AN INTRODUCTION TO MULTI-SPACE AND MULTI-STRUCTURE

by Florentin Smarandache
University of New Mexico
Gallup, NM 87301, USA
E-mail: smarand@unm.edu

Abstract.
The notions of "multi-space" and "multi-structure" are introduced in this paper in order to better model certain real phenomena.

Keywords: Multi-structure, Multi-space (or k-Structured Space), Algebraic structures, Multi-Group, Multi-Ring, Multi-Field, Multi-Lattice, Multi-Module, Infinite-Structured Space, Infinite-Structured Group, S_1-structure with respect to S_2-structure.

1991 MSC: 08A99, 05A18, 03G10, 06D05

Introduction.

Because the reality is not homogeneous, and natural spaces are a mixture of subspaces with various features, it is necessary to define multi-spaces and multi-structures.

Let S_1 and S_2 be two distinct structures, induced by the ensemble of laws L which verify the axiom ensemble A_1 and A_2 respectively, such that A_1 is strictly included in A_2.

Definition of Multi-Structure.

One says that the set M, endowed with the properties:
a) M has an S_1-structure;
b) there is a proper subset P (different from the empty set ø, from the unitary element with respect to S_2, and from M) of the initial set M which has an S_2-structure;
c) M doesn't have an S_2-structure;
is an *S_1-structure with respect to S_2-structure*.

As examples see [4] about “Special Algebraic Structures” applied in the Congruence Theory.

Definition of Multi-Space.

Let $S_1, S_2, \ldots, S_k$ be distinct space-structures. We define the *Multi-Space* (or *k-Structured Space*) as a set M such that for each structure S_i, $1 \le i \le k$, there is a proper (different from the empty set, from the unitary element with respect to S_i, and from M) subset M_i of it which has that structure. The $M_1, M_2, \ldots, M_k$ proper subsets are distinct two by two.
(F.Smarandache, "Mixed Non-euclidean Geometries", 1969; see [2].)

Generalization.

Similarly one can define the *Multi-Group*, *Multi-Ring*, *Multi-Field, Multi-Lattice, Multi-Module,* etc. - which may be generalized to *Infinite-Structured-Space, Infinite-Structured-Group,* and so on.

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