# Representation of Graphs using Intuitionistic Neutrosophic Soft Sets

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**Abstract**

The concept of intuitionistic neutrosophic soft sets can be utilized as a mathematical tool to deal with imprecise and unspecified information. In this paper, we apply the concept of intuitionistic neutrosophic soft sets to graphs. We introduce the concepts of intuitionistic neutrosophic soft graphs, and present applications of intuitionistic neutrosophic soft graphs in a multiple-attribute decision-making problems. We also present an algorithm of our proposed method.

**Key-words**: Intuitionistic neutrosophic graphs, Self complementary intuitionistic neutrosophic soft graph, decision-making.

**Mathematics Subject Classification 2000**: 03E72, 05C72, 05C78

## 1 Introduction

Zadeh [38] introduced the concept of fuzzy set, characterized by a membership function in [0, 1], which is very useful in dealing with uncertainty, imprecision and vagueness. Since then, many higher order fuzzy sets [5, 40] have been introduced in literature to solve many real life problems involving ambiguity and uncertainty. Atanassov [5] introduced the concept of intuitionistic fuzzy sets (IFSs) as a extension of Zadeh's fuzzy set [38]. The concept of IFS can be viewed as an alternative approach for when available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but IFS is described by a membership(truth-membership) function and a non-membership(falsity-membership) function, the only requirement is that the sum of both values is less than and equal to one. However IFSs cannot deal with all types of uncertainty, including indeterminate information and inconsistent information, which exist commonly in different real-world problems. Smarandache[31] introduced the idea of neutrosophic set theory from philosophical point of view. Its prominent characteristic is that a truth-membership degree, an indeterminacy membership degree and a falsity membership degree, in non-standard unit interval $]0^-, 1^+[$, are independently assigned to each element in the set. Moderately, it has been discovered that without a specific description, neutrosophic sets are difficult to apply in the real applications. After analyzing this difficulty, Wang et al.[33] presented the idea of single-valued neutrosophic set (SVNS) from scientific or engineering point of view, as an instance of the neutrosophic set and an extension of IFS, and provide its various properties. SVNSs represent uncertainty, incomplete, imprecise, indeterminate and inconsistent information which exist in real world. On the other hand, Bhowmik and Pal [7] introduced intuitionistic neutrosophic set (INS) and discussed some of its properties.

Molodtsov [26] introduced soft set theory as a new mathematical tool for dealing with imprecision. Soft sets introduced by Molodtsov gave us new technique for dealing with uncertainty after specifying set of parameters. Soft sets has many applications in several fields including operations research, decision-making, probability theory, and smoothness of functions, measurement theory [10, 12, 13]. Some new operations are proposed for defining soft sets [26]. Maji et al [21, 22, 24] proposed fuzzy soft sets, intuitionistic fuzzy soft sets (IFSSs) and neutrosophic soft sets (NSSs) by combining fuzzy, intuitionistic fuzzy and neutrosophic set theories with soft set theory. Said and Smarandache [30] proposed intuitionistic neutrosophic soft set (INSSs) and its application in decision making-problems. Broumi [11] introduced generalized neutrosophic soft set. Sahin and Kucuk [32] defined similarity and entropy of neutrosophic soft set. Ye [37] proposed correlation coefficients of neutrosophic soft set and its application in decision-making problem. Ye [36] also defined multi criteria decision-making method using aggregation operators. Akram and Nawaz [1] have introduced the concept of soft graphs and some operation on soft graphs. Certain concepts of fuzzy soft graphs and intuitionistic fuzzy soft graphs are discussed in [2, 3, 29]. Akram and Shahzadi [4] have introduced neutrosophic soft graphs. In this paper, we apply the concept of intuitionistic neutrosophic soft sets to graphs. We introduce the notions of intuitionistic neutrosophic soft graphs and present applications of intuitionistic neutrosophic soft graphs in a multiple-attribute decision-making problems.

## 2 Intuitionistic neutrosophic soft graphs

**Definition 2.1.** [30] Let $U$ be an initial universe, and let $P$ be the set of all parameters. $\mathcal{N}(U)$ denotes the set of all INSSs of $U$. Let $N$ be a subset of $P$. A pair $(F, N)$ is called an *intuitionistic neutrosophic soft set* INSS over $U$.

Let $\mathcal{N}(V)$ denotes the set of all INSSs of $V$ and $\mathcal{N}(E)$ denotes the set of all INSSs of $E$.

**Definition 2.2.** An intuitionistic neutrosophic soft graph on a nonempty $V$ is an ordered 3-tuple $\mathbb{G} = (F, K, N)$ such that

1. $N$ is a non-empty set of parameters,
2. $(F, N)$ is an INSS over $V$,
3. $(K, N)$ is an intuitionistic neutrosophic soft relation on $V$, i.e., $K : N \to \mathcal{N}(V \times V)$, where $\mathcal{N}(V \times V)$ is intuitionistic neutrosophic power set,
4. $(F(e), K(e))$ is an ING for all $e \in N$.

   That is,
   $$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\},$$
   $$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\},$$
   $$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\},$$
   such that $0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 2 \;\forall\; e \in N,\; x, y \in V$.

The intuitionistic neutrosophic graph (ING) $(F(e), K(e))$ is denoted by $\mathbb{H}(e)$. Note that $T_{K(e)}(xy) = I_{K(e)}(xy) = 0$ and $F_{K(e)}(xy) = 1$ for all $xy \in V \times V - E, e \notin N$. $(F, N)$ is called an intuitionistic neutrosophic soft vertex and $(K, N)$ is called an intuitionistic neutrosophic soft edge.
Thus, $((F, N), (K, N))$ is called an INSG if

$$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\},$$

$$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\},$$
$$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\},$$

such that $0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 2 \ \forall \ e \in N, \ x, y \in V$. In other words, an INSG is a parameterized family of INGs. The class of all INSGs is denoted by $\mathcal{INS}(G^*)$. The *order* of an INSG is

$$O(\mathbb{G}) = \Big( \sum_{e_i \in N} ( \sum_{w \in V} T_{F(e_i)}(w)), \sum_{e_i \in N} ( \sum_{w \in V} I_{F(e_i)}(w)), \sum_{e_i \in N} ( \sum_{v \in V} F_{F(e_i)}(w)) \Big).$$

The *size* of an INSG is

$$S(\mathbb{G}) = \Big( \sum_{e_i \in N} ( \sum_{wv \in E} T_{K(e_i)}(wv)), \sum_{e_i \in N} ( \sum_{wv \in E} I_{K(e_i)}(wv)), \sum_{e_i \in N} ( \sum_{wv \in E} F_{K(e_i)}(wv)) \Big).$$

**Example 2.3.** Consider a simple graph $G^* = (V, E)$ such that $V = \{w_1, w_2, w_3, w_4\}$ and $E = \{w_1w_2, w_2w_3, w_1w_3, w_1w_5, \}$. Let $N = \{e_1, e_2, e_3\}$ be a set of parameters and let $(F, N)$ be an INSS over $V$ with intuitionistic neutrosophic approximation function $F : N \to \mathcal{N}(V)$ defined by
$F(e_1) = \{(w_1, 0.4, 0.5, 0.3), (w_2, 0.5, 0.4, 0.6), (w_3, 0.6, 0.5, 0.4), \}$,
$F(e_2) = \{(w_1, 0.6, 0.2, 0.3), (w_3, 0.6, 0.5, 0.3), (w_5, 0.7, 0.5, 0.4)\}$,
$F(e_3) = \{(w_1, 0.8, 0.5, 0.4), (w_2, 0.5, 0.5, 0.3), (w_5, 0.6, 0.5, 0.4)\}$. Let $(K, N)$ be an INSS over $E$ with intuitionistic neutrosophic approximation function $K : N \to \mathcal{N}(E)$ defined by
$K(e_1) = \{(w_1w_2, 0.3, 0.3, 0.6), (w_2w_3, 0.5, 0.4, 0.6)\}$,
$K(e_2) = \{(w_1w_3, 0.6, 0.2, 0.2), (w_1w_5, 0.6, 0.1, 0.4)\}$,
$K(e_3) = \{(w_1w_2, 0.4, 0.5, 0.4), (w_1w_3, 0.6, 0.5, 0.3)\}$.
Clearly, $\mathbb{H}(e_1) = (F(e_1), K(e_1))$, $\mathbb{H}(e_2) = (F(e_2), K(e_2))$ and $\mathbb{H}(e_3) = (F(e_3), K(e_3))$ are INGs corresponding to the parameters $e_1$, $e_2$ and $e_3$, respectively as shown in Figure 2.1.

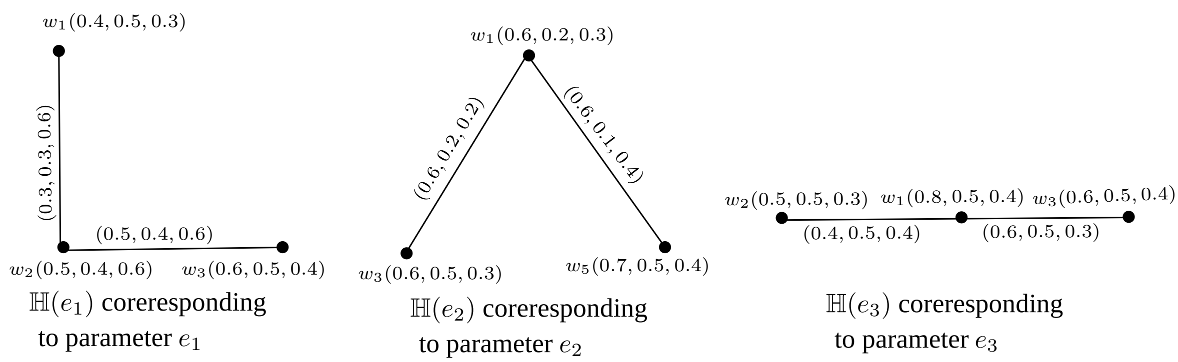


Figure 2.1: Intuitionistic neutrosophic soft graph $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2), \mathbb{H}(e_3)\}$.

Hence $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2), \mathbb{H}(e_3)\}$ is an INSG of $G^*$. Tabular representation of an INSG is given in Table 1.

Table 1: Tabular representation of an intuitionistic neutrosophic soft graph.

| $F$ | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ |
|---|---|---|---|---|---|
| $e_1$ | $(0.4, 0.5, 0.3)$ | $(0.5, 0.4, 0.6)$ | $(0.6, 0.5, 0.4)$ | $(0.0, 0.0, 0.0)$ | (0.0,0.0,0.0) |
| $e_2$ | $(0.6, 0.2, 0.3)$ | $(0.0, 0.0, 0.0)$ | $(0.6, 0.5, 0.3)$ | $(0.0, 0.0, 0.0)$ | (0.7,0.5,0.4) |
| $e_3$ | $(0.8, 0.5, 0.4)$ | $(0.5, 0.5, 0.3)$ | $(0.6, 0.5, 0.4)$ | $(0.0, 0.0, 0.0)$ | (0.0,0.0,0.0) |

| $K$ | $w_1w_2$ | $w_2w_3$ | $w_1w_3$ | $w_1w_5$ |
|---|---|---|---|---|
| $e_1$ | $(0.3, 0.3, 0.6)$ | $(0.5, 0.4, 0.6)$ | $(0.0, 0.0, 0.0)$ | $(0.0, 0.0, 0.0)$ |
| $e_2$ | $(0.0, 0.0, 0.0)$ | $(0.0, 0.0, 0.0)$ | $(0.6, 0.2, 0.2)$ | $(0.6, 0.1, 0.4)$ |
| $e_3$ | $(0.4, 0.5, 0.4)$ | $(0.0, 0.0, 0.0)$ | $(0.6, 0.5, 0.3)$ | $(0.0, 0.0, 0.0)$ |

The order of INSG is $\mathbb{G}$ is $O(\mathbb{G}) = \big((0.4+0.5+0.6)+(0.6+0.6+0.7)+(0.8+0.5+0.6), (0.5+0.4+0.5)+(0.2+0.5+0.5)+(0.5+0.5+0.5), (0.3+0.6+0.4)+(0.3+0.3+0.4)+(0.4+0.3+0.4)\big) = (5.3, 4.1, 3.4)$. The size of intuitionistic neutrosophic soft graph $\mathbb{G}$ is $S(\mathbb{G}) = \big((0.3+0.5)+(0.6+0.6)+(0.4+0.6), (0.3+0.4)+(0.2+0.1)+(0.5+0.5), (0.6+0.6)+(0.2+0.4)+(0.4+0.3)\big) = (3.0, 2.0, 2.5)$.

**Definition 2.4.** Let $\mathbb{G}_1 = (F_1, K_1, N_1)$ and $\mathbb{G}_2 = (F_2, K_2, N_2)$ be two INSGs of $G_1^*$ and $G_2^*$, respectively. The *Cartesian product* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG $\mathbb{G} = \mathbb{G}_1 \times \mathbb{G}_2 = (F, K, N_1 \times N_2)$, where $(F = F_1 \times F_2, N_1 \times N_2)$ is an intuitionistic neutrosophic soft set over $V = V_1 \times V_2$, $(K = K_1 \times K_2, N_1 \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v), (w_2, v)) : v \in V_2, (w_1, w_2) \in E_1\}$ and $(F, K, N_1 \times N_2)$ is intuitionistic neutrosophic soft graph such that

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v) \ \forall \ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1, v_2) \ \forall \ w \in V_1, (v_1, v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1, w_2)$,
$I_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1, w_2)$,
$F_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1, w_2) \ \forall \ v \in V_2, (w_1, w_2) \in E_1$.

$\mathbb{H}(e_1, e_2) = \mathbb{H}_1(e_1) \times \mathbb{H}_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are intuitionistic neutrosophic graphs.

**Definition 2.5.** The *cross product* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG $\mathbb{G} = \mathbb{G}_1 \circledcirc \mathbb{G}_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, (v_1, v_2) \in E_2\}$ and $(F, K, N_1 \times N_2)$ is INSG such that

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v) \ \forall \ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$

(ii) $T_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = T_{K_1(e_1)}(w_1, w_2) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = I_{K_1(e_1)}(w_1, w_2) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = F_{K_1(e_1)}(w_1, w_2) \vee F_{K_2(e_2)}(v_1, v_2) \ \forall \ (w_1, w_2) \in E_1, (v_1, v_2) \in E_2$.

$\mathbb{H}(e_1, e_2) = \mathbb{H}_1(e_1) \circledcirc \mathbb{H}_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are intuitionistic neutrosophic graphs.

**Definition 2.6.** The *lexicographic product* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG $\mathbb{G} = \mathbb{G}_1 \odot \mathbb{G}_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, (v_1, v_2) \in E_2\}$ and $(F, K, N_1 \times N_2)$ are INSGs such that

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ w \in V_1, (v_1, v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = T_{K_1(e_1)}(w_1, w_2) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = I_{K_1(e_1)}(w_1, w_2) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = F_{K_1(e_1)}(w_1, w_2) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ (w_1, w_2) \in E_1, (v_1, v_2) \in E_2$.

$\mathbb{H}(e_1, e_2) = \mathbb{H}_1(e_1) \odot \mathbb{H}_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are INGs.

**Definition 2.7.** The *strong product* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG $\mathbb{G} = \mathbb{G}_1 \otimes \mathbb{G}_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, A \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v), (w_2, v)) : v \in V_2, (w_1, w_2) \in E_1\} \cup \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, (v_1, v_2) \in E_2\}$ and $(F, K, N_1 \times N_2)$ is INSG such that

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ w \in V_1, (v_1, v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1, w_2)$,
$I_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1, w_2)$,
$F_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1, w_2)\ \forall\ v \in V_2, (w_1, w_2) \in E_1$,

(iv) $T_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = T_{K_1(e_1)}(w_1, w_2) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = I_{K_1(e_1)}(w_1, w_2) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = F_{K_1(e_1)}(w_1, w_2) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ (w_1, w_2) \in E_1, (v_1, v_2) \in E_2$.

$\mathbb{H}(e_1, e_2) = \mathbb{H}_1(e_1) \otimes \mathbb{H}_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are INGs.

**Definition 2.8.** The *composition* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG $\mathbb{G} = \mathbb{G}_1[\mathbb{G}_2] = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v), (w_2, v)) : v \in V_2, (w_1, w_2) \in E_1\} \cup \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, v_1 \neq v_2\}$ and $(F, K, N_1 \times N_2)$ is INSG such that

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}((w, v_1), (w, v_2)) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}((w, v_1), (w, v_2)) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}((w, v_1), (w, v_2)) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ w \in V_1, (v_1, v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1,w_2)$,
$I_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1,w_2)$,
$F_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1,w_2)\ \forall\ v \in V_2, (w_1,w_2) \in E_1$,

(iv) $T_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = T_{F_1(e_1)}(w_1,w_2) \wedge T_{F_2(e_2)}(v_1) \wedge T_{F_2(e_2)}(v_2)$,
$I_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = I_{F_1(e_1)}(w_1,w_2) \wedge I_{F_2(e_2)}(v_1) \wedge I_{F_2(e_2)}(v_2)$,
$F_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = F_{F_1(e_1)}(w_1,w_2) \vee F_{F_2(e_2)}(v_1) \vee F_{F_2(e_2)}(v_2)\ \forall\ (w_1,w_2) \in E_1$, where $v_1 \neq v_2$.

$\mathbb{H}(e_1,e_2) = \mathbb{H}_1(e_1)[\mathbb{H}_2(e_2)]$ for all $(e_1,e_2) \in N_1 \times N_2$ are INGs.

**Proposition 2.9.** *The Cartesian product, cross product, lexicographic product, strong product and composition of two INSGs is an ING.*

**Definition 2.10.** Let $\mathbb{G}_1 = (F_1, K_1, N_1)$ and $\mathbb{G}_2 = (F_2, K_2, N_2)$ be two INSGs. The *intersection* of $\mathbb{G}_1$ and $\mathbb{G}_2$ is an INSG denoted by $\mathbb{G} = \mathbb{G}_1 \cap \mathbb{G}_2 = (F, K, N_1 \cup N_2)$, where $(F, N_1 \cup N_2)$ is an INSS over $V = V_1 \cap V_2$, $(K, N_1 \cup N_2)$ is an INSS over $E = E_1 \cap E_2$, the truth-membership, indeterminacy-membership, and falsity-membership functions of $\mathbb{G}$ for all $w, v \in V$ defined by,

(i) $$T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ T_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ T_{F_1(e)}(v) \wedge T_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ I_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ F_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ F_{F_1(e)}(v) \vee F_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

(ii) $$T_{K(e)}(wv) = \begin{cases} T_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ T_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ T_{K_1(e)}(wv) \wedge T_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{K(e)}(wv) = \begin{cases} I_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ I_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ I_{K_1(e)}(wv) \wedge I_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{K(e)}(wv) = \begin{cases} F_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ F_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ F_{K_1(e)}(wv) \vee F_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

**Definition 2.11.** Let $\mathbb{G}_1 = (F_1, K_1, N_1)$ and $\mathbb{G}_2 = (F_2, K_2, N_2)$ be two INSGs. The *union* of $\mathbb{G}_1$ and $\mathbb{G}_2$ may or may not be INSG denoted by $\mathbb{G} = \mathbb{G}_1 \cup \mathbb{G}_2 = (F, K, N_1 \cup N_2)$, where $(F, N_1 \cup N_2)$ is an INSS over $V = V_1 \cup V_2$, $(K, N_1 \cup N_2)$ is an INSS over $E = E_1 \cup E_2$, the truth-membership, indeterminacy-membership, and falsity-membership functions of $\mathbb{G}$ for all $w, v \in V$ defined by,

(i) $$T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ T_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ T_{F_1(e)}(v) \vee T_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ I_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ F_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ F_{F_1(e)}(v) \wedge F_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

(ii) $$T_{K(e)}(wv) = \begin{cases} T_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ T_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ T_{K_1(e)}(wv) \vee T_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{K(e)}(wv) = \begin{cases} I_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ I_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ I_{K_1(e)}(wv) \wedge I_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{K(e)}(wv) = \begin{cases} F_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ F_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ F_{K_1(e)}(wv) \wedge F_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

**Remark 2.12.** Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be two INSG over $G^*$ then $\mathbb{G}_1 \cup \mathbb{G}_2$ may or may not be INSG.

**Definition 2.13.** Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be two INSGs. The *join* of $\mathbb{G}_1$ and $\mathbb{G}_2$ may or may not be intuitionistic neutrosophic soft graph denoted by $\mathbb{G}_1 + \mathbb{G}_2 = (F_1 + F_2, K_1 + K_2, N_1 \cup N_2)$, where $(F_1 + F_2, N_1 \cup N_2)$ is an intuitionistic neutrosophic soft set over $V_1 \cup V_2$, $(K_1 + K_2, N_1 \cup N_2)$ is an INSS over $E_1 \cup E_2 \cup \acute{E}$ defined by

(i) $(F_1 + F_2, N_1 \cup N_2) = (F_1, N_1) \cup (F_2, N_2)$,

(ii) $(K_1 + K_2, N_1 \cup N_2) = (K_1, N_1) \cup (K_2, N_2)$ if $wv \in E_1 \cup E_2$,
where $e \in N_1 \cap N_2, wv \in \acute{E}$, and $\acute{E}$ is the set of all edges joining the vertices of $V_1$ and $V_2$, the truth-membership, indeterminacy-membership, and falsity-membership functions are defined by

$$\begin{aligned} T_{K_1+K_2(e)}(wv) &= \min\{T_{F_1(e)}(w), T_{F_2(e)}(v)\}, \\ I_{K_1+K_2(e)}(wv) &= \min\{I_{F_1(e)}(w), I_{F_2(e)}(v)\}, \\ F_{K_1+K_2(e)}(wv) &= \max\{F_{F_1(e)}(w), F_{F_2(e)}(v)\} \; \forall wv \in \acute{E}. \end{aligned}$$

**Proposition 2.14.** *If $\mathbb{G}_1$ and $\mathbb{G}_2$ are two INSGs then their join $\mathbb{G}_1 + \mathbb{G}_2$ may or may not be intuitionistic neutrosophic soft graph.*

**Definition 2.15.** The *complement* of an INSG $\mathbb{G} = (F, K, N)$ denoted by $\mathbb{G}^c = (F^c, K^c, N^c)$ is defined as follows:

(i) $N^c = N$,

(ii) $F^c(e) = F(e)$,

(iii) $T_{K^c(e)}(w, v) = T_{F(e)}(w) \wedge T_{F(e)}(v) - T_{K(e)}(w, v)$,

(iv) $I_{K^c(e)}(w, v) = I_{F(e)}(w) \wedge I_{F(e)}(v) - I_{K(e)}(w, v)$, and

(v) $F_{K^c(e)}(w, v) = F_{F(e)}(w) \vee F_{F(e)}(v) - F_{K(e)}(w, v)$, for all $w, v \in V, e \in N$.

**Example 2.16.** Let $G^* = (V, E)$ be a crisp graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_1v_3, v_2v_3, v_3v_4\}$. Let $N = \{e_1, e_2\}$ be a set of parameters and let $(F, N)$ be a INSS over $V$ with intuitionistic neutrosophic approximation function $F : N \to \mathcal{N}(V)$ defined by

$F(e_1) = \{(v_1, 0.4, 0.6, 0.1), (v_2, 0.5, 0.4, 0.7), (v_3, 0.5, 0.3, 0.4), (v_4, 0.5, 0.6, 0.2)\}$,
$F(e_2) = \{(v_1, 0.4, 0.2, 0.2), (v_2, 0.5, 0.3, 0.4), (v_3, 0.6, 0.3, 0.5), (v_4, 0.5, 0.4, 0.2)\}$.
Let $(K, N)$ be an INSS over $E$ with intuitionistic neutrosophic approximation function $K : N \to \mathcal{N}(E)$ defined by
$K(e_1) = \{(v_1v_2, 0.3, 0.3, 0.5), (v_1v_4, 0.2, 0.5, 0.2), (v_1v_3, 0.4, 0.3, 0.4), (v_2v_3, 0.5, 0.4, 0.5)\}$,
$K(e_2) = \{(v_1v_3, 0.3, 0.2, 0.5), (v_1v_4, 0.4, 0.1, 0.1), (v_3v_4, 0.5, 0.3, 0.4), (v_3v_2, (0.5, 0.3, 0.5)\}$.
Clearly, $\mathbb{G} = \{\mathbb{H}(e_1) = (F(e_1), K(e_1)), \mathbb{H}(e_2) = (F(e_2), K(e_2))\}$ is intuitionistic neutrosophic soft graphs corresponding to the parameters $e_1$ and $e_2$, respectively as shown in Figure 2.2.

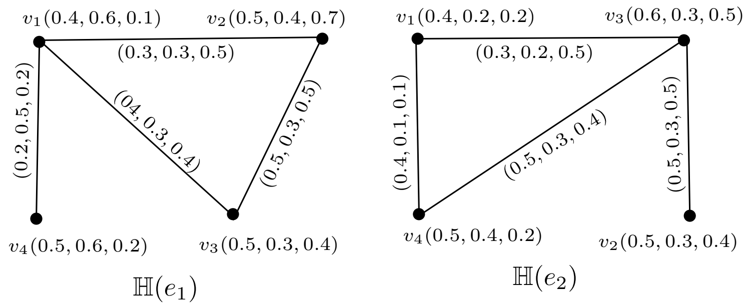


Figure 2.2: INSG $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2)\}$.

Now, the complement of INSG $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2)\}$ is the complement of INGs $\mathbb{H}(e_1)$ and $\mathbb{H}(e_2)$ which are shown in Figure 2.3.

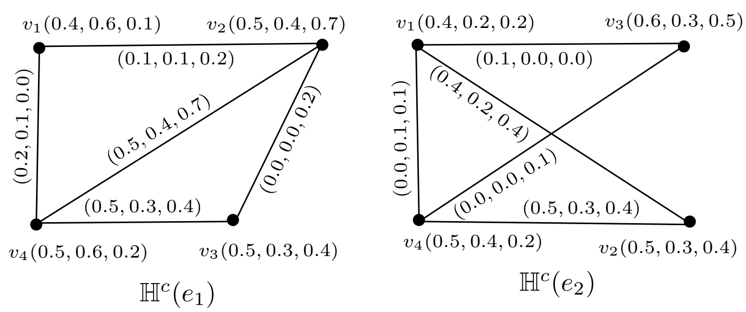


Figure 2.3: Complement of INSG $\mathbb{G}^c = \{\mathbb{H}^c(e_1), \mathbb{H}^c(e_2)\}$

**Definition 2.17.** An INSG $\mathbb{G}$ is a *complete INSG* if $\mathbb{H}(e)$ is a complete ING for all $e \in N$, i.e.,

$$\begin{aligned} T_{K(e)}(wv) &= \min(T_{F(e)}(w), T_{F(e)}(v)), \\ I_{K(e)}(wv) &= \min(I_{F(e)}(w), I_{F(e)}(v)), \\ F_{K(e)}(wv) &= \max(F_{F(e)}(w), F_{F(e)}(v)) \end{aligned}$$

$\forall\ w, v \in V, e \in N$.

**Definition 2.18.** An INSG $\mathbb{G}$ is a *strong INSG* if $\mathbb{H}(e)$ is a strong ING for all $e \in N$.

**Example 2.19.** Consider the simple graph $G^* = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{v_1v_2, v_2v_5, v_3v_5, v_1v_3, v_1v_4, v_3v_6, v_5v_6\}$. Let $N = \{e_1, e_2\}$. Let $(F, N)$ be an INSS over $V$ with its approximation function $F : N \to \mathcal{N}(V)$ defined by

$F(e_1) = \{(v_1, 0.4, 0.5, 0.7), (v_2, 0.6, 0.5, 0.5), (v_3, 0.6, 0.3, 0.5), (v_4, 0.7, 0.5, 0.4), (v_5, 0.7, 0.4, 0.5), (v_6, 0.3, 0.5, 0.7)\}$,
$F(e_2) = \{(v_1, 0.6, 0.4, 0.3), (v_2, 0.5, 0.3, 0.8), (v_3, 0.5, 0.6, 0.3), (v_4, 0.8, 0.5, 0.4), (v_5, 0.6, 0.3, 0.2)\}$.
Let $(K, N)$ be an INSS over $E$ with its approximation function $K : N \to \mathcal{N}(E)$ defined by
$K(e_1) = \{(v_1v_2, 0.4, 0.5, 0.7), (v_1v_3, 0.4, 0.3, 0.7), (v_1v_4, 0.4, 0.5, 0.7), (v_2v_5, 0.6, 0.4, 0.5), (v_3v_5, 0.6, 0.3, 0.5), (v_3v_6, 0.3, 0.3, 0.7), (v_5v_6, 0.3, 0.5, 0.7)\}$,
$K(e_2) = \{(v_1v_3, 0.5, 0.4, 0.3), (v_1v_4, 0.6, 0.4, 0.4), (v_1v_2, 0.5, 0.3, 0.8), (v_2v_3, 0.5, 0.3, 0.8), (v_2v_4, 0.5, 0.3, 0.8), (v_2v_5, 0.5, 0.3, 0.8)\}$.
$\mathbb{H}(e_1) = (F(e_1), K(e_1))$, and $\mathbb{H}(e_2) = (F(e_2), K(e_2))$ are strong INGs corresponding to the parameters $e_1$, and $e_2$, respectively as shown in Figure 2.4.

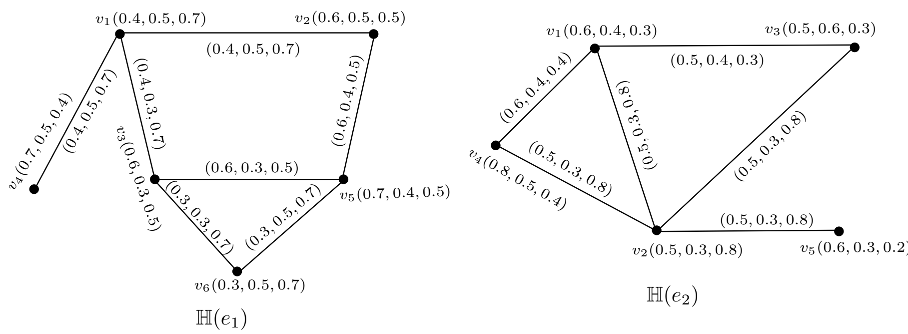


Figure 2.4: Strong INSG $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2)\}$.

Hence $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2)\}$ is a strong INSG of $G^*$.

**Proposition 2.20.** *If $\mathbb{G}_1$ and $\mathbb{G}_2$ are strong INSGs, then $\mathbb{G}_1 \times \mathbb{G}_2$, and $\mathbb{G}_1[\mathbb{G}_2]$ are strong INSGs.*

**Remark 2.21.** The union of two strong INSGs is not necessarily strong INSG.

**Example 2.22.** Let $N_1 = \{e_1\}$ and $N_2 = \{e_1, e_2\}$ be the parameter sets. Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be the two strong INSGs defined as follows:
$\mathbb{G}_1 = \{\mathbb{H}_1(e_1), \mathbb{H}_1(e_2)\} = \{(\{(w_1, 0.5, 0.6, 0.4), (w_2, 0.7, 0.4, 0.5), (w_3, 0.5, 0.8, 0.4)\}, \{(w_1w_2, 0.5, 0.4, 0.5), (w_2w_3, 0.5, 0.4, 0.5)\}), (\{(w_1, 0.4, 0.6, 0.5), (w_3, 0.5, 0.7, 0.4)\}, \{(w_1w_3, 0.4, 0.6, 0.5)\})\}$,
$\mathbb{G}_2 = \{\mathbb{H}_2(e_1)\} = \{(w_1, 0.4, 0.9, 0.3), (w_2, 0.5, 0.6, 0.4), (w_1w_2, 0.4, 0.6, 0.4)\}$.
The union of $\mathbb{G}_1$ and $\mathbb{G}_2$ is $\mathbb{G} = \mathbb{G}_1 \cup \mathbb{G}_2 = (H, N_1 \cup N_2)$, where $N_1 \cup N_2 = \{e_1, e_2\}$, $\mathbb{H}(e_1) = \mathbb{H}_1(e_1) \cup \mathbb{H}_2(e_1)$ and $\mathbb{H}(e_2) = \mathbb{H}_1(e_2)$ are as shown in Figure. 2.5.

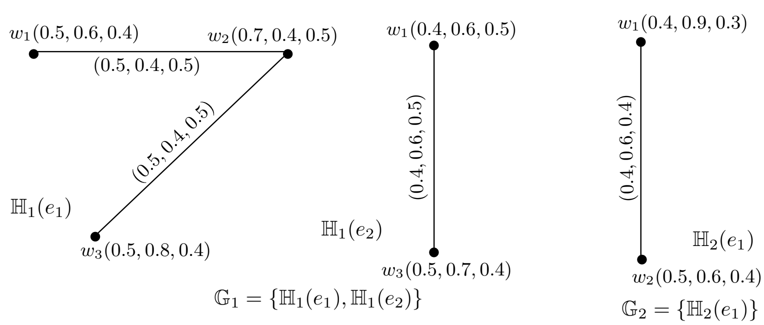


Figure 2.5: Strong INSGs $\mathbb{G}_1$ and $\mathbb{G}_2$.

Clearly, $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2)\}$ is not a strong INSG as shown in Figure. 2.6.

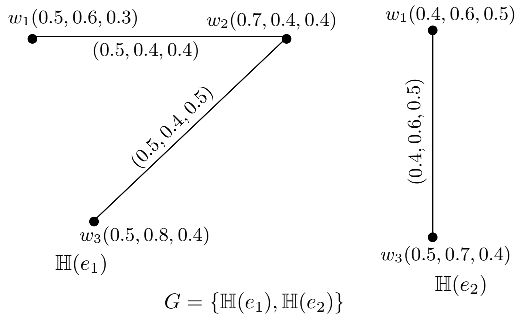


Figure 2.6: Union of two strong intuitionistic neutrosophic soft graphs.

**Proposition 2.23.** *If $\mathbb{G}_1 \times \mathbb{G}_2$ is strong INSG, then at least $\mathbb{G}_1$ or $\mathbb{G}_2$ must be strong INSG.*

**Proposition 2.24.** *If $\mathbb{G}_1[\mathbb{G}_2]$ is strong INSG, then at least $\mathbb{G}_1$ or $\mathbb{G}_2$ must be strong INSG.*

**Definition 2.25.** The complement of a strong INSG $\mathbb{G} = (F, K, N)$ is an INSG $G^c = (F^c, K^c, N^c)$ defined by

(i) $N^c = N$,

(ii) $F^c(e)(w) = F(e)(w)$ for all $e \in N$ and $w \in V$,

(iii) $T_{K^c(e)}(w, v) = \begin{cases} 0 & \text{if } T_{K(e)}(w, v) > 0, \\ \min\{T_{F(e)}(w), T_{F(e)}(v)\}, & \text{if } T_{K(e)}(w, v) = 0, \end{cases}$

$$I_{K^c(e)}(w, v) = \begin{cases} 0 & \text{if } I_{K(e)}(w, v) > 0, \\ \min\{I_{F(e)}(w), I_{F(e)}(v)\}, & \text{if } I_{K(e)}(w, v) = 0, \end{cases}$$

$$F_{K^c(e)}(w, v) = \begin{cases} 0 & \text{if } F_{K(e)}(w, v) > 0, \\ \max\{F_{F(e)}(w), F_{F(e)}(v)\}, & \text{if } F_{K(e)}(w, v) = 0, \end{cases}$$

**Proposition 2.26.** *If $\mathbb{G}$ is a strong INSG over $G^*$, then $\mathbb{G}^c$ is also a strong intuitionistic neutrosophic soft graph.*

**Theorem 2.27.** *If $\mathbb{G}$ and $\mathbb{G}^c$ are strong INSGs of $G^*$. Then $\mathbb{G} \cup \mathbb{G}^c$ is a complete intuitionistic neutrosophic soft graph.*

# 3 Isomorphism of intuitionistic neutrosophic soft graphs

**Definition 3.1.** Let $\mathbb{G}_1 = (F_1, K_1, N)$ and $\mathbb{G}_2 = (F_2, K_2, N)$ be two INSGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. A *homomorphism* $f_N : \mathbb{G}_1 \to \mathbb{G}_2$ is a mapping $f_N : V_1 \to V_2$ which satisfies the following conditions:

(i) $T_{F_1(e)}(v_1) \leq T_{F_2(e)}(f_e(v_1))$, $I_{F_1(e)}(v_1) \leq I_{F_2(e)}(f_e(v_1))$, $F_{F_1(e)}(v_1) \geq F_{F_2(e)}(f_e(v_1))$,

(ii) $T_{K_1(e)}(v_1v_2) \leq T_{K_2(e)}(f_e(v_1)f_e(v_2))$, $I_{K_1(e)}(v_1v_2) \leq I_{K_2(e)}(f_e(v_1)f_e(v_2))$, $F_{K_1(e)}(v_1v_2) \geq F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1 \in V_1, v_1v_2 \in E_1$.

A bijective homomorphism is called a *weak isomorphism* if

$$T_{F_1(e)}(v_1) = T_{F_2(e)}(f_e(v_1)), I_{F_1(e)}(v_1) = I_{F_2(e)}(f_e(v_1)), F_{F_1(e)}(v_1) = F_{F_2(e)}(f_e(v_1)), \forall e \in N, v_1 \in V_1.$$

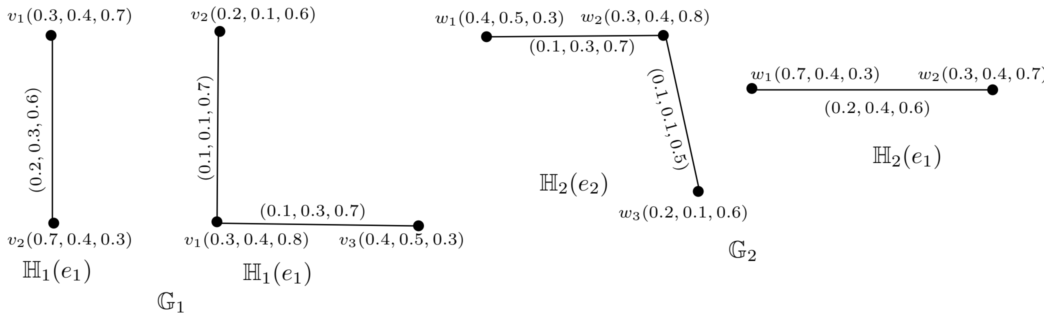


Figure 3.1: $\mathbb{G}_1 = \{\mathbb{H}_1(e_1), \mathbb{H}_1(e_2)\}$, and $\mathbb{G}_2 = \{\mathbb{H}_2(e_1), \mathbb{H}_2(e_2)\}$.

A bijective homomorphism $f_N : \mathbb{G}_1 \to \mathbb{G}_2$ such that

$$T_{K_1(e)}(v_1v_2) = T_{K_2(e)}(f_e(v_1)f_e(v_2)), I_{K_1(e)}(v_1v_2) = I_{K_2(e)}(f_e(v_1)f_e(v_2)), F_{K_1(e)}(v_1v_2) = F_{K_2(e)}(f_e(v_1)f_e(v_2)),$$

for all $e \in N, v_1v_2 \in E_1$ is called a *co-weak isomorphism*.
An *endomorphism* of INSG $\mathbb{G}$ with $V$ as the underlying set is a homomorphism of $\mathbb{G}$ into itself.

**Definition 3.2.** Let $\mathbb{G}_1 = (F_1, K_1, N)$ and $\mathbb{G}_2 = (F_2, K_2, N)$ be two INSGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. An *isomorphism* $f_N : \mathbb{G}_1 \to \mathbb{G}_2$ is a mapping $f_N : V_1 \to V_2$ which satisfies the following conditions:

(i) $T_{F_1(e)}(v_1) = T_{F_2(e)}(f_e(v_1))$, $I_{F_1(e)}(v_1) = I_{F_2(e)}(f_e(v_1))$, $F_{F_1(e)}(v_1) = F_{F_2(e)}(f_e(v_1))$,

(ii) $T_{K_1(e)}(v_1v_2) = T_{K_2(e)}(f_e(v_1)f_e(v_2)), I_{K_1(e)}(v_1v_2) = I_{K_2(e)}(f_e(v_1)f_e(v_2)), F_{K_1(e)}(v_1v_2) = F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1 \in V_1, v_1v_2 \in E_1$.

**Example 3.3.** Let $N = \{e_1, e_2\}$ be a parameter set. $\mathbb{G}_1 = (F_1, K_1, N)$ and $\mathbb{G}_2 = (F_1, K_2, N)$ be two INSGs defined as follows:

$\mathbb{G}_1 = \{\mathbb{H}_1(e_1), \mathbb{H}_1(e_2)\} = \{(\{(v_1, 0.3, 0.4, 0.7), (v_2, 0.7, 0.4, 0.3)\}, \{(v_1v_2, 0.2, 0.3, 0.6)\}), (\{(v_1, 0.3, 0.4, 0.8), (v_2, 0.2, 0.1, 0.6), (v_3, 0.4, 0.5, 0.3)\}, \{(v_1v_2, 0.1, 0.1, 0.7), (v_1v_3, 0.1, 0.3, 0.7)\})\}$,

$\mathbb{G}_2 = \{\mathbb{H}_2(e_1), \mathbb{H}_2(e_2)\} = \{(\{(w_1, 0.7, 0.4, 0.3), (w_2, 0.3, 0.4, 0.7)\}, \{(w_1w_2, 0.2, 0.4, 0.6)\}), (\{(w_1, 0.4, 0.5, 0.3), (w_2, 0.3, 0.4, 0.8), (w_3, 0.2, 0.1, 0.6)\}, \{(w_1w_2, 0.1, 0.3, 0.7), (w_2w_3, 0.1, 0.1, 0.5)\})\}$.

A mapping $f_N : V_1 \to V_2$ defined by $f_{e_1}(v_1) = w_2$, $f_{e_1}(v_2) = w_1$ and $f_{e_2}(v_1) = w_2$, $f_{e_2}(v_2) = w_3$, and $f_{e_2}(v_3) = w_1$, then $T_{F_1(e_1)}(v_1) = T_{F_2(e_1)}(w_2)$, $I_{F_1(e_1)}(v_1) = I_{F_2(e_1)}(w_2)$, $F_{F_1(e_1)}(v_1) = F_{F_2(e_1)}(w_2)$, and $T_{F_1(e_1)}(v_2) = T_{F_2(e_1)}(w_1)$, $I_{F_1(e_1)}(v_2) = I_{F_2(e_1)}(w_1)$, $F_{F_1(e_1)}(v_2) = F_{F_2(e_1)}(w_1)$, but $T_{K_1(e_1)}(v_1v_2) = T_{K_2(e_1)}(w_2w_1)$, $I_{K_1(e_1)}(v_1v_2) \neq I_{K_2(e_1)}(w_2w_1)$, $F_{K_1(e_1)}(v_1v_2) = F_{K_2(e_1)}(w_2w_1)$. Clearly, $\mathbb{H}_1(e_1)$ is weak isomorphic to $\mathbb{H}_2(e_1)$. By routine computation, we can see that $\mathbb{H}_1(e_2)$ is weak isomorphic to $\mathbb{H}_2(e_2)$. Hence $\mathbb{G}_1$ is weak isomorphic to $\mathbb{G}_2$ but not isomorphic as shown in Figure 3.1.

**Example 3.4.** Let $N = \{e_1, e_2\}$ be a parameter set. $\mathbb{G}_1 = (F_1, K_1, N)$ and $\mathbb{G}_2 = (F_1, K_2, N)$ be two INSGs as shown in Figure 3.2. A mapping $f_N : V_1 \to V_2$ defined by $f_{e_1}(w_1) = v_2$, $f_{e_1}(w_2) = v_1$, $f_{e_1}(w_3) = v_4$, $f_{e_1}(w_4) = v_3$ and $f_{e_2}(w_1) = v_1$, $f_{e_2}(w_2) = v_2$, and $f_{e_2}(w_3) = v_3$. By routine computations, we can see that $\mathbb{G}_1$ is co-weak isomorphic to $\mathbb{G}_2$ but not isomorphic as $T_{F_1(e_1)}(w_2) = T_{F_2(e_1)}(v_1)$, $I_{F_1(e_1)}(w_2) \neq I_{F_2(e_1)}(v_1)$, $F_{F_1(e_1)}(w_2) \neq F_{F_2(e_1)}(v_1)$ and $T_{F_1(e_2)}(w_3) \neq T_{F_2(e_2)}(v_3)$, $I_{F_1(e_2)}(w_3) \neq I_{F_2(e_2)}(v_3)$, $F_{F_1(e_2)}(w_3) \neq F_{F_2(e_2)}(v_3)$.

**Theorem 3.5.** *For any two isomorphic INSGs their order and size are same.*

**Definition 3.6.** Let $\mathbb{G}$ be an INSG with $V$ as the underlying set. A one-to-one, onto map $f_N : V \to V$ is an *automorphism* of $\mathbb{G}$ if

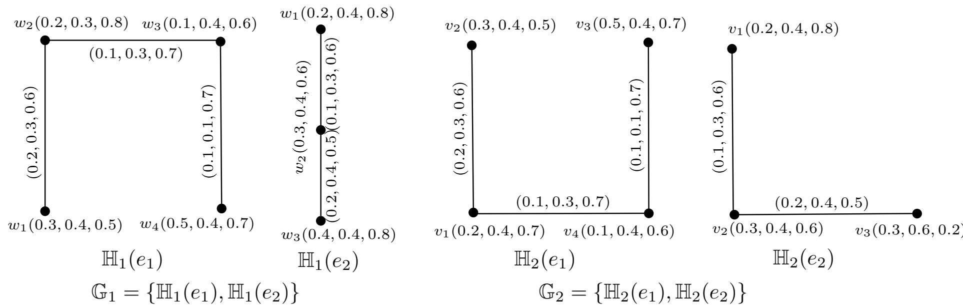


Figure 3.2: $\mathbb{G}_1 = \{\mathbb{H}_1(e_1), \mathbb{H}_1(e_2)\}$, and $\mathbb{G}_2 = \{\mathbb{H}_2(e_1), \mathbb{H}_2(e_2)$.

(i) $T_{F_1(e)}(v_1) = T_{F_2(e)}(f_e(v_1))$, $I_{F_1(e)}(v_1) = I_{F_2(e)}(f_e(v_1))$, $F_{F_1(e)}(v_1) = F_{F_2(e)}(f_e(v_1))$,

(ii) $T_{K_1(e)}(v_1v_2) = T_{K_2(e)}(f_e(v_1)f_e(v_2))$, $I_{K_1(e)}(v_1v_2) = I_{K_2(e)}(f_e(v_1)f_e(v_2))$, $F_{K_1(e)}(v_1v_2) = F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1, v_2 \in V$.

**Definition 3.7.** An INSG $\mathbb{G} = (F, K, N)$ of $G^* = (V, E)$ is an *ordered intuitionistic neutrosophic soft graph* if satisfies the following condition:

$$T_{F(e)}(v_1) \leq T_{F(e)}(v_2), I_{F(e)}(v_1) \leq I_{F(e)}(v_2), F_{F(e)}(v_1) \geq F_{F(e)}(v_2),$$

$$T_{F(e)}(w_1) \leq T_{F(e)}(w_2), I_{F(e)}(w_1) \leq I_{F(e)}(w_2), F_{F(e)}(w_1) \geq F_{F(e)}(w_2),$$

for $v_1, v_2, w_1, w_2 \in V, v_1 \neq w_1,\ v_2 \neq w_2$, for all $e \in N$, imply

$$T_{K(e)}(v_1w_1) \leq T_{K(e)}(v_2w_2), I_{K(e)}(v_1w_1) \leq I_{K(e)}(v_2w_2), F_{K(e)}(v_1w_1) \geq F_{K(e)}(v_2w_2)\ .$$

**Proposition 3.8.** *Let $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_3$ are INSGs. Then the isomorphism between these intuitionistic neutrosophic soft graphs is an equivalence relation.*

*Proof.* Let $\mathbb{G}_1 = (F_1, K_1, N)$, $\mathbb{G}_2 = (F_2, K_2, N)$, and $\mathbb{G}_3 = (F_3, K_3, N)$ are three INSGs with the underlying sets $V_1$, $V_2$ and $V_3$, respectively.

(1) Reflexive: Consider identity mapping $f_N : V_1 \to V_1$, $f_e(v) = v$ for all $v \in V_1$, satisfying
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v))$, $I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v))$, $F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$. Hence $f_N$ is an isomorphism of intuitionistic neutrosophic soft graph to itself.

(2) Symmetric: Let $f_N : V_1 \to V_2$ be an isomorphism of $\mathbb{G}_1$ onto $\mathbb{G}_2$, $f_e(v) = v'$ for all $v \in V_1$, such that
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v))$, $I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v))$, $F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$.
As $f_N$ is a bijective mapping, $f^{-1}(v') = v$ for all $v' \in V_2$, then
$T_{F_2(e)}(v') = T_{F_1(e)}(f^{-1}(v'))$, $I_{F_2(e)}(v') = I_{F_1(e)}(f^{-1}(v'))$, $F_{F_2(e)}(v') = F_{F_1(e)}(f^{-1}(v'))$,
$T_{K_2(e)}(u'v') = T_{K_1(e)}(f^{-1}(u')f^{-1}(v'))$, $I_{K_2(e)}(u'v') = I_{K_1(e)}(f^{-1}(u')f^{-1}(v'))$,
$F_{K_2(e)}(u'v') = F_{K_1(e)}(f^{-1}(u')f^{-1}(v'))$ for all $u', v' \in V_2, e \in N$.
Hence $f^{-1} : V_2 \to V_1$ is an isomorphism from $\mathbb{G}_2$ to $\mathbb{G}_1$, that is $\mathbb{G}_1 \cong \mathbb{G}_2$ implies $\mathbb{G}_2 \cong \mathbb{G}_1$.

(3) Transitive: Let $f_N : V_1 \to V_2$ and $g_N : V_2 \to V_3$ are isomorphisms of the intuitionistic neutrosophic soft graphs $\mathbb{G}_1$ onto $\mathbb{G}_2$ and $\mathbb{G}_2$ onto $\mathbb{G}_3$, respectively. For transitive relation we consider a bijective

mapping $g_N \circ f_N : V_1 \to V_3$ such that $(g_N \circ f_N)(u) = g_e(f_e(u))$ for all $u \in V_1$.
As $f_N : V_1 \to V_2$ is an isomorphism from $\mathbb{G}_1$ onto $\mathbb{G}_2$, such that $f_e(v) = v'$ for all $v \in V_1$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v')$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v')$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v')$, and
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v')$, $I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v')$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v')$, for all $u, v \in V_1, e \in N$.

As $g_N : V_2 \to V_3$ is an isomorphism from $\mathbb{G}_2$ onto $\mathbb{G}_3$ such that $g_e(v') = v''$ for all $v' \in V_2$, then

$T_{F_2(e)}(v') = T_{F_3(e)}(g_e(v')) = T_{F_2(e)}(v'')$, $I_{F_2(e)}(v') = I_{F_3(e)}(g_e(v')) = I_{F_3(e)}(v'')$,
$F_{F_2(e)}(v') = F_{F_3(e)}(g_e(v')) = F_{F_3(e)}(v'')$, and
$T_{K_2(e)}(u'v') = T_{K_3(e)}(g_e(u')g_e(v')) = T_{K_3(e)}(u''v'')$, $I_{K_2(e)}(u'v') = I_{K_3(e)}(g_e(u')g_e(v')) = I_{K_2(e)}(u''v'')$,
$F_{K_2(e)}(u'v') = F_{K_3(e)}(g_e(u')g_e(v')) = F_{K_3(e)}(u''v'')$, for all $u', v' \in V_2, e \in N$.

For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v') = T_{F_3(e)}(g_e(f_e(v)))$,
$I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v') = I_{F_3(e)}(g_e(f_e(v)))$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v') = F_{F_3(e)}(g_e(f_e(v)))$, and
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v') = T_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v') = I_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v') = F_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$ for all $u, v \in V_1, e \in N$.
Therefore $g_N \circ f_N$ is an isomorphism between $\mathbb{G}_1$ and $\mathbb{G}_3$.

Hence isomorphism between INSGs by (1), (2) and (3) is an equivalence relation. □

**Proposition 3.9.** *Let $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_3$ are INSGs. Then the weak isomorphism between these INGs is a partial order relation*

*Proof.* Let $\mathbb{G}_1 = (F_1, K_1, N)$, $\mathbb{G}_2 = (F_2, K_2, N)$, and $\mathbb{G}_3 = (F_3, K_3, N)$ are three INSGs with the underlying sets $V_1$, $V_2$ and $V_3$, respectively.

(1) Reflexive: Consider identity mapping $f_N : V_1 \to V_1$, $f_e(v) = v$ for all $v \in V_1$, satisfying
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v))$, $I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v))$, $F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$. Hence $f_N$ is a weak isomorphism of intuitionistic neutrosophic soft graph to itself. Thus $\mathbb{G}_1$ is a weak isomorphic to itself.

(2) Anti symmetric: Let $f_N : V_1 \to V_2$ be an isomorphism of $\mathbb{G}_1$ onto $\mathbb{G}_2$, $f_e(v) = v'$ for all $v \in V_1$, such that
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,
$T_{K_1(e)}(uv) \leq T_{K_2(e)}(f_e(u)f_e(v))$, $I_{K_1(e)}(uv) \leq I_{K_2(e)}(f_e(u)f_e(v))$, $F_{K_1(e)}(uv) \geq F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$.
Let $g_N : V_2 \to V_1$ be an isomorphism of $\mathbb{G}_2$ onto $\mathbb{G}_1$, $g_e(v') = v$ for all $v' \in V_2$, such that
$T_{F_2(e)}(v') = T_{F_1(e)}(g_e(v'))$, $I_{F_2(e)}(v') = I_{F_1(e)}(g_e(v'))$, $F_{F_2(e)}(v') = F_{F_2(e)}(g_e(v'))$,
$T_{K_2(e)}(u'v') \leq T_{K_1(e)}(g_e(u')g_e(v'))$, $I_{K_2(e)}(u'v') \leq I_{K_1(e)}(g_e(u')g_e(v'))$, $F_{K_2(e)}(u'v') \geq F_{K_1(e)}(g_e(u')g_e(v'))$,
for all $u', v' \in V_2, e \in N$.
Both weak isomorphisms $f_N$ from $\mathbb{G}_1$ onto $\mathbb{G}_2$ and $g_N$ from $\mathbb{G}_2$ onto $\mathbb{G}_3$, are holds when $\mathbb{G}_1$ and $\mathbb{G}_2$ have same number of edges and the corresponding edges have same truth-membership degree, indeterminacy-membership degree and falsity-membership degree corresponding to the parameter to the set of parameters. Hence $\mathbb{G}_1$ and $\mathbb{G}_2$ are identical.

(3) Transitive: Let $f_N : V_1 \to V_2$ and $g_N : V_2 \to V_3$ are weak isomorphisms of the intuitionistic neutrosophic soft graphs $\mathbb{G}_1$ onto $\mathbb{G}_2$ and $\mathbb{G}_2$ onto $\mathbb{G}_3$, respectively. For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$ such that $(g_N \circ f_N)(u) = g_e(f_e(u))$ for all $u \in V_1$.
As $f_N : V_1 \to V_2$ is a weak isomorphism from $\mathbb{G}_1$ onto $\mathbb{G}_2$, such that $f_e(v) = v'$ for all $v \in V_1$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v')$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v')$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v')$, and
$T_{K_1(e)}(uv) \le T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v')$, $I_{K_1(e)}(uv) \le I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v')$,
$F_{K_1(e)}(uv) \ge F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v')$, for all $u, v \in V_1, e \in N$.

As $g_N : V_2 \to V_3$ is an isomorphism from $\mathbb{G}_2$ onto $\mathbb{G}_3$ such that $g_e(v') = v''$ for all $v' \in V_2$, then

$T_{F_2(e)}(v') = T_{F_3(e)}(g_e(v')) = T_{F_3(e)}(v'')$, $I_{F_2(e)}(v') = I_{F_3(e)}(g_e(v')) = I_{F_3(e)}(v'')$,
$F_{F_2(e)}(v') = F_{F_3(e)}(g_e(v')) = F_{F_3(e)}(v'')$, and
$T_{K_2(e)}(u'v') \le T_{K_3(e)}(g_e(u')g_e(v')) = T_{K_3(e)}(u''v'')$, $I_{K_2(e)}(u'v') \le I_{K_3(e)}(g_e(u')g_e(v')) = I_{K_3(e)}(u''v'')$,
$F_{K_2(e)}(u'v') \ge F_{K_3(e)}(g_e(u')g_e(v')) = F_{K_3(e)}(u''v'')$, for all $u', v' \in V_2, e \in N$.

For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v') = T_{F_3(e)}(g_e(f_e(v)))$,
$I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v') = I_{F_3(e)}(g_e(f_e(v)))$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v') = F_{F_3(e)}(g_e(f_e(v)))$, and
$T_{K_1(e)}(uv) \le T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v') \le T_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$I_{K_1(e)}(uv) \le I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v') \le I_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$F_{K_1(e)}(uv) \ge F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v') \ge F_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$ for all $u, v \in V_1, e \in N$.
Therefore $g_N \circ f_N$ is a weak isomorphism between $\mathbb{G}_1$ and $\mathbb{G}_3$, i.e., weak isomorphism satisfying transitivity.

Hence isomorphism between INSGs by (1), (2) and (3) is a partial order relation. □

**Definition 3.10.** An INSG $\mathbb{G}$ is *self complementary* if $\mathbb{G} \approx G^c$.

**Proposition 3.11.** *Let $\mathbb{G}_1$ and $\mathbb{G}_2$ are INSGs. Then $\mathbb{G}_1 \cong \mathbb{G}_2$ if and only if $\mathbb{G}_1^c \cong \mathbb{G}_2^c$.*

*Proof.* Let $\mathbb{G}_1$, $\mathbb{G}_2$ be the two INSGs. Suppose that $\mathbb{G}_1 \cong \mathbb{G}_2$, then there exist a bijective mapping $f_N : V_1 \to V_2$ such that $f_e(v) = v'$ for all $v \in V_1$, $T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)), I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)), F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$, and $T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)), I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v))$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$, for all $u, v \in V_1, e \in N$. By the definition of complement of INSGs

$$\begin{aligned}
T^c_{K_1(e)}(uv) &= T_{F_1(e)}(u) \wedge T_{F_1(e)}(v) - T_{K_1(e)}(uv),\\
&= T_{F_2(e)}(f_e(u)) \wedge T_{F_2(e)}(f_e(v)) - T_{K_2(e)}(f_e(u)f_e(v))\\
&= T^c_{K_2(e)}(f_e(u)f_e(v)),\\
I^c_{K_1(e)}(uv) &= I_{F_1(e)}(u) \wedge I_{F_1(e)}(v) - I_{K_1(e)}(uv),\\
&= I_{F_2(e)}(f_e(u)) \wedge T_{F_2(e)}(f_e(v)) - I_{K_2(e)}(f_e(u)f_e(v))\\
&= I^c_{K_2(e)}(f_e(u)f_e(v)),\\
F^c_{K_1(e)}(uv) &= F_{F_1(e)}(u) \vee F_{F_1(e)}(v) - F_{K_1(e)}(uv),\\
&= F_{F_2(e)}(f_e(u)) \wedge F_{F_2(e)}(f_e(v)) - F_{K_2(e)}(f_e(u)f_e(v))\\
&= F^c_{K_2(e)}(f_e(u)f_e(v))
\end{aligned}$$

Hence $\mathbb{G}_1^c \cong G_2^c$.
Conversely, assume that $\mathbb{G}_1^c \cong G_2^c$, then there exist an isomorphism $g_N : V_1 \to V_2$ such that $g_e(v) = v'$, $T_{F_1(e)}(v) = T_{F_2(e)}(g_e(v)), I_{F_1(e)}(v) = I_{F_2(e)}(g_e(v)), F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$, for all $v \in V_1, e \in N$,$T^c_{K_1(e)}(uv) = T^c_{K_2(e)}(g_e(u)g_e(v)), I^c_{K_1(e)}(uv) = I^c_{K_2(e)}(g_e(u)g_e(v)), F^c_{K_1(e)}(uv) = F^c_{K_2(e)}(g_e(u)g_e(v))$, for all $u, v \in V_1, e \in N$.
By using the definition of complement of intuitionistic neutrosophic soft graph

$$\begin{aligned}
T^c_{K_1(e)}(uv) &= T^c_{F_1(e)}(u) \wedge T^c_{F_1(e)}(v) - T_{K_1(e)}(uv),\\
T^c_{K_2(e)}(g_e(u)g_e(v)) &= T^c_{F_2(e)}(g_e(u)) \wedge T^c_{F_2(e)}(g_e(v)) - T_{K_2(e)}(g_e(u)g_e(v)),\\
I^c_{K_1(e)}(uv) &= I^c_{F_1(e)}(u) \wedge I^c_{F_1(e)}(v) - I_{K_1(e)}(uv),\\
I^c_{K_2(e)}(g_e(u)g_e(v)) &= I^c_{F_2(e)}(g_e(u)) \wedge I^c_{F_2(e)}(g_e(v)) - I_{K_2(e)}(g_e(u)g_e(v)),\\
F^c_{K_1(e)}(uv) &= F^c_{F_1(e)}(u) \vee F^c_{F_1(e)}(v) - F_{K_1(e)}(uv),\\
F^c_{K_2(e)}(g_e(u)g_e(v)) &= F^c_{F_2(e)}(g_e(u)) \vee F^c_{F_2(e)}(g_e(v)) - F_{K_2(e)}(g_e(u)g_e(v)).
\end{aligned}$$

As $T^c_{K_1(e)}(uv) = T^c_{K_2(e)}(g_e(u)g_e(v)), I^c_{K_1(e)}(uv) = I^c_{K_2(e)}(g_e(u)g_e(v)), F^c_{K_1(e)}(uv) = F^c_{K_2(e)}(g_e(u)g_e(v))$, for all $u, v \in V_1, e \in N$, $g_N : V_1 \to V_2$ is an isomorphism between $\mathbb{G}_1$ and $\mathbb{G}_2$, that is $\mathbb{G}_1 \cong \mathbb{G}_2$. □

**Proposition 3.12.** *If $\mathbb{G}_1$ is co-weak isomorphic to $\mathbb{G}_2$, then there can be a homomorphism between $\mathbb{G}_1^c$ and $\mathbb{G}_2^c$.*

**Proposition 3.13.** *If $\mathbb{G}_1$ is weak isomorphic to $\mathbb{G}_2$, then $\mathbb{G}_1^c$ and $\mathbb{G}_2^c$ are weak isomorphic intuitionistic neutrosophic soft graphs.*

# 4 Applications

Intuitionistic neutrosophic soft graph has several applications in decision making problems and used to deal with uncertainties from our different daily life problems. In this section we apply the concept of INSSs in a decision making problems. Many practical problems can be represented by graphs. We present an application of INSG to a multiple criteria decision-making problem. We present an algorithm for most appropriate selection of an object in a multiple criteria decision-making problem.

**Algorithm 4.1.**

1. Input the set of parameters $e_1, e_2, \ldots, e_k$ .
2. Input the INSSs $(F, N)$ and $(K, N)$.
3. Input the INGs $\mathbb{H}(e_1), \mathbb{H}(e_2), \ldots, \mathbb{H}(e_k)$.
4. Calculate the score values of INGs $\mathbb{H}(e_1), \mathbb{H}(e_2), \ldots, \mathbb{H}(e_k)$ using formula
$$S_{ij} := \sqrt{(T_j)^2 + (I_j)^2 + (1 - F_j)^2} \tag{4.1}$$
Tabular representation of score values of INGs $\mathbb{H}(e_k),\ \forall\ k$.
5. Compute the choice values of $C_p = \sum_j S_{ij}$ for all $i = 1, 2, \ldots, n$ and $p = 1, 2, \ldots, k$.
6. The decision is $S_i$ if $S_i = \max_{i=1}^{n}\{\min_{p=1}^{k} C_p\}$.
7. If $i$ has more than one value then any one of $S_i$ may be chosen.

An algorithm for the selection of optimal object based upon given set of information.

1. An appropriate selection of a machine for a specific task is an important decision-making problem for a machine manufacturing corporation. The performance of a manufacturing corporation is badly effected by the wrong selection. The main purpose in machine selection is that machine will achieve the require tasks within possible short time and minimum cost. The main purpose is select the machine that will complete the required task within the time available for the lowest possible cost. Rate of productivity, automatic system and price are important aspects consider in selection of a machine. The rate of productivity, value of product and charge of manufacturing depends upon the performance of machine. Mr. X should be an expert or at least familiar with the machine properties, to select a best machine among the parameters (alternatives), i.e., “price”, “rate of productivity” and “automatic system”. Let $V = \{m_1, m_2, m_3, m_4, m_5, m_6\}$, set of six machines to be consider as the universal set and $N = \{e_1, e_2, e_3\}$ be the set of parameters that characterize the machine, the parameters $e_1$, $e_2$ and $e_3$ stands for “price”, “rate of productivity” and “automatic system”, respectively. Consider the INSS $(F, N)$ over $V$ which define the “efficiency of machines” corresponding to the given parameters that Mr. X want to select. $(K, N)$ is an INSS over $E = \{m_1m_2, m_2m_3, m_6m_1, m_1m_3, m_1m_4, m_1m_5, m_2m_4, m_2m_5, m_2m_6, m_3m_4, m_3m_5, m_3m_6, m_4m_5, m_4m_6, m_5m_6\}$ define degree of truth membership, degree of indeterminacy, and degree of falsity membership of the connection between two machines corresponding to the selected attributes $e_1$, $e_2$ and $e_3$. The INGs $\mathbb{H}(e_1)$, $\mathbb{H}(e_2)$ and $\mathbb{H}(e_3)$ of INSG $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2), \mathbb{H}(e_3)\}$ corresponding to the parameters “price”, “machinery size” and “automatic system”, respectively are shown in Figure 4.1.

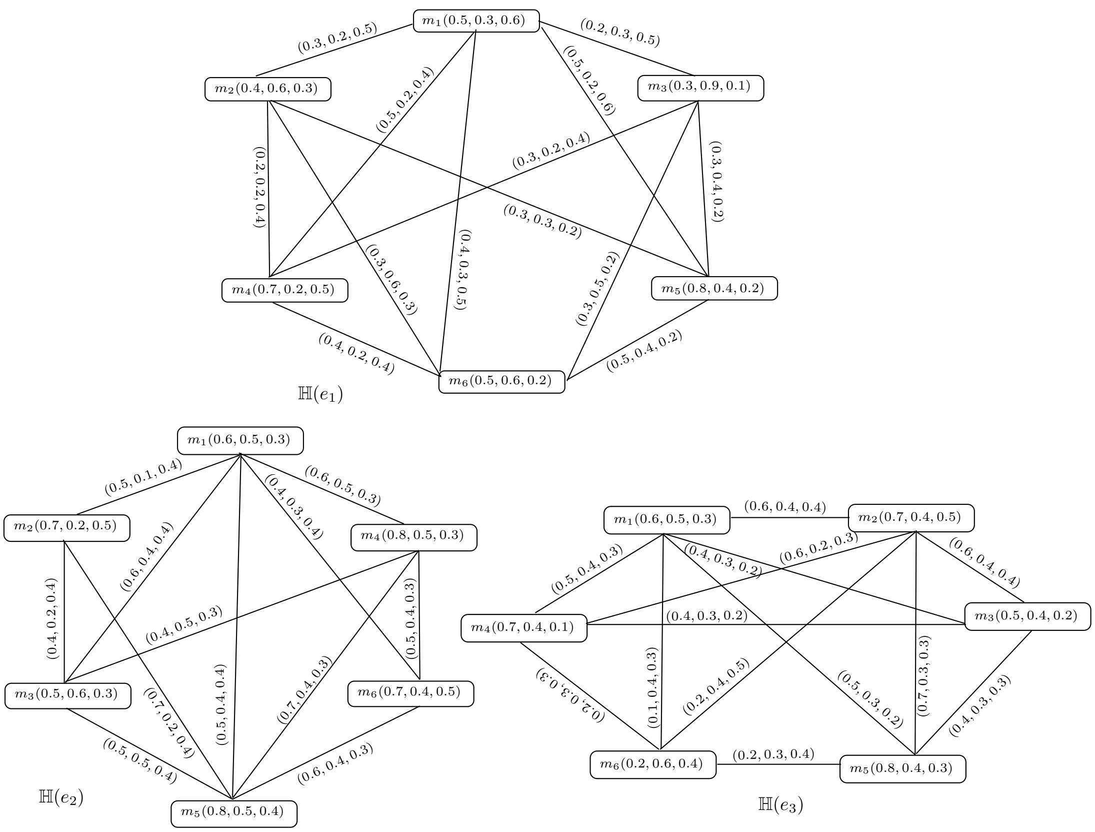


Figure 4.1: Intuitionistic neutrosophic soft graph $\mathbb{G} = \{\mathbb{H}(e_1), \mathbb{H}(e_2), \mathbb{H}(e_3)\}$

Tabular representation of score values of INGs $\mathbb{H}(e_1)$, $\mathbb{H}(e_2)$, and $\mathbb{H}(e_3)$ with normalized score function $S_{ij} = \sqrt{(T_j)^2 + (I_j)^2 + (1 - F_j)^2}$ and choice value for each machine $m_i$ for $i = 1, 2, 3, 4, 5, 6$.

Table 2: Tabular representation of score values and choice values of $\mathbb{H}(e_1)$.

| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $\acute{m}_k$ |
|---|---|---|---|---|---|---|---|
| $m_1$ | 0 | 0.62 | 0.62 | 0.80 | 0.67 | 0.71 | 3.42 |
| $m_2$ | 0.62 | 0 | 0 | 0.66 | 0.91 | 0.97 | 3.16 |
| $m_3$ | 0.62 | 0 | 0 | 0.70 | 0.94 | 0.99 | 3.25 |
| $m_4$ | 0.80 | 0.66 | 0.70 | 0 | 0 | 0.75 | 2.91 |
| $m_5$ | 0.67 | 0.91 | 0.94 | 0 | 0 | 1.0 | 3.52 |
| $m_6$ | 0.71 | 0.97 | 0.94 | 0.75 | 1.0 | 0 | 4.37 |

Table 3: Tabular representation of score values and choice values of $\mathbb{H}(e_2)$.

| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $\acute{m}_k$ |
|---|---|---|---|---|---|---|---|
| $m_1$ | 0 | 0.79 | 0.94 | 1.0 | 0.88 | 0.78 | 4.39 |
| $m_2$ | 0.79 | 0 | 0.75 | 0 | 0.94 | 0 | 2.48 |
| $m_3$ | 0.94 | 0.75 | 0 | 0.95 | 0.93 | 0 | 3.57 |
| $m_4$ | 1.0 | 0 | 0.95 | 0 | 1.0 | 0.95 | 3.9 |
| $m_5$ | 0.88 | 0.94 | 0.93 | 1.0 | 0 | 1.0 | 4.75 |
| $m_6$ | 0.78 | 0 | 0 | 0.95 | 1.0 | 0 | 2.73 |

Table 4: Tabular representation of score values and choice values of $\mathbb{H}(e_3)$.

| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $\acute{m}_k$ |
|---|---|---|---|---|---|---|---|
| $m_1$ | 0 | 0.94 | 0.94 | 0.95 | 0.99 | 0.81 | 4.63 |
| $m_2$ | 0.94 | 0 | 0.94 | 0.94 | 1.0 | 0.67 | 4.49 |
| $m_3$ | 0.94 | 0.94 | 0 | 0.94 | 0.86 | 0 | 3.68 |
| $m_4$ | 0.95 | 0.94 | 0.94 | 0 | 0 | 0.79 | 3.62 |
| $m_5$ | 0.99 | 1.0 | 0.86 | 0 | 0 | 0.70 | 3.55 |
| $m_6$ | 0.81 | 0.67 | 0 | 0.79 | 0.70 | 0 | 2.97 |

The decision is $S_i$ if $S_i = \max_{i=1}^{6}\{\min_{p=1}^{3} m_p\} = \max_{i=1}^{6}\{3.42, 2.48, 3.25, 2.91, 3.52, 2.73\} = 3.52$. Clearly, the maximum score value is 3.52, scored by the $m_5$. Mr. X will buy the machine $m_5$.

2. We present a multi-criteria decision making problem for product marketing if there are multiple brands of a product, product marketing has intuitionistic neutrosophic behaviour. Consider Mr. X who is a retail owner wants to maximize his profit by selling some electronic items which meets all the requirements set by a retail outlet owner. Let $V = \{S_1, S_2, S_3, S_4, S_5\}$ be a set of five brands of an item to be sold in an international market, and let $N = \{e_1 =$"price", $e_2 =$"quality"$\}$ be a set of parametric factors in product marketing. Let $(F, N)$ be the INSS over $V$, which describe the effectiveness of the brands, $T_{F(e_k)}(S_i)$, $T_{F(e_k)}(S_i)$, and $T_{F(e_k)}(S_i)$, for $i = 1, 2, \ldots, 5, k = 1, 2$ represents the degree of membership (goodness), degree of indeterminacy and degree of non-membership (poorness) of the brands corresponding to the parameters $e_1 =$"price" and $e_2 =$"quality", respectively and $(K, N)$ be the INSS on $E = \{S_1S_2, S_1S_4, S_1S_3, S_2S_3, S_3S_4, S_2S_5, S_3S_5, S_1S_5, S_4S_5\}$ describe the relationship between brands corresponding to the parameters $e_1 =$"price" and $e_2 =$"quality". The INSG is shown in Figure 4.2. The method for selection of brand in product marketing is presented in Algorithm 4.2.

**Algorithm 4.2.**

(a) Input the set of parameters $e_1, e_2, \dots, e_k$ .

(b) Input the INSSs $(F, N)$ and $(K, N)$.

(c) Construct ING $\mathbb{H}(e_1) \cap \mathbb{H}(e_2) \cap \dots \cap \mathbb{H}(e_k)$.

(d) Calculate the average score values of INGs $\mathbb{H}(e)$ using formula

$$\zeta_{ij} := \frac{T_{j\,F(e)} + I_{j\,F(e)} + 1 - F_{j\,F(e)}}{3}, \tag{4.2}$$

Tabular representation of score values of INGs $\mathbb{H}(e)$.

(e) Compute the choice values of $C_i = \sum_j \zeta_{ij}$ for all $i = 1, 2, \dots, n$.

(f) The decision is $S_i$ if $S_i = \max_{i=1}^{n} C_i$.

(g) If $i$ has more than one value then any one of $S_i$ may be chosen.

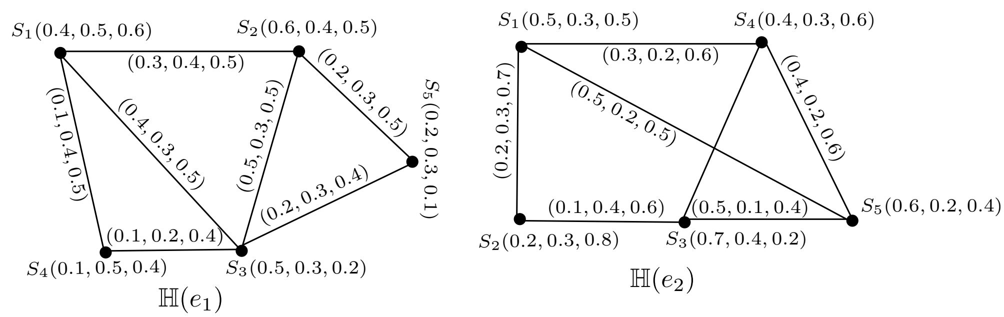


Figure 4.2: Intuitionistic neutrosophic soft graph.

The ING $\mathbb{H}(e_1) \cap \mathbb{H}(e_2)$ is shown in Figure 4.3. and tabular representation of average score values of ING is shown in Table 5.

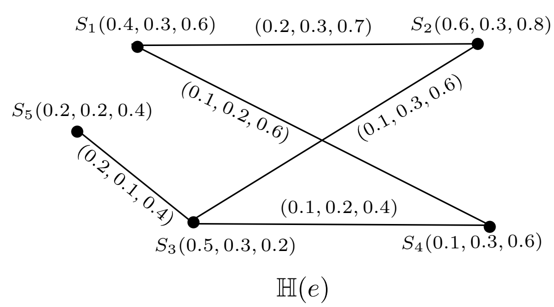


Figure 4.3: $\mathbb{H}(e_1) \cap \mathbb{H}(e_2)$

Table 5: Tabular representation of score values with choice values.

| | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $\acute{C}_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 0 | 0.27 | 0 | 0.23 | 0 | 0.5 |
| $S_2$ | 0.27 | 0 | 0.27 | 04 | 0 | 0.54 |
| $S_3$ | 0 | 0.27 | 0 | 0.30 | 0.30 | 0.87 |
| $S_4$ | 0.23 | 0 | 0.30 | 0 | 0 | 0.53 |
| $S_5$ | 0 | 0 | 0.30 | 0 | 0 | 0.30 |

Clearly, the maximum score value is 0.87, scored by the $S_3$. Mr. X will choose the brand $S_3$.

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