement of $B = \langle \{a,b,c\},\{\phi\},\{d,e\} \rangle$, $B^c = \langle \{d,e\},\{\phi\},\{a,b,c\} \rangle$ NCS-Type1. The complement of $C = \langle \{a,b\},\{c,d\},\{e,f\} \rangle$ may be defined as three types:

Type 1: $C^c = \langle \{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\} \rangle$.

Type 2: $C^c = \langle \{e,f\},\{c,d\},\{a,b\} \rangle$,

Type 3: $C^c = \langle \{e,f\},\{a,b,e,f\},\{a,b\} \rangle$,

Proposition 3.1

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets on X, then

1) $\cap A_j$ may be defined two types as :

Type1: $\cap A_j = \left\langle \cap Aj_1, \cap A_{j_2}, \cup A_{j_3} \right\rangle$,or

Type2: $\cap A_j = \left\langle \cap Aj_1, \cup A_{j_2}, \cup A_{j_3} \right\rangle$.

2) $\cup A_j$ may be defined two types as :

Type1: $\cup A_j = \left\langle \cup Aj_1, \cap A_{j_2}, \cap A_{j_3} \right\rangle$ or

Type2: $\cup A_j = \left\langle \cup Aj_1, \cup A_{j_2}, \cap A_{j_3} \right\rangle$.

Definition 3.9

(a) If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.

(b) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3)^c) \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.2

Let A, $\{A_i : i \in J\}$, be a family of NCS in X, and B, $\{B_j : j \in K\}$ NCS in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,
$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$,

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$,

(d) $f^{-1}(\cup B_i) = f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_{ii}) = \cup f(A_{ii})$; $f(\cap A_{ii}) \subseteq \cap f(A_{ii})$; and if f is injective, then $f(\cap A_{ii}) = \cap f(A_{ii})$;

(f) $f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N$, $f(X_N) = Y_N$, if f is subjective.

Proof

Obvious

IV. Neutrosophic Crisp Relations

Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three ordinary nonempty sets

Definition 4.1

Let X and Y are two non-empty crisp sets and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y. Then

i) The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by $A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$ on $X \times Y$.

ii) We will call a neutrosophic crisp relation $R \subseteq A \times B$ on the direct product $X \times Y$.

The collection of all neutrosophic crisp relations on $X \times Y$ is denoted as $NCR(X \times Y)$

Definition 4.2

Let R be a neutrosophic crisp relation on $X \times Y$, then the inverse of R is denoted by R^{-1} where $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$ on $Y \times X$.

Example 4.1

Let $X = \{a,b,c,d\}$, $A = \langle \{a,b\},\{c\},\{d\} \rangle$ and $B = \langle \{a\},\{c\},\{d,b\} \rangle$ then the product of two neutrosophic crisp sets given by

$A \times B = \langle \{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\} \rangle$

and

$B \times A = \langle \{(a,a),(a,b)\},\{(c,c)\},\{(d,d),(b,d)\} \rangle$,

and

$R_1 = \langle \{(a,a)\},\{(c,c)\},\{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\},\{(c,c)\},\{(d,d),(b,d)\} \rangle$

$R_2 \subseteq B \times A$ on $X \times X$.

Example 4.2

From the Example 3.1

$R_1^{-1} = \langle \{(a,a)\},\{(c,c)\},\{(d,d)\} \rangle \subseteq B \times A$ and

$R_2^{-1} = \langle \{(b,a)\},\{(c,c)\},\{(d,d),(d,b)\} \rangle \subseteq B \times A$.

Example 4.3

Let $X = \{a,b,c,d,e,f\}$,

$A = \langle \{a,b,c,d\},\{e\},\{f\} \rangle$,

$D = \langle \{a,b\},\{e,c\},\{f,d\} \rangle$ be a NCS-Type 2,

$B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$ be a NCS-Type1.

$C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ be a NCS-Type 3. Then

$$A\times D=\left\langle \begin{matrix}\{(a,a),(a,b),(b,a),(b,b),(b,b),(c,a),(c,b)\\ ,(d,a),(d,b)\},\{(e,e),(e,c)\},\{(f,f),(f,d)\}\end{matrix}\right\rangle$$

$$D\times C=\left\langle \begin{matrix}\{(a,a),(a,b),(b,a),(b,b)\},\{(e,c),(e,d),\\ (c,c),(c,d)\},\{(f,e),(f,f),(d,e),(d,f)\}\end{matrix}\right\rangle$$

we can construct many types of relations on products.
We can define the operations of neutrosophic crisp relation.

Definition 4.4

Let R and S be two neutrosophic crisp relations between X and Y for every $(x,y)\in X\times Y$ and NCSS A and B in the form $A=\langle A_1,A_2,A_3\rangle$ on X, $B=\langle B_1,B_2,B_3\rangle$ on Y, Then we can defined the following operations

i) $R\subseteq S$ may be defined as two types

a)Type1: $R\subseteq S \Leftrightarrow A_{1_R}\subseteq B_{1_S}$, $A_{2_R}\subseteq B_{2_S}$,

$A_{3R}\supseteq B_{3S}$

b)Type2: $R\subseteq S \Leftrightarrow A_{1_R}\subseteq B_{1_S}$, $A_{2R}\supseteq B_{2S}$,

$B_{3S}\subseteq A_{3R}$

ii) $R\cup S$ may be defined as two types

a)Type1:

$R\cup S=\langle A_{1R}\cup B_{1S}, A_{2R}\cup B_{2S}, A_{3R}\cap B_{3S}\rangle$,

b)Type2:

$R\cup S$

$=\langle A_{1R}\cup B_{1S}, A_{2R}\cap B_{2S}, A_{3R}\cap B_{3S}\rangle$.

iii) $R\cap S$ may be defined as two types

a)Type1: $R\cap S$

$=\langle A_{1R}\cap B_{1S}, A_{2R}\cup B_{2S}, A_{3R}\cup B_{3S}\rangle$,

b)Type2:

$R\cap S$

$=\langle A_{1R}\cap B_{1S}, A_{2R}\cap B_{2S}, A_{3R}\cup B_{3S}\rangle$.

Theorem 4.1

Let R, S and Q be three neutrosophic crisp relations between X and Y for every $(x,y)\in X\times Y$, then

i) $R\subseteq S\Rightarrow R^{-1}\subseteq S^{-1}$.

ii) $(R\cup S)^{-1}\Rightarrow R^{-1}\cup S^{-1}$.

iii) $(R\cap S)^{-1}\Rightarrow R^{-1}\cap S^{-1}$.

iv) $(R^{-1})^{-1}=R$.

v) $R\cap(S\cup Q)=(R\cap S).\cup(R\cap Q)$.

vi) $R\cup(S\cap Q)=(R\cup S).\cap(R\cup Q)$.

vii)If $S\subseteq R$, $Q\subseteq R$, then $S\cup Q\subseteq R$

Proof

Clear

Definition 4.5

The neutrosophic crisp relation $I\in NCR(X\times X)$, the neutrosophic crisp relation of identity may be defined as two types

i)Type1: $I=\{<\{A\times A\},\{A\times A\},\phi>\}$

ii)Type2: $I=\{<\{A\times A\},\phi,\phi>\}$

Now we define two composite relations of neutrosophic crisp sets.

Definition 4.6

Let R be a neutrosophic crisp relation in $X\times Y$, and S be a neutrosophic crisp relation in $Y\times Z$. Then the composition of R and S, $R\circ S$ be a neutrosophic crisp relation in $X\times Z$ as a definition may be defined as two types

i)Type1:

$R\circ S\leftrightarrow(R\circ S)(x,z)$

$=\cup\{<\{(A_1\times B_1)_R\cap(A_2\times B_2)_S\}$,

$\{(A_2\times B_2)_R\cap(A_2\times B_2)_S\}$,

$\{(A_3\times B_3)_R\cap(A_3\times B_3)_S\}>$.

ii)Type2 :

$R\circ S\leftrightarrow(R\circ S)(x,z)$

$=\cap\{<\{(A_1\times B_1)_R\cup(A_2\times B_2)_S\}$,

$\{(A_2\times B_2)_R\cup(A_2\times B_2)_S\}$,

$\{(A_3\times B_3)_R\cup(A_3\times B_3)_S\}>$.

Example 4.5

Let $X=\{a,b,c,d\}$, $A=\langle\{a,b\},\{c\},\{d\}\rangle$ and $B=\langle\{a\},\{c\},\{d,b\}\rangle$ then the product of two events given by $A\times B=\langle\{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\}\rangle$, and $B\times A=\langle\{(a,a),(a,b)\},\{(c,c)\},\{(d,d),(b,d)\}\rangle$, and $R_1=\langle\{(a,a)\},\{(c,c)\},\{(d,d)\}\rangle$, $R_1\subseteq A\times B$ on $X\times X$,

$R_2=\langle\{(a,b)\},\{(c,c)\},\{(d,d),(b,d)\}\rangle$

$R_2\subseteq B\times A$ on $X\times X$.

$R_1 \circ R_2 = \cup\langle \{(a,a)\} \cap \{(a,b)\}, \{(c,c)\}, \{(d,d)\}\rangle$
$= \langle \{\phi\}, \{(c,c)\}, \{(d,d)\}\rangle$ and
$I_{A1} = \langle \{(a,a).(a,b).(b.a)\}, \{(a,a).(a,b).(b,a)\}, \{\phi\}\rangle$
, $I_{A2} = \langle \{(a,a).(a,b).(b.a)\}, \{\phi\}, \{\phi\}\rangle$

Theorem 4.2
Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof
Let $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$, $S \subseteq B \times D$ on $Y \times Z$ then $S^{-1} \subseteq D \times B$, from Definition 3.6 and similarly we can $I_{(R\circ S)^{-1}}(x,z) = I_{S^{-1}}(x,z)$ and $I_{R^{-1}}(x,z)$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

V. Conclusion

In our work, we have put forward some new types of neutrosophic crisp sets and neutrosophic crisp continuity relations. Some related properties have been established with example. It ‘s hoped that our work will enhance this study in neutrosophic set theory.

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