Relations on neutrosophic multi sets with properties

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Abstract

In this paper, we first give the cartesian product of two neutrosophic multi sets(NMS). Then, we define relations on neutrosophic multi sets to extend the intuitionistic fuzzy multi relations to neutrosophic multi relations. The relations allows to compose two neutrosophic sets. Also, various properties like reflexivity, symmetry and transitivity are studied.

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1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness such as probability set theory, fuzzy set theory[51], intuitionistic fuzzy set theory [7], rough set theory[44] etc. These theories are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. But, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, inspired from the sport games (wining/tie/defeating), from votes (yes/ NA/ No), from decision making (making a decision/ hesitating/not making) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, Smarandache[41] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. This theory and their hybrid structures has proven useful in many different fields such as control theory[1], databases[4, 5], medical diagnosis problem[2], decision making problem [16, 24], physics[31], topology [25], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [3, 6, 11, 46]).

After Molodotsov[30] proposed the theory of soft set combining fuzzy, intuitionistic fuzzy set models with other mathematical models has attracted the attention of many researchers (e.g. [23, 28, 48]. Also, Maji et al.[26] presented the concept of neutrosophic soft sets which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache[9, 12] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets and soft sets. The works on neutrosophic sets combining soft sets, in theories and applications, have been progressing rapidly (e.g. [10, 13, 14, 20, 21, 27]).

The notion of multisets was formulated first in [47] by Yager as generalization of the concept of set theory and then the multisets developed in [15] by Calude et al.. Several authors from time to time made a number of generalization of set theory. For example, Sebastian and Ramakrishnan[38, 39] introduced a new notion called multi fuzzy sets, which is a generalization of multiset. Since then, several researcher[29, 37, 43, 45] discussed more properties on multi fuzzy set. [22, 40] made an extension of the concept of fuzzy multisets by an intuitionstic fuzzy set, which called intuitionstic fuzzy multisets(IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researchers [18, 32, 33, 34, 35, 36]. An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multisets allows the repeated occurrences of membership and non–membership values. The concepts of FMS and IFMS fails to deal with indeterminatcy. In 2013 Smarandache [42] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively T_1, T_2, ..., T_m, and I_1, I_2, ..., I_p, and F_1, F_2, ..., F_r. Recently, Ye et al. [49], Ye and Ye [50] and Chatterjee et al.[17] presented single valued neutrosophic multi sets in detail. The concept of neutrosophic multi set (NMS)is a generalisation of fuzzy multisets and intuitionistic fuzzy multi sets.

The purpose of this paper is an attempt to extend the neutrosophic relations to neutrosophic multi relations(NMR). This paper is arranged in the following manner. In section 2, we present the basic definitions and results of neutrosophic set theory and neutrosophic multi(or refined) set theory that are useful for subsequent discussions. In section 3, we study the concept of neutrosophic multi relations and their operations. Finally, we conclude the paper.

2 Preliminary

In this section, we present the basic definitions and results of neutrosophic set theory [41, 46] and neutrosophic multi(or refined) set theory [19] that are useful for subsequent discussions. See especially [4, 5, 2, 3, 6, 11, 16, 19, 20, 24, 25, 31] for further details and background.

Smarandache[42] refine T , I, F to T_1, T_2,..., T_m and I_1, I_2,..., I_p and F_1, F_2,..., F_r where all T_m, I_p and F_r can be subset of [0,1]. In the following sections, we considered only the case when T ,I and F are split into the same number of subcomponents 1,2,...p, and T_A^j I_A^j,F_A^j are single valued neutrosophic number.

Definition 2.1 *[41] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set(N-set) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of* $]^-0, 1^+[$.

It can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in U, T_A(u), I_A(x), F_A(x) \subseteq [0, 1]\}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $^-0 \leq supT_A(u)+supI_A(u)+supF_A(u) \leq 3^+$.

Here, $1^+ = 1+\varepsilon$, *where 1 is its standard part and ε its non-standard part. Similarly,* $^-0 = 1+\varepsilon$, *where 0 is its standard part and ε its non-standard part.*

For application in real scientific and engineering areas,Wang et al.[46] proposed the concept of an SVNS, which is an instance of neutrosophic set. In the following, we introduce the definition of SVNS.

Definition 2.2 *[46] Let U be a space of points (objects), with a generic element in U denoted by u. An SVNS A inX is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ belongs to [0,1] for each point u in U. Then, an SVNS A can be expressed as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq supT_A(x)+supI_A(x)+supF_A(x) \leq 3$.

Definition 2.3 *[49] Let E be a universe. A neutrosophic multi set (NMS or Nm-set) A on E can be defined as follows:*

$$A = \{< x, (T_A^1(x), T_A^2(x), ..., T_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), (F_A^1(x), F_A^2(x), ..., F_A^P(x)) >: x \in E\}$$

where, $T_A^1(x), T_A^2(x), ..., T_A^P(x) : E \to [0,1]$, $I_A^1(x), I_A^2(x), ..., I_A^P(x) : E \to [0,1]$ *and* $F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \to [0,1]$ *such that* $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3 (i = 1, 2, ..., P)$ *and* $T_A^1(x) \le T_A^2(x) \le ... \le T_A^P(x)$ *for any* $x \in E$. $(T_A^1(x), T_A^2(x), ..., T_A^P(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^P(x))$ *and* $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$ *is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension of NMS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order. The Cardinality of the membership function Tc(x) ,the indterminacy function Ic(x) and non-membership Fc(x) is the the lenght of an element x in a NMs A denoted as P(A), defined as*

$$P(A) = |Tc(x)| = |Ic(x)| = |Fc(x)|$$

if A,B,C are the NMS defined on E, then their cardinality is

$$P = Max[P(A), P(B), P(C)\}.$$

set of all Neutrosophic multi sets on E is denoted by NMS(E).

Definition 2.4 *[49] Let* $A, B \in NMS(E)$. *Then,*

1. *A is said to be NM subset of B is denoted by* $A \tilde{\subseteq} B$ *if* $T_A^i(x) \le T_B^i(x)$, $I_A^i(x) \ge I_B^i(x)$,$F_A^i(x) \ge F_B^i(x)$, $\forall x \in E$.
2. *A is said to be neutrosophic equal of B is denoted by* $A = B$ *if* $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$, $F_A^i(x) = F_B^i(x)$, $\forall x \in E$.
3. *the complement of A denoted by* $A^{\tilde{c}}$ *and is defined by*

$$A^{\tilde{c}} = \{< x, (F_A^1(x), F_A^2(x), ..., F_A^P(x)), (1 - I_A^1(x), 1 - I_A^2(x), ..., 1 - I_A^P(x)), (T_A^1(x), T_A^2(x), ..., T_A^P(x)) >: x \in E\}$$

Definition 2.5 *[49] Let* $A, B \in NMS(E)$. *Then,*

1. *The union of A and B is denoted by* $A \tilde{\cup} B = C$ *and is defined by*

$$C = \{< x, (T_C^1(x), T_C^2(x), ..., T_C^P(x)), (I_C^1(x), I_C^2(x), ..., I_C^P(x)), (F_C^1(x), F_C^2(x), ..., F_C^P(x)) >: x \in E\}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$,$F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

2. *The intersection of A and B is denoted by* $A \tilde{\cap} B = D$ *and is defined by*

$$D = \{< x, (T_D^1(x), T_D^2(x), ..., T_D^P(x)), (I_D^1(x), I_D^2(x), ..., I_D^P(x)), (F_D^1(x), F_D^2(x), ..., F_D^P(x)) >: x \in E\}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$,$F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

3. *The addition of A and B is denoted by* $A \tilde{+} B = E_1$ *and is defined by*

$$E_1 = \{< x, (T_{E_1}^1(x), T_{E_1}^2(x), ..., T_{E_1}^P(x)), (I_{E_1}^1(x), I_{E_1}^2(x), ..., I_{E_1}^P(x)), (F_{E_1}^1(x), F_{E_1}^2(x), ..., F_{E_1}^P(x)) >: x \in E\}$$

where $T_{E_1}^i = T_A^i(x) + T_B^i(x) - T_A^i(x).T_B^i(x)$, $I_{E_1}^i = I_A^i(x).I_B^i(x)$,$F_{E_1}^i = F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

4. *The multiplication of A and B is denoted by* $A \tilde{\times} B = E_2$ *and is defined by*

$$E_2 = \{< x, (T_{E_2}^1(x), T_{E_2}^2(x), ..., T_{E_2}^P(x)), (I_{E_2}^1(x), I_{E_2}^2(x), ..., I_{E_2}^P(x)), (F_{E_2}^1(x), F_{E_2}^2(x), ..., F_{E_2}^P(x)) >: x \in E\}$$

where $T_{E_2}^i = T_A^i(x).T_B^i(x)$, $I_{E_2}^i = I_A^i(x) + I_B^i(x) - I_A^i(x).I_B^i(x)$,$F_{E_2}^i = F_A^i(x) + F_B^i(x) - F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

3 Relations on Neutrosophic Multi Sets

In this section, after given the cartesian product of two neutrosophic multi sets(NMS), we define relations on neutrosophic multi sets and study their desired properties. The relation extend the concept of intuitionistic multi relation [34] to neutrosophic multi relation. Some of it is quoted from [19, 20, 34, 41].

Definition 3.1 *Let $\emptyset \neq A, B \in NMS(E)$. Then, cartesian product of A and B is a Nm-set in $E \times E$, denoted by $A \times B$, defined as*

$$\begin{aligned} A \times B = \ & \{< (x,y), (T^1_{A\times B}(x,y), T^2_{A\times B}(x,y), ..., T^n_{A\times B}(x,y)), \\ & (I^1_{A\times B}(x,y), I^2_{A\times B}(x,y), ..., I^n_{A\times B}(x,y)), \\ & (F^1_{A\times B}(x,y), F^2_{A\times B}(x,y), ..., F^n_{A\times B}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^j_{A\times B}, I^j_{A\times B}, F^j_{A\times B} : E \to [0,1],$$

$$T^j_{A\times B}(x,y) = \min\left\{T^j_A(x), T^j_B(y)\right\},$$

$$I^j_{A\times B}(x,y) = \max\left\{I^j_A(x), I^j_B(y)\right\}$$

and

$$F^j_{A\times B}(x,y) = \max\left\{F^j_A(x), F^j_B(y)\right\}$$

for all $x, y \in E$ and $j \in \{1, 2, ..., n\}$ ($n = max\{P(A), P(B)\}$).

Remark 3.2 *A cartesian product on A is a Nm-set in $E \times E$, denoted by $A \times A$, defined as*

$$\begin{aligned} A \times A = \ & \{< (x,y), (T^1_{A\times A}(x,y), T^2_{A\times A}(x,y), ..., T^n_{A\times A}(x,y)), \\ & (I^1_{A\times A}(x,y), I^2_{A\times A}(x,y), ..., I^n_{A\times A}(x,y)), \\ & (F^1_{A\times A}(x,y), F^2_{A\times A}(x,y), ..., F^n_{A\times A}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^j_{A\times A}, I^j_{A\times A}, F^j_{A\times A} : E \times E \to [0,1],$$

$$T^j_{A\times A}(x,y) = \min\left\{T^j_A(x), T^j_A(y)\right\},$$

$$I^j_{A\times A}(x,y) = \max\left\{I^j_A(x), I^j_A(y)\right\}$$

and

$$F^j_{A\times A}(x,y) = \max\left\{F^j_A(x), F^j_A(y)\right\}$$

$j \in \{1, 2, ..., n\}$ ($n = max\{P(A)\}$)

Example 3.3 *Let $E = \{x_1, x_2\}$ be a universal set and A and B be two Nm-sets over E as;*

$$\begin{aligned} A = \ & \{< x_1, \{0.3, 0.5, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.9\} >, \\ & < x_2, \{0.4, 0.5, 0.7\}, \{0.4, 0.5, 0.1\}, \{0.6, 0.2, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} B = \ & \{< x_1, \{0.4, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.4\} >, \\ & < x_2, \{0.6, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.6\} >\} \end{aligned}$$

Then, the cartesian product of A and B is obtained as follows

$$\begin{aligned} A \times B = \ & \{< (x_1, x_1), \{0.3, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.7, 0.8\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.7, 0.9\} >, \\ & < (x_2, x_1), \{0.4, 0.5, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.3, 0.8, 0.7\} >, \\ & < (x_2, x_2), \{0.4, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.7\} >\} \end{aligned}$$

Definition 3.4 *Let $\emptyset \neq A, B \in NMS(E)$ and $j \in \{1, 2, ..., n\}$. Then, a neutrosophic multi relation from A to B is a Nm-subset of $A \times B$. In other words, a neutrosophic multi relation from A to B is of the form (R, C), $(C \subseteq E \times E)$ where $R(x, y) \subseteq A \times B$ $\forall (x, y) \in C$.*

Example 3.5 *Let us consider the Example 3.3. Then, we define a neutrosophic multi relation R and S, from A to B, as follows*

$$\begin{aligned} R = \; & \{< (x_1, x_1), \{0.2, 0.6, 0.9\}, \{0.2, 0.4, 0.5\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.7\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.2, 0.8, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} S = \; & \{< (x_1, x_1), \{0.1, 0.7, 0.9\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.9, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.8\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.7\}, \{0.2, 0.9, 0.4\}, \{0.2, 0.8, 0.9\} >\} \end{aligned}$$

Definition 3.6 *Let $A, B \in NMS(E)$ and, R and S be two neutrosophic multi relation from A to B. Then, the operations $R\tilde{\cup}S$, $R\tilde{\cap}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are defined as follows;*

1.

$$\begin{aligned} R\tilde{\cup}S = \; & \{< (x, y), (T^1_{R\tilde{\cup}S}(x, y), T^2_{R\tilde{\cup}S}(x, y), ..., T^n_{R\tilde{\cup}S}(x, y)), \\ & (I^1_{R\tilde{\cup}S}(x, y), I^2_{R\tilde{\cup}S}(x, y), ..., I^n_{R\tilde{\cup}S}(x, y)), \\ & (F^1_{R\tilde{\cup}S}(x, y), F^2_{R\tilde{\cup}S}(x, y), ..., F^n_{R\tilde{\cup}S}(x, y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^j_{R\tilde{\cup}S}(x, y) = T^j_R(x) \vee T^j_S(y),$$
$$I^j_{R\tilde{\cup}S}(x, y) = I^j_R(x) \wedge I^j_S(y),$$
$$F^j_{R\tilde{\cup}S}(x, y) = F^j_R(x) \wedge F^j_S(y)$$

$\forall x, y \in E$ *and* $j = 1, 2, ..., n$.

2.

$$\begin{aligned} R\tilde{\cap}S \; = & \{< (x, y), (T^1_{R\tilde{\cap}S}(x, y), T^2_{R\tilde{\cap}S}(x, y), ..., T^n_{R\tilde{\cap}S}(x, y)), \\ & (I^1_{R\tilde{\cap}S}(x, y), I^2_{R\tilde{\cap}S}(x, y), ..., I^n_{R\tilde{\cap}S}(x, y)), \\ & (F^1_{R\tilde{\cap}S}(x, y), F^2_{R\tilde{\cap}S}(x, y), ..., F^n_{R\tilde{\cap}S}(x, y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^j_{R\tilde{\cap}S}(x, y) = T^j_R(x) \wedge T^j_S(y),$$
$$I^j_{R\tilde{\cap}S}(x, y) = I^j_R(x) \vee I^j_S(y),$$
$$F^j_{R\tilde{\cap}S}(x, y) = F^j_R(x) \vee F^j_S(y)$$

$\forall x, y \in E$ *and* $j = 1, 2, ..., n$.

3.

$$\begin{aligned} R\tilde{+}S = \; & \{< (x, y), (T^1_{R\tilde{+}S}(x, y), T^2_{R\tilde{+}S}(x, y), ..., T^n_{R\tilde{+}S}(x, y)), \\ & (I^1_{R\tilde{+}S}(x, y), I^2_{R\tilde{+}S}(x, y), ..., I^n_{R\tilde{+}S}(x, y)), \\ & (F^1_{R\tilde{+}S}(x, y), F^2_{R\tilde{+}S}(x, y), ..., F^n_{R\tilde{+}S}(x, y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^j_{R\tilde{+}S}(x, y) = T^j_R(x) + T^j_S(y) - T^j_R(x).T^j_S(y),$$
$$I^j_{R\tilde{+}S}(x, y) = I^j_R(x).I^j_S(y),$$
$$F^j_{R\tilde{+}S}(x, y) = F^j_R(x).F^j_S(y)$$

$\forall x, y \in E$ *and* $j = 1, 2, ..., n$.

4.

$$R\tilde{\times}S = \{<(x,y),(T^1_{R\tilde{\times}S}(x,y),T^2_{R\tilde{\times}S}(x,y),...,T^n_{R\tilde{\times}S}(x,y)),(I^1_{R\tilde{\times}S}(x,y),I^2_{R\tilde{\times}S}(x,y),...,I^n_{R\tilde{\times}S}(x,y)),(F^1_{R\tilde{\times}S}(x,y),F^2_{R\tilde{\times}S}(x,y),...,F^n_{R\tilde{\times}S}(x,y))>: x,y\in E\}$$

where

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x).T^j_S(y),$$

$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x)+I^j_S(y)-I^j_R(x).I^j_S(y),$$

$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x)+F^j_S(y)-F^j_R(x).F^j_S(y)$$

$\forall x,y\in E$ *and* $j=1,2,...,n$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

Example 3.7 *Let us consider the Example 3.5. Then,*

$$R\tilde{\cup}S = \{<(x_1,x_1),\{0.2,0.6,0.9\},\{0.2,0.4,0.5\},\{0.3,0.8,0.9\}>, <(x_1,x_2),\{0.3,0.9,0.8\},\{0.2,0.8,0.7\},\{0.1,0.8,0.9\}>, <(x_2,x_1),\{0.1,0.9,0.6\},\{0.2,0.5,0.4\},\{0.2,0.8,0.7\}>\}$$

and

$$R\tilde{\cap}S = \{<(x_1,x_1),\{0.1,0.7,0.9\},\{0.2,0.5,0.7\},\{0.1,0.9,0.9\}>, <(x_1,x_2),\{0.3,0.9,0.8\},\{0.2,0.8,0.8\},\{0.1,0.8,0.9\}>, <(x_2,x_1),\{0.1,0.9,0.7\},\{0.2,0.9,0.4\},\{0.2,0.8,0.9\}>\}$$

Similarly, $R\widetilde{+}S$ *and* $R\widetilde{\times}S$ *can be computed.*

Assume that $\emptyset\neq A,B,C\in NMS(E)$. Two neutrosophic multi relations under a suitable composition, could too yield a new neutrosophic multi relation with a useful significance. Composition of relations is important for applications, because of the reason that if a relation on A and B is known and if a relation on B and C is known then the relation on A and C could be computed and defined as follows;

Definition 3.8 *Let R(A→ B) and S (B→ C) be two neutrosophic multi relations. The composition* $S\circ R$ *is a neutrosophic multi relation from A to C, defined by*

$$S\circ R = \{<(x,z),(T^1_{S\circ R}(x,z),T^2_{S\circ R}(x,z),...,T^n_{S\circ R}(x,z)),(I^1_{S\circ R}(x,z),I^2_{S\circ R}(x,z),...,I^n_{S\circ R}(x,z)),(F^1_{S\circ R}(x,z),F^2_{S\circ R}(x,z),...,F^n_{S\circ R}(x,z))>: x,z\in E\}$$

where

$$T^j_{S\circ R}(x,z) = \bigvee_y \left\{T^j_R(x,y)\wedge T^j_S(y,z)\right\}$$

$$I^j_{S\circ R}(x,z) = \bigwedge_y \left\{I^j_R(x,y)\vee I^j_S(y,z)\right\}$$

and

$$F^j_{S\circ R}(x,z) = \bigwedge_y \left\{F^j_R(x,y)\vee F^j_S(y,z)\right\}$$

for every (x, z) $E\times E$, *for every* $y\in E$ *and* $j=1,2,...,n$.

Definition 3.9 *A neutrosophic multi relation R on A is said to be;*

1. *Reflexive if* $T^j_R(x,x)=1$, $I^j_R(x,x)=0$ *and* $F^j_R(x,x)=0$ *for all* $x\in E$,
2. *Symmetric if* $T^j_R(x,y)=T^j_R(y,x)$, $I^j_R(x,y)=I^j_R(y,x)$ *and* $F^j_R(x,y)=F^j_R(y,x)$ *for all* $x,y\in E$,
3. *Transitive if* $R\circ R\subseteq R$,

4. neutrosophic multi equivalence relation if the relation R satisfies reflexive, symmetric and transitive.

Definition 3.10 *The transitive closure of a neutrosophic multi relation R on $E \times E$ is $\hat{R} = R\tilde{\cup}R^2\tilde{\cup}R^3\tilde{\cup}...$*

Definition 3.11 *If R is a neutrosophic multi relation from A to B then R^{-1} is the inverse neutrosophic multi relation R from B to A, defined as follows:*

$$R^{-1} = \left\{ \left\langle (y,x), T^j_{R^{-1}}(x,y)), I^j_{R^{-1}}(x,y), F^j_{R^{-1}}(x,y) \right\rangle : (x,y) \in E \times E \right\}$$

where
$T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$, $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ *and* $j = 1,2,...,n$.

Proposition 3.12 *If R and S are two neutrosophic multi relation from A to B and B to C, respectively. Then,*

1. $(R^{-1})^{-1} = R$
2. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof

1. Since R^{-1} is a neutrosophic multi relation from B to A, we have
$T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$ and $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$
Then,

$$T^j_{(R^{-1})^{-1}}(x,y) = T^j_{R^{-1}}(y,x) = T^j_R(x,y),$$
$$I^j_{(R^{-1})^{-1}}(x,y) = I^j_{R^{-1}}(y,x) = I^j_R(x,y)$$

and

$$F^j_{(R^{-1})^{-1}}(x,y) = F^j_{R^{-1}}(y,x) = F^j_R(x,y)$$

therefore $(R^{-1})^{-1} = R$.

2. If the composition $S \circ R$ is a neutrosophic multi relation from A to C, then the compostion $R^{-1} \circ S^{-1}$ is a neutrosophic multi relation from C to A. Then,

$$\begin{aligned} T^j_{(S\circ R)^{-1}}(z,x) &= T^j_{(S\circ R)}(x,z) \\ &= \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\} \\ &= \bigvee_y \left\{ T^j_{R^{-1}}(y,x) \wedge T^j_{S^{-1}}(z,y) \right\} \\ &= \bigvee_y \left\{ T^j_{S^{-1}}(z,y) \wedge T^j_{R^{-1}}(y,x) \right\} \\ &= T^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned} ,$$

$$\begin{aligned} I^j_{(S\circ R)^{-1}}(z,x) &= I^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ I^j_{R^{-1}}(y,x) \vee I^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ I^j_{S^{-1}}(z,y) \vee I^j_{R^{-1}}(y,x) \right\} \\ &= I^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{(S\circ R)^{-1}}(z,x) &= F^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ F^j_{R^{-1}}(y,x) \vee F^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ F^j_{S^{-1}}(z,y) \vee F^j_{R^{-1}}(y,x) \right\} \\ &= F^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

Finally; proof is valid.

Proposition 3.13 *If R is symmetric, then* R^{-1}*is also symmetric.*

Proof: Assume that R is Symmetric then we have

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

Also if R^{-1} is an inverse relation, then we have

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x),$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

To prove R^{-1} is symmetric, it is enough to prove

$$T^j_{R^{-1}}(x,y) = T^j_{R^{-1}}(y,x),$$

$$I^j_{R^{-1}}(x,y) = I^j_{R^{-1}}(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_{R^{-1}}(y,x)$$

for all $x, y \in E$

Therefore;

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y) = T^j_{R^{-1}}(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y) = I^j_{R^{-1}}(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y) = F^j_{R^{-1}}(y,x)$$

Finally; proof is valid.

Proposition 3.14 *If R is symmetric ,if and only if* $R = R^{-1}$.

Proof: Let R be symmetric, then

$$T^j_R(x,y) = T^j_R(y,x);$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

and

R^{-1} is an inverse relation, then

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

Therefore; $T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y)$.

Similarly

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y)$$

for all $x, y \in E$.

Hence $R = R^{-1}$

Conversely, assume that $R = R^{-1}$ then, we have

$$T^j_R(x,y) = T^j_{R^{-1}}(x,y) = T^j_R(y,x).$$

Similarly

$$I^j_R(x,y) = I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_{R^{-1}}(x,y) = F^j_R(y,x).$$

Hence R is symmetric.

Proposition 3.15 *If R and S are symmetric neutrosophic multi relations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{\cap}S$,
3. $R\tilde{+}S$
4. $R\tilde{\times}S$

are also symmetric.

Proof: R is symmetric, then we have;

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

similarly S is symmetric, then we have

$$T^j_S(x,y) = T^j_S(y,x),$$

$$I^j_S(x,y) = I^j_S(y,x)$$

and

$$F^j_S(x,y) = F^j_S(y,x)$$

Therefore,

1.
$$\begin{aligned} T^j_{R\widetilde{\cup}S}(x,y) &= \max\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ &= \max\left\{T^j_R(y,x), T^j_S(y,x)\right\} \\ &= T^j_{R\widetilde{\cup}S}(y,x) \end{aligned},$$

$$\begin{aligned} I^j_{R\widetilde{\cup}S}(x,y) &= min\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ &= \min\left\{I^j_R(y,x), I^j_S(y,x)\right\} \\ &= I^j_{R\widetilde{\cup}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cup}S}(x,y) &= \min\left\{F^j_R(x,y), F^j_S(x,y)\right\} \\ &= \min\left\{F^j_R(y,x), F^j_S(y,x)\right\} \\ &= F^j_{R\widetilde{\cup}S}(y,x) \end{aligned}$$

therefore, $R\widetilde{\cup}S$ is symmetric.

2.

$$\begin{aligned} T^j_{R\tilde{\cap}S}(x,y) &= \min\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ &= \min\left\{T^j_R(y,x), T^j_S(y,x)\right\} \\ &= T^j_{R\tilde{\cap}S}(y,x), \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\cap}S}(x,y) &= max\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ &= \max\left\{I^j_R(y,x), I^j_S(y,x)\right\} \\ &= I^j_{R\tilde{\cap}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{\cap}S}(x,y) &= \max\left\{F^j_R(x,y), F^j_S(x,y)\right\} \\ &= \max\left\{F^j_R(y,x), F^j_S(y,x)\right\} \\ &= F^j_{R\tilde{\cap}S}(y,x) \end{aligned}$$

therefore; $R\tilde{\cap}S$ is symmetric.

3.

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x) + T^j_S(y,x) - T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{+}S}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{+}S}(y,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{+}S}(y,x) \end{aligned}$$

therefore, $R\tilde{+}S$ is also symmetric

4.

$$\begin{aligned} T^j_{R\tilde{\times}S}(x,y) &= T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{\times}tS}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\times}S}(x,y) &= I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x) + I^j_S(y,x) - I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

$$\begin{aligned} F^j_{R\tilde{\times}S}(x,y) &= F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x) + F^j_S(y,x) - F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

hence, $R\tilde{\times}S$ is also symmetric.

Remark 3.16 *$R \circ S$ in general is not symmetric, as*

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y) \wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x) \wedge T^j_R(z,y)\right\} \\ &\neq T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \underset{y}{\wedge}\left\{I^j_S(x,y)\vee I^j_R(y,z)\right\}\\ &= \underset{y}{\wedge}\left\{I^j_S(y,x)\vee I^j_R(z,y)\right\}\\ &\neq I^j_{(R\circ S)}(z,x)\end{aligned}$$

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \underset{y}{\wedge}\left\{F^j_S(x,y)\vee F^j_R(y,z)\right\}\\ &= \underset{y}{\wedge}\left\{F^j_S(y,x)\vee F^j_R(z,y)\right\}\\ &\neq F^j_{(R\circ S)}(z,x)\end{aligned}$$

but $R\circ S$ is symmetric, if $R\circ S = S\circ R$, for R and S are symmetric relations.

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \underset{y}{\vee}\left\{T^j_S(x,y)\wedge T^j_R(y,z)\right\}\\ &= \underset{y}{\vee}\left\{T^j_S(y,x)\wedge T^j_R(z,y)\right\}\\ &= \underset{y}{\vee}\left\{T^j_R(y,x)\wedge T^j_R(z,y)\right\}\\ &T^j_{(R\circ S)}(z,x)\end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \underset{y}{\wedge}\left\{I^j_S(x,y)\vee I^j_R(y,z)\right\}\\ &= \underset{y}{\wedge}\left\{I^j_S(y,x)\vee I^j_R(z,y)\right\}\\ &= \underset{y}{\wedge}\left\{I^j_R(y,x)\vee I^j_R(z,y)\right\}\\ &I^j_{(R\circ S)}(z,x)\end{aligned}$$

and

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \underset{y}{\wedge}\left\{F^j_S(x,y)\vee F^j_R(y,z)\right\}\\ &= \underset{y}{\wedge}\left\{F^j_S(y,x)\vee F^j_R(z,y)\right\}\\ &= \underset{y}{\wedge}\left\{F^j_R(y,x)\vee F^j_R(z,y)\right\}\\ &F^j_{(R\circ S)}(z,x)\end{aligned}$$

for every $(x,z)\in E\times E$ and for $y\in E$.

Proposition 3.17 *If R is transitive relation, then R^{-1} is also transitive.*

Proof : R is transitive relation, if $R\circ R\subseteq R$, hence if $R^{-1}\circ R^{-1}\subseteq R^{-1}$, then R^{-1} is transitive. Consider;

$$\begin{aligned} T^j_{R^{-1}}(x,y) &= T^j_R(y,x)\geq T^j_{R\circ R}(y,x)\\ &= \underset{z}{\vee}\left\{T^j_R(y,z)\wedge T^j_R(z,x)\right\}\\ &= \underset{z}{\vee}\left\{T^j_{R^{-1}}(x,z)\wedge T^j_{R^{-1}}(z,y)\right\}\\ &= T^j_{R^{-1}\circ R^{-1}}(x,y)\end{aligned}$$

$$\begin{aligned} I^j_{R^{-1}}(x,y) &= I^j_R(y,x)\leq I^j_{R\circ R}(y,x)\\ &= \underset{z}{\wedge}\left\{I^j_R(y,z)\vee I^j_R(z,x)\right\}\\ &= \underset{z}{\wedge}\left\{I^j_{R^{-1}}(x,z)\vee I^j_{R^{-1}}(z,y)\right\}\\ &= I^j_{R^{-1}\circ R^{-1}}(x,y)\end{aligned}$$

and

$$\begin{aligned} F^j_{R^{-1}}(x,y) &= F^j_R(y,x)\leq F^j_{R\circ R}(y,x)\\ &= \underset{z}{\wedge}\left\{F^j_R(y,z)\vee F^j_R(z,x)\right\}\\ &= \underset{z}{\wedge}\left\{F^j_{R^{-1}}(x,z)\vee F^j_{R^{-1}}(z,y)\right\}\\ &= F^j_{R^{-1}\circ R^{-1}}(x,y)\end{aligned}$$

hence, proof is valid.

Proposition 3.18 *If R is transitive relation, then $R \cap S$ is also transitive.*

Proof: As R and S are transitive relations, $R \circ R \subseteq R$ and $S \circ S \subseteq S$. Also

$$T^j_{R\tilde{\cap}S}(x,y) \geq T^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y)$$
$$I^j_{R\tilde{\cap}S}(x,y) \leq I^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y)$$
$$F^j_{R\tilde{\cap}S}(x,y) \leq F^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y)$$

implies $R\tilde{\cap}S) \circ (R\tilde{\cap}S) \subseteq R \cap S$, hence $R \cap S$ is transitive.

Proposition 3.19 *If R and S are transitive relations, then*

1. *$R\tilde{\cup}S$,*
2. *$R\tilde{+}S$*
3. *$R\tilde{\times}S$*

are not transitive.

Proof:

1. As

$$T^j_{R\tilde{\cup}S}(x,y) = \max\left\{T^j_R(x,y), T^j_S(x,y)\right\}$$
$$I^j_{R\tilde{\cup}S}(x,y) = \min\left\{I^j_R(x,y), I^j_S(x,y)\right\}$$
$$F^j_{R\tilde{\cup}S}(x,y) = \min\left\{F^j_R(x,y), F^j_S(x,y)\right\}$$

and

$$T^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \geq T^j_{R\tilde{\cup}S}(x,y)$$
$$I^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \leq I^j_{R\tilde{\cup}S}(x,y)$$
$$F^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \leq F^j_{R\tilde{\cup}S}(x,y)$$

2. As

$$T^j_{R\tilde{+}S}(x,y) = T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{+}S}(x,y) = I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{+}S}(x,y) = F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \geq T^j_{R\tilde{+}S}(x,y)$$
$$I^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq I^j_{R\tilde{+}S}(x,y)$$
$$F^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq F^j_{R\tilde{+}S}(x,y)$$

3. As

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \geq T^j_{R\tilde{\times}S}(x,y)$$
$$I^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq I^j_{R\tilde{\times}S}(x,y)$$
$$F^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq F^j_{R\tilde{\times}S}(x,y)$$

Hence $R\tilde{\cup}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are not transitive.

Proposition 3.20 *If R is transitive relation, then R^2 is also transitive.*

Proof: R is transitive relation, if $R \circ R \subseteq R$, therefore if $R^2 \circ R^{-2} \subseteq R^2$, then R^2 is transitive.

$$T^j_{R\circ R}(y,x) = \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \geq \bigvee_z \left\{T^j_{R\circ R}(y,z) \wedge T^j_{R\circ R}(z,x)\right\} = T^j_{R^2\circ R^2}(y,x),$$

$$I^j_{R\circ R}(y,x) = \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee I^j_{R\circ R}(z,x)\right\} = I^j_{R^2\circ R^2}(y,x)$$

and

$$F^j_{R\circ R}(y,x) = \bigwedge_z \left\{F(y,z) \vee F^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee F^j_{R\circ R}(z,x)\right\} = F^j_{R^2\circ R^2}(y,x)$$

Finally, the proof is valid.

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5 Conclusion

In this paper, we have firstly defined the neutrosophic multi relations(NMR). The NMR are the extension of neutrosophic soft relation(NR)[20] and intuitionistic multi relation [34]. Then, some notions such as; inverse, symmetry, reflexivity and transitivity on neutrosophic multi relations are studied. The future work will cover the application of the MNR in decision making, pattern recogntion and in medical diagnosis.

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