Unit Graph of Some Finite Group Zn, Cn and Dn

RESEARCH - JULY 2015
DOI: 10.13140/RG.2.1.4726.3843

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Unit Graph of Some Finite Group $\mathbb{Z}_n$, $C_n$ and $D_n$

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Abstract
We represent finite group in the form of a graph, these graphs are called unit graph. Since the main role in obtaining the graph is played by the unit element of the group, this study is innovative. Also study of different properties like the subgroups of a group, normal subgroups of a group are carried out using the unit graph of the group.

Mathematics Subject Classification: 20F65, 05C25

Keywords: Finite Group, Commutative Ring, Graphs.

1. INTRODUCTION:
The phenomenon of representing Groups using Graphs has been studied theoretically by number of researchers [3-8]. In a series of investigations Akbari, S and Mohammadian [9, 10] discussed the zero divisor graphs of finite rings. In [12] maximum finite Groups are represented as graphs with examples, diagrams and theorems. In this article we give unit graph of finite groups such as $\mathbb{Z}_n$, $C_n$ and $D_n$.

2. UNIT GRAPH OF GROUP:

2.1 Definition: A graph $G (V, E)$ is Unit Graph of a Group of group $(G, .)$ if
\begin{itemize}
  \item[i.] Elements $v_i$ and $v_j$ are adjacent in graph $G (V, E)$ if $v_i . v_j = e$ in group $(G, .)$.
  \item[ii.] Every element of group $(G, .)$ is adjoined with the unity of group $(G, .)$.
\end{itemize}

2.2 UNIT GRAPH OF GROUP $\mathbb{Z}_n$:
2.2.1 (Unit graph of group $\mathbb{Z}_2$:) Let $\mathbb{Z}_2 = \{0, 1\}$ be the group under addition modulo 2. The unit graph of $\mathbb{Z}_2$ is

\[
\begin{array}{c}
0 \\
1
\end{array}
\]
2.2.2 (Unit graph of group $\mathbb{Z}_3$) Let $\mathbb{Z}_3 = \{0, 1, 2\}$ be the group under addition modulo three. The unit graph of $\mathbb{Z}_3$ is

2.2.3 (Unit graph of group $\mathbb{Z}_4$) Let $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ be the group under addition modulo four. The unit graph of the group $\mathbb{Z}_4$ is

2.2.4 (Unit graph of group $\mathbb{Z}_7$) Let $\mathbb{Z}_7 = \{0, 1, 2, \ldots, 6\}$, the group of integers modulo 7 under addition. The unit graph of $\mathbb{Z}_7$ is

2.2.5 (Unit graph of group $\mathbb{Z}_{12}$) Let $\mathbb{Z}_{12} = \{0, 1, 2, \ldots, 11\}$ be the group under addition modulo 12. The unit graph of $\mathbb{Z}_{12}$ is

2.2.6 (Unit graph of group $\mathbb{Z}_{17}$) Let $\mathbb{Z}_{17} = \{0, 1, 2, \ldots, 16\}$ be the group under addition modulo 17. The unit graph of $\mathbb{Z}_{17}$ is
2.2.7 (Unit graph of group $\mathbb{Z}_{17}$) Let $G = \mathbb{Z}_{17} \setminus \{0\} = \{1, 2, \ldots, 16\}$ be the group under multiplication modulo 17. The unit graph associated with $G$ is

![Unit graph of $\mathbb{Z}_{17}$]

2.2.8 (Unit graph of group $\mathbb{Z}_{30}$) Let $G = \mathbb{Z}_{30} = \{0, 1, 2, \ldots, 29\}$ be the group under addition modulo 30. The unit graph associated with $G$ is as follows.

![Unit graph of $\mathbb{Z}_{30}$]

2.3 UNIT GRAPH OF CYCLIC GROUP $C_n$:

2.3.1 (Unit graph of cyclic group $C_6$) Let $G = C_6 = \{g \mid g^6 = 1\}$ the cyclic group of order 6 under multiplication. The unit graph of $G$ is

![Unit graph of $C_6$]
2.3.2 *(Unit graph of cyclic group $C_8$ :)* Let $G = C_8 = \{g \mid g^8 = 1\}$ the cyclic group of order 8 under multiplication. The unit graph of $G$ is

2.3.3 *(Unit graph of cyclic group $C_7$ :)* Let $G = C_7 = \{g \mid g^7 = 1\}$ the cyclic group of order 7 under multiplication. The unit graph of $G$ is

2.3.4 *(Unit graph of cyclic group $C_{12}$ :)* Let $G = C_{12} = \{g \mid g^{12} = 1\}$ the cyclic group of order 12 under multiplication. The unit graph of $G$ is

2.3.5 *(Unit graph of cyclic group $C_{14}$ :)* Let $G = C_{14} = \{g \mid g^{14} = 1\}$ the cyclic group of order 14 under multiplication. The unit graph of $G$ is
2.3.6 (Unit graph of cyclic group $C_{16}$) Let $G = C_{16} = \{g \mid g^{16} = 1\}$ the cyclic group of order 16 under multiplication. The unit graph of $G$ is

2.3.7 (Unit graph of cyclic group $C_2 \times C_8$) Let $G = C_2 \times C_8 = \{g \mid g^2 = 1\} \times \{g \mid g^8 = 1\} = \{(1, 1), (1, h), (1, h^2), (1, h^3), (1, h^4), (1, h^5), (1, h^6), (1, h^7), (g, h), (g, h^2), (g, h^3), (g, h^4), (g, h^5), (g, h^6), (g, h^7)\}$. The cyclic group of order 16 under multiplication. The unit graph of $G$ is

2.3.8 (Unit graph of cyclic group $C_4 \times C_4$) Let $G = C_4 \times C_4 = \{g \mid g^4 = 1\} \times \{g \mid g^4 = 1\} = \{(1, 1), (1, h), (1, h^2), (1, h^3), (g, h) (g, 1) (g, h^2), (g, h^3), (g^2, 1) (g^2, h^2), (g^2, h) (g^2, h^3) (g^3, 1) (g^3, h) (g^3, h^2) (g^3, h^3)\}$. The cyclic group of order 16 under multiplication. The unit graph of $G$ is
2.3.9 (Unit graph of cyclic group $C_2 \times C_3 \times C_2$) Let $G = C_2 \times C_3 \times C_2 = \{g \mid g^2 = 1\} \times \{g \mid g^3 = 1\} = \{(1,1,1), (g,1,1), (1,h,1), (1,h^2,1), (g,h,1), (g,h^2,1), (1,1,p), (g,h,p), \}$. The cyclic group of order 12 under multiplication. The unit graph of $G$ is

![Unit graph of cyclic group $C_2 \times C_3 \times C_2$](image1.png)

2.3.9 (Unit graph of cyclic group $C_2 \times C_6$) Let $G = C_2 \times C_6 = \{g \mid g^2 = 1\} \times \{g \mid g^6 = 1\} = \{(1,1), (g,1), (1,h), (1,h^2), (1,h^3), (1,h^4), (1,h^5), (g,h), (g,h^2), (g,h^3), (g,h^4), (g,h^5)\}$. The cyclic group of order 12 under multiplication. The unit graph of $G$ is

![Unit graph of cyclic group $C_2 \times C_6$](image2.png)

**THEOREM 2.3.1:** If $G = C_p = \{g \mid g^p = 1\}$ be a cyclic group of order $p$, $p$ a prime. Then the unit graph formed by $G$ has only triangles in fact $(p - 1)/2$ triangles.

**COROLLARY 2.4:** If $G$ is a cyclic group of odd order then also $G$ has the unit graph $G_i$ which is formed only by triangles with no lines.

2.4 UNIT GRAPH OF DIHEDRAL GROUP $D_{m,n}$:

2.4.1 (Unit graph of Dihedral group $D_{2,3}$) Let $D_{2,3} = \{a, b \mid a^2 = b^3 = 1; b \ a \ b = a\}$ be the dihedral group. $O(D_{2,3}) = 6$. The identity graph associated with $D_{2,3}$ is given below.

![Unit graph of Dihedral group $D_{2,3}$](image3.png)
2.4.2 (Unit graph of Dihedral group $D_{2,6}$) The dihedral group $D_{2,6} = \langle a, b \mid a^2 = b^6 = 1, bab = a \rangle = \{1, a, b, ab, ab^2, ab^3, ab^4, ab^5, b^2, b^3, b^4, b^5\}$. The identity graph of $D_{2,6}$ is

![Unit graph of $D_{2,6}$](image1)

2.4.3 (Unit graph of Dihedral group $D_{2,7}$) The identity graph of the dihedral group $D_{2,7} = \langle a, b \mid a^2 = b^7 = 1, bab = a \rangle$ is as follows:

![Unit graph of $D_{2,7}$](image2)

2.4.4 (Unit graph of Dihedral group $D_{2,8}$) Let $D_{2,8} = \langle a, b \mid a^2 = b^8 = 1, bab = a \rangle = \{1, a, b, b^2, b^3, b^4, b^5, b^6, b^7, ab, ab^2, ab^3, ab^4, ab^5, ab^6, ab^7\}$ be the dihedral group of order 16. The identity graph of $D_{2,8}$ is

![Unit graph of $D_{2,8}$](image3)
CONCLUSION:
The first time we represent finite group in the form of a graphs so this is very exciting area of research & relatively new in mathematics with a range of applications including coding & digital communications.

REFERENCES: