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Surfaces family with a common Mannheim geodesic curve

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Abstract

In this paper, we analyzed surfaces family possessing a Mannheim partner curve of a given curve as a geodesic. Using the Frenet frame of the curve in Euclidean 3-space, we express the family of surfaces as a linear combination of the components of this frame and derive the necessary and sufficient conditions for coefficients to satisfy both the geodesic and isoparametric requirements. The extension to ruled surfaces is also outlined. Finally, examples are given to show the family of surfaces with common Mannheim geodesic curve.

Keywords

Geodesic curve; Mannheim partner; Frenet Frame; Ruled Surface.

1. Introduction

At the corresponding points of associated curves, one of the Frenet vectors of a curve coincides with one of the Frenet vectors of other curve. This has attracted the attention of many mathematicians. One of the well-known curves is the Mannheim curve, where the principal normal line of a curve coincides with the binormal line of another curve at the corresponding points of these curves. The first study of Mannheim curves has been presented by Mannheim in 1878 and has a special position in the theory of curves (Blum, 1966). Other studies have been revealed, which introduce some characterized properties in the Euclidean and Minkowski space (Lee, 2011; Liu & Wang, 2008; Orbay &Kasap, 2009; Öztekin & Ergüt, 2011). Liu and Wang called these new curves as Mannheim partner curves: Let x and x_1 be two curves in the three dimensional Euclidean E^3. If there exists a corresponding relationship between the space curves x and x_1 such that, at the corresponding points of the curves, the principal normal lines of x coincides with the binormal lines of x_1, then x is called a Mannheim curve, and x_1 is called a Mannheim partner curve of x. The pair $\{x, x_1\}$ is said to be a Mannheim pair. They showed that the curve: $x_1(s_1)$ is the Mannheim partner curve of the curve $x(s)$ if and only if the curvature κ_1 and the torsion τ_1 of $x_1(s_1)$ satisfy following equation

$$\tau' = \frac{d\tau}{ds_1} = \frac{\kappa_1}{\lambda}\left(1+\lambda^2\tau_1^{\ 2}\right)$$

for some non-zero constant λ. They also study the Mannheim curves in Minkowski 3-space. The generalizations of the Mannheim curves in the 4-dimensional spaces have been given (Matsuda & Yorozu, 2009; Akyiğit, *et al*. 2011). Later, Mannheim offset the ruled surfaces and dual Mannheim curves have been defined in Orbay *et al*.2009; Özkaldı *et al.* 2009; Güngör & Tosun, 2010). Apart from these, some properties of Mannheim curves have been analyzed according to different frames such as the weakened Mannheim curves, quaternionic Mannheim curves and quaternionic Mannheim curves of Aw(k) - type (Karacan, 2011; Okuyucu, 2013; Önder & Kızıltuğ, 2012; Kızıltuğ & Yaylı, 2015). In differential geometry, there are many important consequences and properties of curves (O'Neill, 1966; do Carmo, 1976). Researches follow labours about the curves. One of most significant curve on a surface is geodesic curve. Geodesics are important in the relativistic description of gravity. Einstein's principle of equivalence tells us that geodesics represent the

paths of freely falling particles in a given space. (Freely falling in this context means moving only under the influence of gravity, with no other forces involved). The geodesics principle states that the free trajectories are the geodesics of space. It plays a very important role in a geometric-relativity theory, since it means that the fundamental equation of dynamics is completely determined by the geometry of space, and therefore has not to be set as an independent equation. In architecture, some special curves have nice properties in terms of structural functionality and manufacturing cost. One example is planar curves in vertical planes, whichcan be used as support elements. Another example is geodesic curves, Deng described methods to create patterns of special curves on surfaces, which find applications in design and realization of freeform architecture. (Deng, B. 2011). He presented an evolution approach to generate a series of curves which are either geodesic or piecewise geodesic, starting from a given source curve on a surface. The concept of family of surfaces having a given characteristic curve was first introduced by Wang et.al. in Euclidean 3-space. Kasap et.al. generalized the work of Wang by introducing new types of marching-scale functions, coefficients of the Frenet frame appearing in the parametric representation of surfaces. Atalay and Kasap , studied the problem: given a curve (with Bishop frame), how to characterize those surfaces that posess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space. Also they studied the problem: given a curve (with Frenet frame), how to characterize those surfaces that posess this curve as a common isogeodesic and Smarandache curve in Euclidean 3-space.
As is well-known, a surface is said to be “ruled” if it is generated by moving a straight line continuously in Euclidean space E^3 (O'Neill, 1997). Ruled surfaces are one of the simplest objects in geometric modeling. One important fact about ruled surfaces is that they can be generated by straight lines. A practical application of this type surfaces is that they are used in civil engineering and physics (Guan et al.,1997). Since building materials such as wood are straight, they can be considered as straight lines. The result is that if engineers are planning to construct something with curvature, they can use a ruled surface since all the lines are straight (Orbay et al., 2009).
In this paper, we analyzed surfaces family possessing an Mannheim partner of a given curve as a geodesic. Using the Frenet frame of the curve in Euclidean 3-space, we express the family of surfaces as a linear combination of the components of this frame, and derive the necessary and sufficient conditions for coefficents to satisfy both the geodesic and isoparametric requirements. The extension to ruled surfaces is also outlined. Finally, examples are given to show the family of surfaces with common Mannheim geodesic curve.

2. Preliminaries

Let E^3 be a 3-dimensional Euclidean space provided with the metric given by

$$\langle\, ,\rangle = dx_1^{\,2} + dx_2^{\,2} + dx_3^{\,2}$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. Recall that, the norm of a arbitrary vector $X \in E^3$ is given by $\|X\| = \sqrt{\langle X, X\rangle}$. Let $\alpha = \alpha(s) : I \subset IR \to E^3$ is an arbitrary curve of arc-length parameter s. The curve α is called a unit speed curve if velocity vector α' of a satisfies $\|\alpha'\| = 1$. Let $\{\mathrm{T(s)}, \mathrm{N(s)}, \mathrm{B(s)}\}$ be the moving Frenet frame along α, Frenet formulas is given by

$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{pmatrix}\begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix},$$

where the function $\kappa(s)$ and $\tau(s)$ are called the curvature and torsion of the curve $\alpha(s)$, respectively.

Let C: α (s) be the Mannheim curve in E^3 parameterized by its arc length s and C*: $\alpha^*(s^*)$ is the Mannheim partner curve of C with an arc length parameter s^*. Denote by $\{T(s), N(s), B(s)\}$ the Frenet frame field along C: α (s), that is; $T(s)$ is the tangent vector field, $N(s)$ is the normal vector field, and $B(s)$ is the binormal vector field of the curve C respectively also denote by $\{T^*(s), N^*(s), B^*(s)\}$ the Frenet frame field along C^*: α^* (s), that is; $T^*(s)$ is the tangent vector field, $N^*(s)$ is the normal vector field, and $B^*(s)$ is the binormal vector field of the curve C^* respectively. If there exists one to one correspondence between the points of the space curves C and C^* such that the binormal vector of C is in the direction of the principal normal vector of the curve C^*, then the (C, C^*) curve couple is called Mannheim pairs (Liu & Wang, 2008).

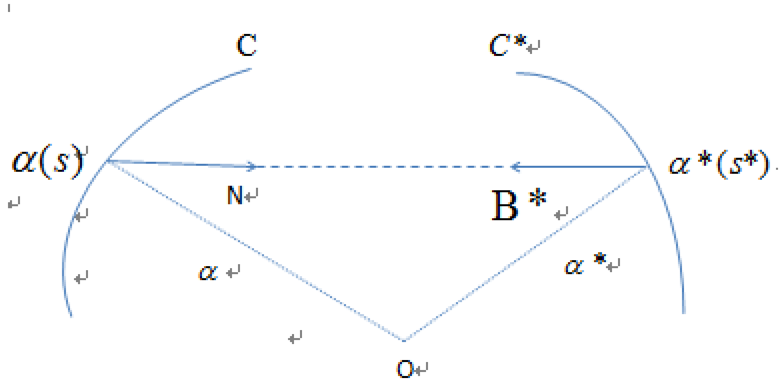

Figure 1. The Mannheim partner curves.

From the figure1, we can write $\alpha(s)=\alpha^*(s^*)+\lambda(s^*)B^*(s^*)$.

A curve on a surface is geodesic if and only if the normal vector to the curve is everywhere parallel to the local normal vector of the surface. Another criterion for a curve in a surface M to be geodesic is that its geodesic curvature vanishes. An isoparametric curve α(s) is a curve on a surface Ψ=Ψ(s,t) is that has a constant s or t-parameter value. In other words, there exist a parameter s_0 or t_0 such that α(s)= Ψ(s,t_0) or α(t)= Ψ(s_0 ,t). Given a parametric curve α(s), we call α(s) an isogeodesic of a surface Ψ if it is both a geodesic and an isoparametric curve on Ψ.

3. Surfaces with common Mannheim geodesic curve

Suppose we are given a 3-dimensional parametric curve $\alpha(s)$, $L_1 \le s \le L_2$, in which s is the arc length and $\|\alpha''(s)\| \ne 0$. Let $\bar{\alpha}(s)$, $L_1 \le s \le L_2$, be the Mannheim partner of the given curve $\alpha(s)$.

Surface family that interpolates $\bar{\alpha}(s)$ as a common curve is given in the parametric form as

$$\varphi(s,v)=\bar{\alpha}(s)+[x(s,v)\bar{T}(s)+y(s,v)\bar{N}(s)+z(s,v)\bar{B}(s)],\ L_1 \le s \le L_2,\ T_1 \le v \le T_2, \tag{3.1}$$

where $x(s,v), y(s,v)$ and $z(s,v)$ are C^1 functions. The values of the marching-scale functions $x(s,v), y(s,v)$ and $z(s,v)$ indicate, respectively; the extension-like, flexion-like and retortion-like effects, by the point unit through the time v, starting from $\bar{\alpha}(s)$ and $\left\{\bar{T}(s), \bar{N}(s), \bar{B}(s)\right\}$ is the Frenet frame associated with the curve $\bar{\alpha}(s)$.

Our goal is to find the necessary and sufficient conditions for which the Mannheim partner curve of the unit space curve $\alpha(s)$ is an parametric curve and a geodesic curve on the surface $\varphi(s,v)$.

Firstly, since Mannheim partner curve of the curve $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0 \in [T_1, T_2]$ such that

$$x(s,v_0)=y(s,v_0)=z(s,v_0)=0,\ L_1 \le s \le L_2,\ T_1 \le v_0 \le T_2. \tag{3.2}$$

Secondly, since Mannheim partner curve of $\alpha(s)$ is a geodesic curve on the surface $\varphi(s,v)$, there exist a parameter $v_0 \in [T_1, T_2]$ such that

$$n(s,v_0)\,//\,\overline{\mathrm{N}}(\mathrm{s})\,. \tag{3.3}$$

where n is a normal vector of $\varphi=\varphi(s,v)$ and $\overline{\mathrm{N}}$ is a normal vector of $\overline{\alpha}(s)$.

Theorem 3.1: Let $\alpha(s)$, $L_1 \le s \le L_2$, be a unit speed curve with nonvanishing curvature and $\overline{\alpha}(s)$, $L_1 \le s \le L_2$, be a Mannheim partner curve. $\overline{\alpha}$ is a geodesic curve on the surface (3.1) if and only if

$$\begin{cases} \mathrm{x(s,v_0)=y(s,v_0)=z(s,v_0)=0}, \\ \dfrac{\partial \mathrm{y(s,v_0)}}{\partial \mathrm{v}}=0 \text{ and } \dfrac{\partial \mathrm{z(s,v_0)}}{\partial \mathrm{v}}\neq 0 \end{cases}$$

Proof: Let $\overline{\alpha}(s)$ be a Mannheim partner of the curve $\alpha(s)$. From (3.1) , $\varphi(s,v)$ parametric surface is defined by as follows:

$$\varphi(\mathrm{s,v})=\overline{\alpha}(\mathrm{s})+[\mathrm{x(s,v)}\,\overline{\mathrm{T}}(\mathrm{s})+\mathrm{y(s,v)}\,\overline{\mathrm{N}}(\mathrm{s})+\mathrm{z(s,v)}\,\overline{\mathrm{B}}(\mathrm{s})]\,.$$

Let $\overline{\alpha}(s)$ Mannheim partner curve of the curve $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0\in[T_1,T_2]$ such that,

$$x(s,v_0)=y(s,v_0)=z(s,v_0)\equiv 0,\ L_1\le s\le L_2,\ T_1\le v_0\le T_2\ (v_0\ \mathit{fixed}) \tag{3.4}$$

Secondly, since Mannheim partner curve of $\alpha(s)$ is an geodesic curve on the surface $\varphi(s,v)$, there exist a parameter $v_0\in[T_1,T_2]$ such that $n(s,v_0)\,//\,\overline{\mathrm{N}}(\mathrm{s})$ where n is a normal vector of $\varphi=\varphi(s,v)$ and $\overline{\mathrm{N}}$ is a normal vector of $\overline{\alpha}(s)$.

The normal vector can be expressed as

$$\begin{aligned} n(s,v)=&\left[\frac{\partial z(s,v)}{\partial v}\left(\frac{\partial y(s,v)}{\partial s}+\overline{\kappa}(s)x(s,v)-\overline{\tau}(s)z(s,v)\right)-\frac{\partial y(s,v)}{\partial v}\left(\frac{\partial z(s,v)}{\partial s}+\overline{\tau}(s)y(s,v)\right)\right]\overline{T}(s)\\ &+\left[\frac{\partial x(s,v)}{\partial v}\left(\frac{\partial z(s,v)}{\partial s}+\overline{\tau}(s)y(s,v)\right)-\frac{\partial z(s,v)}{\partial v}\left(1+\frac{\partial x(s,v)}{\partial s}-\overline{\kappa}(s)y(s,v)\right)\right]\overline{N}(s)\\ &+\left[\frac{\partial y(s,v)}{\partial v}\left(1+\frac{\partial x(s,v)}{\partial s}-\overline{\kappa}(s)y(s,v)\right)-\frac{\partial x(s,v)}{\partial v}\left(\frac{\partial y(s,v)}{\partial s}+\overline{\kappa}(s)x(s,v)-\overline{\tau}(s)z(s,v)\right)\right]\overline{B}(s) \end{aligned} \tag{3.5}$$

Thus, if we let

$$\begin{cases} \phi_1(s,v_0)=\dfrac{\partial z(s,v_0)}{\partial v}\left(\dfrac{\partial y(s,v_0)}{\partial s}+\overline{\kappa}(s)x(s,v_0)-\overline{\tau}(s)z(s,v_0)\right)-\dfrac{\partial y(s,v_0)}{\partial v}\left(\dfrac{\partial z(s,v_0)}{\partial s}+\overline{\tau}(s)y(s,v_0)\right), \\ \phi_2(s,v_0)=\dfrac{\partial x(s,v_0)}{\partial v}\left(\dfrac{\partial z(s,v_0)}{\partial s}+\overline{\tau}(s)y(s,v_0)\right)-\dfrac{\partial z(s,v)}{\partial v}\left(1+\dfrac{\partial x(s,v_0)}{\partial s}-\overline{\kappa}(s)y(s,v_0)\right), \\ \phi_3(s,v_0)=\dfrac{\partial y(s,v_0)}{\partial v}\left(1+\dfrac{\partial x(s,v_0)}{\partial s}-\overline{\kappa}(s)y(s,v_0)\right)-\dfrac{\partial x(s,v_0)}{\partial v}\left(\dfrac{\partial y(s,v_0)}{\partial s}+\overline{\kappa}(s)x(s,v_0)-\overline{\tau}(s)z(s,v_0)\right). \end{cases}$$

We obtain

$$n(s,v_0)=\phi_1(s,v_0)\overline{T}(s)+\phi_2(s,v_0)\overline{N}(s)+\phi_3(s,v_0)\overline{B}(s) \tag{3.6}$$

We know that $\overline{\alpha}(s)$ is a geodesic curve if and only if

$$\phi_1(s,v_0)=\phi_3(s,v_0)=0,\ \phi_2(s,v_0)\neq 0 \tag{3.7}$$

From (3.4),

$$\begin{cases} \phi_1(s,v_0)=0 \\ \phi_3(s,v_0)=0, we\ have\ \dfrac{\partial y(s,v_0)}{\partial v}=0 \\ \phi_2(s,v_0)\neq 0, we\ have\ -\dfrac{\partial z(s,v_0)}{\partial v}\neq 0 \end{cases} \tag{3.8}$$

Combining the conditions (3.4) and (3.8), we have found the necessary and sufficient conditions for the $\varphi(s,v)$ to have the Mannheim partner curve of the curve $\alpha(s)$ is an isogeodesic.

Now let us consider other types of the marching-scale functions. In the Eqn. (3.1) marching-scale functions $x(s,v), y(s,v)$ and $z(s,v)$ can be choosen in two different forms:

1) If we choose

$$\begin{cases} x(s,v)=\sum_{k=1}^{p} a_{1k} l(s)^k x(v)^k, \\ y(s,v)=\sum_{k=1}^{p} a_{2k} m(s)^k y(v)^k, \\ z(s,v)=\sum_{k=1}^{p} a_{3k} n(s)^k z(v)^k, \end{cases}$$

then we can simply express the sufficient condition for which the curve $\overline{\alpha}(s)$ is a geodesic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0)=y(v_0)=z(v_0)=0, \\ a_{21}=0\ or\ m(s)=0\ \text{ or }\ \dfrac{dy(v_0)}{dv}=0, \\ a_{31}\neq 0\ , n(s)\neq 0\, and\ \dfrac{dz(v_0)}{dv}=const\neq 0 \end{cases} \tag{3.9}$$

where $l(s), m(s), n(s), x(v), y(v)$ and $z(v)$ are C^1 functions, $a_{ij} \in IR$, $i = 1,2,3,$ $j = 1,2,..., p.$

2) If we choose

$$\begin{cases} x(s,v) = f\left(\sum_{k=1}^{p} a_{1k} l(s)^k x(v)^k\right), \\ y(s,v) = g\left(\sum_{k=1}^{p} a_{2k} m(s)^k y(v)^k\right), \\ z(s,v) = h\left(\sum_{k=1}^{p} a_{3k} n(s)^k z(v)^k\right), \end{cases}$$

then we can write the sufficient condition for which the curve $\overline{\alpha}(s)$ is a geodesic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0) = y(v_0) = z(v_0) = f(0) = g(0) = h(0) = 0, \\ a_{21} = 0 \ or \ m(s) = 0 \ \text{ or } \ \dfrac{dy(v_0)}{dv} = 0 \ or \ g'(0) = 0, \\ a_{31} \neq 0, n(s) \neq 0, h'(0) \neq 0 \ and \ \dfrac{dz(v_0)}{dv} = const \neq 0. \end{cases} \tag{3.10}$$

where $l(s)$, $m(s)$, $n(s)$, $x(v)$, $y(v)$, $z(v)$, f, g and h are C^1 functions.

Also conditions for different types of marching-scale functions can be obtained by using the Eqn. (3.4) and (3.8).

4. Ruled surfaces with common Mannheim geodesic curve

Ruled surfaces are one of the simplest objects in geometric modelling as they are generated basically by moving a line in space. A surface φ is a called a ruled surface in Euclidean space, if it is a surface swept out by a straight line l moving alone a curve α. The generating line l and the curve α are called the rulings and the base curve of the surface, respectively.

We show how to derive the formulations of a ruled surfaces family such that the common Mannheim geodesic is also the base curve of ruled surfaces.

Let $\varphi = \varphi(s,v)$ be a ruled surface with the Mannheim isogeodesic base curve. From the definition of ruled surface, there is a vector $R = R(s)$ such that;

$$\varphi(s,v) - \varphi(s,v_0) = (v - v_0)R(s)$$

and where 3.1 is used , we get

$$(v - v_0)R(s) = x(s,v)\overline{T}(s) + y(s,v)\overline{N}(s) + z(s,v)\overline{B}(s)$$

For the solutions of three unknows $x(s,v)$, $y(s,v)$ and $z(s,v)$ we have,

$$x(s,v)=(v-v_0)\langle \overline{T}(s),R(s)\rangle$$

$$y(s,v)=(v-v_0)\langle \overline{N}(s),R(s)\rangle \tag{4.1}$$

$$z(s,v)=(v-v_0)\langle \overline{B}(s),R(s)\rangle .$$

From (3.8) and (4.1), we have

$$\langle \overline{N}(s),R(s)\rangle =0 \text{ and } \langle \overline{B}(s),R(s)\rangle \neq 0 \tag{4.2}$$

Including, $R(s)=x(s)\overline{T}(s)+y(s)\overline{N}(s)+z(s)\overline{B}(s)$ using (4.2) we obtain,

$$y(s)=0 \quad \text{and} \quad z(s)\neq 0 \tag{4.3}$$

So, the ruled surfaces family with common Mannheim isoasymptotic given by;

$$\varphi(s,v)=\overline{\alpha}(s)+v[x(s)\overline{T}(s)+z(s)\overline{B}(s)];z(s)\neq 0 \tag{4.4}$$

5. Examples of generating simple surfaces with common Mannheim geodesic curve

Example 5.1. Let $\alpha(s)=\left(3\cos\left(\frac{s}{5}\right),-3\sin\left(\frac{s}{5}\right),\frac{4}{5}s\right)$ be a unit speed curve. Then it is easy to show that

$$\begin{cases} T(s)=\left(-\frac{3}{5}\sin\left(\frac{s}{5}\right),-\frac{3}{5}\cos\left(\frac{s}{5}\right),\frac{4}{5}\right), \\ N(s)=\left(-\cos\left(\frac{s}{5}\right),\sin\left(\frac{s}{5}\right),0\right), \\ B(s)=\left(-\frac{4}{5}\sin\left(\frac{s}{5}\right),-\frac{4}{5}\cos\left(\frac{s}{5}\right),-\frac{3}{5}\right). \end{cases}$$

and the curvatures of this curve is $\kappa=\frac{3}{25}$ *and* $\tau=\frac{4}{25}$.

The parametric representation of the Mannheim partner (with respect to Frenet frame) curve $\overline{\alpha}(s)$ of the curve $\alpha(s)$ is obtained as (where $\lambda=\text{constant}$, for $\lambda=1$)

$$\begin{aligned}\overline{\alpha}(s)&=\alpha(s)-\lambda N(s)\\ &=\left(4\cos\left(\frac{s}{5}\right),-4\sin\left(\frac{s}{5}\right),\frac{4}{5}s\right)\end{aligned}$$

The Frenet vectors of the curve $\overline{\alpha}$ are found as

$$\begin{cases}\bar{T}(s)=\left(\frac{\sqrt{2}}{10}\sin\left(\frac{s}{5}\right),\frac{\sqrt{2}}{10}\cos\left(\frac{s}{5}\right),\frac{7\sqrt{2}}{10}\right),\\ \bar{N}(s)=\left(-\frac{7\sqrt{2}}{10}\sin\left(\frac{s}{5}\right),-\frac{7\sqrt{2}}{10}\cos\left(\frac{s}{5}\right),\frac{\sqrt{2}}{10}\right),\\ \bar{B}(s)=\left(-\cos\left(\frac{s}{5}\right),\sin\left(\frac{s}{5}\right),0\right).\end{cases}$$

If we take $x(s,v)=y(s,v)\equiv 0,\ z(s,v)=v$ and $v_0=0$ then the Eqn. (3.4) and (3.8) are satisfied. Thus, we obtain a member of the surface with common Mannheim geodesic curve $\bar{\alpha}(s)$ as

$$\bar{\varphi}(s,v)=\left(4\cos\left(\frac{s}{5}\right)-v\cos\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right)+v\sin\left(\frac{s}{5}\right),\ \frac{4s}{5}\right)$$

Also, for $x(s,v)=y(s,v)\equiv 0,\ z(s,v)=v$ and $v_0=0$, we obtain a member of the surface with common geodesic curve $\alpha(s)$ as

$$\varphi(s,v)=\left(3\cos\left(\frac{s}{5}\right)-\frac{4}{5}v\cos\left(\frac{s}{5}\right),\ -3\sin\left(\frac{s}{5}\right)-\frac{4}{5}v\cos\left(\frac{s}{5}\right),\ \frac{4s}{5}-\frac{3v}{5}\right)$$

where $-\pi\le s\le\pi$, $-1\le v\le 1$ (Fig.2).

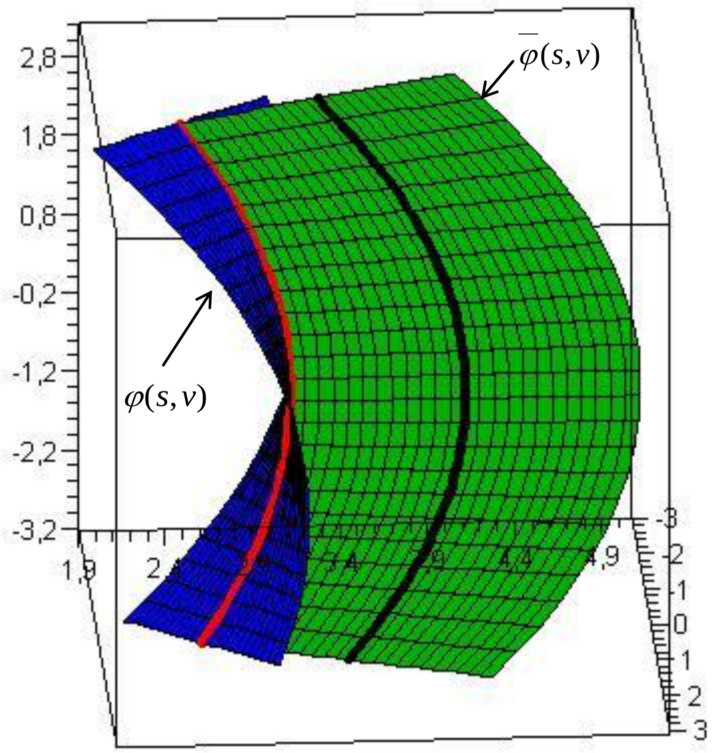

Figure 2. $\varphi(s,v)$ surface with curve $\alpha(s)$ and $\bar{\varphi}(s,v)$ Mannheim offset surface with Mannheim partner curve ($\bar{\alpha}(s)$) of the curve $\alpha(s)$

If we take $x(s,v)=v\cos(s/5),\ y(s,v)=0,\ z(s,v)=e^{v}-1$ and $v_0=0$ then the Eqn. (3.4) and (3.8) are satisfied. Thus, we obtain a member of the surface with common Mannheim geodesic curve as

$$\bar{\varphi}(s,v)=\begin{pmatrix} 4\cos\left(\frac{s}{5}\right)+\frac{\sqrt{2}}{10}v\cos\left(\frac{s}{5}\right)\sin\left(\frac{s}{5}\right)-(e^{v}-1)\cos\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right)+\frac{\sqrt{2}}{10}v\cos^{2}\left(\frac{s}{5}\right) \\ +(e^{v}-1)\cos\left(\frac{s}{5}\right),\frac{4s}{5}+\frac{7\sqrt{2}}{10}v\cos\left(\frac{s}{5}\right) \end{pmatrix}$$

Also, for $x(s,v)=\cos(s/5),\ y(s,v)=0,\ z(s,v)=e^{v}-1$ and $v_0=0$, we obtain a member of the surface with common geodesic curve as

$$\varphi(s,v)=\begin{pmatrix} 3\cos\left(\frac{s}{5}\right)-\frac{4}{5}v\cos\left(\frac{s}{5}\right)\sin\left(\frac{s}{5}\right)-\frac{4}{5}(e^{v}-1)\sin\left(\frac{s}{5}\right),\ -\frac{4}{5}(e^{v}-1)\cos\left(\frac{s}{5}\right), \\ \frac{4s}{5}+\frac{4v}{5}\cos\left(\frac{s}{5}\right)-\frac{3}{5}(e^{v}-1) \end{pmatrix}$$

where $\frac{\pi}{3}\leq s\leq 2\pi,\ -1\leq v\leq 1$ (Fig. 3).

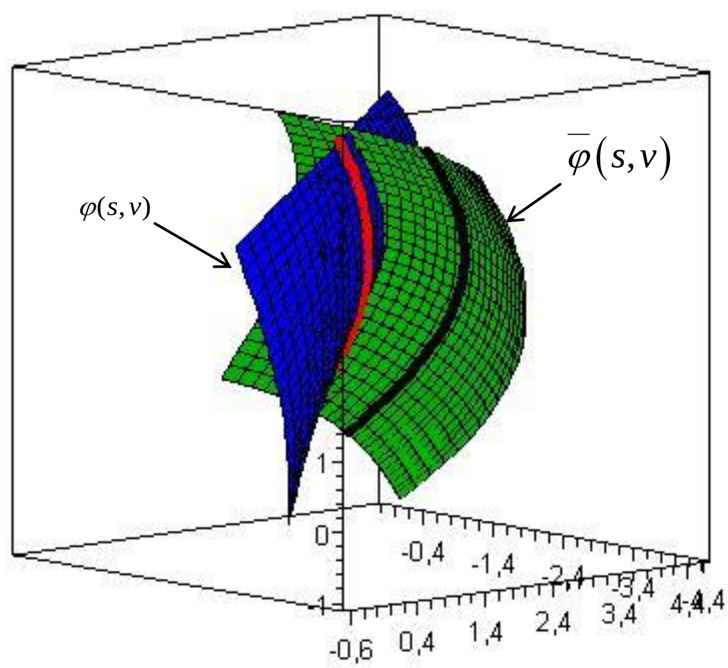

Figure 3. $\varphi(s,v)$ surface with curve $\alpha(s)$ and $\bar{\varphi}(s,v)$ Mannheim offset surface with Mannheim partner curve ($\bar{\alpha}(s)$) of the curve $\alpha(s)$

$x(s)=y(s)\equiv 0,\ z(s)=s$ and $v_0=0$ then the Eqn. (4.4) is satisfied. Thus, we obtain a member of the ruled surface with common Mannheim geodesic curve as

$$\overline{\varphi}(s,v)=\left(4\cos\left(\frac{s}{5}\right)-vs\cos\left(\frac{s}{5}\right),\ -4\sin\left(\frac{s}{5}\right)+vs\sin\left(\frac{s}{5}\right),\ \frac{4s}{5}\right)$$

Also, for $x(s)=y(s)\equiv 0,\ z(s)=s$ and $v_0=0$, we obtain a member of the ruled surface with common geodesic curve as

$$\varphi(s,v)=\left(3\cos\left(\frac{s}{5}\right)-\frac{4}{5}vs\cos\left(\frac{s}{5}\right),\ -3\sin\left(\frac{s}{5}\right)-\frac{4}{5}vs\cos\left(\frac{s}{5}\right),\ \frac{4s}{5}-\frac{3vs}{5}\right)$$

where $-\pi\le s\le\pi$, $-1\le v\le 1$ (Fig.4).

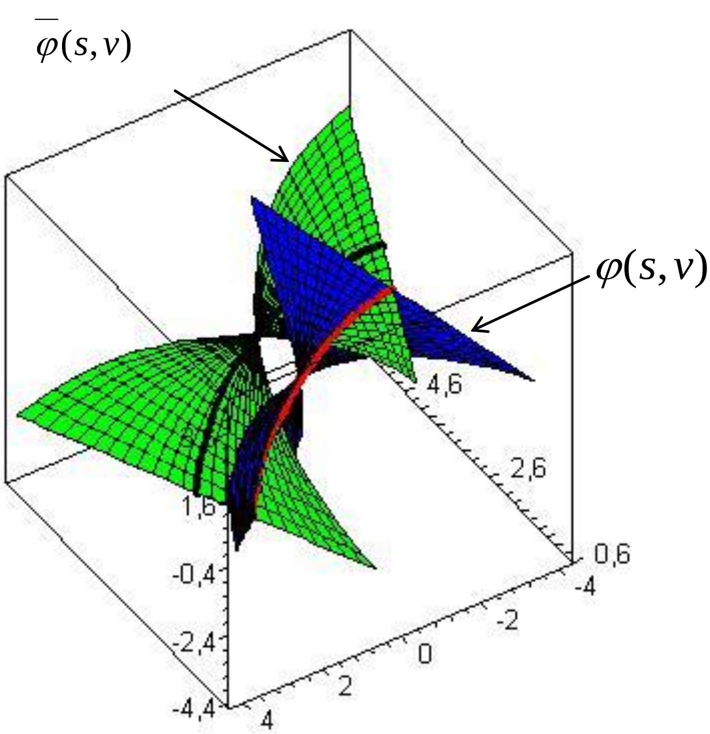

Figure 4. $\varphi(s,v)$ as a member of the ruled surface and its Mannheim geodesic curve.

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