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4−Remainder Cordial Labeling of Some Graphs

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Abstract: Let G be a (p, q) graph. Let f be a function from $V(G)$ to the set $\{1, 2, \cdots, k\}$ where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, \cdots, k\}$, where $v_f(x)$ denote the number of vertices labeled with x and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labeled with even integers and number of edges labelled with odd integers. A graph with admits a k-remainder cordial labeling is called a k-remainder cordial graph. In this paper we investigate the 4- remainder cordial behavior of grid, subdivision of crown, Subdivision of bistar, book, Jelly fish, subdivision of Jelly fish, Mongolian tent graphs.

Key Words: k-Remainder cordial labeling, Smarandache k-remainder cordial labeling, grid, subdivision of crown, subdivision of bistar, book, Jelly fish, subdivision of Jelly fish, Mongolian tent.

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§1. Introduction

We considered only finite and simple graphs.The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv. The product graph G_1XG_2 is defined as follows:

Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$. The graph $P_m \times P_n$ is called the planar grid. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. A mongolian tent $M_{m,n}$ is a graph obtained from $P_m \times P_n$ by adding one extra vertex above the grid and joining every other of the top row of $P_m \times P_n$ to the new vertex. Cahit [1], introduced the concept of cordial labeling of graphs. Ponraj et al. [4, 6], introduced remainder cordial labeling of graphs and investigate the remainder cordial labeling behavior of path, cycle,

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star, bistar, complete graph, $S(K_{1,n})$, $S(B_{n,n})$, $S(W_n)$, P_n^2, $P_n^2 \bigcup K_{1,n}$, $P_n^2 \bigcup B_{n,n}$, $P_n \cup B_{n,n}$, $P_n \bigcup K_{1,n}$, $K_{1,n} \bigcup S(K_{1,n})$, $K_{1,n} \bigcup S(B_{n,n})$, $S(K_{1,n}) \bigcup S(B_{n,n})$, etc., and also the concept of k-remainder cordial labeling introduced in [5]. In this paper we investigate the 4-remainder cordial labeling behavior of Grid, Subdivision of crown, Subdivision of bistar, Book, Jelly fish, Subdivision of Jelly fish, Mongolian tent, etc,. Terms are not defined here follows from Harary [3] and Gallian [2].

§2. k-Remainder Cordial Labeling

Definition 2.1 *Let G be a (p,q) graph. Let f be a function from $V(G)$ to the set $\{1, 2, \cdots, k\}$ where k is an integer $2 < k \leq |V(G)|$. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ (or) $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The labeling f is called a k-remainder cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, otherwise, Smarandachely if $|v_f(i) - v_f(j)| \geq 1$ or $|e_f(0) - e_f(1)| \geq 1$ for integers $i, j \in \{1, \cdots, k\}$, where $v_f(x)$ and $e_f(0)$, $e_f(1)$ respectively denote the number of vertices labeled with x, the number of edges labeled with even integers and the number of edges labelled with odd integers. Such a graph with a k-remainder cordial labeling is called a k-remainder cordial graph.*

First we investigate the 4−remainder cordial labeling behavior of the planar grid.

Theorem 2.2 *The planar grid $P_m \times P_n$ is 4−remainder cordial.*

Proof Clearly this grid has $m-$rows and $n-$columns. We assign the labels to the vertices by row wise.

Case 1. $m \equiv 0 \pmod 4$

Let $m = 4t$. Then assign the label 1 to the vertices of $1^{st}, 2^{nd}, \cdots, t^{th}$ rows. Next we move to the $(t+1)^{th}$ row. Assign the label 4 to the vertices of $(t+1)^{th}, (t+2)^{th}, \ldots, (2t)^{th}$ rows. Next assign the label to the vertices $(2t+1)^{th}$ row. Assign the labels 2 and 3 alternatively to the vertices of $(2t+1)^{th}$ row. Next move to $(2t+2)^{th}$ row. Assign the labels 3 and 2 alternatively to the vertices of $(2t+2)^{th}$ row. In general i^{th} row is called as in the $(i-2)^{th}$ row, where $2t+1 \leq i \leq 3t$. This procedure continued until we reach the $(4t)^{th}$ row.

Case 2. $m \equiv 1 \pmod 4$

As in Case 1, assign the labels to the vertices of the first, second, $\cdots, (m-1)^{th}$ row. We give the label to the m^{th} row as in given below.

Subcase 2.1 $n \equiv 0 \pmod 4$

Rotate the row and column and result follows from Case 1.

Subcase 2.2 $n \equiv 1 \pmod 4$

Assign the labels $4, 3, 4, 3, \cdots, 4, 3$ to the vertices of the first, second, $\cdots$, $\left(\frac{n-1}{2}\right)^{th}$ columns. Next assign the label 2 to the vertices of $(\frac{n+1}{2})^{th}$ column. Then next assign the labels $2, 1, 2, 1, \cdots,$

$2, 1$ to the vertices of $\frac{n+3}{2}, \frac{n+5}{2}, \cdots, (\frac{2n}{2} - 2)^{th}$ columns. Assign the remaining vertices.

Subcase 2.3 $n \equiv 2 \pmod 4$

Assign the labels $4, 3, 4, 3, \cdots, 4, 3$ to the vertices of $1^{st}, 2^{nd}, \cdots, (\frac{n-2}{2})^{th}$ columns. Next assign the label 2 to the vertices of $\left(\frac{n}{2}\right)^{th}$ column. Then next assign the labels $2, 1, 2, 1, \cdots, 2, 1$ to the vertices of $\frac{n}{2}+1, \frac{n}{2}+2, \cdots, \left(\frac{2n}{2} - 1\right)^{th}$ columns. Finally assign the label 1 to the remaining vertices of n^{th}column.

Subcase 2.4 $n \equiv 3 \pmod 4$

Assign the labels $4, 3, 4, 3, \cdots, 4, 3$ alternatively to the vertices of $1^{st}, 2^{nd}, \cdots, \left(\frac{n+1}{2}\right)^{th}$ columns. Then next assign the labels $1, 2, 1, 2, \cdots$ to the vertices of $\frac{n+3}{2}, \frac{n+5}{2}, \cdots, \left(\frac{2n}{2} - 1\right)^{th}$ columns. Finally assign the label 1 to the remaining vertices of n^{th} column. Hence f is a $4-$remainder cordial labeling of $P_m \times P_n$.

All other cases follow by symmetry. □

Next is the graph $K_2 + mK_1$.

Theorem 2.3 *If $m \equiv 0, 1, 3 \pmod 4$ then $K_2 + mK_1$ is $4-$remainder cordial.*

Proof It is easy to verify that $K_2 + mK_1$ has $m + 2$ vertices and $2m$ edges. Let $V(K_2 + mK_1) = \{u, u_i, v : 1 \leq i \leq m\}$ and $E(K_2 + mK_1) = \{uv, uu_i, vu_i : 1 \leq i \leq m\}$.

Case 1. $m \equiv 0 \pmod 4$

Let $m = 4t$. Then assign the label $3, 3$ respectively to the vertices u, v. Next assign the label 1 to the vertices $u_1, u_2, \cdots, u_{t+1}$. Then next assign the label 2 to the vertices $u_{t+2}, u_{t+3}, \cdots, u_{2t+1}$. Then followed by assign the label 3 to the vertices $u_{2t+2}, u_{2t+3}, \ldots, u_{3t}$. Finally assign the label 4 to the remaining non-labelled vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{4t}$.

Case 2. $m \equiv 1 \pmod 4$

As in Case 1, assign the labels to the vertices $u, v, u_i, (1 \leq i \leq m - 1)$. Next assign the label 2 to the vertex u_m.

Case 3. $m \equiv 3 \pmod 4$

Assign the labels to the vertices $u, v, u_i, (1 \leq i \leq m - 2)$ as in case(ii). Finally assign the labels $3, 4$ respectively to the vertices u_{m-1}, u_m. The table given below establish that this labeling f is a $4-$remainder cordial labeling.

Nature of m	$e_f(0)$	$e_f(1)$
$m \equiv 0 \pmod 4$	$m + 1$	m
$m \equiv 1 \pmod 4$	m	$m + 1$
$m \equiv 3 \pmod 4$	m	$m + 1$

Table 1

This completes the proof. □

The next graph is the book graph B_n.

Theorem 2.4 *The book B_n is 4−remainder cordial for all n.*

Proof Let $V(B_n) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_n) = \{uv, uu_i, vv_i, u_iv_i : 1 \leq i \leq n\}$.

Case 1. n is even

Assign the labels 3, 4 to the vertices u and v respectively. Assign the label 1 to the vertices $u_1, u_2, \cdots, u_{\frac{m}{2}}$ and assign 4 to the vertices $u_{\frac{m}{2}+1}, u_{\frac{m}{2}+2}, \cdots, u_n$. Next we consider the vertices v_i. Assign the label 2 to the vertices $v_1, v_2, \cdots, v_{\frac{n}{2}}$. Next assign the label 3 to the remaining vertices $v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \cdots, u_n$, respectively.

Case 2. n is odd

Assign the labels 3, 4 to the vertices u and v respectively. Fix the labels 4, 2, 1 to the vertices $u_1, u_2, \cdots, u_{\frac{n}{2}+1}$ and also fix the labels 3, 1, 2 respectively to the vertices $v_1, v_2, \cdots, v_{\frac{n}{2}+1}$. Assign the labels to the vertices $u_4, u_5, \cdots, u_n$ as in the sequence $2, 1, 2, 1 \ldots, 2, 1$. In similar fashion, assign the labels to the vertices $v_4, v_5, \cdots, v_n$ as in the sequence $3, 4, 3, 4 \ldots, 3, 4$. The table 2 shows that this vertex labeling f is a 4− remainder cordial labeling.

Nature of n	$e_f(0)$	$e_f(1)$
n is even	$m+1$	m
n is odd	m	$m+1$

Table 2

This completes the proof. □

Now we consider the subdivision of $B_{n,n}$.

Theorem 2.5 *The subdivision of $B_{n,n}$ is 4−remainder cordial.*

Proof Let $V(S(B_{n,n})) = \{u, v, u_i, v_i, w_i, x, x_i : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{uu_i, vv_i, u_iw_i, v_ix_i, ux, xv : 1 \leq i \leq n\}$.It is clearly to verify that $S(B_{n,n})$ has $4n+3$ vertices and $4n+2$ edges.

Assign the labels 1, 4, 3 to the vertices u, x and v respectively. Assign the labels 1, 3 alternatively to the vertices $u_1, u_2, \cdots, u_n$. Next assign the labels 2, 4 alternatively to the vertices $w_1, w_2, \cdots, w_n$. We now consider the vertices v_i and x_i. Assign the labels 2, 4 alternatively to the vertices $v_1, v_2, \cdots, v_n$. Then finally assign the labels 3, 1 alternatively to the vertices $x_1, x_2, \cdots, x_n$. Obviously this vertex labeling is a 4−remainder cordial labeling. □

Next, we consider the subdivision of crown graph.

Theorem 2.5 *The subdivision of $C_n \odot K_1$ is 4−remainder cordial.*

Proof Let $u_1u_2 \ldots u_n$ be a cycle C_n. Let $V(C_n \odot K_1) = V(C_n \bigcup \{v_i : 1 \leq i \leq n\})$ and $E(C_n \odot K_1) = E(C_n \bigcup \{u_i, v_i : 1 \leq i \leq n\})$.The subdivide edges u_iu_{i+1} and u_iv_i by x_i and y_i respectively. Assign the label 2 to the vertices $u_i, (1 \leq i \leq n)$ and 3 to the vertices

$x_i, (1 \le i \le n)$. Next assign the label 1 to the vertices $y_i, (1 \le i \le n)$. Finally assign the label 4 to the vertices $v_i, (1 \le i \le n)$. Clearly, this labeling f is a 4−remainder cordial labeling. □

Now we consider the Jelly fish $J(m, n)$.

Theorem 2.6 *The Jelly fish $J(m, n)$ is 4−remainder cordial.*

Proof Let $V(J(m, n)) = \{u, v, x, y, u_i, v_j : 1 \le i \le m \text{ and } 1 \le j \le n\}$ and $E(J(m, n)) = \{uu_i, vv_j, ux, uy, vx, vy : 1 \le i \le m \text{ and } 1 \le j \le n\}$. Clearly $J(m, n)$ has $m + n + 4$ vertices and $m + n + 5$ edges.

Case 1. $m = n$ and m is even.

Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{\frac{n}{2}}$ and assign the label 4 to the vertices $u_{\frac{n}{2}+1}, u_{\frac{n}{2}+2}, \cdots, u_n$. Next assign the label 1 to the vertices $v_1, v_2, \cdots, v_{\frac{n}{2}}$ and assign 3 to the vertices $v_{\frac{n}{2}+1}, v_{\frac{n}{2}+2}, \ldots, v_n$. Finally assign the labels 3, 4, 2, 1 respectively to the vertices u, x, y, v.

Case 2. $m = n$ and m is odd.

In this case assign the labels to the vertices $u_i, v_i (1 \le i \le m - 1)$ and u, v, x, y as in Case 1. Next assign the labels 2, 1 respectively to the vertices u_n and v_n.

Case 3. $m \ne n$ and assume $m > n$.

Assign the labels 3, 4, 1, 2 to the vertices u, x, y, v respectively. As in Case 1 and 2, assign the labels to the vertices $u_i, v_i (1 \le i \le n)$.

Subcase 3.1 $m - n$ is even. Assign the labels to the vertices $u_{n+1}, u_{n+2}, \cdots, u_m$ as in the sequence $3, 4, 2, 1; 3, 4, 2, 1; \cdots$. It is easy to verify that u_n is received the label 1 if $m - n \equiv 0 \pmod 4$.

Subcase 3.2 $m - n$ is odd. Assign the labels to the vertices $u_i (n \le i \le m)$ as in the sequence $4, 3, 2, 1; 4, 3, 2, 1; \cdots$. Clearly, u_n is received the label 1 if $m - n \equiv 0 \pmod 4$. □

For illustration, a 4−remainder cordial labeling of Jelly fish $J(m, n)$ is shown in Figure 1.

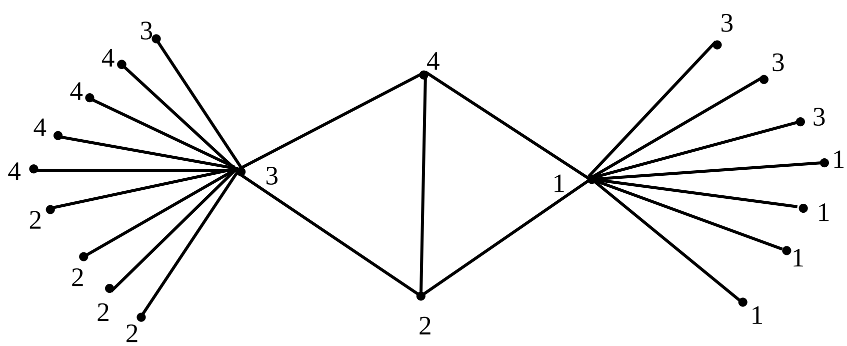

Figure 1

Theorem 2.8 *The subdivision of the Jelly fish $J(m, n)$ is 4−remainder cordial.*

Proof Let $V(S(J(m,n))) = \{u, u_i, x_i, v, v_j, y_j : 1 \le i \le m, 1 \le j \le n\} \bigcup \{w_i : 1 \le i \le 7\}$ and $E(S(J(m,n))) = \{uu_i, u_ix_i, vv_j, v_jy_j : 1 \le i \le m, 1 \le j \le n\} \bigcup \{uw_1, uw_2, w_1w_5, w_5w_6, w_6w_7, w_5w_3, w_3v, vw_4, w_4w_7, w_2w_7\}$.

Case 1. $m = n$.

Assign the label 2 to the vertices $u_1, u_2, \cdots, u_m$ and 3 to the vertices $x_1, x_2, \cdots, x_m$. Next assign the label 1 to the vertices $v_1, v_2, \cdots, v_m$ and assign the label 4 to the vertices $y_1, y_2, \cdots, y_m$. Finally assign the labels $3, 2, 3, 2, 3, 1, 4, 4$ and 1 respectively to the vertices $u, w_1, w_5, w_6, w_7, w_2, w_3, v$ and w_4.

Case 2. $m > n$.

Assign the labels to the vertices $u, u_i, v, v_i, x_i, y_i, w_1, w_2, w_3, w_4, w_5, w_6, w_7, (1 \le i \le n)$ as in case(i). Next assign the labels $1, 4$ to the next two vertices x_{n+1}, x_{n+2} respectively. Then next assign the labels $1, 4$ respectively to the vertices x_{n+3}, x_{n+4}. Proceeding like this until we reach the vertex x_n.That is the vertices $x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}, \cdots$ are labelled in the pattern $1, 4; 1, 4; 1, 4; 1, 4; \cdots$. Similarly the vertices $u_{n+1}, u_{n+2}, \cdots$ are labelled as $2, 3; 2, 3; 2, 3; \cdots, 2, 3$. The Table 3, establish that this vertex labeling f is a 4−remainder cordial labeling of $S(J(m,n))$.

Nature of m and n	$e_f(0)$	$e_f(1)$
$m = n$	$2n + 5$	$2n + 5$
$m > n$	$m + n + 5$	$m + n + 5$

This completes the proof. □

Theorem 2.9 *The graph* $C_3^{(t)}$ *is* 4−*remainder cordial.*

Proof Let $V(C_3^{(t)}) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(C_3^{(t)}) = \{uu_i, vv_i, u_iv_i : 1 \le i \le n\}$. Assume $t \ge 3$. Fix the label 3 to the central vertex u and fix the labels $1, 2, 2, 4, 3, 4$ respectively to the vertices u_1, u_2, u_3, v_1, v_2 and v_3. Next assign the labels $1, 2$ to the vertices u_4, u_5. Then assign the labels $1, 2$ respectively to the next two vertices u_6, u_7 and so on. That is the vertices u_4, u_5, u_6, u_7 are labelled as in the pattern $1, 2, 1, 2 \cdots, 1, 2$. Note that the vertex u_n received the label 1 or 2 according as n is even or odd. In a similar way assign the labels to the vertices v_4, v_5, v_6, v_7 as in the sequence $4, 3, 4, 3, 4, 3, \cdots$. Clearly 4 is the label of u_n according as n is even or odd. The Table 4 establish that this vertex labeling is a 4−remainder cordial labeling of $C_3^{(t)}$, $t \ge 3$.

Nature of n	$e_f(0)$	$e_f(1)$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$n + 1$	$n + 2$

Table 4

For $t = 1, 2$ the remainder cordial labeling of graphs $C_3^{(1)}$ and $C_3^{(2)}$ are given below in Figure 2.

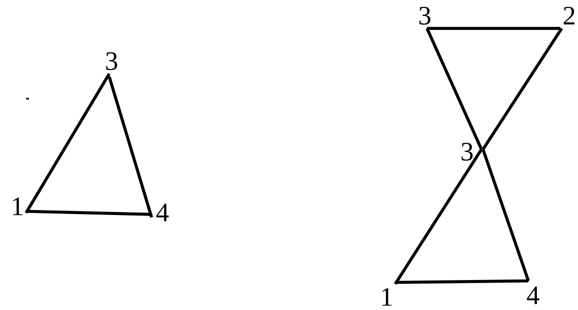

Figure 2

This completes the proof. □

Theorem 2.10 *The Mongolian tent* $M_{m,n}$ *is* $4-$*remainder cordial.*

Proof Assign the label 3 to the new vertex.

Case 1. $m \equiv 0 \pmod 4$ and $n \equiv 0, 2 \pmod 4$.

Consider the first row of M_n. Assign the labels $2, 3, 2, 3, \cdots 2, 3$ to the vertices in the first row. Next assign the labels $3, 2, 3, 2 \cdots 3, 2$ to the vertices in the second row. This procedure is continue until reach the $\frac{n}{2}^{th}$ row. Next assign the labels $1, 4, 1, 4, \cdots, 1, 4$ to the vertices in the $\frac{n}{2} + 1^{th}$ row. Then next assign the labels $4, 1, 4, 1, \cdots, 4, 1$ to the vertices in the $\frac{n}{2} + 2^{th}$ row. This proceedings like this assign the labels continue until reach the last row.

Case 2. $m \equiv 2 \pmod 4$ and $n \equiv 0, 2 \pmod 4$.

In this case assign the labels to the vertices as in Case 1.

Case 3. $m \equiv 1 \pmod 4$ and $n \equiv 0, 2 \pmod 4$.

Here assign the labels by column wise to the vertices of M_n. Assign the labels $2, 3, 2, 3, \cdots 2, 3$ to the vertices in the first column. Next assign the labels $3, 2, 3, 2 \cdots 3, 2$ to the vertices in the second column. This method is continue until reach the $\frac{n}{2}^{th}$ column. Next assign the labels $1, 4, 1, 4, \cdots, 1, 4$ to the vertices in the $\frac{n}{2} + 1^{th}$ column. Then next assign the labels $4, 1, 4, 1, \cdots, 4, 1$ to the vertices in the $\frac{n}{2} + 2^{th}$ column. This procedure is continue until reach the last column.

Case 4. $m \equiv 3 \pmod 4$ and $n \equiv 0, 2 \pmod 4$.

As in Case 3 assign the labels to the vertices in this case. The remainder cordial labeling of graphs $M_{7,4}$ is given below in Figure 3..

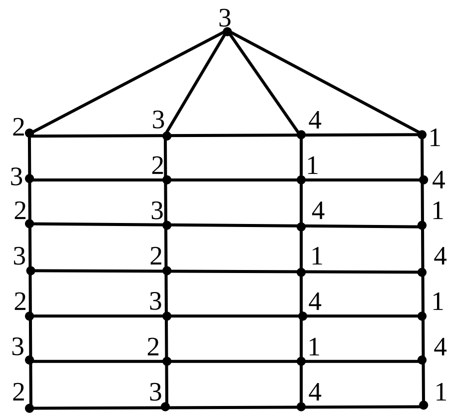

Figure 3

This completes the proof. □

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