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Several Trigonometric Hamming Similarity Measures of Rough Neutrosophic Sets and their Applications in Decision Making

Abstract

In 2014, Broumi et al. (S. Broumi, F. Smarandache, M. Dhar, Rough neutrosophic sets, Italian Journal of Pure and Applied Mathematics, 32 (2014), 493-502.) introduced the notion of rough neutrosophic set by combining neutrosophic sets and rough sets, which has been a mathematical tool to deal with problems involving indeterminacy and incompleteness. The real world is full of indeterminacy. Naturally, real world decision making problem involves indeterminacy. Rough neutrosophic set is capable of describing and handling imprecise, indeterminate and inconsistent and incomplete information. This paper is devoted to propose several new similarity measures based on trigonometric hamming similarity operators of rough neutrosophic sets and their applications in decision making. We prove the required properties of the proposed similarity measures. To illustrate the applicability of the proposed similarity measures in decision making, an illustrative problem is solved.

Keywords

Neutrosophic set, rough set, rough neutrosophic set, Hamming distance, similarity measure.

1. Introduction

L. A. Zadeh [1] introduced the degree of membership in 1965 and defined the concept of fuzzy set to deal with uncertainty. K. T. Atanassov [2] introduced the degree of non-membership as independent component in 1986 and defined the intuitionistic fuzzy set. F. Smarandache [3, 4] introduced the degree of indeterminacy as independent component and defined the neutrosophic set in 1998.

To use the concept of neutrosophic set in practical fields such as real scientific and engineering applications, Wang et al. [5] presented an instance of neutrosophic set, called single valued neutrosophic set (SVNS).

In many applications, due to lack of knowledge or data about the problem domains, the decision information may be provided with intervals, instead of real numbers. To deal with the situation Wang et al. [6] introduced interval valued neutrosophic sets (IVNS), which is characterized by a membership function, non-membership function and an indeterminacy function, whose values are intervals rather than real numbers. Also, the interval valued neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world.

In 2014, Broumi et al. [7, 8] introduced the concept of rough neutrosophic set (RNS). It is derived by hybridizing the concepts of rough set proposed by Pawlak [9] and neutrosophic set originated by F. Smarandache [3, 4]. Neutrosophic sets and rough sets are both capable of dealing with uncertainty and partial information. Rough neutrosophic set [7, 8] is the generalization of rough fuzzy sets [10], [11] and rough intuitionistic fuzzy sets [12].

Mondal and Pramanik [13] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis in 2015. S. Pramanik and K. Mondal [14] also studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis in 2015. Mondal and Pramanik [15] proposed multi attribute decision making using rough accuracy score function. Pramanik and Mondal [16] proposed cotangent similarity measure under rough neutrosophic environment. Pramanik and Mondal [17] further proposed some similarity measures namely Dice similarity measure and Jaccard similarity measure in rough neutrosophic environment. Mondal et al. [18] proposed rough neutrosophic variational coefficient similarity measure and presented its application in multi attribute decision making. Mondal et al. [19] presented rough neutrosophic TOPSIS for multi-attribute group decision making problem. Mondal and Pramanik [20] studied tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Mondal et al. [21] further proposed rough neutrosophic hyper-complex set and its application to multi-attribute decision making.

Literature review reflects that no studies have been made on multi-attribute decision making using trigonometric Hamming similarity measures under rough neutrosophic environment. In this paper, we propose cosine, sine and cotangent Hamming similarity measures under rough neutrosophic environment. We also present a numerical example to show the effectiveness and applicability of the proposed similarity measures.

2. Mathematical Preliminaries

2.1 Neutrosophic set [3, 4]

Let U be a universe of discourse. Then the neutrosophic set A is presented in the form:

$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions T, I, F: $U \rightarrow]^-0,1^+[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set P satisfying the following the condition.

$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$

2.2 Single valued neutrosophic sets [6]

Definition 2.2 [6]

Wang et al. [6] mentioned that the neutrosophic set assumes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$ is difficult to apply in the real applications such as scientific and engineering problems.

Assume that X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_x \frac{<T_A(x), I_A(x), F_A(x)>}{x} : x \in X$$

When X is discrete, a SVNS A can be written as follows:

$$A = \sum_{i=1}^{n} \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i} : x_i \in X .$$

For two SVNSs, $A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\}$ and $B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$, $A_{SVNS} \subseteq B_{SVNS}$ and $A_{SVNS} = B_{SVNS}$ are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$

2.3 Hamming distance [17]

Hamming distance [17] between two neutrosophic sets $A(T_A(x), I_A(x), F_A(x))$ and $B(T_B(x), I_B(x), F_B(x))$ is defined as

$$H(A,B) = \frac{1}{2} \sum_{i=1}^{n} (|T_A(x) - T_B(x)| + |I_A(x) - I_B(x)| + |F_A(x) - F_B(x)|) \quad (1)$$

2.4 Rough neutrosophic set (RNS)

Definition 2.2.1 [1], [2]: Let Z be a non-null set and R be an equivalence relation on Z. Let A be a neutrosophic set in Z with the membership function T_A, indeterminacy function I_A and non-membership function F_A. The lower and the upper approximations of A in the approximation (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \langle <x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x)> / z \in [x]_R, x \in Z \rangle$$

$$\overline{N}(A) = \langle <x, T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x)> / z \in [x]_R, x \in Z \rangle \quad (2)$$

where, $T_{\underline{N}(A)}(x) = \wedge_{z \in [x]_R} T_A(z)$, $I_{\underline{N}(A)}(x) = \wedge_{z \in [x]_R} I_A(z)$, $F_{\underline{N}(A)}(x) = \wedge_{z \in [x]_R} F_A(z)$,

$T_{\overline{N}(A)}(x) = \vee_{z \in [x]_R} T_A(z)$, $I_{\overline{N}(A)}(x) = \vee_{z \in [x]_R} T_A(z)$, $F_{\overline{N}(A)}(x) = \vee_{z \in [x]_R} I_A(z)$.

So, $0 \leq T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \leq 3$ and $0 \leq T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \leq 3$ hold. Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively. $T_A(z)$, $I_A(z)$ and $F_A(z)$ are the membership, indeterminacy and non-membership degrees of z with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic sets in Z.

Thus, NS mappings $\underline{N}, \overline{N} : N(Z) \rightarrow N(Z)$ denote respectively the lower and upper rough NS approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic set in (Z, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence class of R, if $\underline{N}(A) = \overline{N}(A)$; i.e. $T_{\underline{N}(A)}(x) = T_{\overline{N}(A)}(x)$, $I_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x)$, $F_{\underline{N}(A)}(x) = F_{\overline{N}(A)}(x)$.

For any *x* belongs to *Z, P* is said to be a definable neutrosophic set in the approximation (Z, *R*). Obviously, zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.2.2 [1], [2]: Let *N*(*A*) = ($\underline{N}(A), \overline{N}(A)$) is a rough neutrosophic set in (Z, *R*). The rough complement of *N*(*A*) is denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c, \overline{N}(A)^c$ are the complements of neutrosophic sets of $\underline{N}(A), \overline{N}(A)$ respectively.

$$\underline{N}(A)^c = \left\langle < x, F_{\underline{N}(A)}(x), 1 - I_{\underline{N}(A)}(x), T_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \text{ and}$$

$$\overline{N}(A)^c = \left\langle < x, F_{\underline{N}(A)}(x), 1 - I_{\overline{N}(A)}(x), T_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \quad (3)$$

Definition 2.2.3 [1], [2]: Let $N(A)$ and $N(B)$ are two rough neutrosophic sets respectively in *Z*, then the following definitions hold good:

$N(A) = N(B) \Leftrightarrow \underline{N}(A) = \underline{N}(B) \wedge \overline{N}(A) = \overline{N}(B)$

$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \wedge \overline{N}(A) \subseteq \overline{N}(B)$

$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B), \overline{N}(A) \cup \overline{N}(B) >$

$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B), \overline{N}(A) \cap \overline{N}(B) >$

$N(A) + N(B) = < \underline{N}(A) + \underline{N}(B), \overline{N}(A) + \overline{N}(B) >$

$N(A) \,.\, N(B) = < \underline{N}(A) \,.\, \underline{N}(B), \overline{N}(A) \,.\, \overline{N}(B) >$

If *A*, *B*, *C* are the rough neutrosophic sets in (*Z*, *R*), then the following propositions can be stated from definitions.

Proposition 1 [1], [2]:

1. $\sim A(\sim A) = A$
2. $A \cup B = B \cup A, A \cap B = B \cap A$
3. $(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C)$
4. $(A \cup B) \cap C = (A \cup B) \cap (A \cup C), (A \cap B) \cup C = (A \cap B) \cup (A \cap C)$

Proposition 2 [1], [2]:

De Morgan‘s Laws are satisfied for rough neutrosophic sets *N*(*A*) and *N*(*B*)

1. $\sim (N(A) \cup N(B)) = (\sim N(A)) \cap (\sim N(B))$
2. $\sim (N(A) \cap N(B)) = (\sim N(A)) \cup (\sim N(B))$

For the proofs of the propositions, see [1, 2]

Proposition 3[1], [2]:

If *A* and *B* are two neutrosophic sets in *U* such that $A \subseteq B, then\ N(A) \subseteq N(B)$

1. $N(A \cap B) \subseteq N(A) \cap N(B)$
2. $N(A \cup B) \supseteq N(A) \cup N(B)$

For the proofs of the propositions, see [1, 2]

Proposition 4 [1], [2]:

1. $\underline{N}(A) = \sim \overline{N}(\sim A)$
2. $\overline{N}(A) = \sim \underline{N}(\sim A)$
3. $\underline{N}(A) \subseteq \overline{N}(A)$

For the proofs of the propositions, see [1, 2]

3. Cosine Hamming Similarity Measures of RNS

Assume that $A = \left\langle \left(\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)\right), \left(\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)\right) \right\rangle$ and $B = \left\langle \left(\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)\right), \left(\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)\right) \right\rangle$ in *X* = {x_1, x_2, …, x_n} be any two rough neutrosophic sets. A cosine Hamming similarity operator between rough neutrosophic sets *A* and *B* is defined as follows:

$C_{CHSO}(A, B)=$

$$\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\pi}{6}\left(\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|+\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|+\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|\right)\right) \quad (4)$$

Here, $\Delta T_A(x_i)=\left(\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}\right)$, $\Delta T_B(x_i)=\left(\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}\right)$,

$\Delta I_A(x_i)=\left(\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}\right)$, $\Delta I_B(x_i)=\left(\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}\right)$,

$\Delta F_A(x_i)=\left(\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}\right)$, $\Delta F_B(x_i)=\left(\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}\right)$.

Also, $[\Delta T_A(x), \Delta I_A(x), \Delta F_A(x)]\neq[0, 0, 0]$ and $[\Delta T_B(x), \Delta I_B(x), \Delta F_B(x)]\neq[0, 0, 0]$, $i = 1, 2, \ldots, n$.

Proposition 3.1

The defined rough neutrosophic cosine hamming similarity operator $C_{CHSO}(A, B)$ between RNSs *A* and *B* satisfies the following properties:

1. $0\leq C_{RCHSO}(A, B)\leq 1$
2. $C_{CHSO}(A, B) = 1$ if and only if $A = B$
3. $C_{CHSO}(A, B) = C_{CHSO}(B, A)$

Proof of the property 1.

Since the functions $\Delta T_A(x)$, $\Delta I_A(x)$, $\Delta F_A(x)$, $\Delta T_B(x)$, $\Delta I_B(x)$, and $\Delta F_B(x)$, and the value of the cosine function are within [0,1], the similarity measure based on rough neutrosophic cosine hamming similarity function also lies within [0,1].

Hence $0\leq C_{CHSO}(A, B)\leq 1$.

This completes thee proved.

Proof of the property 2.

For any two RNSs *A* and *B*, if $A = B$, then the following relations hold $\Delta T_A(x_i)=\Delta T_B(x_i)$, $\Delta I_A(x_i)=\Delta I_B(x_i)$, $\Delta F_A(x_i)=\Delta F_B(x_i)$. Hence

$\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|=0$, $\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|=0$, $\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|=0$.

Thus $C_{CHSO}(A, B) = 1$

Conversely,

If $C_{CHSO}(A, B) = 1$, then $\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|=0$, $\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|=0$, $\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|=0$. since cos(0) = 1. So we can write $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, $F_A(x_i)=F_B(x_i)$

Hence $A = B$**.**

4. Sine Hamming Similarity Measures of RNS

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in X = {x_1, x_2, …, x_n} be any two rough neutrosophic sets. A sine Hamming similarity operator between two rough neutrosophic sets *A* and *B* is defined as follows:

$S_{CHSO}(A, B)=$

$$1-\left[\frac{1}{n}\sum_{i=1}^{n}\sin\left(\frac{\pi}{6}\left(\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|+\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|+\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|\right)\right)\right] \quad (4)$$

Also, $[\Delta T_A(x), \Delta I_A(x), \Delta F_A(x)] \neq [0, 0, 0]$ and $[\Delta T_B(x), \Delta I_B(x), \Delta F_B(x)] \neq [0, 0, 0]$, $i = 1, 2, \ldots, n$.

Proposition 4.1

The defined rough neutrosophic sine Hamming similarity operator $S_{CHSO}(A, B)$ between RNSs *A* and *B* satisfies the properties 4, 5, 6 as follows.

1. $0 \leq S_{CHSO}(A, B) \leq 1$
1. $S_{CHSO}(A, B) = 1$ if and only if $A = B$
2. $S_{CHSO}(A, B) = S_{CHSO}(B, A)$

Proof of the property 1.

Since the functions $\Delta T_A(x)$, $\Delta I_A(x)$, $\Delta F_A(x)$, $\Delta T_B(x)$, $\Delta I_B(x)$, and $\Delta F_B(x)$, and the value of the sine function are within [0 ,1], the similarity measure based on rough neutrosophic sine hamming similarity function also lies within [0,1].

Hence $0 \leq S_{CHSO}(A, B) \leq 1$.

Proof of the property 2.

For any two RNSs *A* and *B* if *A* = *B*, then the following relations hold $\Delta T_A(x_i)=\Delta T_B(x_i)$, $\Delta I_A(x_i)=\Delta I_B(x_i)$, $\Delta F_A(x_i)=\Delta F_B(x_i)$. Hence

$\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|=0$, $\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|=0$, $\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|=0$. Thus $S_{CHSO}(A, B) = 1$

Conversely,

If $S_{CHSO}(A, B) = 1$, then $\left|\Delta T_A(x_i)-\Delta T_B(x_i)\right|=0$, $\left|\Delta I_A(x_i)-\Delta I_B(x_i)\right|=0$, $\left|\Delta F_A(x_i)-\Delta F_B(x_i)\right|=0$. since sin(0) = 0. So we can write $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, $F_A(x_i)=F_B(x_i)$

Hence *A* = ***B*.**

Proof of the property 3.

This proof is obvious.

5. Cotangent Hamming Similarity Measures of RNS

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in $X = \{x_1, x_2, \ldots, x_n\}$ be any two rough neutrosophic sets. A cotangent Hamming similarity operator between two rough neutrosophic sets A and B can be defined as follows:

$$COT_{CHSO}(A,\ B)= \frac{1}{n}\sum_{i=1}^{n}\cot\left(\frac{\pi}{4}+\frac{\pi}{12}\left(|\Delta T_A(x_i)-\Delta T_B(x_i)|+|\Delta I_A(x_i)-\Delta I_B(x_i)|+|\Delta F_A(x_i)-\Delta F_B(x_i)|\right)\right)$$

(5)

Also, $[\Delta T_A(x), \Delta I_A(x), \Delta F_A(x)] \neq [0, 0, 0]$ and $[\Delta T_B(x), \Delta I_B(x), \Delta F_B(x)] \neq [0, 0, 0]$, $i = 1, 2, \ldots, n$.

Proposition 5.1

The defined rough neutrosophic cotangent Hamming similarity operator COT_{CHSO}(A, B) between RNSs A and B satisfies the properties 7, 8, 9.

1. $0 \leq COT_{CHSO}(A, B) \leq 1$
2. $COT_{CHSO}(A, B) = 1$ if and only if $A = B$
3. $COT_{CHSO}(A, B) = COT_{CHSO}(B, A)$

Proof of the property 1:

Proof: Since the functions $\Delta T_A(x)$, $\Delta I_A(x)$, $\Delta F_A(x)$, $\Delta T_B(x)$, $\Delta I_B(x)$, and $\Delta F_B(x)$, and the value of the cotangent function are within [0 ,1], the similarity measure based on rough neutrosophic cotangentHamming similarity function also lies within [0,1].

Hence $0 \leq COT_{CHSO}(A, B) \leq 1$

Proof of the property 2:

For any two RNSs A and B if $A = B$, we have

$\Delta T_A(x_i)=\Delta T_B(x_i)$, $\Delta I_A(x_i)=\Delta I_B(x_i)$, $\Delta F_A(x_i)=\Delta F_B(x_i)$.

Hence

$|\Delta T_A(x_i)-\Delta T_B(x_i)|=0$, $|\Delta I_A(x_i)-\Delta I_B(x_i)|=0$, $|\Delta F_A(x_i)-\Delta F_B(x_i)|=0$. Thus $COT_{CHSO}(A, B) = 1$

Conversely,

If $COT_{CHSO}(A, B) = 1$, then $|\Delta T_A(x_i)-\Delta T_B(x_i)|=0$, $|\Delta I_A(x_i)-\Delta I_B(x_i)|=0$, $|\Delta F_A(x_i)-\Delta F_B(x_i)|=0$.

Since $\cot(\frac{\pi}{4}) = 1$, we can write $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, $F_A(x_i)=F_B(x_i)$

Hence A = B.

Proof of the property 3:

This proof is obvious.

6. Decision making under trigonometric rough neutrosophic Hamming similarity measures

In this section, we apply rough cosine, sine and cotangent Hamming similarity measures between RNSs to the multi-criteria decision making problem. Assume that $S = \{S_1, S_2, \ldots, S_m\}$ be a set of alternatives and $A = \{A_1, A_2, \ldots, A_n\}$ be a set of attributes.

The proposed decision making method is described using the following steps.

Step 1: **Construction of the decision matrix with rough neutrosophic number**

Decision maker considers the decision matrix with respect to m alternatives and n attributes in terms of rough neutrosophic numbers as follows.

Table1: *Rough neutrosophic decision matrix*

$$D=\langle \underline{d}_{ij}, \overline{d}_{ij}\rangle_{m\times n} = \begin{array}{c|cccc} & A_1 & A_2 & \cdots & A_n \\ \hline S_1 & \langle \underline{d}_{11}, \overline{d}_{11}\rangle & \langle \underline{d}_{12}, \overline{d}_{12}\rangle & \ldots & \langle \underline{d}_{1n}, \overline{d}_{1n}\rangle \\ S_2 & \langle \underline{d}_{21}, \overline{d}_{21}\rangle & \langle \underline{d}_{22}, \overline{d}_{22}\rangle & \ldots & \langle \underline{d}_{2n}, \overline{d}_{2n}\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ S_m & \langle \underline{d}_{m1}, \overline{d}_{m1}\rangle & \langle \underline{d}_{m2}, \overline{d}_{m2}\rangle & \ldots & \langle \underline{d}_{mn}, \overline{d}_{mn}\rangle \end{array} \quad (6)$$

Here $\langle \underline{d}_{ij}, \overline{d}_{ij}\rangle$ is the rough neutrosophic number according to the *i*-th alternative and the *j*-th attribute.

Step 2: **Determination of the weights of attribute**

Assume that the weight of the attributes A_j ($j = 1, 2, \ldots, n$) considered by the decision-maker be w_j (($j = 1, 2, \ldots, n$)) such that $\forall w_j \in [0, 1]$ ($j = 1, 2, \ldots, n$) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: **Determination of the benefit type attribute and cost type attribute**

Generally, the evaluation attribute can be categorized into two types: benefit type attribute and cost type attribute. Let K be a set of benefit type attributes and M be a set of cost type attributes. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit type attribute and a minimum operator for the cost type attribute to determine the best value of each criterion among all alternatives. We define an ideal alternative S* as follows:

S* = $\{S_1^*, S_2^*, \ldots, S_m^*\}$, where benefit attribute is presented as

$$S_j^* = \left[\max_i T_{A_j}^{(S_i)}, \min_i I_{A_j}^{(S_i)}, \min_i F_{A_j}^{(S_i)}\right]$$

and cost type attribute is presented as

$$S_j^* = \left[\min_i T_{A_j}^{(S_i)}, \max_i I_{A_j}^{(S_i)}, \max_i F_{A_j}^{(S_i)}\right].$$

Step 4: **Determination of the overall weighted rough trigonometric neutrosophic Hamming similarity function (WRTNHSF) of the alternatives**

We define weighted rough trigonometric neutrosophic similarity function as follows.

$$C_{WCHSO}(A, B) = \sum_{j=1}^{n} w_j C_{CHSO}(A, B) \quad (7)$$

$$S_{WCHSO}(A, B) = \sum_{j=1}^{n} w_j S_{CHSO}(A, B) \quad (8)$$

$$COT_{WCHSO}(A, B) = \sum_{j=1}^{n} w_j COT_{CHSO}(A, B) \quad (9)$$

where $\sum_{j=1}^{n} w_j = 1$, j = 1, 2, …, n.

Step 5: **Ranking the alternatives**

Using the weighted rough trigonometric neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be selected with the highest similarity value.

Step 6: End

7. Numerical Example

Assume that a decision maker intends to select the most suitable smart phone for rough use from the three initially chosen smart phones (S_1, S_2, S_3) by considering four attributes namely: features A_1, reasonable price A_2, customer care A_3, risk factor A_4. Based on the proposed approach discussed in section 5, the considered problem is solved using the following steps:

Step 1: **Construction of the decision matrix with rough neutrosophic numbers**

The decision maker forms a decision matrix with respect to three alternatives and four attributes in terms of rough neutrosophic numbers (see the Table 2).

Table 2. *Decision matrix with rough neutrosophic number*

$d_S = \langle \underline{N}(P), \overline{N}(P) \rangle_{3\times 4}$ = (10)

	A_1	A_2	A_3	A_4
S_1	$\langle (0.6, 0.3, 0.3), (0.8, 0.1, 0.1) \rangle$	$\langle (0.6, 0.4, 0.4), (0.8, 0.2, 0.2) \rangle$	$\langle (0.6, 0.4, 0.4), (0.8, 0.2, 0.2) \rangle$	$\langle (0.7, 0.4, 0.4), (0.9, 0.2, 0.2) \rangle$
S_2	$\langle (0.7, 0.3, 0.3), (0.9, 0.1, 0.3) \rangle$	$\langle (0.6, 0.3, 0.3), (0.8, 0.3, 0.3) \rangle$	$\langle (0.6, 0.2, 0.2), (0.8, 0.4, 0.2) \rangle$	$\langle (0.7, 0.3, 0.3), (0.9, 0.3, 0.3) \rangle$
S_3	$\langle (0.6, 0.2, 0.2), (0.8, 0.0, 0.2) \rangle$	$\langle (0.7, 0.3, 0.3), (0.9, 0.1, 0.1) \rangle$	$\langle (0.7, 0.4, 0.6), (0.9, 0.2, 0.4) \rangle$	$\langle (0.6, 0.3, 0.2), (0.8, 0.1, 0.2) \rangle$

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.32, 0.28, 0.28 and 0.12 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here three benefit types attributes A_1, A_2, A_3 and one cost type attribute A_4.

S^* = [(0.8, 0.1, 0.2), (0.8, 0.2, 0.2), (0.8, 0.3, 0.3), (0.0.7, 0.3, 0.3)]

Step 4: Determination of the overall weighted rough trigonometric neutrosophic Hamming similarity function (WRHNHSF) of the alternatives

We calculate weighted rough trigonometric neutrosophic Hamming similarity values as follows.

$C_{WCHSO}(S_1, S^*) = 0.99554$, $C_{WCHSO}(S_2, S^*) = 0.99253$, $C_{WCHSO}(S_3, S^*) = 0.99799$

$S_{WCHSO}(S_1, S^*) = 0.89455$, $S_{WCHSO}(S_2, S^*) = 0.89233$, $S_{WCHSO}(S_3, S^*) = 0.91729$

$COT_{WCHSO}(S_1, S^*) = 0.92114$, $COT_{WCHSO}(S_2, S^*) = 0.90322$, $COT_{WCHSO}(S_3, S^*) = 0.93009$

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative.

Here,

$C_{WCHSO}(S_3, S^*) \succ C_{WCHSO}(S_1, S^*) \succ C_{WCHSO}(S_2, S^*)$

$S_{WCHSO}(S_3, S^*) \succ S_{WCHSO}(S_1, S^*) \succ S_{WCHSO}(S_2, S^*)$

$COT_{WCHSO}(S_3, S^*) \succ COT_{WCHSO}(S_1, S^*) \succ COT_{WCHSO}(S_2, S^*)$

Hence, the smartphone S_3 is the best alternative for rough use.

Step 6: End

7.1 Comparison

All the three similarity measures provided the same ranking order.

8. Conclusion

In this paper, we propose rough trigonometric Hamming similarity measures based multi-attribute decision making of rough neutrosophic environment and prove some of their basic properties. We provide an application, namely selection of the most suitable smart phone for rough use. We also present comparison with the three rough neutrosophic similarity measures. The concept presented in this paper can be applied other multiple attribute decision making problems in rough neutrosophic environment.

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