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(1,N)-Arithmetic Labelling of Ladder and Subdivision of Ladder

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Abstract: A (p,q)-graph G is said to be $(1,N)$-arithmetic labelling if there is a function ϕ from the vertex set $V(G)$ to $\{0,1,N,(N+1),2N,(2N+1),\cdots,N(q-1),N(q-1)+1\}$ so that the values obtained as the sums of the labelling assigned to their end vertices, can be arranged in the arithmetic progression $\{1,N+1,2N+1,\cdots,N(q-1)+1\}$. In this paper we prove that ladder and subdivision of ladder are $(1,N)$-arithmetic labelling for every positive integer $N>1$.

Key Words: Ladder, subdivision of ladder, one modulo N graceful, Smarandache k modulo N graceful.

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§1. Introduction

V.Ramachandran and C. Sekar [8, 9] introduced one modulo N graceful where N is any positive integer. In the case $N=2$, the labelling is odd graceful and in the case $N=1$ the labelling is graceful. A graph G with q edges is said to be one modulo N graceful (where N is a positive integer) if there is a function ϕ from the vertex set of G to $\{0,1,N,(N+1),2N,(2N+1),\cdots,N(q-1),N(q-1)+1\}$ in such a way that (i) ϕ is $1-1$ (ii) ϕ induces a bijection ϕ^* from the edge set of G to $\{1,N+1,2N+1,\cdots,N(q-1)+1\}$ where $\phi^*(uv)=|\phi(u)-\phi(v)|$. Generally, a graph G with q edges is called to be *Smarandache k modulo N graceful* if one replacing N by kN in the definition of one modulo N graceful graph. Clearly, a graph G is Smarandache k modulo N graceful if and only if it is one modulo kN graceful by definition.

B. D. Acharya and S. M. Hegde [2] introduced (k,d)- arithmetic graphs. A (p,q)- graph G is said to be (k,d)- arithmetic if its vertices can be assigned distinct nonnegative integers so that the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression $k,k+d,k+2d,\ldots,k+(q-1)d$. Joseph A. Gallian [4] surveyed numerous graph labelling methods.

V.Ramachandran and C. Sekar [10] introduced $(1,N)$-Arithmetic labelling. We proved that stars, paths, complete bipartite graph $K_{m,n}$, highly irregular graph $H_i(m,m)$ and cycle C_{4k} are $(1,N)$-Arithmetic labelling,C_{4k+2} is not $(1,N)$-Arithmetic labelling. We also proved that no graph G containing an odd cycle is $(1,N)$-arithmetic labelling for every positive integer

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N. A (p,q)-graph G is said to be $(1,N)$-Arithmetic labelling if there is a function $\phi : V(G) \to \{0, 1, N, (N+1), 2N, (2N+1), \cdots, N(q-1), N(q-1)+1\}$.

In this situation the induced mapping ϕ^* to the edges is given by $\phi^*(uv)$=$\phi(u) + \phi(v)$. If the values of $\phi(u) + \phi(v)$ are $1, N+1, 2N+1, \ldots, N(q-1)+1$ all distinct, then we call the labelling of vertices as $(1,N)$- Arithmetic labelling. In case if the induced mapping ϕ^* is defined as $\phi^*(uv)$=$|\phi(u) - \phi(v)|$ and if the resulting edge labels are are distinct and equal to $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$. We call it as one modulo N graceful. In this paper we prove that Ladder and Subdivision of Ladder are $(1,N)$-Arithmetic labelling for every positive integer $N > 1$.

§2. Main Results

Definition 2.1 *A graph G with q edges is said to be one modulo N graceful (where N is a positive integer) if there is a function ϕ from the vertex set of G to $\{0, 1, N, (N+1), 2N, (2N+1), \cdots, N(q-1), N(q-1)+1\}$ in such a way that (i) ϕ is $1-1$ and (ii) ϕ induces a bijection ϕ^* from the edge set of G to $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$ where $\phi^*(uv) = |\phi(u) - \phi(v)|$.*

Definition 2.2 *A (p,q)-graph G is said to be $(1,N)$-Arithmetic labelling if there is a function ϕ from the vertex set $V(G)$ to $\{0, 1, N, (N+1), 2N, (2N+1), \cdots, N(q-1), N(q-1)+1\}$ so that the values obtained as the sums of the labelling assigned to their end vertices, can be arranged in the arithmetic progression $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$.*

Definition 2.3 *A (p,q)- graph G is said to be (k,d)- arithmetic if its vertices can be assigned distinct nonnegative integers so that the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression $k, k+d, k+2d, \cdots, k+(q-1)d$.*

Definition 2.4([7]) *Let G be a graph with p vertices and q edges. A graph H is said to be a subdivision of G if H is obtained from G by subdividing every edge of G exactly once. H is denoted by $S(G)$.*

Definition 2.5 *The ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with $\times$ denotes the cartesian product. L_n has $2n$ vertices and $3n-2$ edges.*

Theorem 2.6 *For every positive integer n, ladder L_n is $(1,N)$-Arithmetic labelling, for every positive integer $N > 1$.*

Proof Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of L_n, respectively, and let $u_iv_{i+1}, i = 1, 2, \cdots, n-1$. $v_iu_{i+1}, i = 1, 2, \cdots, n-1$ and $u_iv_i, i = 1, 2, \cdots, n$ be the edges of L_n. The ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with $\times$ denotes the cartesian product. Then the ladder L_n has $2n$vertices and $3n-2$ edges as shown in figures following. Define $\phi(u_i) = N(i-1)$ for $i = 1, 2, 3, \cdots, n$, $\phi(v_i) = 2N(i-1)+1$ for $i = 1, 2, 3, \cdots, n$.

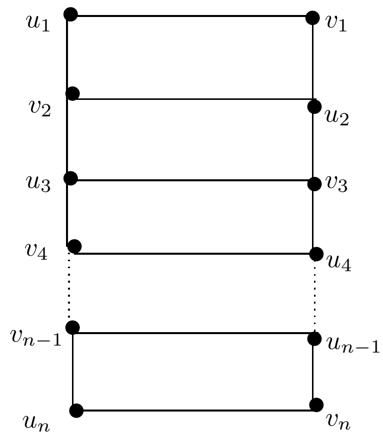

Figure 1 Ladder L_n where n is odd

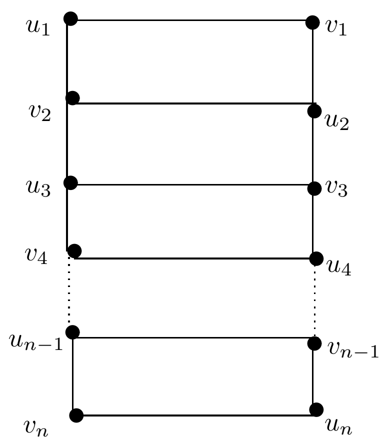

Figure 2 Ladder L_n where n is even

From the definition of ϕ it is clear that

$$\{\phi(u_i), i = 1, 2, \cdots, n\} \bigcup \{\phi(v_i), i = 1, 2, \cdots, n\}$$
$$= \{0, N, 2N, \ldots, N(n-1)\} \cup \{1, 2N+1, 4N+1, \cdots, 2N(n-1)+1\}$$

It is clear that the vertices have distinct labels. Therefore ϕ is $1-1$. We compute the edge labels as follows:

for $i = 1, 2, \cdots, n$, $\phi^*(v_i u_i) =\mid \phi(v_i) + \phi(u_i) \mid = 3N(i-1)+1$; for $i = 1, 2, \cdots, n-1$, $\phi^*(v_{i+1}u_i) =\mid \phi(v_{i+1}) + \phi(u_i) \mid = N(3i-1)+1$, $\phi^*(v_i u_{i+1}) =\mid \phi(v_i) + \phi(u_{i+1}) \mid = N(3i-2)+1$.

This shows that the edges have the distinct labels $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$, where $q = 3n-2$. Hence L_n is $(1, N)$-Arithmetic labelling for every positive integer $N > 1$. □

Example 2.7. A $(1, 5)$-Arithmetic labelling of L_6 is shown in Figure 3.

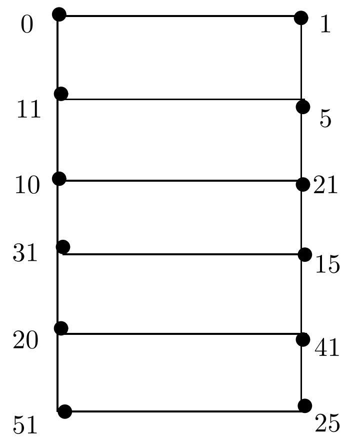

Figure 3

Example 2.8 A $(1, 2)$-Arithmetic labelling of L_7 is shown in Figure 4.

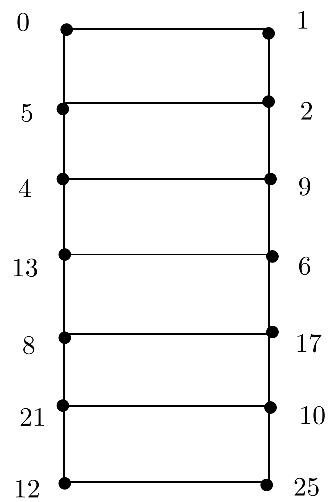

Figure 4

Theorem 2.9 *A subdivision of ladder L_n is $(1, N)$-Arithmetic labelling for every positive integer $N > 1$.*

Proof Let $G = L_n$. The ladder graph L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with $\times$ denotes the cartesian product. L_n has $2n$ vertices and $3n - 2$ edges. A graph H is said to be a subdivision of G if H is obtained from G by subdividing every edge of G exactly once. H is denoted by $S(G)$. Then the subdivision of ladder L_n has $5n - 2$ vertices and $6n - 4$ edges as shown in Figure 5. Let $H = S(L_n)$.

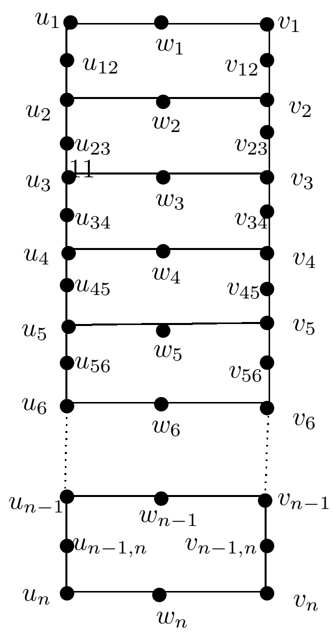

Figure 5 Subdivision of ladder L_n

Define the following functions:

$\eta : N \to N$ by

$$\eta(i) = \begin{cases} N(2i-1) & \text{if } i \text{ is even} \\ 2N(i-1) & \text{if } i \text{ is odd} \end{cases}$$

and $\gamma : N \to N$ by

$$\gamma(i) = \begin{cases} 2N(i-1) & \text{if } i \text{ is even} \\ N(2i-1) & \text{if } i \text{ is odd} \end{cases}$$

Define $\phi : V \to \{0, 1, 2, \cdots, q\}$ by

$$\phi(u_i) = \eta(i), i = 1, 2, \cdots, n$$
$$\phi(v_i) = \gamma(i), i = 1, 2, \cdots, n.$$

Define

$$\phi(u_{i,i+1}) = \begin{cases} 1 + (i-1)4N & \text{if } i \text{ is odd} \\ (4i-1)N + 1 & \text{if } i \text{ is even.} \end{cases}$$

For $i = 1, 2, \cdots, n-2$, define

$$\phi(v_{i,i+1}) = \phi(u_{i+1,i+2}) - 4N, \quad \phi(v_{n-1,n}) = \phi(u_{n-2,n-1}) + 4N,$$

$$\phi(w_i) = \begin{cases} 1 + (4i-3)N & \text{if } i = 1, 2, \cdots, n-1 \\ 4Nn - 4N + 1 & \text{if } i = n\ . \end{cases}$$

It is clear that the vertices have distinct labels. Therefore ϕ is $1-1$. We compute the edge labels as follows:

$$\phi^*(w_n u_n) = \mid \phi(w_n) + \phi(u_n) \mid = 6Nn - 6N + 1,$$

$$\phi^*(w_n v_n) = \mid \phi(w_n) + \phi(v_n) \mid = 6Nn - 5N + 1,$$

$$\phi^*(v_{n-1,n} v_{n-1}) = \mid \phi(v_{n-1,n}) + \phi(v_{n-1}) \mid = \begin{cases} 6Nn - 12N + 1 & \text{if } n \text{ is odd} \\ 6Nn - 8N + 1 & \text{if } n \text{ is even.} \end{cases},$$

$$\phi^*(v_{n-1,n} v_n) = \mid \phi(v_{n-1,n}) + \phi(v_n) \mid = \begin{cases} 6Nn - 9N + 1 & \text{if } n \text{ is odd} \\ 6Nn - 7N + 1 & \text{if } n \text{ is even.} \end{cases}$$

For $i = 1, 2, \cdots, n-1$,

$$\phi^*(w_i u_i) = \mid \phi(w_i) + \phi(u_i) \mid = \begin{cases} N(6i-4) + 1 & \text{if } i \text{ is even} \\ N(6i-5) + 1 & \text{if } i \text{ is odd.} \end{cases}$$

$$\phi^*(w_i v_i) = \mid \phi(w_i) + \phi(v_i) \mid = \begin{cases} N(6i-5) + 1 & \text{if } i \text{ is even} \\ N(6i-4) + 1 & \text{if } i \text{ is odd.} \end{cases}$$

For $i = 1, 2, \cdots, n-1$,

$$\phi^*(u_{i,i+1} u_i) = \mid \phi(u_{i,i+1}) + \phi(u_i) \mid = \begin{cases} N(6i-2) + 1 & \text{if } i \text{ is even} \\ N(6i-6) + 1 & \text{if } i \text{ is odd.} \end{cases}$$

$$\phi^*(u_{i,i+1} u_{i+1}) = \mid \phi(u_{i,i+1}) + \phi(u_{i+1}) \mid = \begin{cases} N(6i-3) + 1 & \text{if } i \text{ is odd} \\ N(6i-1) + 1 & \text{if } i \text{ is even.} \end{cases}$$

For $i = 1, 2, \cdots, n-2$,

$$\phi^*(v_{i,i+1} v_i) = \mid \phi(v_{i,i+1}) + \phi(v_i) \mid = \begin{cases} N(6i-6) + 1 & \text{if } i \text{ is even} \\ N(6i-2) + 1 & \text{if } i \text{ is odd.} \end{cases}$$

$$\phi^*(v_{i,i+1} v_{i+1}) = \mid \phi(v_{i,i+1}) + \phi(v_{i+1}) \mid = \begin{cases} N(6i-3) + 1 & \text{if } i \text{ is even} \\ N(6i-1) + 1 & \text{if } i \text{ is odd.} \end{cases}$$

This shows that the edges have distinct labels $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$ with $q = 6n - 4$. Hence $S(L_n)$ is $(1, N)$-Arithmetic labelling for every positive integer $N > 1$. □

Example 2.10 A $(1, 3)$-Arithmetic labelling of $S(L_5)$ is shown in Figure 6.

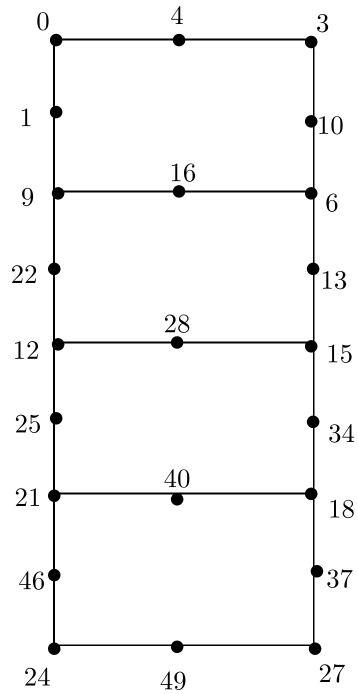

Figure 6

Example 2.11 A $(1, 10)$-Arithmetic labelling of $S(L_6)$ is shown in Figure 7.

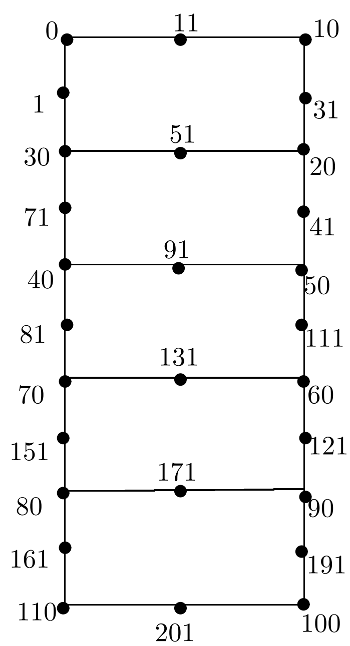

Figure 7

References

[1] B. D. Acharya, On d-sequential graphs, *J. Math. Phys. Sci.*, 17 (1983), 21-35.

[2] B. D. Acharya and S. M. Hegde, Arithmetic graphs, *Journal of Graph Theory*, 14 (1990) 275-299.

[3] B. D. Acharya and S. M. Hegde, On certain vertex valuations of a graph I, *Indian J. Pure Appl. Math.*, 22 (1991) 553-560.

[4] Joseph A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics*, #DS6 (2016).

[5] S.W.Golomb, *How to Number a Graph in Graph theory and Computing*, R.C. Read, ed., Academic Press, New york (1972)23-27.

[6] R. B. Gnanajothi, *Topics in Graph theory*, Ph.D. thesis, Madurai Kamaraj University, 1991.

[7] KM. Kathiresan, Subdivisions of ladders are graceful, *Indian J. Pure Appl. Math.*, (1992), 21–23.

[8] V. Ramachandran, C. Sekar, One modulo N gracefullness of arbitrary supersubdivisions of graphs, *International J. Math. Combin.*, Vol.2 (2014), 36-46.

[9] V. Ramachandran, C. Sekar, One modulo N gracefulness of supersubdivision of ladder, *Journal of Discrete Mathematical Sciences and Cryptography*, Vol.18 (3) (2015), 265-274.

[10] V. Ramachandran, C. Sekar, $(1, N)$-arithmetic graphs, *International Journal of Computers and Applications*, Vol.38 (1) (2016) 55-59.

[11] C. Sekar, *Studies in Graph Theory*, Ph.D. thesis, Madurai Kamaraj University, 2002.