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# Path Related n-Cap Cordial Graphs

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**Abstract**: Let G = (V,E) be a graph with p vertices and q edges. A *n-cap* $\overline{(\bigwedge)}$ cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label

$$f(uv) = \begin{cases} 0, & \text{if } f(u) = f(v) = 1 \\ 1, & \text{otherwise.} \end{cases}$$

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a $\overline{\bigwedge}$ cordial labeling is called a $\overline{\bigwedge}$ cordial graph (nCCG). In this paper, we proved that Path $P_n$, Comb $(P_n \odot K_1)$, $P_m \odot 2K_1$ and Fan $(F_n = P_n + K_1)$ are $\overline{\bigwedge}$ cordial graphs.

**Key Words**: $\overline{\bigwedge}$ cordial labeling, Smarandachely cordial labeling, $\overline{\bigwedge}$ cordial labeling graph.

**AMS(2010)**: 05C78.

## §1. Introduction

A graph $G$ is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of $G$ which is called edges. Each pair $e = \{uv\}$ of vertices in $E$ is called an edge or a line of G. In this paper, we proved that Path $P_n$, Comb $(P_n \odot_1)$, $P_m \odot 2K_1$ and Fan $(F_n = P_n + K_1)$ are $\overline{\bigwedge}$ cordial graphs.

## §2. Preliminaries

Let $G = (V, E)$ be a graph with $p$ vertices and q edges. A *n-cap* $(\overline{\bigwedge})$ cordial labeling of a graph $G$ with vertex set $V$ is a bijection from $V$ to $\{0, 1\}$ such that if each edge $uv$ is assigned the

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label

$$f(uv) = \begin{cases} 0, & \text{if } f(u) = f(v) = 1 \\ 1, & \text{otherwise.} \end{cases}$$

with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1, and it is said to be a Smarandachely cordial labeling if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at least 1 and the number of edges labeled with 0 or the number of edges labeled with 1 differ by at least 1.

The graph that admits a $\overline{\bigwedge}$ cordial labeling is called a $\overline{\bigwedge}$ cordial graph. We proved that Path $P_n$, Comb ($P_n \odot K_1$), $P_m \odot 2K_1$ and Fan ($F_n = P_n + K_1$) are $\overline{\bigwedge}$ cordial graphs.

**Definition** 2.1 *A path is a graph with sequence of vertices $u_1, u_2, \cdots, u_n$ such that successive vertices are joined with an edge, denoted by $P_n$, which is a path of length $n-1$.*

*A closed path of length $n$ is cycle $C_n$.*

**Definition** 2.2 *A comb is a graph obtained from a path $P_n$ by joining a pendent vertex to each vertices of $P_n$, it is denoted by $P_n \odot K_1$*

**Definition** 2.3 *A graph obtained from a path $P_m$ by joining two pendent vertices at each vertices of $P_m$ is denoted by $P_m \odot 2K_1$*

**Definition** 2.4 *A fan is a graph obtained from a path $P_n$ by joining each vertices of $P_n$ to a pendent vertex, it is denoted by $F_n = P_n + K_1$*

## §3. Main Results

**Theorem** 3.1 *A path $P_n$ is a $\overline{\bigwedge}$ cordial graph*

*Proof* Let $V(P_n) = \{u_i : 1 \leq i \leq n\}$ and $E(P_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\}$ Define $f : V(P_n) \to \{0, 1\}$ with the vertex labeling determined following.

**Case 1.** $n$ is odd.

Define

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n-1}{2}, \\ 1, & \frac{n+1}{2} \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2} \leq i \leq n. \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $e_0(f) = e_1(f)$. Clearly, it satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$.

**Case 2.** $n$ is even.

Define

$$f(u_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2}+1 \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$f^*(u_iu_{i+1}) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2}+1 \leq i \leq n. \end{cases}$$

Here $V_0(f) = V_1(f)$ and $e_0(f)+1 = e_1(f)$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, a path $P_n$ is a $\overline{\bigwedge}$ cordial graph. $\square$

For example, $P_5$ and $P_6$ are $\overline{\bigwedge}$ cordial graph shown in the Figure 1.

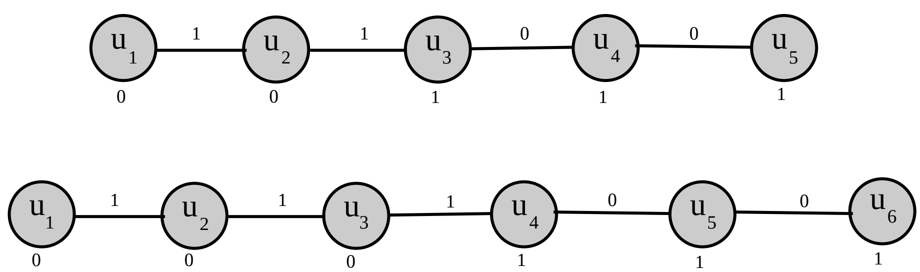


**Figure 1**

**Theorem** 3.2 *A comb $P_n \odot K_1$ is a $\overline{\bigwedge}$ cordial graph*

*Proof* Let G be a comb $P_n \odot K_1$ and let $V(G) = \{(u_i, v_i) : 1 \leq i \leq n\}$ and $E(G) = \{[(u_iu_{i+1}) : 1 \leq i \leq n-1] \bigcup [(u_iv_i) : 1 \leq i \leq n]\}$. Define $f : V(G) \to \{0, 1\}$ with a vertex labeling

$$\begin{aligned} f(u_i) &= 1,\ 1 \leq i \leq n, \\ f(v_i) &= 0,\ 1 \leq i \leq n. \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_iu_{i+1}) &= 1,\ 1 \leq i < n, \\ f^*(u_iv_i) &= 0,\ 1 \leq i \leq n. \end{aligned}$$

Here $V_0(f) = V_1(f)$ and $e_0(f) = e_1(f)+1$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, a comb $P_n \odot K_1$ is a $\overline{\bigwedge}$ cordial graph. $\square$

For example, $P_5 \odot K_1$ is a $\overline{\bigwedge}$ cordial graph shown in Figure 2.

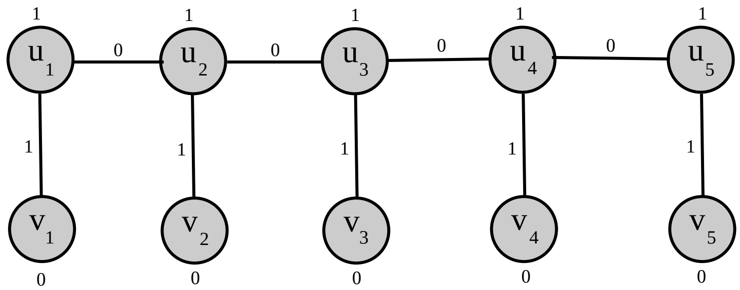


**Figure 2**

**Theorem** 3.3 *A graph $P_m \odot 2K_1$ is a $\overline{\bigwedge}$ cordial graph.*

*Proof* Let $G$ be a $P_m \odot 2K_1$ with $V(G) = \{u_i, v_{1i}, v_{2i}, 1 \leq i \leq n\}$ and $E(G) = \{[(u_i u_{i+1}) : 1 \leq i < n] \bigcup [(u_i v_{1i}) : 1 \leq i \leq n] \bigcup [(u_i v_{2i}) : 1 \leq i \leq n]\}$. Define $f : V(C_n) \to \{0,1\}$ by a vertex labeling $f(u_i) = \{1, 1 \leq i \leq n\}$, $f(v_{1i}) = \{0, 1 \leq i \leq n\}$ and if $n$ is even,

$$f(v_{2i}) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2} + 1 \leq i \leq n, \end{cases}$$

if $n$ is odd

$$f(v_{2i}) = \begin{cases} 1, & 1 \leq i \leq \frac{n+1}{2}, \\ 0, & \frac{n+1}{2} + 1 \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_i u_{i+1}) &= \{0, 1 \leq i \leq n\}, \\ f^*(u_i v_{1i}) &= \{1, 1 \leq i \leq n\} \end{aligned}$$

and if $n$ is even

$$f^*(u_i v_{2i}) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{2}, \\ 1, & \frac{n}{2} + 1 \leq i \leq n. \end{cases}$$

Here $V_0(f) = V_1(f)$ and $e_0(f) + 1 = e_1(f)$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$, and if $n$ is odd

$$f^*(u_i v_{2i}) = \begin{cases} 0, & 1 \leq i \leq \frac{n+1}{2}, \\ 1, & \frac{n+1}{2} \leq i \leq n. \end{cases}$$

Here $V_0(f) + 1 = V_1(f)$ and $e_0(f) = e_1(f)$ which satisfies the condition $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, $P_m \odot 2K_1$ is a $\overline{\bigwedge}$ cordial graph. $\square$

For example, $P_5 \odot 2K_1$ is a $\overline{\bigwedge}$ cordial graph shown in the Figures 3.

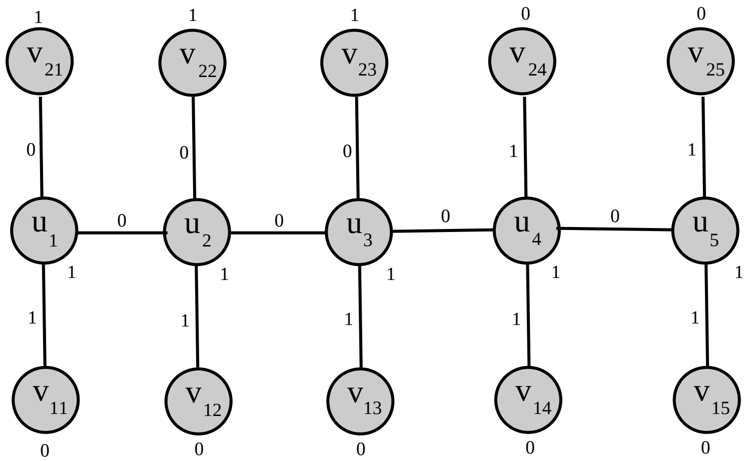


**Figure 3**

**Theorem** 3.4 *A fan $F_n = P_n + K_1$ is a $\overline{\bigwedge}$ cordial graph if $n$ is even.*

*Proof* Let G be a fan $F_n = P_n + K_1$ and $n$ is even with $V(G) = \{u, v_i : 1 \leqslant i \leqslant n\}$ and $E(G) = \{(u, v_i) : 1 \leqslant i \leqslant n\}$. Define $f : V(G) \to \{0, 1\}$ with a vertex labeling $f(u) = \{1\}$ and

$$f(v_i) = \begin{cases} 1, & 1 \leq i \leq \frac{n}{2}, \\ 0, & \frac{n}{2} + 1 \leq i \leq n. \end{cases}$$

The induced edge labeling are

$$f^*(uv_i) = \begin{array}{l} 0,\ 1 \leq i \leq \frac{n}{2}, \\ 1,\ \frac{n}{2} + 1 \leq i \leq n, \end{array} \qquad \text{and} \qquad f^*(v_i v_{i+1}) = \begin{array}{l} 0,\ 1 \leq i \leq \frac{n}{2}, \\ 1,\ \frac{n}{2} \leq i \leq n. \end{array}$$

Here $V_0(f) + 1 = V_1(f)$ and $e_0(f) + 1 = e_1(f)$ which satisfies the conditions $|V_0(f) - V_1(f)| \leq 1$ and $|e_0(f) - e_1(f)| \leq 1$. Hence, a fan $F_n = P_n + K_1$ is a $\overline{\bigwedge}$ cordial graph if $n$ is even. □

For example, a fan $F_6 = P_6 + K_1$ is $\overline{\bigwedge}$ cordial shown in Figure 4.

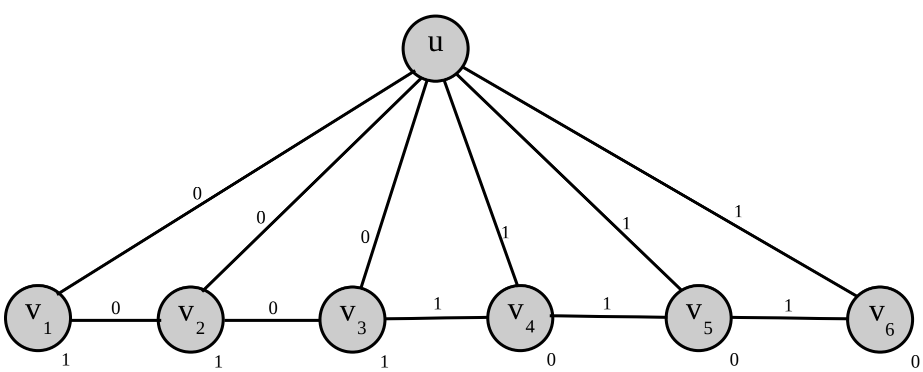


**Figure 4**

## References


[1] J. A. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 6(2001), #DS6.

[2] F. Harary, *Graph Theory*, Addition - Wesley Publishing Company Inc, USA, 1969.

[3] A.Nellai Murugan and V.Baby Suganya, Cordial labeling of path related splitted graphs, *Indian Journal of Applied Research* , Vol.4, 3(2014), 1-8.

[4] A.Nellai Murugan and M. Taj Nisha, A study on divisor cordial labelling of star attached paths and cycles, *Indian Journal of Research*, Vol.3, 3(20140, 12-17.

[5] A.Nellai Murugan and V.Brinda Devi, A study on path related divisor cordial graphs, *International Journal of Scientific Research*, Vol.3, 4(2014), 286 - 291.

[6] A.Nellai Murugan and A Meenakshi Sundari, On cordial graphs, *International Journal of Scientific Research*, Vol.3, 7(20140, 54-55.

[7] A.Nellai Murugan and P. Iyadurai Selvaraj, Path related cup cordial graphs, *Indian Journal of Applied Research*, Vol.4, 8(2014).

[8] A.Nellai Murugan and P. Iyadurai Selvaraj, Cycle and Armed Cap Cordial Graphs, *International Journal on Mathematical Combinatorics*, Vol.2(2016), 144-152.

[9] A.Nellai Murugan and P. Iyadurai Selvaraj, Cycle and armed cup cordial graphs, *International Journal of Innovative Science, Engineering and Technology*, Vol. I, 5(2014), 478-485.

[10] A.Nellai Murugan and P. Iyadurai Selvaraj, Path related cap cordial graphs, *OUTREACH, Multidisciplinary Research Journal*, Vol. VII (2015), 100-106.

[11] A.Nellai Murugan and P. Iyadurai Selvaraj, Additive square mean labeling of path related graphs, *OUTREACH, Multidisciplinary Research Journal*, Vol. IX (2016), 168-174.

[12] A.Nellai Murugan and P. Iyadurai Selvaraj, Path related n-cup cordial craphs, *ACTA VELIT*, Vol.3, 3, 12-17.