THE SMARANDACHE-PĂTRAŞCU THEOREM OF ORTHOHOMOLOGICAL TRIANGLES

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The Smarandache-Pătrașcu Theorem of Orthohomological Triangles is the folllowing:

If P_1, P_2 are isogonal points in the triangle ABC, and if $A_1B_1C_1$ and $A_2B_2C_2$ are their pedal triangles such that the triangles ABC and $A_1B_1C_1$ are homological (the lines AA_1, BB_1, CC_1 are concurrent), then the triangles ABC and $A_2B_2C_2$ are also homological.

Proof

It is known that the projections of the isogonal points on the sides of the triangle ABC are 6 concyclic points. Therefore $A_1, A_2, B_1, B_2, C_1, C_2$ are concyclic (the *Circle of Six Points*).

It is also known the following:

Theorem: If in the triangle ABC the Cevianes AA_1, BB_1, CC_1 are concurrent in the point F_1 and the circumscribed circle to the triamgle $A_1B_1C_1$ intersects the sides of the triangle ABC in A_2, B_2, C_2, then the lines AA_2, BB_2, CC_2 are concurrent in a point F_2 (The Terquem's Theorem, in *Nouvelles Annales de Mathématiques*, de Terquem et Gérono, 1842).

Note

The points F_1 and F_2 were named the Terquem's points by Candido from Pisa in 1900.

From these two theorems it results the theorem from above.

The homologic centers of the triangles ABC, $A_1B_1C_1$ and ABC, $A_2B_2C_2$ being the Terquem's Points in the triangle ABC.