

## Shortest Path Problem under Bipolar Neutrosophic Setting

Said Broumi<sup>1,a</sup>, Assia Bakali<sup>2,b</sup>, Mohamed Talea<sup>3,c</sup>, Florentin Smarandache<sup>4,d</sup>,  
Mumtaz Ali<sup>5,e</sup>

<sup>1,3</sup>Laboratory of Information processing, Faculty of Science Ben M'Sik, University Hassan II,  
B.P 7955, Sidi Othman, Casablanca, Morocco

<sup>2</sup>Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco

<sup>4</sup>Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup,  
NM 87301, USA

<sup>5</sup>Department of Mathematics, Quaid-i-azam University Islamabad 45320, Pakistan

<sup>a</sup>broumisaid78@gmail.com, <sup>b</sup>assiabakali@yahoo.fr, <sup>c</sup>taleamohamed@yahoo.fr,  
<sup>d</sup>fsmarandache@gmail.com, <sup>e</sup>Mumtazali7288@gmail.com

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**Abstract.** This main purpose of this paper is to develop an algorithm to find the shortest path on a network in which the weights of the edges are represented by bipolar neutrosophic numbers. Finally, a numerical example has been provided for illustrating the proposed approach.

### Introduction

Smarandache [1, 2] introduced neutrosophic set and neutrosophic logic by considering the non-standard analysis. The concept of neutrosophic sets generalized the concepts of fuzzy sets [3] and intuitionistic fuzzy set [4] by adding an independent indeterminacy-membership. Neutrosophic set is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world, which have attracted the widespread concerns for researchers. The concept of neutrosophic set is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). From scientific or engineering point of view, the neutrosophic set and set-theoretic operator will be difficult to apply in the real application. The subclass of the neutrosophic sets called single-valued neutrosophic sets [5] (SVNS for short) was studied deeply by many researchers. The concept of single valued neutrosophic theory has proven to be useful in many different field such as the decision making problem, medical diagnosis and so on. Additional literature on neutrosophic sets can be found in [6]. Recently, Deli et al. [7] introduced the concept of bipolar neutrosophic sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. The bipolar neutrosophic set (BNS) is an important concept to handle uncertain and vague information existing in real life, which consists of three membership functions including bipolarity. Also, they give some operations including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets and operators on the bipolar neutrosophic sets. The shortest path problem is a fundamental algorithmic problem, in which a minimum weight path is computed between two nodes of a weighted, directed graph. The shortest path problem has been widely studied in the fields of operations research, computer science, and transportation engineering. In literature, there are many publications which deal with shortest path problems [8-13] that have been studied with different types of input data, including fuzzy set, intuitionistic fuzzy sets, trapezoidal intuitionistic fuzzy sets and vague set. Recently, Broumi et al. [14-17] presented the concept of neutrosophic graphs, interval valued neutrosophic graphs and bipolar single valued neutrosophic graphs. Smarandache [18-19] proposed another variant of neutrosophic graphs based on literal indeterminacy component (I). Also Kandasamy et al. [20] studied the concept of neutrosophic graphs, To do best of our knowledge, few research papers deal with shortest path in neutrosophic environment. Broumi et al. [21] proposed an algorithm for solving neutrosophic shortest path problem based on score function. The same authors in [22] proposed a study of neutrosophic shortest path with interval valued

neutrosophic number on a network. Till now, there is no study in the literature for computing shortest path problem in bipolar neutrosophic environment.

The structure of the paper is as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets and bipolar neutrosophic sets. In section 3, we give the network terminology. In Section 4, an algorithm is proposed for finding the shortest path and shortest distance in bipolar neutrosophic graph. In section 5 an illustrative example is provided to find the shortest path and shortest distance between the source node and destination node. Finally, in Section 6 we provide conclusion and proposal for further research.

### Preliminaries

In this section, some basic concepts and definitions on neutrosophic sets, single valued neutrosophic sets and bipolar neutrosophic sets are reviewed from the literature.

**Definition 2.1 [1-2].** Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ ; then the neutrosophic set  $A$  (NS  $A$ ) is an object having the form  $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$ , where the functions  $T, I, F: X \rightarrow ]0, 1^+[$  define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element  $x \in X$  to the set  $A$  with the condition:

$$-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  are real standard or nonstandard subsets of  $]0, 1^+[$ .

Since it is difficult to apply NSs to practical problems, Wang et al. [14] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

**Definition 2.2 [3].** Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ . A single valued neutrosophic set  $A$  (SVNS  $A$ ) is characterized by truth-membership function  $T_A(x)$ , an indeterminacy-membership function  $I_A(x)$ , and a falsity-membership function  $F_A(x)$ . For each point  $x$  in  $X$   $T_A(x), I_A(x), F_A(x) \in [0, 1]$ . A SVNS  $A$  can be written as

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \} \quad (2)$$

Deli et al. [15] proposed the concept of bipolar neutrosophic set, which is an instance of a neutrosophic set, and introduced the definition of an BNS.

**Definition 2.3 [4].** A bipolar neutrosophic set  $A$  in  $X$  is defined as an object of the form  $A = \{ \langle x, T^p(x), I^p(x), F^p(x), T^n(x), I^n(x), F^n(x) \rangle: x \in X \}$ , where  $T^p, I^p, F^p: X \rightarrow [1, 0]$  and  $T^n, I^n, F^n: X \rightarrow [-1, 0]$ . The positive membership degree  $T^p(x), I^p(x), F^p(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $\in X$  corresponding to a bipolar neutrosophic set  $A$  and the negative membership degree  $T^n(x), I^n(x), F^n(x)$  denotes the truth membership, indeterminate membership and false membership of an element  $\in X$  to some implicit counter-property corresponding to a bipolar neutrosophic set  $A$ .

**Definition 2.4 [4].** An empty bipolar neutrosophic set  $\tilde{A}_1 = \langle T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n \rangle$  is defined as

$$T_1^p = 0, I_1^p = 0, F_1^p = 1 \quad \text{and} \quad T_1^n = -1, I_1^n = 0, F_1^n = 0 \quad (4)$$

**Definition 2.5** [7]. Let  $\tilde{A}_1 = \langle T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n \rangle$  and  $\tilde{A}_2 = \langle T_2^p, I_2^p, F_2^p, T_2^n, I_2^n, F_2^n \rangle$  be two bipolar neutrosophic numbers and  $\lambda > 0$ . Then, the operations of these numbers defined as below;

$$(i) \quad \tilde{A}_1 \oplus \tilde{A}_2 = \langle T_1^p + T_2^p - T_1^p T_2^p, I_1^p I_2^p, F_1^p F_2^p - T_1^n T_2^n, -(-I_1^p - I_2^p - I_1^p I_2^p), -(-F_1^p - F_2^p - F_1^p F_2^p) \rangle \quad (5)$$

$$(ii) \quad \tilde{A}_1 \otimes \tilde{A}_2 = \langle T_1^p T_2^p, I_1^p + I_2^p - I_1^p I_2^p, F_1^p + F_2^p - F_1^p F_2^p - (-T_1^n - T_2^n - T_1^n T_2^n), -I_1^n I_2^n, -F_1^n F_2^n \rangle \quad (6)$$

$$(iii) \quad \lambda \tilde{A}_1 = \langle 1 - (1 - T_1^p)^\lambda, (I_1^p)^\lambda, (F_1^p)^\lambda, -(T_1^n)^\lambda, -(I_1^n)^\lambda, -(1 - (1 - F_1^n)^\lambda) \rangle \quad (7)$$

$$(iv) \quad \tilde{A}_1^\lambda = \langle (T_1^p)^\lambda, 1 - (1 - I_1^p)^\lambda, 1 - (1 - F_1^p)^\lambda, -(1 - (1 - T_1^n)^\lambda), -(I_1^n)^\lambda, -(F_1^n)^\lambda \rangle \text{ where } \lambda > 0 \quad (8)$$

**Definition 2.6** [7]. In order to make a comparisons between two BNN. Deli et al. [7], introduced a concept of score function. The score function is applied to compare the grades of BNS. This function shows that greater is the value, the greater is the bipolar neutrosophic sets and by using this concept paths can be ranked. Let  $\tilde{A} = \langle T^p, I^p, F^p, T^n, I^n, F^n \rangle$  be a bipolar neutrosophic number. Then, the score function  $s(\tilde{A})$ , accuracy function  $a(\tilde{A})$  and certainty function  $c(\tilde{A})$  of an BNN are defined as follows:

$$(i) \quad s(\tilde{A}) = \left(\frac{1}{6}\right) \times [T^p + 1 - I^p + 1 - F^p + 1 + T^n - I^n - F^n] \quad (9)$$

$$(ii) \quad a(\tilde{A}) = T^p - F^p + T^n - F^n \quad (10)$$

$$(iii) \quad c(\tilde{A}) = T^p - F^n \quad (11)$$

**Comparison of bipolar neutrosophic numbers**

Let  $\tilde{A}_1 = \langle T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n \rangle$  and  $\tilde{A}_2 = \langle T_2^p, I_2^p, F_2^p, T_2^n, I_2^n, F_2^n \rangle$  be two bipolar neutrosophic numbers then

- i. If  $s(\tilde{A}_1) > s(\tilde{A}_2)$ , then  $\tilde{A}_1$  is greater than  $\tilde{A}_2$ , that is,  $\tilde{A}_1$  is superior to  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 \succ \tilde{A}_2$
- ii. If  $s(\tilde{A}_1) = s(\tilde{A}_2)$ , and  $a(\tilde{A}_1) > a(\tilde{A}_2)$  then  $\tilde{A}_1$  is greater than  $\tilde{A}_2$ , that is,  $\tilde{A}_1$  is superior to  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 \succ \tilde{A}_2$
- iii. If  $s(\tilde{A}_1) = s(\tilde{A}_2)$ ,  $a(\tilde{A}_1) = a(\tilde{A}_2)$ , and  $c(\tilde{A}_1) > c(\tilde{A}_2)$  then  $\tilde{A}_1$  is greater than  $\tilde{A}_2$ , that is,  $\tilde{A}_1$  is superior to  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 \succ \tilde{A}_2$
- iv. If  $s(\tilde{A}_1) = s(\tilde{A}_2)$ ,  $a(\tilde{A}_1) = a(\tilde{A}_2)$ , and  $c(\tilde{A}_1) = c(\tilde{A}_2)$  then  $\tilde{A}_1$  is equal to  $\tilde{A}_2$ , that is,  $\tilde{A}_1$  is indifferent to  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 = \tilde{A}_2$

**Network Terminology**

Consider a directed network  $G(V, E)$  consisting of a finite set of nodes  $V = \{1, 2, \dots, n\}$  and a set of  $m$  directed edges  $E \subseteq V \times V$ . Each edge is denoted by an ordered pair  $(i, j)$  where  $i, j \in V$  and  $i \neq j$ . In this network, we specify two nodes, denoted by  $s$  and  $t$ , which are the source node and the destination node, respectively. We define a path  $P_{ij} = \{i = i_1, (i_1, i_2), i_2, \dots, i_{l-1}, (i_{l-1}, i_l), i_l = j\}$  of alternating nodes and edges. The existence of at least one path  $P_{si}$  in  $G(V, E)$  is assumed for every  $i \in V - \{s\}$ .

$d_{ij}$  denotes bipolar neutrosophic number associated with the edge (i, j), corresponding to the length necessary to traverse (i, j) from i to j. the bipolar neutrosophic distance along the path P is denoted as  $d(P)$  is defined as

$$d(P) = \sum_{(i,j) \in P} d_{ij} \quad (12)$$

### Bipolar Neutrosophic Path Problem

In this paper the arc length in a network is considered to be a neutrosophic number, namely, bipolar neutrosophic number.

The algorithm for the shortest path proceeds in 6 steps.

**Step 1** Assume  $\tilde{d}_1 = \langle 0, 1, 1, -1, 0, 0 \rangle$  and label the source node (say node 1) as  $[\tilde{d}_1 = \langle 0, 1, 1, -1, 0, 0 \rangle, -]$ .

**Step 2** Find  $\tilde{d}_j = \text{minimum} \{ \tilde{d}_i \oplus \tilde{d}_{ij} \}; j=2,3,\dots,n$ .

**Step 3** If minimum occurs corresponding to unique value of i i.e.,  $i=r$  then label node j as  $[\tilde{d}_j, r]$ . If minimum occurs corresponding to more than one values of i then it represents that there are more than one bipolar neutrosophic path between source node and node j but bipolar neutrosophic distance along path is  $\tilde{d}_j$ , so choose any value of i.

**Step 4** Let the destination node (node n) be labeled as  $[\tilde{d}_n, l]$ , then the bipolar neutrosophic shortest distance between source node is  $\tilde{d}_n$ .

**Step 5** Since destination node is labeled as  $[\tilde{d}_n, l]$ , so, to find the bipolar neutrosophic shortest path between source node and destination node, check the label of node l. Let it be  $[\tilde{d}_l, p]$ , now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

**Step 6** Now the bipolar neutrosophic shortest path can be obtained by combining all the nodes obtained by the step 5.

**Remark:** Let  $\tilde{A}_i; i=1, 2, \dots, n$  be a set of bipolar neutrosophic numbers, if  $S(\tilde{A}_k) < S(\tilde{A}_i)$ , for all i, the bipolar neutrosophic number is the minimum of  $\tilde{A}_k$

### Illustrative Example

In order to illustrate the above procedure consider a small example network shown in Fig1, where each arc length is represented as bipolar neutrosophic number as shown in Table 1. The problem is to find the shortest distance and shortest path between source node and destination node on the network.

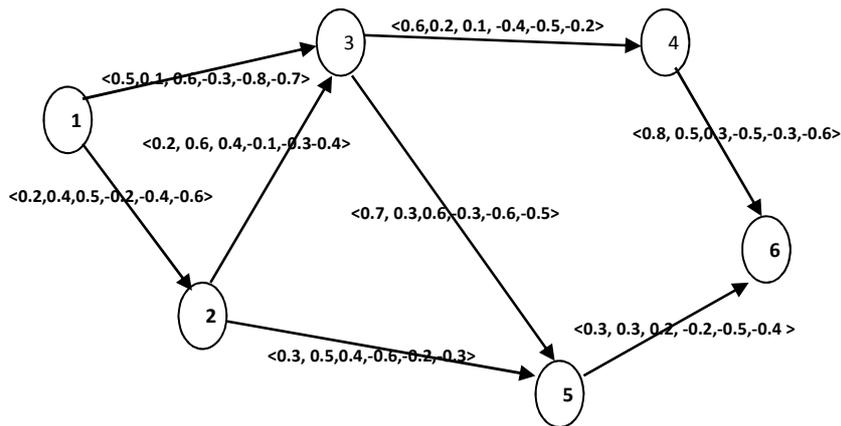


Fig.1. A network with bipolar neutrosophic edges

In this network each edge has been assigned to bipolar neutrosophic number as follows:

Table 1. Weights of the bipolar neutrosophic graphs

Edges	Bipolar Neutrosophic distance
1-2	$\langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle$
1-3	$\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle$
2-3	$\langle 0.2, 0.6, 0.4, -0.1, -0.3, -0.4 \rangle$
2-5	$\langle 0.3, 0.5, 0.4, -0.6, -0.2, -0.3 \rangle$
3-4	$\langle 0.6, 0.2, 0.1, -0.4, -0.5, -0.2 \rangle$
3-5	$\langle 0.7, 0.3, 0.6, -0.3, -0.6, -0.5 \rangle$
4-6	$\langle 0.8, 0.5, 0.3, -0.5, -0.3, -0.6 \rangle$
5-6	$\langle 0.3, 0.3, 0.2, -0.2, -0.5, -0.4 \rangle$

Solution since node 6 is the destination node, so  $n=6$ .

assume  $\tilde{d}_1 = \langle 0, 1, 1, -1, 0, 0 \rangle$  and label the source node ( say node 1) as  $[\langle 0, 1, 1, -1, 0, 0 \rangle, -]$ , the value of  $\tilde{d}_j$ ;  $j=2, 3, 4, 5, 6$  can be obtained as follows:

**Iteration 1** Since only node 1 is the predecessor node of node 2, so putting  $i=1$  and  $j=2$  in step2 of the proposed algorithm, the value of  $\tilde{d}_2$  is

$\tilde{d}_2 = \text{minimum} \{ \tilde{d}_1 \oplus \tilde{d}_{12} \} = \text{minimum} \{ \langle 0, 1, 1, -1, 0, 0 \rangle \oplus \langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle = \langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle$  Since minimum occurs corresponding to  $i=1$ , so label node 2 as  $[\langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle, 1]$

**Iteration 2** The predecessor node of node 3 are node 1 and node 2, so putting  $i=1, 2$  and  $j=3$  in step 2 of the proposed algorithm, the value of  $\tilde{d}_3$  is  $\tilde{d}_3 = \text{minimum} \{ \tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23} \} = \text{minimum} \{ \langle 0, 1, 1, -1, 0, 0 \rangle \oplus \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle, \langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle \oplus \langle 0.2, 0.6, 0.4, -0.1, -0.3, -0.4 \rangle \} = \text{minimum} \{ \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle, \langle 0.36, 0.24, 0.2, -0.02, -0.58, -0.76 \rangle \}$

$$S(\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle) = \left(\frac{1}{6}\right) \times [T^p + 1 - I^p + 1 - F^p + 1 + T^n - I^n - F^n] = 0.66$$

$$S(\langle 0.36, 0.24, 0.2, -0.02, -0.58, -0.76 \rangle) = 0.70$$

Since  $S(\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle) < S(\langle 0.36, 0.24, 0.2, -0.02, -0.58, -0.76 \rangle)$

So minimum  $\{ \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle, \langle 0.36, 0.24, 0.2, -0.02, -0.58, -0.76 \rangle \} = \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle$

Since minimum occurs corresponding to  $i=1$ , so label node 3 as  $[\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle, 1]$

**Iteration 3.** The predecessor node of node 4 is node 3, so putting  $i=3$  and  $j=4$  in step 2 of the proposed algorithm, the value of  $\tilde{d}_4$  is  $\tilde{d}_4 = \text{minimum} \{ \tilde{d}_3 \oplus \tilde{d}_{34} \} = \text{minimum} \{ \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle \oplus \langle 0.6, 0.2, 0.1, -0.4, -0.5, -0.2 \rangle \} = \langle 0.8, 0.02, 0.06, -0.12, -0.9, -0.76 \rangle$

So minimum  $\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle \oplus \langle 0.6, 0.2, 0.1, -0.4, -0.5, -0.2 \rangle = \langle 0.8, 0.02, 0.06, -0.12, -0.9, -0.76 \rangle$

Since minimum occurs corresponding to  $i=3$ , so label node 4 as  $[\langle 0.8, 0.02, 0.06, -0.12, -0.9, -0.76 \rangle, 3]$

**Iteration 4.** The predecessor node of node 5 are node 2 and node 3, so putting  $i=2, 3$  and  $j=5$  in step 2 of the proposed algorithm, the value of  $\tilde{d}_5$  is  $\tilde{d}_5 = \text{minimum} \{ \tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35} \} = \text{minimum} \{ \langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle \oplus \langle 0.3, 0.5, 0.4, -0.6, -0.2, -0.3 \rangle, \langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle \oplus \langle 0.7, 0.3, 0.6, -0.3, -0.6, -0.5 \rangle \} =$

Minimum  $\{ \langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle, \langle 0.85, 0.03, 0.36, -0.09, -0.92, -0.85 \rangle \}$

$S(\langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle) = 0.69$

$S(\langle 0.85, 0.03, 0.36, -0.09, -0.92, -0.85 \rangle) = 0.85$

Since  $S(\langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle) < S(\langle 0.85, 0.03, 0.36, -0.09, -0.92, -0.85 \rangle)$

Minimum  $\{ \langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle, \langle 0.85, 0.03, 0.36, -0.09, -0.92, -0.85 \rangle \}$

$= \langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle$

$\tilde{d}_5 = \langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle$

Since minimum occurs corresponding to  $i=2$ , so label node 5 as  $[\langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle, 2]$

**Iteration 5** The predecessor node of node 6 are node 4 and node 5, so putting  $i=4, 5$  and  $j=6$  in step 2 of the proposed algorithm, the value of  $\tilde{d}_6$  is  $\tilde{d}_6 = \text{minimum} \{ \tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56} \} = \text{minimum} \{ \langle 0.8, 0.02, 0.06, -0.12, -0.9, -0.76 \rangle \oplus \langle 0.8, 0.5, 0.3, -0.5, -0.3, -0.6 \rangle, \langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle \oplus \langle 0.3, 0.3, 0.2, -0.2, -0.5, -0.4 \rangle \} = \text{minimum} \{ \langle 0.96, 0.01, 0.018, -0.06, -0.93, -0.904 \rangle, \langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle \}$

$S(\langle 0.96, 0.01, 0.018, -0.06, -0.93, -0.904 \rangle) = 0.95$

$S(\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle) = 0.85$

Since  $S(\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle) < S(\langle 0.96, 0.01, 0.018, -0.06, -0.93, -0.904 \rangle)$

So minimum  $\{ \langle 0.96, 0.01, 0.018, -0.06, -0.93, -0.904 \rangle, \langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle \}$

$= \langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle$

$\tilde{d}_6 = \langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle$

Since minimum occurs corresponding to  $i=5$ , so label node 6 as  $[\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle, 5]$

Since node 6 is the destination node of the given network, so the bipolar neutrosophic shortest distance between node 1 and node 6 is  $\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle$

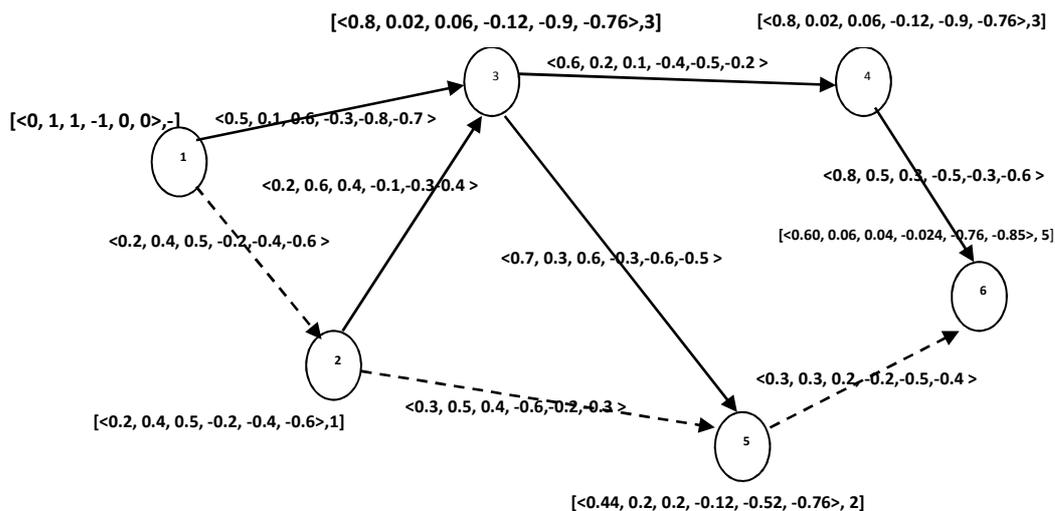
Now the bipolar neutrosophic shortest path between node 1 and node 6 can be obtained by using the following procedure:

Since node 6 is labeled by  $[\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle, 5]$ , which represents that we are coming from node 5. Node 5 is labeled by  $[\langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle, 2]$ , which represents that we are coming from node 2. Node 2 is labeled by  $[\langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle, 1]$  which represents that we are coming from node 1. Now the bipolar neutrosophic shortest path between node 1 and node 6 is obtained by joining all the obtained nodes. Hence the bipolar neutrosophic shortest path is  $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

The bipolar neutrosophic shortest distance and the neutrosophic shortest path of all nodes from node 1 is shown in the table 2 and the labeling of each node is shown in figure 4

**Table 2.** Tabular representation of different bipolar neutrosophic shortest path

Node No.(j)	$\tilde{d}_i$	Bipolar neutrosophic shortest path between $j^{\text{th}}$ and 1st node
2	$\langle 0.2, 0.4, 0.5, -0.2, -0.4, -0.6 \rangle$	$1 \rightarrow 2$
3	$\langle 0.5, 0.1, 0.6, -0.3, -0.8, -0.7 \rangle$	$1 \rightarrow 3$
4	$\langle 0.8, 0.02, 0.06, -0.12, -0.9, -0.76 \rangle$	$1 \rightarrow 3 \rightarrow 4$
5	$\langle 0.44, 0.2, 0.2, -0.12, -0.52, -0.76 \rangle$	$1 \rightarrow 2 \rightarrow 5$
6	$\langle 0.60, 0.06, 0.04, -0.024, -0.76, -0.85 \rangle$	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$



**Fig. 2.** Network with bipolar neutrosophic shortest distance of each node from node 1

**Conclusion**

In this paper we developed an algorithm for solving shortest path problem on a network with bipolar neutrosophic arc lengths. The process of ranking the path is very useful to make decisions in choosing the best of all possible path alternatives. We have explained the method by an example with the help of a hypothetical data. Further, we plan to extend the following algorithm of bipolar neutrosophic shortest path problem in an interval valued bipolar fuzzy neutrosophic environment.

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