Single Valued Neutrosophic Information Systems Based on Rough Set Theory

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Abstract: The theory of rough sets was firstly proposed by Pawlak. Later on, Smarandache introduced the concept of neutrosophic (NS) sets in 1998. In this paper based on the concept of rough neutrosohic set, we define the concept of single valued neutrosophic information systems. In addition, we will discuss the knowledge reduction and extension of the single valued neutrosophic information systems.

1 Introduction

The rough sets theory introduced by Pawlak [27] is an excellent mathematical tool for modeling and processing incomplete and inconsistent information in information systems. The basic philosophy of rough sets is based upon the approximation of sets by pair of sets known as lower approximation and upper approximation. Here, the lower and upper approximation operators are based on equivalence relation. However, in many real life problems, rough set model cannot be applied due to the restrictive condition of requirement of equivalence relation. The combination of fuzzy sets and rough sets lead to two concepts: rough fuzzy sets and fuzzy rough sets. Rough fuzzy sets [5] are the fuzzy sets approximated in the crisp approximation spaces and fuzzy rough sets [3] are the crisp sets approximated in the fuzzy approximation space. In addition, the concept of rough fuzzy sets and fuzzy rough sets are generalized to intuitionistic fuzzy environment such as rough intuitionstic fuzzy sets [4, 14]. Different applications in this direction are studied by various authors [3, 9, 10, 11, 12, 27, 28, 29].

As a generalization of fuzzy sets [17] and intuitionistic fuzzy sets [13], the concept of neutrosophic sets (NS for short) was introduced by Smarandache [6]. The concept of neutrosophic set theory is a new mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data (useful to deal with indeterminacy). According to Smarandache, a neutrosophic set is characterized by a triple of functions valued in [0, 1], the membership function, indeterminacy function and the non-membership function. The evaluation degrees of membership, indeterminate and non-membership are independent. From scientific or engineering point of view, the neutrosophic set and set-theoretic view, operators need to be defined. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al. [7] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Later on, Wang et al. [8] introduced the concept of interval neutrosophic sets. The original rough sets approach and their hybrid structures such as rough fuzzy sets and rough intuitionistic fuzzy sets are not able to deal with indeterminate and inconsistent data. For this reason, Broumi et al. [21] introduced a new hybrid mathematical structure called rough neutrosophic sets, handling incomplete and indeterminate information, then defined and studied the operations and properties of rough neutrosphic sets and presented some examples. The concept of rough neutrosophic sets combines neutrosophic set theory and rough set theory. The concept of rough neutrosophic set is the generalization of rough fuzzy sets and rough intuitionistic fuzzy sets. Based on the equivalence relation on the universe of discourse, Salama and Broumi [2] defined rough neutrosophic sets in another way and added some important properties. Broumi et al. [22] proposed a new mathematical model named "lower and upper soft interval valued neutrosophic rough approximations of an IVNSS-Relation" by extending the concept of lower and upper soft interval valued intuitionistic fuzzy rough approximations of an IVIFSS-relation to the case of IVNSS and investigated some of their properties. In the same year, Broumi and Smarandache [23] extended the concept of interval valued intuitionistic fuzzy soft rough sets [1] to the case of interval valued neutrosophic soft rough sets. Based on the concept of interval neutrosophic sets and rough set, Broumi and Smarandache [24] extended the rough neutrosophic sets to the interval rough neutrosophic sets. Monadal and Pramanik [14] applied the concept of rough neutrosophic set in multi-attribute decision-making based on grey relational analysis. The same authors in [16] also studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis.

Literature review reflects that no studies have been made on information system using rough neutrosophic sets.

In this paper, based on the combination of the classical Pawlak rough set theory with the neutrosophic set theory presented in [21] we develop the single valued neutrosophic information systems. Finally, we define the neutrosophic reduction on the classical Pawlak information systems, and then discuss the knowledge reduction of the single valued neutrosophic information systems.

2 Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of single valued neutrosophic sets and rough neutrosophic sets. For more details, the reader may refer to [6, 21].

Definition 2.1 [6]

Let U be an universe of discourse then the neutrosophic set A is an object having the form $A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\}$, where the functions μ, ν, ω: $U \rightarrow]^-0,1^+[$define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+.$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A_{NS} = \{<x, \mu_A(x), \nu_A(x), \omega_A(x) > | x \in X\}$$

and

$$B_{NS} = \{<x, \mu_B(x), \nu_B(x), \omega_B(x) > | x \in X\}$$

then,

1. $A_{NS} \subseteq B_{NS}$ if and only if

$$\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x), \omega_A(x) \geq \omega_B(x).$$

2. $A_{NS} = B_{NS}$ if and only if

$\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$.

3. The complement of A_{NS} is denoted by A_{NS}^o and is defined by

$A_{NS}^o = \{<x, \omega_A(x), 1 - \nu_A(x), \mu_A(x) \mid x \in X\}$

4. $A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}>: x \in X\}$
5. $A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}>: x \in X\}$

Definition 2.2 [21]

Let U be a non-null set and R be an equivalence relation on U. Let A be a single valued neutrosophic set in U with the membership function μ_R, indeterminacy function ν_R and non-membership function ω_R. The lower and the upper approximations of F in the approximation (U, R) denoted by $\underline{R}$ (A) and $\overline{R}$(A) are respectively defined as follows:

$\underline{R}(A) = \{ <x, \mu_{\underline{R}(A)}(x), \nu_{\underline{R}(A)}(x), \omega_{\underline{R}(A)}(x)> \mid y \in [x]_R, x \in U\}$, $\overline{R}(A) =$
$\{<x, \mu_{\overline{R}(A)}(x), \nu_{\overline{R}(A)}(x), \omega_{\overline{R}(A)}(x)> \mid y \in [x]_R, x \in U\}$,

where:

$\mu_{\underline{R}(A)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y)$, $\nu_{\underline{R}(A)}(x) = \bigvee_{y \in [x]_R} \nu_F(y)$, $\omega_{\underline{R}(A)} = \bigvee_{y \in [x]_R} \omega_F(y)$,
$\mu_{\overline{R}(A)}(x) = \bigvee_{y \in [x]_R} \mu_F(y)$, $\nu_{\overline{R}(A)}(x) = \bigwedge_{y \in [x]_R} \nu_F(y)$, $\omega_{\overline{R}(A)} = \bigwedge_{y \in [x]_R} \omega_F(y)$,

so $0 \leq \mu_{\overline{R}(A)}(x) + \nu_{\overline{R}(A)}(x) + \omega_{\overline{R}(A)}(x) \leq 3$

and $0 \leq \mu_{\underline{R}(A)}(x) + \nu_{\underline{R}(A)}(x) + \omega_{\underline{R}(A)}(x) \leq 3$,

where " $\vee$ " and " $\wedge$ " mean "max" and "min " operators respectively, $\mu_A(y)$, $\nu_A(y)$ and $\omega_A(y)$ are the membership, indeterminacy and non-membership of y with respect to F. It is easy to see that $\overline{R}$(A) and $\underline{R}$(A) are two single valued neutrosophic sets in U, thus SVNS mapping $\overline{R}$, $\underline{R}$:N(U) → N(U) are, respectively, referred to as the upper and lower rough SVNS approximation operators, and the pair ($\underline{R}$(A), $\overline{R}$(A)) is called the rough neutrosophic set in (U, R).

From the above definition, we can see that $\underline{R}$(A) and $\overline{R}$(A) have constant membership on the equivalence classes of U. If $\underline{R}(A) = \overline{R}(A)$; i.e $\mu_{\underline{R}(A)} = \mu_{\overline{R}(A)}$, $\nu_{\underline{R}(A)} = \nu_{\overline{R}(A)}$ and $\omega_{\underline{R}(A)} = \omega_{\overline{R}(A)}$ for any $x \in U$, we call A a definable neutrosophic set in the approximation (U, R). It is easily to be proved that zero 0_N neutrosophic set and unite neutrosophic sets 1_N are definable neutrosophic sets.

Definition 2.3 [19]

If R(A)= ($\underline{R}$(A), $\overline{R}$(A) is a rough neutrosophic set in (U, R), the rough complement of R(A) is the rough neutrosophic set denoted $\sim$ R(A)=($\underline{R}(A)^c$, $\overline{R}(A)^c$),where $\underline{R}(A)^c$, $\overline{R}(A)^c$ are the complements of neutrosophic sets $\underline{R}$(F) and $\overline{R}$(F) respectively.

$\underline{R}(A)^c = \{ <x, \omega_{\underline{R}(A)}, 1 - \nu_{\underline{R}(A)}(x), \mu_{\underline{R}(A)}(x) > \mid x \in U\}$,

and

$\overline{R}(A) = \{ <x, \omega_{\overline{R}(A)}(x), 1 - \nu_{\overline{R}(A)}(x), \mu_{\overline{R}(A)}(x)> \mid x \in U\}$.

3 The Single Valued Neutrosophic Information Systems

3.1 The Knowledge discovery in the single valued neutrosophic Information Systems

In this section, applying the basic theory of the rough set proposed, we will give some results about the knowledge discovery for a single valued neutrosophic information system. According to [29] we have the owing definitions.

Let (U, A, F) be a classical information system. Here U is the set of objects, i.e. U= $\{x_1, x_2,\ldots, x_n\}$. Every element $x_i \in$ U, i $\leq$ n, is called an object, and A is the attribute set, i.e. A =$\{a_1, a_2,\ldots, a_n\}$. Every element $a_j \in$ A, j$\leq$ m, is an attribute, F is the relation set of U and A, i.e. F=$\{f_j$: j $\leq$ m$\}$, (f_j : U $\rightarrow V_j$), and V_j is the domain of the attribute a_j .We call (U, A, F, D, G) an information system or decision table, where (U, A, F) is the classical information system, A is the condition attribute set and D the decision attribute set, i.e. D=$\{d_1, d_2,\ldots, d_p\}$. G is the relation set of the U and D, G =$\{g_i$, j$\leq$ p$\}$, where g_j:U$\rightarrow V_j$ is the domain of the decision attribute d_i.

Let (U, A, D, G) be the information system. If $R_A \subset R_B$, i.e; $U/_{R_A} \leq U/_{R_D}$ (or for any $[\mathrm{x}]_A$, x $\in$ U , there exists $[\mathrm{x}]_D$ such that $[\mathrm{x}]_A \subset [\mathrm{x}]_D$), then the information system is called a consistent information systems, or called an inconsistent information system [29].

Definition 3.1

Let (U, A, F, D, G) be an information system or decision table, where (U, A, F) is the classical information system, D = { $\widetilde{D}_k$ | k=1,2,… }, $\widetilde{D}_k$ be the neutrosophic set of U, G be the relation sets from U to D. Then the information system is called a single valued neutrosophic information system.

Theorem 3.2

Let (U, A, F, D) be a single valued neutrosophic information system and B $\subseteq$A. If for any x $\in$U: ($\mu_{\widetilde{D}_i}$(x) , $\nu_{\widetilde{D}_i}$(x), $\omega_{\widetilde{D}_i}$(x)) =(α(x), β(x), γ(x)) =$\underline{R}_B(\widetilde{D}_i$)(x) > $\underline{R}_B(\widetilde{D}_j$)(x) (j $\neq$ i), then $[x]_B \cap (\sim \widetilde{D}_j)_{\alpha(x),\beta(x),\gamma(x)} \neq \emptyset$, $[x]_B \subseteq (\sim \widetilde{D}_i)_{\alpha(x),\beta(x),\gamma(x)}$ where $(\alpha,\beta,\gamma) \in$ SVNS(U).

Proof. Since $(\widetilde{D}_i)_{\alpha(x),\beta(x),\gamma(x)}$={y $\in$ U| $\mu_{\widetilde{D}_i}$(y) $\geq$ α(x), $\nu_{\widetilde{D}_i}$(y) $\leq \beta$(x), $\omega_{\widetilde{D}_i}$(y) $\leq$ γ(x)}, and (α(x),β(x),γ(x)) = $\underline{R}_B(\widetilde{D}_i)$(x), we have $\alpha(x) = \wedge_{y \in [x]_R} \mu_{\widetilde{D}_i}(y)$, $\beta(x) = \vee_{y \in [x]_R} \nu_{\widetilde{D}_i}(y)$ and $\gamma(x) = \vee_{y \in [x]_R} \omega_{\widetilde{D}_i}(y)$. Then for any x $\in$ U, y $\in$ $[x]_B$; $\mu_{\widetilde{D}_i}$(y) $\geq$ α(x), $\nu_{\widetilde{D}_i}$(y) $\leq$ β(x) and $\omega_{\widetilde{D}_i}$(y) $\leq$ γ(x), hence y $\in (\widetilde{D}_i)_{\alpha(x),\beta(x),\gamma(x)}$

Now, since for any x $\in$ U, (α(x), β(x), γ(x)) =$\underline{R}_B(\widetilde{D}_i$)(x) > $\underline{R}_B(\widetilde{D}_j$)(x), then there exists y $\in [x]_B$ such that ($\widetilde{D}_j$)(y) < (α(x),β(x),γ(x)) ,i.e $\mu_{\widetilde{D}_i}$ (y) < α(x), $\nu_{\widetilde{D}_i}$ (y) > β(x), and $\omega_{\widetilde{D}_i}$ (y) > γ(x), hence $[x]_B \cap (\sim \widetilde{D}_i)_{\alpha(x),\beta(x),\gamma(x)} \neq \emptyset$.

Theorem 3.3

Let (U, A, F, D) be a single valued neutrosophic information system and B $\subseteq$A and the cardinal of the set D is finite. If for any x $\in$ U, there exist a natural number k_x, such that

(α(x),β(x),γ(x)) =$\underline{R}_B(\widetilde{D}_{k_x}$)(x) > $\underline{R}_B(\widetilde{D}_j$)(x) (j $\neq k_x$) ,

then $\{(\widetilde{D}_{k_x})_{\alpha(x),\beta(x),\gamma(x)}$ | x $\in$ U$\}$ is a partition of the universe U.

Proof. For any x $\in$ U we have $[x]_B \subseteq (\widetilde{D}_{k_x})_{\alpha(x),\beta(x),\gamma(x)}$.Then there exists $[\mathrm{x}]_B$ such that:

$[\mathrm{x}]_B \subseteq \{(\underline{R}_B(\widetilde{D}_{k_x}))_{\alpha(x),\beta(x),\gamma(x)}\}$.

For the reason that $\{[\mathrm{x}]_B$| x $\in$ U$\}$ made up a partition of U and $[\mathrm{x}]_B \cap (\sim \widetilde{D}_i)_{\alpha(x),\beta(x),\gamma(x)} \neq \emptyset$, then $\{(\widetilde{D}_j)_{\alpha(x),\beta(x),\gamma(x)}$ | x $\in$ U$\}$ made up a partition of U also.

Let (U, A, F, D) be the single valued neutrosophic information system, R_A be the equivalence classes which induced by the condition attribute set A, and the universe is divided by R_A as following:

$$U/_{R_A} = \{X_1, X_2, X_3, \ldots, X_k\},$$

denoted as

$$\underline{R}_A(\tilde{D}_k)(x) = (\underline{R}_A(\tilde{D}_1)(X_i), \underline{R}_A(\tilde{D}_2)(X_i), \ldots, \underline{R}_A(\tilde{D}_k)(X_i)). \qquad (1)$$

Example 1

Table 1 gives a single valued neutrosophic information system, where the object sets be $\{x_1, x_2, x_3, \ldots, x_{10}\}$, condition attribute set is A= $\{a_1, a_2, a_3\}$, and the objection attribute set D is $\{\tilde{D}_1, \tilde{D}_2, \tilde{D}_3\}$,where $\tilde{D}_i$ (i=1, 2, 3), serve as a single valued neutrosophic set. i.e., $\tilde{D}_i \in$ SVNS(U), (i=1,2,3).

Obviously, the universe U can be divided into five basic classes by their conditional attribute set A= $\{a_1, a_2, a_3\}$, then:

$$U/_{R_A} = \{\{x_1, x_3, x_9\},\{x_2, x_7, x_{10}\},\{x_4\},\{x_5, x_8\},\{x_6\}\}.$$

Using the definition (2.3), we obtain the approximation of the single valued neutrosophic sets, which are given in Table 1.

Table 1: The single valued neutrosophic information system

$U/_{R_A}$	a_1	a_2	a_3	D_1	D_2	D_3
x_1	2	1	3	(0.2 , 0.5 , 0.6)	(0.15 , 0.2 , 0.5)	(0.4 , 0.5 , 0.3)
x_2	3	2	1	(0.3 , 0.5 , 0.4)	(0.3 , 0.3 , 0.4)	(0.35 , 0.4, 0.6)
x_3	2	1	3	(0.6 , 0.45 , 0.3)	(0.3 , 0.6 , 0.7)	(0.1 , 0.2 , 0.6)
x_4	2	2	3	(0.15 , 0.7 , 0.4)	(0.1 , 0.8 , 0.3)	(0.2 , 0.3 , 0.4)
x_5	1	1	4	(0.05 , 0.7 , 0.1)	(0.2 , 0.3 , 0.3)	(0.05 , 0.5 , 0.7)
x_6	1	2	4	(0.1 , 0.5 , 0.2)	(0.2 , 0.4 , 0.1)	(1.0 , 0.0 , 0.0)
x_7	3	2	1	(0.25 , 0.4, 0.6)	(1.0 , 0.0 , 0.0)	(0.3 , 0.4 , 0.5)
x_8	1	1	4	(0.1, 0.2 , 0.5)	(0.25 , 0.4 , 0.3)	(0.4 , 0.6 , 0.4)
x_9	2	1	3	(0.45 , 0.45 , 0.5)	(0.25 , 0.3 , 0.8)	(0.2 , 0.3 , 0.3)
x_{10}	3	2	1	(0.05 , 0.9 , 0.6)	(0.4 , 0.3 , 0.9)	(0.05 , 0.2 , 0.3)

It is easy to test that the conditions of Theorem 3.3 is satisfied, and obtain the maximum of $\underline{R}_A(\tilde{D}_i)(x)$, {i=1, 2, 3} for every equivalence as it follows:

$y \in X_1 \longrightarrow k_y=1, (\alpha(x), \beta(x), \gamma(x)) = (0.2, 0.5, 0.6)$

$y \in X_2 \longrightarrow k_y=3, (\alpha(x), \beta(x), \gamma(x)) = (0.3, 0.3, 0.6)$

$y \in X_3 \longrightarrow k_y=3, (\alpha(x), \beta(x), \gamma(x)) = (0.2, 0.3, 0.3)$

$y \in X_4 \longrightarrow k_y=2, (\alpha(x), \beta(x), \gamma(x)) = (0.2, 0.4, 0.3)$

$y \in X_5 \longrightarrow k_y=1, (\alpha(x), \beta(x), \gamma(x)) = (1, 0, 0)$

It follows that:

$\{(\tilde{D}_{k_x})_{\alpha(x),\beta(x),\gamma(x)} \mid x \in U\} = \{y \in U \mid \mu_{\tilde{D}_{k_x}}(x) \geq \alpha(x), \nu_{\tilde{D}_{k_x}}(x) \leq \beta(x), \omega_{\tilde{D}_{k_x}}(x) \leq \gamma(x)\}$
$=\{x_1, x_2, x_1, x_1, x_1\} = \{X_1, X_2, X_1\}$.

Therefore, the approximation of the single valued neutrosophic decision is as it follows:

Table 2: The approximation of the single valued neutrosophic decision

U/R_A	$\underline{R}_A(\tilde{D}_1)$	$\underline{R}_A(\tilde{D}_2)$	$\underline{R}_A(\tilde{D}_3)$
X_1	(0.2 , 0.5 , 0.6)	(0.15 , 0.6 , 0.3)	(0.1 , 0.5 , 0.6)
X_2	(0.05, 0.9, 0.6)	(0.3 , 0.3 , 0.9)	(0.05 , 0.4 , 0.6)
X_3	(0.15 , 0.7 , 0.4)	(0.1 , 0.8 , 0.3)	(0.2 , 0.3 , 0.4)
X_4	(0.05 , 0.7 , 0.5)	(0.2 , 0.4 , 0.3)	(0.05 , 0.6 , 0.7)
X_5	(0.1 , 0.5 , 0.2)	(0.2 , 0.4 , 0.1)	(1 , 0 , 0)

3.2 The knowledge reduction and extension of the single valued neutrosophic information

Definition 3.5

Let (U, A, F) be a classical information system, and B be subset of A. Then, B is called the single valued neutrosophic reduction of the classical information system (U, A, F), if B is the minimum set in the inclusion set which satisfies the following relations: for any X ∈ SVNS (U), and for any x ∈U;

$$\underline{R}_A(X)(x)=\underline{R}_B(X)(x), \qquad \overline{R}_A(X)(x)=\overline{R}_B(X)(x),$$

where $\underline{R}_A(X)(x)$, $\underline{R}_B(X)(x)$, $\overline{R}_A(X)(x)$, $\overline{R}_B(X)(x)$ are defined as the single valued neutrosophic rough sets. B is called the single valued neutrosophic lower approximation reduction of the classical information system (U, A, F) if B is the minimum set that satisfies the following relations:

$\forall$ X ∈ SVNS (U), x ∈ U,

$\underline{R}_A(X)(x) = \underline{R}_B(X)(x)$

B is called the single valued neutrosophic upper approximation reduction of the classical information system (U, A, F) if B is the minimum set that satisfies the following relations:

$\forall$ X ∈ SVNS (U), x ∈ U,

$\overline{R}_A(X)(x)=\overline{R}_B(X)(x)$

If X is the crisp set of U, then the set B is the reduction of the classical information system (U, A, F) [3, 18].

In the following section, we present the knowledge reduction of the single valued neutrosophic information system by introducing the discernibility matrix.

Definition 3.6

Let **(**U, A, F, D) be the single valued neutrosophic information system

$$D_{ij} = \begin{cases} \{a_l \in A: f_l(X_i) \neq f_l(X_j)\}, if\ g_{X_i}(D_k) \neq g_{X_j}(D_k) \\ A\ , if\ \ g_{X_i}(D_k) = g_{X_j}(D_k) \end{cases} \tag{2}$$

is called the discernibility matrix of (U, A, F, D), where g_{Xi} (D_k) denotes the maximum value of $\underline{R}_A(D)(x_i)$ at the line of K, i.e., the rows i and j of Eq. (1).

Definition 3.7

Let (U, A, F, D) be the single valued neutrosophic information system, for any B ⊆A, if the following relations hold:

$$\mu_{\underline{R}_B}(D_i)(x) > \mu_{\underline{R}_B}(D_j)(x) \Leftrightarrow \mu_{\underline{R}_A}(D_i)(x) > \mu_{\underline{R}_A}(D_j)(x) \quad (i\neq j) \tag{3}$$

$$\nu_{\underline{R}_B}(D_i)(x) < \nu_{\underline{R}_B}(D_j)(x) \Leftrightarrow \nu_{\underline{R}_A}(D_i)(x) < \nu_{\underline{R}_A}(D_j)(x) \quad (i \neq j) \tag{4}$$

and

$$\omega_{\underline{R}_B}(D_i)(x) < \omega_{\underline{R}_B}(D_j)(x) \Leftrightarrow \omega_{\underline{R}_A}(D_i)(x) < \omega_{\underline{R}_A}(D_j)(x) \quad (i \neq j) \tag{5}$$

then B is called the consistent of A.

Theorem 3.8

Let **(**U, A, F, D) be the single valued neutrosophic information system. If there exists a subset B $\subseteq$A such that B$\cap$ $D_{ij} \neq \emptyset$, then B is the consistent set of A.

Proof. The proof is similar to the interval-valued sets, see [3].

Definition 3.9

Let **(**U, A, F, D) be a single valued neutrosophic neutrosophic information system. Then

$$D_{ij}^{c} = \begin{cases} \{a_l \in A : f_l(X_i) = f_l(X_j)\}, & if\, g_{X_i}(D_k) \neq g_{X_j}(D_k) \\ \emptyset, & if\ g_{X_i}(D_k) = g_{X_j}(D_k) \end{cases} \tag{6}$$

is called the complement of discernibility matrix of (U, A, F, D) (where $g_{X_i}(D_k)$ denotes the maximum value of $\underline{R}_A(D)(X_i)$ at the line of K, i.e., the rows i and j of Eq. (1).

Theorem 3.10

Let **(**U, A, F, D) be a single valued neutrosophic information system. If there exists a subset B $\subseteq$ A such that B $\cap$ D_{ij}^{c}=$\emptyset$, then B is the consistent set of A.

Proof. If B $\cap$ D_{ij}^{c}=$\emptyset$, then B $\subseteq$ D_{ij}. So according to theorem 3.8, B is the consistent set of A.

Definition 3.11

Let (U, A, F) be a classical information system, for any set B (A $\subseteq$ B). B is called the single valued neutrosophic extension of the classical information system (U, A, F), if for any X $\in$ SVNS(U), and for any x $\in$ U:

$\underline{R}_A(X)(x)= \underline{R}_B(X)(x)$, $\overline{R}_A(X)(x)=\overline{R}_B(X)(x)$

B is called the single valued neutrosophic lower approximation extension of the classical information system (U, A, F), if for any X $\in$ SVNS(U), and for any x $\in$ U:

$\underline{R}_A(X)(x)= \underline{R}_B(X)(x)$

B is called the single valued neutrosophic upper approximation extension of the classical information system (U, A, F), if for any X $\in$ SVNS(U), and for any x $\in$ U:

$\overline{R}_A(X)(x)=\overline{R}_B(X)(x)$

Using this definition, the following theorem can be easily derived.

Theorem 3.12

Let (U, A, F) be a classical information system. For any hyper set B, such that A $\subseteq$ B, if A is the single valued neutrosophic reduction of the classical information system (U, B), then B is the single valued neutrosophic extension of the classical system (U, B, F), but not conversely necessary.

Example 2

Considering the approximation of the single valued neutrosophic decision in table 1 and 2, therefore let B = $\{a_1, a_2\}$; then:

U/R_h = { X_1, X_2, X_3, X_4, X_5 }.

We can obtain an approximation value given in Table 3. It is easy to see that B satisfies the definition 3.5, i.e. B is the single valued neutrosophic lower approximation reduction of the classical information system (U, A, F)

Table 3: The approximation of the single valued neutrosophic decision

U/R_B	$\underline{R_A}(\widetilde{D}_1)$	$\underline{R_A}(\widetilde{D}_2)$	$\underline{R_A}(\widetilde{D}_3)$
X_1	(0.2 , 0.5 , 0.6)	(0.15 , 0.6 , 0.9)	(0.1 , 0.5, 0.6)
X_2	(0.05 , 0.9 , 0.6)	(0.3 , 0.3 , 0.9)	(0.05 , 0.4 , 0.6)
X_3	(0.15 , 0.7 , 0.4)	(0.1 , 0.8 , 0.3)	(0.2 , 0.3 , 0.4)
X_4	(0.05 , 0.7 , 0.5)	(0.2 , 0.4 , 0.3)	(0.05 , 0.6 , 0.7)
X_5	(0.1 , 0.5 , 0.2)	(0.2 , 0.4 , 0.1)	(1, 0, 0)

It is clear that B satisfies the theorem 3.8,i.e, B is the consistent of A.

Table 4: The discernibility matrix of the single valued neutrosophic objection

U/R_B	X_1	X_2	X_3	X_4	X_5
X_1	A				
X_2	A	A			
X_3	$\{a_2\}$	$\{a_1, a_3\}$	A		
X_4	$\{a_1, a_3\}$	A	A	A	
X_5	$\{a_1, a_3, a_2\}$	A	A	A	A

4 Conclusions

In this paper, we defined a single valued neutrosophic information system. Then a rough approximation of every single valued neutrosophic set in the single valued neutrosophic information system was presented. Finally, the knowledge reduction and extension of the single valued neutrosophic information system were investigated.

However, we hope that the concept presented here will open new avenue of research in current single valued neutrosophic information systems.

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