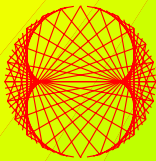


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Unification of Interactions in Discrete Spacetime

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I assume that both spacetime and the internal symmetry space of the Standard Model (SM) of leptons and quarks are discrete. If lepton and quark states represent specific finite binary rotational subgroups of the SM gauge group, unification with gravitation is accomplished by combining finite subgroups of the Lorentz group $SO(3,1)$ with the specific finite SM subgroups. The unique result is a particular finite subgroup of $SO(9,1)$ in discrete 10-D spacetime related to $E_8 \times E_8$ of superstring theory. A physical model of particles based upon the finite subgroups and the discrete geometry is proposed. Evidence for discreteness might be the appearance of a b' quark at about 80–100 GeV decaying via FCNC to a b quark plus a photon at the Large Hadron Collider.

1 Introduction

I consider both spacetime and the internal symmetry space of the Standard Model (SM) of leptons and quarks to be discrete instead of continuous. Using specific finite subgroups of the SM gauge group, a unique finite group in discrete 10-D spacetime unifies the fundamental interactions, including gravitation. This finite group is a special subgroup of the continuous group $E_8 \times E_8$ that in superstring theory (also called M-theory) is considered to be the most likely group for unifying gravitation with the SM gauge group.

This unique result follows directly from two fundamental assumptions: (1) the internal symmetry space is discrete, requiring specific finite binary rotational subgroups of the SM gauge group to dictate the physical properties of the lepton and quark states, and (2) spacetime is discrete, and therefore its discrete symmetries correspond to finite subgroups of the Lorentz group. Presumably, this discreteness must occur as one approaches the Planck scale of about 10^{-35} meters.

I suggest a particular physical model of fundamental fermions based upon these finite subgroups in the discrete geometry. Further evidence for this discreteness might be the appearance of a b' quark at about 80–100 GeV decaying via FCNC to a b quark plus a photon at the Large Hadron Collider.

2 Motivation

The Standard Model (SM) of leptons and quarks successfully describes their electromagnetic, weak and color interactions in terms of symmetries dictated by the $SU(2)_L \times U(1)_Y \times SU(3)_C$ continuous gauge group. These fundamental fermions and their antiparticles are defined by their electroweak isospin states in two distinct but gauge equivalent unitary planes in an internal symmetry space “attached” at a space-

time point. Consequently, particle states and antiparticle states have opposite-signed physical properties but their masses are the same sign.

In an earlier 1994 paper [1] I discussed how the SM continuous gauge group could be acting like a “cover group” for its finite binary rotational subgroups, thereby hiding any important underlying discrete rotational symmetries of these fundamental particle states. From group theory, one knows that the continuous SM gauge group contains thousands of elements of finite order including, for example, all the elements of the finite binary rotational subgroups in their 3-dimensional and 4-dimensional representations. I showed that these subgroups were very important because they are connected to the j -invariant of elliptic modular functions from which one can predict the mass ratios for the lepton and quark states.

The mathematical properties of these finite subgroups of the SM dictate the same physical properties of the leptons and quarks as achieved by the SM. However, electroweak symmetry breaking to these specific finite binary rotational subgroups occurs without a Higgs particle. More importantly, some additional physical properties are dictated also, such as their mass ratios, why more than one generation is present, the important family relationships, and the dimensionalities of the particle states because they are no longer point particles.

The gravitational interaction is not included explicitly in the SM gauge group. However, because the finite binary rotational subgroup approach determined the lepton and quark mass ratios, one suspects that the gravitational interaction is included already in the discretized version of the gauge group. Or, equivalently, since mass/energy is the source of the gravitational interaction, the gravitational interaction arises from the discrete symmetries associated with the finite rotational subgroups.

Therefore, I make some conjectures. If leptons and quarks actually represent the specific discrete symmetries of the finite subgroups of the SM gauge group as proposed, the internal symmetry space may be discrete instead of being continuous. Going one step further, then not only the internal symmetry space might be discrete but also spacetime itself may be discrete, since gravitation determines the spacetime metric. Spacetime would appear to be continuous only at the low resolution scales of experimental apparatus such as the present particle colliders. Unification of the fundamental interactions then requires combining these finite groups mathematically.

3 4-D internal symmetry space?

I take the internal symmetry space of the SM to be discrete, but we need to know how many dimensions there are. Do we need two complex spatial dimensions for a unitary plane as suggested by SU(2), or do we need three as suggested by the SU(3) symmetry of the color interaction, or do we need more?

The lepton and quark particle states are defined as electroweak isospin states by the electroweak part of the SM gauge group, with particles in the normal unitary plane C^2 and antiparticles in the conjugate unitary plane C'^2 . Photon, W^+ , W^- , and Z^0 interactions of the electroweak $SU(2)_L \times U(1)_Y$ gauge group rotate the two particle states (i. e., the two complex basis spinors in the unitary plane) into one another. For example, $e^- + W^+ \rightarrow \nu_e$. These electroweak rotations can be considered to occur also in an equivalent 4-dimensional real euclidean space R^4 and in an equivalent quaternion space Q , both these spaces being useful for a better geometrical understanding of the SM.

The quark states are defined also by the color symmetries of $SU(3)_C$, i. e., each quark comes in one of three possible colors, red R, green G, or blue B, while the lepton states have no color charge. Normally, one would consider $SU(3)_C$ operating in a space of three complex dimensions, or its equivalent six real dimensions. In fact, $SU(3)_C$ can operate successfully in the smaller unitary plane C^2 , because each SU(3) operation can be written as the product of three specific SU(2) operations [2]. An alternative geometrical explanation has the gluon operations of the color interaction rotate one color state into another in a 4-dimensional real space, as discussed in my 1994 article. Briefly, real 4-dimensional space R^4 has four orthogonal coordinates (w, x, y, z) , and its 4-D rotations occur simultaneously in two orthogonal planes. There being only three distinct pairs of orthogonal planes, $[wx, yz]$, $[xy, zw]$, and $[yw, xz]$, each color R, G, or B is assigned to a specific pair, thereby making color an exact geometrical symmetry. Consequently, the gluon operations of $SU(3)_C$ occur in the 4-D real space R^4 that is equivalent to the unitary plane. Detailed matrix operations confirm that

hadrons with quark-antiquark pairs, three quarks, or three antiquarks, are colorless combinations.

Therefore I take the internal symmetry space to be a discrete 4-dimensional real space because this space is the minimum dimensional space that allows the SM gauge group to operate completely. One does not need a larger space, e. g., a 6-dimensional real space, for its internal symmetry space.

4 Dimensions of spacetime?

I take physical spacetime to be 4-dimensional with its one time dimension. Spacetime is normally considered to be continuous and 4-dimensional, with three spatial dimensions and one time dimension. However, in the last two decades several approaches toward unifying all fundamental interactions have considered additional mathematical spatial dimensions and/or more time dimensions. For example, superstring theory [3] at the high energy regime, i. e., at the Planck scale, proposes 10 or 11 spacetime dimensions in its present mathematical formulation, including the one time dimension. These extra spatial dimensions may correspond to six or seven dimensions “curled up” into an internal symmetry space for defining fundamental particle states at each spacetime point in order to accommodate the SM in the low energy regime. The actual physical spacetime itself may still have three spatial dimensions and one time dimension.

I take 4-D spacetime to be discrete. We do not know whether spacetime is continuous or discrete. If the internal symmetry space is indeed discrete, then perhaps spacetime itself might be discrete also. Researchers in loop quantum gravity [4] at the Planck scale divide spacetime into discrete subunits, considering a discrete 4-D spacetime with its discrete Lorentz transformations to be a viable approach.

The goal now is to combine the finite subgroups of the gauge group of the SM and the finite group of discrete Lorentz boosts and discrete spacetime rotations into one unified group. All four known fundamental interactions would be unified. Although many unification schemes for the fundamental interactions have been attempted over the past three decades utilizing continuous groups, I believe this attempt is the first one that combines finite groups. Mathematically, the result must be unique, otherwise different fundamental laws could exist in different parts of the universe.

5 Discrete internal space

The most important finite symmetry groups in the 4-D discrete internal symmetry space are the 3-D binary rotational subgroups $[3, 3, 2]$, $[4, 3, 2]$, and $[5, 3, 2]$ of the SM gauge group because they are the symmetry groups I have assigned to the three lepton families. They contain discrete rotations and inversions and operate in the 3-D subspace R^3 of R^4 and C^2 .

Being subgroups of $SU(2)_L \times U(1)_Y$, they have group operations represented by 2×2 unitary matrices or, equivalently, by unit quaternions. Quaternions provide the more obvious geometrical connection [5], because quaternions perform the dual role of being a group operation and of being a vector in R^3 and in R^4 . One can think visually about the 3-D group rotations and inversions for these three subgroups as quaternions operating on the Platonic solids, with the same quaternions also defining the vertices of regular geometrical objects in R^4 .

The two mathematical entities, the unit quaternion \mathbf{q} and the $SU(2)$ matrix, are related by

$$\mathbf{q} = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \iff \begin{pmatrix} w + iz & x + iy \\ -x + iy & w - iz \end{pmatrix} \quad (1)$$

where the \mathbf{i} , \mathbf{j} , and \mathbf{k} are unit imaginaries, their coefficients are real, and $w^2 + x^2 + y^2 + z^2 = 1$. The conjugate quaternion $\mathbf{q}' = w - x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$ and its corresponding matrix would represent the same group operation in the conjugate unitary plane for the antiparticles. Recall that Clifford algebra and Bott periodicity dictate that only R^4 , R^8 , and other real spaces R^n with dimensions divisible by four have two equivalent conjugate spaces, the specific mathematical property that accommodates both particle states and antiparticle states. The group $U(1)_Y$ for weak hypercharge Y then reduces the symmetry to being gauge equivalent so that particles and antiparticles have the same positive mass.

One might expect that we need to analyze each of the three binary rotational subgroups separately when the discrete internal symmetry space is combined with discrete spacetime. Fortunately, the largest binary rotational group [5, 3, 2] of icosahedral symmetries can accommodate the two other groups, and a discussion of its 120 quaternion operations is all inclusive mathematically. The elements of this icosahedral group, rotations and inversions, can be represented by the appropriate unit quaternions.

The direct connection between the 3-D and 4-D spaces is realized when one equates the 120 group operations on the regular icosahedron (3, 5) to the vectors for the 120 vertices of the 600-cell hypericosahedron (3, 3, 5) in 4-D space in a particular way. These operations of the binary icosahedral group [5, 3, 2] and the vertices of the hypericosahedron are defined by 120 special unit quaternions q_i known as isosians [6], which have the mathematical form

$$q_i = \left(e_1 + e_2\sqrt{5} \right) + \left(e_3 + e_4\sqrt{5} \right) \mathbf{i} \\ + \left(e_5 + e_6\sqrt{5} \right) \mathbf{j} + \left(e_7 + e_8\sqrt{5} \right) \mathbf{k}, \quad (2)$$

where the eight e_j are special rational numbers. Specifically, the 120 icosians are obtained by permutations of

$$\left(\pm 1, 0, 0, 0 \right), \left(\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2 \right), \\ \left(0, \pm 1/2, \pm g/2, \pm G/2 \right), \quad (3)$$

where $g = G^{-1} = G - 1 = (-1 + \sqrt{5})/2$. Notice that in each pair, such as $(e_3 + e_4\sqrt{5})$, only one of the e_j is nonzero, reminding us that the hypericosahedron is really a 4-D object even though we can now define this object in terms of icosians that are expressed in the much larger R^8 euclidean real space.

So the quaternion's dual role allows us to identify the 120 group operations of the icosahedron with the 120 vertices of the hypericosahedron expressed both in R^4 and in R^8 , essentially telescoping from 3-D rotational operations all the way to their representations in an 8-D space. These special 120 icosians are to be considered as special octonions, 8-tuples of rational numbers which, with respect to a particular norm, form part of a special lattice in R^8 .

Now consider the two other subgroups. The 24 quaternions of the binary tetrahedral group [3, 3, 2] are contained already in the above 120 icosians. So we are left with accommodating the binary octahedral group [4, 3, 2] into the same icosian format. We need 48 special quaternions for its 48 operations, the 24 quaternions defining the vertices of the 4-D object known as the 24-cell contained already in the hypericosahedron above and another 24 quaternions for the reciprocal 24-cell. The 120 unit quaternions reciprocal to the ones above will meet this requirement as well as define an equivalent set for the reciprocal hypericosahedron, and this second set of 120 octonions also forms part of a special lattice in R^8 . Together, these two lattice parts of 120 icosians in each combine to form the 240 octonions of the famous E_8 lattice in R^8 , well known for being the densest lattice packing of spheres in 8-D.

Recall that the three binary rotation groups above are assigned to the lepton families because, as subgroups of the SM gauge group, they predict the correct physical properties of the lepton states, including the correct mass ratios. Therefore, the lepton states as I have defined them span only the 3-D real subspace R^3 of the unitary plane. That is why leptons are color neutral and do not participate in the color interaction, a physical property that requires the ability to undergo complete 4-D rotations.

So how do quark states fit into the icosian picture? I have the quark states in the SM spanning the whole 4-D real space, i. e., the whole unitary plane, because they are the basis states of the 4-D finite binary rotational subgroups of the SM gauge group. But free quarks in spacetime do not exist because they are confined according to QCD, forming the colorless quark-antiquark, three-quark, or three-antiquark combinations called hadrons. Mathematically, these colorless hadron states span the 3-D subspace only, so their resultant discrete symmetry group must be isomorphic to one of the three binary rotational subgroups we have just considered. Consequently, the icosians enumerated above account for all the lepton states and for all the quark states in their allowed hadronic combinations.

6 Discrete spacetime

Linear transformations in discrete spacetime are the discrete rotations and the discrete Lorentz boosts. Before considering these discrete transformations, however, I discuss the continuous transformations of the “heavenly sphere” as a useful mathematical construct before reducing the symmetry to discrete transformations in a discrete spacetime.

The continuous Lorentz group $SO(3,1)$ contains all the rotations and Lorentz boosts, both continuous and discrete, for the 4-D continuous spacetime with the Minkowski metric. Its operations are quaternions because there exists the isomorphism

$$SO(3, 1) = \text{PSL}(2, \mathbb{C}). \quad (4)$$

The group $\text{PSL}(2, \mathbb{C})$ consists of unit quaternions and is the quotient group $\text{SL}(2, \mathbb{C})/Z$ formed by its center Z , those elements of $\text{SL}(2, \mathbb{C})$ which commute with all the rest of the group. Its 2×2 matrix representation has complex numbers as entries.

The continuous Lorentz transformations (including the spatial rotations) operate on the “heavenly sphere” [7], i. e., the famous Riemann sphere formed by augmenting the complex plane \mathbb{C} by the “point at infinity”. The Riemann sphere is also the space of states of a spin-1/2 particle. For the Lorentz transformations in spacetime, if you are located at the center of this “heavenly sphere” so that the light rays from stars overhead each pass through unique points on a unit celestial sphere surrounding you, then the Lorentz boost is a conformal transformation of the star locations. The constellations will look distorted because the apparent lengths of the lines connecting the stars will change but the angles between these connecting lines will remain the same.

These conformal transformations are called fractional linear transformations, or Möbius transformations, of the Riemann sphere, expressed by the general form [8]

$$w \mapsto \frac{\alpha w + \beta}{\gamma w + \delta}, \quad (5)$$

with α, β, γ , and δ complex, and $\alpha\delta - \beta\gamma \neq 0$. The 2×2 matrix representation for transformation of a spinor ν as the map $\nu \mapsto M\nu$ is

$$M = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}. \quad (6)$$

Thus, M is the spinor representation of the Lorentz transformation. M acts on a vector $A = \nu\nu^\dagger$ via $A \mapsto MAM^\dagger$ [9]. All these relationships are tied together by the group isomorphisms in continuous 4-D spacetime

$$SO(3, 1) = \text{Möbius group} = \text{PSL}(2, \mathbb{C}). \quad (7)$$

Discrete spacetime has discrete Lorentz transformations, not continuous ones. These discrete rotations and discrete Lorentz boosts are contained already in $SO(3,1)$, and they

tessellate the Riemann sphere. That is, they form regular polygons on its surface that correspond to the discrete symmetries of the binary tetrahedral, binary octahedral, and binary icosahedral rotation groups [3, 3, 2], [4, 3, 2], and [5, 3, 2], the same groups I used in the internal symmetry space for the discrete symmetries. Therefore, the 240 quaternions defined previously are required also for the discrete rotations and discrete Lorentz boosts in the discrete 4-D spacetime. Again, there are the same 240 icosian connections to octonions in R^8 to form a second E_8 lattice.

Thus, the Lorentz group $SO(3,1)$ with its linear transformations in a continuous 4-D spacetime, when reduced to its discrete transformations in a 4-D discrete spacetime, is connected mathematically by icosians to the E_8 lattice in R^8 , telescoping the transformations from a smaller discrete spacetime to a larger one. Hence all linear transformations for the particles in a 4-D discrete spacetime have become represented by 240 discrete transformations in the 8-D discrete spacetime.

7 Resultant spacetime

The discrete transformations in the 4-D discrete internal symmetry space and in the 4-D discrete spacetime are each represented by an E_8 lattice in the 8-D space R^8 . The finite group of the discrete symmetries of the E_8 lattice is the Weyl group E_8 , not to be confused with the continuous exceptional Lie group E_8 . Thus, the Weyl E_8 is a finite subgroup of $SO(8)$, the continuous group of all rotations of the unit sphere in R^8 with determinant unity. In this section I combine the two Weyl E_8 groups to form a bigger group that operates in a discrete spacetime, and then in the next section I suggest a simple physical model for fundamental fermions that would fit the geometry.

I have now two sets of 240 icosians each forming E_8 lattices in R^8 , each obeying the symmetry operations of the finite group Weyl E_8 . Each finite group of octonions acts as rotations and as vectors in R^8 . I identify their direct product as the elements of a discrete subgroup of the continuous group $\text{PSL}(2, \mathbb{O})$, where \mathbb{O} represents all the unit octonions. That is, if all the unit octonions in each were present, not just the subset of unit octonions that form the E_8 lattice, their direct product group would be the continuous group of 2×2 matrices in which all matrix entries are unit octonions. So the spinors in R^8 are octonions.

The 8-D result is analogous to the 4-D result but different. Recall that in the 4-D case, one has $\text{PSL}(2, \mathbb{C})$, the group of 2×2 matrices with complex numbers as entries, with $\text{PSL}(2, \mathbb{C}) = SO(3,1)$, the Lorentz group in 4-D spacetime. Here in 8-D one has a surprise, for the final combined spacetime is bigger, being isomorphic to a 10-dimensional spacetime instead of 8-dimensional spacetime because

$$\text{PSL}(2, \mathbb{O}) = SO(9, 1), \quad (8)$$

the Lorentz group in 10-D spacetime.

Applied to the discrete case, the combined group is the finite subgroup

$$\text{finite PSL}(2, \mathbb{O}) = \text{finite SO}(9, 1), \quad (9)$$

that is, the finite Lorentz group in discrete 10-D spacetime. The same results, expressed in terms of the direct product of Weyl E_8 groups, is

$$\text{Weyl } E_8 \times \text{Weyl } E_8 = \text{“Weyl” SO}(9, 1), \quad (10)$$

where “Weyl” $\text{SO}(9,1)$ is defined by the direct product on the left and is a finite subgroup of $\text{SO}(9,1)$.

Working in reverse, the discrete 10-D spacetime divides into two parts as a 4-D discrete spacetime plus a 4-D discrete internal symmetry space. There is a surprise in this result: combining a discrete 4-D internal symmetry space with a discrete 4-D spacetime creates a discrete 10-D spacetime, not a discrete 8-D spacetime. Therefore, a continuous 10-D spacetime, when “discretized”, is not required to partition into a 4-D spacetime plus a 6-D “curled up” space as proposed in superstring theory.

8 A physical particle model

In the 1994 paper I proposed originally that leptons have the symmetries of the 3-D regular polyhedral groups and that quarks have the symmetries of the 4-D regular polytope groups. Now that I have combined the discrete 4-D internal symmetry space with a discrete 4-D spacetime to achieve mathematically a discrete 10-D spacetime, the fundamental question arises: Are the leptons and quarks really 3-D and 4-D objects physically, or are they something else, perhaps 8-D or 10-D objects?

In order to answer this question I need to formulate a reasonable physical model of fundamental particles in this discrete spacetime environment. The simplest mathematical viewpoint is that discrete spacetime is composed of identical entities, call them nodes, which have no *measureable* physical properties until they collectively distort spacetime to form a fundamental particle such as the electron, for example. The collection of nodes and its distortion of the surrounding spacetime exhibit the discrete symmetry of the appropriate finite binary rotation group for the specific particle. For example, the electron family has the discrete symmetry of the binary tetrahedral group and the electron is one of its two possible orthogonal basis states. So the distortion for the collection of nodes called the electron will exhibit the discrete symmetries of its $[3, 3, 2]$ group as all of its physical properties emerge for this specific collection and did not exist beforehand. The positron forms in the conjugate space.

One can begin with a regular lattice of nodes in both the normal unitary plane and in its conjugate unitary plane, or one can consider the equivalent R^4 spaces, and then

imagine that a spacetime distortion appears in both to form a particle-antiparticle pair. Mathematically, one begins with an isotropic vector, also called a zero length vector, which is orthogonal to itself, that gets divided into two unit spinors corresponding to the creation of the particle-antiparticle pair. No conservation laws are violated because their quantum numbers are opposite and the sum of the total mass energy plus their total potential energy is zero. The spacetime distortion that is the particle and its “field” mathematically brings the nodes closer together locally with a corresponding adjustment to the node spacing all the way out to infinite distance, all the while keeping the appropriate discrete rotational symmetry intact. The gravitational interaction associated with this discrete symmetry therefore extends to infinite distance.

This model of particle geometry must treat leptons as 3-D objects and quarks as 4-D objects in a discrete 4-D spacetime. We know that there are no isolated quarks, for they immediately form 3-D objects called hadrons. These lepton states and hadron states are described by quaternions of the form $w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, so these 3-D objects “live” in the three imaginary dimensions, and the 4th dimension can be called time. Therefore, leptons and hadrons each experience the “passage of time”, while individual quarks do not have this characteristic until they form hadrons in the 3-D subspace.

If this physical model is a reasonable approximation to describing the world of fundamental particles, why are superstring researchers working in 10-dimensions or more? Because one desires a single symmetry group that includes both the group of spacetime transformations of particles and the group of internal symmetries for the particle interactions. At the Planck scale, if one has a continuous group, then the smallest dimensional continuous spacetime one can use is 10-D in order to have a viable Lagrangian. Reducing this 10-D spacetime to the low energy regime of the SM in 4-D spacetime, the 10-D continuous spacetime has been postulated to divide into 4-D spacetime plus an additional 6-dimensional “curled up” space in which to accommodate the SM. In M-theory, one may be considering an 11-D spacetime dividing into a 4-D spacetime plus a 7-D “curled up” space. But this approach using continuous groups to connect back to the SM has proven difficult, although some significant advances have been achieved.

The analysis presented above for combining the two finite Weyl E_8 groups shows that the combined group operates in 10-D discrete spacetime with all the group operations being discrete. The particles are 3-D objects “traveling” in spacetime. No separate “curled up” space is required at the low energy limit corresponding to a distance scale of about 10^{-23} meters or larger. The discreteness at the Planck scale and the “hidden” discreteness postulated for all larger distance scales is the mathematical feature that permits the direct unique connection through icosians from the high energy world to the familiar lower energy world of the SM.

9 Mathematical connections

The mathematical connections of these binary polyhedral groups to number theory, geometry, and algebra are too numerous to list and discuss in this short article. In fact, according to B. Kostant [10], if one were to choose groups in mathematics upon which to construct the symmetries of the universe, one couldn't choose a better set, for "... in a very profound way, the finite groups of symmetries in 3-space 'see' the simple Lie groups (and hence literally Lie theory) in all dimensions." Therefore, I provide a brief survey of a few important connections here and will discuss them in more detail in future articles.

Geometrical connections are important for these groups. The continuous group $PSL(2, \mathbb{C})$ defines a torus, as does $PSL(2, \mathbb{O})$. In the discrete environment, finite $PSL(2, \mathbb{C})$ and finite $PSL(2, \mathbb{O})$ have special symmetry points on each torus corresponding to the elements of the finite binary polyhedral groups. An important mathematical property of the binary polyhedral groups is their connection to elliptic modular functions, the doubly periodic functions, and their famous j -invariant function, which has integer coefficients in its series expansion related to the largest of the finite simple groups called the Monster.

The binary tetrahedral, octahedral and icosahedral rotation groups are the finite groups of Möbius transformations $PSL(2, Z_3)$, $PSL(2, Z_4)$, and $PSL(2, Z_5)$, respectively, where Z_n denotes integers mod (n) . $PSL(2, Z_n)$ is often called the modular group $\Gamma(n)$. $PSL(2, Z_n) = SL(2, Z_n) / \{\pm I\}$, so these three binary polyhedral groups (along with the cyclic and dihedral groups) are the finite modular subgroups of $PSL(2, \mathbb{C})$ and are also discrete subgroups of $PSL(2, \mathbb{R})$. $PSL(2, Z_n)$ is simple in only three cases: $n = 5, 7, 11$. And these three cases are the Platonic groups again: A_5 and its subgroup A_4 , S_4 , and A_5 , respectively [11].

An important mathematical property for physics is that our binary polyhedral groups, the $\Gamma(n)$, are generated by the two transformations

$$X : \tau \mapsto -1/\tau \quad Y : \tau \mapsto \tau + 1, \quad (11)$$

with τ being the lattice parameter for the plane associated with forming the tessellations of the toroidal Riemann surface. The j -invariant function $j(\tau)$ of elliptic modular functions exhibits this transformation behavior. Consequently, functions describing the physical properties of the fundamental leptons and quarks will exhibit these same transformation properties. So here is where the duality theorems of M-theory, such as the S duality relating the theory at physical coupling g to coupling at $1/g$, arise naturally from mathematical properties of the finite binary polyhedral groups.

One can show also that octonions and the triality connection for spinors and vectors in R^8 are related to the fundamental interactions. In 8-D, the fundamental matrix representations both for left- and right-handed spinors and for

vectors are the same dimension, 8×8 [12], leading to many interesting mathematical properties. For example, an electron represented by a left-handed octonionic spinor interacting with a W^+ boson represented by an octonionic vector becomes an electron neutrino, again an octonionic spinor. Geometrically, this interaction looks like three E_8 lattices combining momentarily to form the famous 24-dimensional Leech lattice!

By using a discrete spacetime, we have begun to suspect that Nature has established a universe based upon fundamental mathematics that dictates unique fundamental physics principles. Moreover, one might expect that all physical constants will be shown to arise from fundamental mathematical relationships, dictating one universe with unique constant values for a unique set of fundamental laws.

10 Experimental tests

There is no direct test yet devised for discrete spacetime. However, my discrete internal symmetry space approach dictates a fourth quark family with a b' quark state at about 80 GeV and a t' quark at about 2600 GeV. The production of this b' quark with the detection of its decay to a b quark and a high energy photon seems at present to be the only attainable empirical test for discreteness. Its appearance in collider decays would be an enormously important event in particle physics, strongly suggesting that the internal symmetry space and its "surrounding" spacetime are discrete.

However, the b' quark has remained hidden among the collision debris at Fermilab because its flavor changing neutral current (FCNC) decay channel has a very low probability compared to all the other particle decays in this energy regime. This b' quark decay may even be confused with the decay of the Higgs boson, should such a particle exist, until all the quantum numbers are established. The t' quark at around 2600 GeV has too great a mass to have been produced directly at Fermilab.

I expect the production of b' quarks at the Large Hadron Collider in a few years to be the acid test for discreteness and to verify the close connection of fundamental physics to the mathematical properties of the finite simple groups.

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References

1. Potter F. Geometrical basis for the Standard Model. *International Journal of Theoretical Physics*, 1994, v.33(2), 279–305.

2. Rowe D.J., Sanders B.C., de Guise H. Representations of the Weyl group and Wigner functions for SU(3). *Journal of Mathematical Physics*, 1999, v. 40(7), 3604–3615.
 3. Schwarz J.H. Update on String Theory. arXiv: astro-ph/0304507.
 4. Urrutia L.F. Flat space modified particle dynamics induced by Loop Quantum Gravity. arXiv: hep-ph/0402271.
 5. Coxeter H.S.M. Regular complex polytopes. Cambridge University Press, Cambridge, 1974, pp. 89–97.
 6. Conway J.H., Sloane N.J.A. Sphere packings, lattices and groups. 3rd ed. Springer-Verlag, New York, 1998.
 7. Penrose R., Rindler W. Spinors and space-time, Volume 1, Reprint edition. Cambridge University Press, Cambridge, 1987, pp. 26–28.
 8. Jones, G., Singerman, D. Complex functions: an algebraic and geometric viewpoint. Cambridge University Press, Cambridge, 1987, pp. 17–53.
 9. Manogue C.A., Dray T. Octonionic Möbius transformations. *Modern Physics Letters v.A14*, 1999, 1243–1256. arXiv: math-ph/9905024.
 10. Kostant B. In *Asterisque*, (Proceedings of the Conference “Homage to Elie Cartan”, Lyons), 1984, p. 13.
 11. Kostant B. The graph of the truncated icosahedron and the last letter of Galois. *Notices of the American Mathematical Society*, 1995, v. 42, 959–968.
 12. Baez J. The octonions. arXiv: math.RA/0105155.
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On Interpretations of Hubble's Law and the Bending of Light

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Currently, Hubble's law is often considered as the observational evidence of an expanding universe. It is shown that Hubble's Law need not be related to the notion of Doppler redshifts of the light from receding Galaxies. In the derivation of the receding velocity, an implicit assumption, which implies no expansion, must be used. Moreover, the notion of receding velocity is incompatible with the local light speeds used in deriving the light bending. The notion of an expanding universe is based on an unverified assumption that a local distance in a physical space is similar to that of a mathematical Riemannian space embedded in a higher dimensional flat space, and thus the physical meaning of coordinates would necessarily depend on the metric. However, this assumption has been proven as theoretically invalid. In fact, a physical space necessarily has a frame of reference, which has a Euclidean-like structure that is independent of the yet to be determined physical metric and thus cannot be such an embedded space. In conclusion, the notion of an expanding universe could be just a mathematical illusion.

1 Introduction

Currently, Hubble's law is often considered as the observational evidence of the expanding universe. This is done by considering Hubble's law essentially as a manifestation of the Doppler red shift of the light from the receding Galaxies [1]. Thus, the further a galaxies is from the Milky Way, the faster it appears to receding. However, Hubble himself rejected this interpretation and concluded in 1936 that the Galaxies are actually stationary [2]. In view of the fact that this interpretation of relating to the receding velocities is far from perfect [3], perhaps, it would be useful to reexamine how solid is such an interpretation in terms of general relativity and physics.

It will be shown that Hubble's Law need not be related to the Doppler redshifts of the light from receding Galaxies (section 2). It is pointed out, in the derivation of the receding velocity, an implicit assumption, which implies no expansion, must be used (section 3). Moreover, the receding velocity is incompatible with the light speeds used in deriving the light bending (section 4). In short, the notion of expanding universe is a production due to confusing notion of the coordinates and also due to inadequate understanding of a physical space. Thus, such a universe is unlikely related to the reality (section 5).

2 Hubble's law

Hubble discovered from light emitted by near by galaxies that the redshifts S are linearly proportion to the present distance L from the Milky Way as follows:

$$S = HL \quad (1)$$

where H is the Hubble constant although the redshifts of distant galaxies will deviate from this linear law with a slightly different constant. In terms of general relativity, it is well known that this law can be derived with the following metric [1, 3],

$$ds^2 = -d\tau^2 + a^2(\tau)(dx^2 + dy^2 + dz^2), \quad (2)$$

since

$$S = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\omega_1}{\omega_2} - 1 = \frac{a(\tau_2)}{a(\tau_1)} - 1, \quad (3)$$

where ω_1 is the frequency of a photon emitted at event P_1 at time τ_1 , and ω_2 is the frequency of the photon observed at P_2 at time τ_2 [1]. Furthermore, for nearby galaxies, one has

$$a(\tau_2) \simeq a(\tau_1) + (\tau_2 - \tau_1)\dot{a}. \quad (4)$$

If

$$(\tau_2 - \tau_1) = L = \int_1^2 \sqrt{dx^2 + dy^2 + dz^2}, \quad (5)$$

then

$$S = \frac{\dot{a}}{a} L = HL, \quad \text{and} \quad H = \frac{\dot{a}}{a}. \quad (6)$$

Formula (5) is compatible with the calculation in the bending of light. Please note that Hubble's Law need not be related to the Doppler redshifts. Understandably, Hubble rejected such an interpretation himself [2]. In fact, there is actually no receding velocity since L is fixed (i. e., $dL/d\tau = 0$).

3 Hubble's law and the Doppler redshifts

On the other hand, if one chooses to define the distance between two points as

$$R = \int_1^2 a(\tau)\sqrt{dx^2 + dy^2 + dz^2} = a(\tau)L, \quad (7)$$

then

$$v = \frac{dR}{d\tau} = \frac{da}{d\tau}L + \frac{dL}{d\tau}a = \frac{da}{d\tau} \frac{R}{a} = HR, \quad \text{if } \frac{dL}{d\tau} = 0. \quad (8)$$

According to relation (7), v would be the receding velocity. Note also that according to (7), (5) would have to change into $(\tau_2 - \tau_1) = R$, and (1) into $S = HR$. Thus,

$$v = S. \quad (9)$$

This means that the redshifts could be superficially considered as a Doppler effect. Thus, whether Hubble's Law represents the effects of an expanding universe is a matter of the interpretation of the local distance. From the above analysis, the crucial point is what is a valid physical velocity in a physical space.

It should be noted that $dL/dt = 0$ means that the space coordinates are independent of the metric. In other words, the physical space has a Euclidean-like structure [4], which is independent of time. However, since L between any two space-points is fixed, the notion of an expanding universe, if it means anything, is just an illusion. Moreover, the validity of (7) as the physical distance has no known experimental supports since it is not really measurable (see section 5). Moreover, a problem is that the notion of velocity in (8) would be incompatible with the light speeds in the calculation of light bending experiment.

4 The coordinates of an Einstein physical space

In mathematics, the Riemannian space is often embedded in a higher dimensional flat space [5]. Then the coordinates dx^μ are determined by the metric through the metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \text{or} \quad -g_{00}dt^2 + g_{ij} dx^i dx^j \quad (10)$$

such as the surface of a sphere in a three-dimensional Euclidean space. For a physical space, however, there are insufficient conditions to do so. Since the metric is a variable function, it is impossible to determine the coordinates with the metric. In view of this, the coordinates must be physically independent of the metric. As shown in metric (2), a physical space has a Euclidean-like structure as a frame of reference.⁽¹⁾ Moreover, it has been proven from the theoretical framework of general relativity [4] that a frame of reference with the Euclidean-like structure must exist for a physical space.

For a spherical mass distribution with the center at the origin, the metric with the isotropic gauge is,

$$ds^2 = -[(1 - Mk/2r)^2 / (1 + Mk/2r)^2] c^2 dt^2 + (1 + Mk/2r)^4 (dx^2 + dy^2 + dz^2), \quad (11)$$

where $k = G/c^2$ ($G = 6.67 \times 10^{-8} \text{erg} \times \text{cm} / \text{gm}^2$), M is the total mass, and $r = \sqrt{x^2 + y^2 + z^2}$. Then, if the equivalence

principle is satisfied, the light speeds are determined by $ds^2 = 0$ [6, 7], i. e.,

$$\frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt} = c \frac{1 - M\kappa/2r}{[1 + M\kappa/2r]^3}. \quad (12)$$

However, such a definition of light speeds is incompatible with the definition of velocity (8) although compatible with (5). Since this light speed is supported by observations, (8) is invalid in physics. Nevertheless, Liu [8] has defined light speeds, which is more compatible with (8), as

$$\frac{\sqrt{g_{ij} dx^i dx^j}}{dt} = c \frac{1 - M\kappa/2r}{1 + M\kappa/2r} \quad (13)$$

for metric (11). However, (13) implies only half of the deflection implied by (12) [6, 7].

The above analysis also explains why many current theorists insist on that the light speeds are not defined even though Einstein defined them clearly in his 1916 paper as well as in his book, *The Meaning of Relativity*. They might argued that the light speeds are not well defined since diffeomorphic metrics give different sets of light speeds for the same frame of reference. However, they should note that Einstein defines light speeds after the assumption that his equivalence principle is satisfied [6, 7]. Different metric for the same frame of reference means only that at most only one of such metrics is physically valid [4], and therefore the definition of light speeds are, in principle, uniquely well-defined.

However, since the problem of a physical valid metric has not been solved, whether a light speed is valid remains a question. Nevertheless, it has been proven that the Maxwell-Newton Approximation gives the valid first order approximation of the physical metric, the first order of the physically valid light speeds are solved [4]. Since metric (11) is compatible with the Maxwell-Newton approximation, the first order of light speed (12) is valid in physics.

Thus, the groundless speculation that local light speeds are not well defined is proven incorrect. In essence, the velocity definition (8), which leads to the notion of the Doppler redshifts, has been rejected by experiments. Nevertheless, some skeptics might prefer to accept formula (6) after light speed (12) is confirmed by the experiment of local light speeds [4].

5 Discussions and Conclusions

A major problem in Einstein's theory, as pointed out by Whitehead [9] and Fock [10], the physical meaning of coordinates is ambiguous and confusing. In view of this, it is understandable that the notion in an embedded Riemannian space is used when the physical nature of the problem is not yet clear.⁽²⁾ A major difference between physics and mathematics is that the coordinates in physics must have physical meaning. Since Einstein is not a mathematician,

his natural step would be to utilize the existing theory of Riemannian space. However, as Whitehead [9] saw, this created a seemingly irreconcilable problem between coordinates of a curved space-time and physics.

Under such a circumstances, the notion of an expanding universe is created while an implicit assumption that restricts the universe as static is also used. This kind of inconsistency is expectedly inevitable because of contradictory principles, Einstein's equivalence principle that requires space-time coordinates have physical meaning and the "principle of covariance" that necessarily means that coordinates are arbitrary, are concurrently used in Einstein's theory [11]. Recently, it is proven [12] that Einstein's "principle of covariance" has no theoretical basis in physics or observational support beyond what is allowed by the principle of general relativity.⁽³⁾

This analysis demonstrates that the Hubble's Law is not necessarily related to the Doppler redshifts. It is also pointed out that the notion of an expanding universe is related to contradictory assumptions and thus is unlikely a physical possibility. Moreover, this kind notion of velocity is incompatible with the light speeds used in the calculation of light bending [6, 7].

In Einstein's theory of measurement, a local distance in a physical space is assumed to be similar to that of a mathematical Riemannian space embedded in a higher dimensional flat space, and thus the physical meaning of coordinates would necessarily depend on the metric. Recently, this unverified assumption is proven to be inconsistent with Einstein's notion of space contractions [13]. In other words, this unverified assumption contradicts Einstein's equivalence principle that the local space of a particle at free falling must be locally Minkowskian [7], from which he obtained the time dilation and space contractions.

In conclusion, the notion of an expanding universe is unlikely a physical reality, although metric (2) is only a model among other possibilities. Currently, there are three theoretical explanations for the cause to observed red shifts. They are: (1) the expanding universe; (2) Doppler redshifts; and (3) gravitational redshifts. In this paper, it has been shown that the current receding velocity of an expanding universe is only a theoretical illusion and is unrelated to the Doppler redshifts. If the notion of expanding universe cannot be explained satisfactorily, it is difficult to imagine that Doppler effects are the cause of observed Hubble's law. Moreover, this law also cannot be explained in terms of gravitational redshifts.

Then, one may ask if the observed gravitational redshifts are not due to an expanding universe, what causes such redshifts that are roughly proportional to the distances from the observer. One possibility is that the scatterings of a light ray along its path to the observer. In physics, it is known that different scatterings are common causes for losing energy of a particle, and for the case of photons it means redshifts. Since such an effect is so small, it must be the scattering of

a weak field. In fact, the inelastic scattering of light by the gravitational field has been speculated [14]. Unfortunately, to test such a conjecture is not possible because no current theory of gravity is capable of handling the inelastic scatterings of lights.

At present, Einstein's equation even does not have any dynamic solution [15, 16]. Thus, to solve this puzzle rigorously seems surely in the remote future. Nevertheless, the assumption that observed redshifts could be due to inelastic scatterings may help to explain some puzzles of observed facts [17]. For instance, it is known that younger objects such as star forming galaxies have higher intrinsic redshifts, and objects with the same path length to the observer have much different redshifts while all parts of the object have about the same amount of redshifts.⁽⁴⁾

A noted advancement of the Euclidean-like structure [4] is that notions used in a Euclidean space could be adapted much easier in general relativity. Many things would be calculated as if in a Euclidean space. On the other hand, the speculations related to the notion of an expanding universe [1] would cease to function, and physics should return to normal. Nevertheless, when a transformation between different frames of reference is considered, the physical space is clearly Riemannian as Einstein discovered.

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Endnotes

- ⁽¹⁾ A common problem is overlooking that the metric of a Riemannian space can actually be compatible with the space coordinates with the Euclidean-like structure. For example, the Schwarzschild solution in quasi-Minkowskian coordinates [18; p. 181] is,

$$ds^2 = -(1 - 2M\kappa/r)c^2 dt^2 + (1 - 2M\kappa/r)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1a)$$

where (r, θ, φ) transforms to (x, y, z) by,

$$\begin{aligned} x &= r \sin \theta \cos \varphi, & y &= r \sin \theta \sin \varphi, \\ \text{and } z &= r \cos \theta. \end{aligned} \quad (1b)$$

Coordinate transformation (1b) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace determined by the metric. Metric (1a) and Euclidean-like structure (1b) are complementary to each other in the Einstein space. These space-time coordinates form not just a mathematical coordinate system

since a light speed ($ds^2 = 0$) is defined in terms of dx/dt , dy/dt , and dz/dt [19].

- (2) In the initial development of Riemannian geometry, the metric was identified formally with the notion of distance in analogy as the case of the Euclidean space. Such geometry is often illustrated with the surface of a sphere, a subspace embedded in a flat space [5]. Then, the distance is determined by the flat space and can be measured with a static method. For a general case, however, the issue of measurement was not addressed before Einstein's theory. In general relativity, according to Einstein's equivalence principle, the local distance represents the space contraction [7, 19], which is actually measured in a free fall local space [13]. Thus, this is a dynamic measurement since the measuring instrument is in a free fall state under the influence of gravity, while the Euclidean-like structure determines the static distance between two points in a frame of reference. Einstein's error is that he overlooked the free fall state, and thus has mistaken this dynamic local measurement as a static measurement.
- (3) If the "covariance principle" was valid, it has been shown that the "event of horizon" for a black hole could be just any arbitrary constant [20]. Zhou [21] is probably the earliest who spoke out against the "principle of covariance" and he pointed out, "The concept that coordinates don't matter in the interpretation of Einstein's theory necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory." More recently, Morrison [12] commented that Einstein's "covariance principle" discontinuously separates special relativity from general relativity.
- (4) These two types of puzzles would be very difficult to be explained in terms of an expanding universe alone. One might object the scattering of gravitational field on the ground that the photon flight path would be deviated and the images blurred. However, although the scattering by random objects would make blurred images, it is not clear this is the case for a scattering by a weak field. Moreover, since the scattering in the path of photons by the weak gravitational field is very weak, the deviation from the path would not be noticeable, and this is different from the gravitational lenses effects that can be directly observed.

7. Einstein A. The Meaning of Relativity (1921). Princeton Univ. Press, 1954.
8. Liu Liao. General Relativity. High Education Press, Shanghai, 1987, pp.26–30.
9. Whitehead A. N. The Principle of Relativity. Cambridge Univ. Press, Cambridge, 1922.
10. Fock V.A. The Theory of Space Time and Gravitation. Pergamon Press, New York, 1964.
11. Lo C. Y. *Phys. Essays*, 2002, v.15(3), 303–321.
12. Lo C. Y. *Phys. Essays*, 2005, v.18(1).
13. Lo C. Y. *Phys. Essays*, 2005, v.18(4).
14. Lo C. Y. *Phys. Essays*, 1999, v.12(2), 226–241.
15. Lo C. Y. *Astrophys. J.* 1995, v.455, 421–428.
16. Lo C. Y. *Phys. Essays*, 2000, v.13(4), 527–539.
17. Arp H. *Progress in Physics*, 2005, v.3, 3–6.
18. Weinberg S. Gravitation and Cosmology. John Wiley Inc., New York, 1972.
19. Einstein A., Lorentz H. A., Minkowski H., and Weyl H. The Principle of Relativity. Dover, New York, 1923.
20. Lo C. Y. *Chin. Phys.*, 2004, v.13(2), 159.
21. Zhou (Chou) Peiyuan. On coordinates and coordinate transformation in Einstein's theory of gravitation. *Proc. of the Third Marcel Grossmann Meeting on Gen. Relativ.*, ed. Hu Ning, Science Press and North Holland, 1983, 1–20.

References

1. Wald R. M. General Relativity. University of Chicago Press, Chicago, 1984.
2. Whitrow G. J. Edwin Powell Hubble. *Dictionary of Scientific Biography*, New York, Charles Scribner's Sons, v.5, 1972, p.532.
3. Ohanian H. C. and Ruffini R. Gravitation and Spacetime. Norton, New York, 1994.
4. Lo C. Y. *Chinese J. of Phys.*, 2003, v.41(4), 332.
5. Dirac P. A. M. General Theory of Relativity. John Wiley, New York, 1975.
6. Lorentz H. A. *Proc. K. Ak. Amsterdam*, 1900, v.8, 603.

Unmatter Entities inside Nuclei, Predicted by the Brightsen Nucleon Cluster Model

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Applying the R. A. Brightsen Nucleon Cluster Model of the atomic nucleus we discuss how unmatter entities (the conjugations of matter and antimatter) may be formed as clusters inside a nucleus. The model supports a hypothesis that antimatter nucleon clusters are present as a parton (*sensu* Feynman) superposition within the spatial confinement of the proton (${}^1\text{H}_1$), the neutron, and the deuteron (${}^1\text{H}_2$). If model predictions can be confirmed both mathematically and experimentally, a new physics is suggested. A proposed experiment is connected to orthopositronium annihilation anomalies, which, being related to one of known unmatter entity, orthopositronium (built on electron and positron), opens a way to expand the Standard Model.

1 Introduction

According to Smarandache [1, 2, 3], following neutrosophy theory in philosophy and set theory in mathematics, the union of matter $\langle A \rangle$ and its antimatter opposite $\langle \text{Anti}A \rangle$ can form a neutral entity $\langle \text{Neut}A \rangle$ that is neither $\langle A \rangle$ nor $\langle \text{Anti}A \rangle$. The $\langle \text{Neut}A \rangle$ entity was termed “unmatter” by Smarandache [1] in order to highlight its intermediate physical constitution between matter and antimatter. Unmatter is formed when matter and antimatter baryons intermingle, regardless of the amount of time before the conjugation undergoes decay. Already Bohr long ago predicted the possibility of unmatter with his principle of complementarity, which holds that nature can be understood in terms of concepts that come in complementary pairs of opposites that are inextricably connected by a Heisenberg-like uncertainty principle. However, not all physical union of $\langle A \rangle$ with $\langle \text{Anti}A \rangle$ must form unmatter. For instance, the charge quantum number for the electron (e^-) and its antimatter opposite positron (e^+) make impossible the formation of a charge neutral state — the quantum situation must be either (e^-) or (e^+).

Although the terminology “unmatter” is unconventional, unstable entities that contain a neutral union of matter and antimatter are well known experimentally for many years (e. g. pions, pentaquarks, positronium, etc.). Smarandache [3] presents numerous additional examples of unmatter that conform to formalism of quark quantum chromodynamics, already known since the 1970’s. The basis that unmatter does exist comes from the 1970’s experiments done at Brookhaven and CERN [4–8], where unstable unmatter-like entities were found. Recently “physicists suspect they have created the first molecules from atoms that meld matter with antimatter. Allen Mills of the University of California, Riverside, and his colleagues say they have seen telltale signs of positronium molecules, made from two positronium atoms” [9, 10]. A bound and quasi-stable unmatter baryon-

ium has been verified experimentally as a weak resonance between a proton and antiproton using a Skyrme-type model potential. Further evidence that neutral entities derive from union of opposites comes from the spin induced magnetic moment of atoms, which can exist in a quantum state of both spin up and spin down at the same time, a quantum condition that follows the superposition principal of physics. In quantum physics, virtual and physical states that are mutually exclusive while simultaneously entangled, can form a unity of opposites $\langle \text{Neut}A \rangle$ via the principle of superposition.

Our motivation for this communication is to the question: would the superposition principal hold when mass symmetrical and asymmetrical matter and antimatter nucleon wavefunctions become entangled, thus allowing for possible formation of macroscopic “unmatter” nucleon entities, either stable or unstable? Here we introduce how the novel Nucleon Cluster Model of the late R. A. Brightsen [11–17] does predict formation of unmatter as the product of such a superposition between matter and antimatter nucleon clusters. The model suggests a radical hypothesis that antimatter nucleon clusters are present as a hidden parton type variable (*sensu* Feynman) superposed within the spatial confinement of the proton (${}^1\text{H}_1$), the neutron, and the deuteron (${}^1\text{H}_2$). Because the mathematics involving interactions between matter and antimatter nucleon clusters is not developed, theoretical work will be needed to test model predictions. If model predictions can be experimentally confirmed, a new physics is suggested.

2 The Brightsen Nucleon Cluster Model to unmatter entities inside nuclei

Of fundamental importance to the study of nuclear physics is the attempt to explain the macroscopic structural phenomena of the atomic nucleus. Classically, nuclear structure mathematically derives from two opposing views: (1) that the proton [P] and neutron [N] are independent (unbound) interacting

Matter Clusters → Antimatter Clusters ↓	[NP] Deuteron i Stable	[NPN] Triton j Beta-unstable	[PNP] Helium-3 k Stable	[NN] Di-Neutron l	[PP] Di-Proton m	[NNN] Tri-Neutron n	[PPP] Tri-Proton o
[N̂P̂] a Stable		[N] [NP] [N̂P̂]	[P] [NP] [N̂P̂]	Pions (q q̂)	Pions (q q̂)	[N] [NN] [N̂P̂]	[P] [N̂P̂] [PP]
[N̂P̂N̂] b Beta-unstable	[N̂] [NP] [N̂P̂]		Pions (q q̂)	[P̂] [NN] [N̂N̂]	[N̂] [N̂P̂] [PP]	Pions (q q̂)	Tetraquarks (q q q̂ q̂)
[P̂N̂P̂] c Stable	[P̂] [NP] [N̂P̂]	Pions (q q̂)		[P̂] [N̂P̂] [NN]	[N̂] [PP] [P̂P̂]	Tetraquarks (q q q̂ q̂)	Pions (q q̂)
[N̂N̂] d	Pions (q q̂)	[N] [NN] [N̂N̂]	[P] [NP] [N̂N̂]		Tetraquarks (q q q̂ q̂)	[N] [NN] [N̂N̂]	[P] [PP] [N̂N̂]
[P̂P̂] e	Pions (q q̂)	[N] [NP] [P̂P̂]	[P] [NP] [P̂P̂]	Tetraquarks (q q q̂ q̂)		[N] [P̂P̂] [NN]	[P] [PP] [P̂P̂]
[N̂N̂N̂] f	[N̂] [NP] [N̂N̂]	Pions (q q̂)	Tetraquarks (q q q̂ q̂)	[N̂] [NN] [N̂N̂]	[N̂] [N̂N̂] [PP]		Hexaquarks (q q q q̂ q̂ q̂)
[P̂P̂P̂] g	[P̂] [NP] [P̂P̂]	Tetraquarks (q q q̂ q̂)	Pions (q q̂)	[P̂] [P̂P̂] [NN]	[P̂] [P̂P̂] [PP]	Hexaquarks (q q q q̂ q̂ q̂)	

Table 1: Unmatter entities (stable, quasi-stable, unstable) created from union of matter and antimatter nucleon clusters as predicted by the gravity-antigravity formalism of the Brightsen Nucleon Cluster Model. Shaded cells represent interactions that result in annihilation of mirror opposite two- and three- body clusters. Top nucleons within cells show superposed state comprised of three valance quarks; bottom structures show superposed state of hidden unmatter in the form of nucleon clusters. Unstable pions, tetraquarks, and hexaquark unmatter are predicted from union of mass symmetrical clusters that are not mirror opposites. The symbol $\hat{}$ = antimatter, N = neutron, P = proton, q = quark. (Communication with R. D. Davic).

fermions within nuclear shells, or (2) that nucleons interact collectively in the form of a liquid-drop. Compromise models attempt to cluster nucleons into interacting [NP] boson pairs (e.g., Interacting Boson Model-IBM), or, as in the case of the Interacting Boson-Fermion Model (IBFM), link boson clusters [NP] with un-paired and independent nucleons [P] and [N] acting as fermions.

However, an alternative view, at least since the 1937 Resonating Group Method of Wheeler, and the 1965 Close-Packed Spheron Model of Pauling, holds that the macroscopic structure of atomic nuclei is best described as being composed of a small number of interacting boson-fermion nucleon “clusters” (e. g., helium-3 [PNP], triton [NPN], deuteron [NP]), as opposed to independent [N] and [P] nucleons acting as fermions, either independently or collectively. Mathematically, such clusters represent a spatially localized mass-charge-spin subsystem composed of strongly correlated nucleons, for which realistic two- and three body wave functions can be written. In this view, quark-gluon dynamics are

confined within the formalism of 6-quark bags [NP] and 9-quark bags ([PNP] and [NPN]), as opposed to valance quarks forming free nucleons. The experimental evidence in support of nucleons interacting as boson-fermion clusters is now extensive and well reviewed.

One novel nucleon cluster model is that of R. A. Brightsen, which was derived from the identification of mass-charge symmetry systems of isotopes along the Z-N Serge plot. According to Brightsen, all beta-stable matter and antimatter isotopes are formed by potential combinations of two- and three nucleon clusters; e.g., ([NP], [PNP], [NPN], [NN], [PP], [NNN], [PPP], and/or their mirror antimatter clusters [N̂P̂], [P̂N̂P̂], [N̂P̂N̂], [N̂N̂], [P̂P̂], [P̂P̂P̂], [N̂N̂N̂], where the symbol $\hat{}$ here is used to denote antimatter. A unique prediction of the Brightsen model is that a stable union must result between interaction of mass asymmetrical matter (positive mass) and antimatter (negative mass) nucleon clusters to form protons and neutrons, for example the interaction between matter [PNP] + antimatter

[N⁺P⁻]. Why union and not annihilation of mass asymmetrical matter and antimatter entities? As explained by Brightsen, independent (unbound) neutron and protons do not exist in nuclear shells, and the nature of the mathematical series of cluster interactions (3 [NP] clusters = 1[NPN] cluster + 1 [PNP] cluster), makes it impossible for matter and antimatter clusters of identical mass to coexist in stable isotopes. Thus, annihilation cannot take place between mass asymmetrical two- and three matter and antimatter nucleon clusters, only strong bonding (attraction).

Here is the Table that tells how unmatter may be formed from nucleon clusters according to the Brightsen model.

3 A proposed experimental test

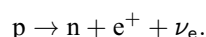
As known, Standard Model of Quantum Electrodynamics explains all known phenomena with high precision, aside for anomalies in orthopositronium annihilation, discovered in 1987.

The Brightsen model, like many other models (see References), is outside the Standard Model. They all pretend to expand the Standard Model in one or another way. Therefore today, in order to judge the alternative models as true or false, we should compare their predictions to orthopositronium annihilation anomalies, the solely unexplained by the Standard Model. Of those models the Brightsen model has a chance to be tested in such way, because it includes unmatter entities (the conjugations of particles and anti-particles) inside an atomic nucleus that could produce effect in the forming of orthopositronium by β^+ -decay positrons and its annihilation.

In brief, the anomalies in orthopositronium annihilation are as follows.

Positronium is an atom-like orbital system that includes an electron and its anti-particle, positron, coupled by electrostatic forces. There are two kinds of that: parapositronium ^SPs, in which the spins of electron and positron are oppositely directed and the summary spin is zero, and orthopositronium ^TPs, in which the spins are co-directed and the summary spin is one. Because a particle-antiparticle (unmatter) system is unstable, life span of positronium is rather small. In vacuum, parapositronium decays in $\tau \simeq 1.25 \times 10^{-10}$ s, while orthopositronium is $\tau \simeq 1.4 \times 10^{-7}$ s after the birth. In a medium the life span is even shorter because positronium tends to annihilate with electrons of the media.

In laboratory environment positronium can be obtained by placing a source of free positrons into a matter, for instance, one-atom gas. The source of positrons is β^+ -decay, self-triggered decays of protons in neutron-deficient atoms*

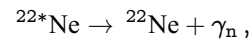


Some of free positrons released from β^+ -decay source

*It is also known as positron β^+ -decay. During β^- -decay in nucleus neutron decays $n \rightarrow p + e^- + \bar{\nu}_e$.

into gas quite soon annihilate with free electrons and electrons in the container's walls. Other positrons capture electrons from gas atoms thus producing orthopositronium and parapositronium (in 3:1 statistical ratio). Time spectrum of positrons (number of positrons vs. life span) is the basic characteristic of their annihilation in matter.

In inert gases the time spectrum of annihilation of free positrons generally reminds of exponential curve with a plateau in its central part, known as "shoulder" [29, 30]. In 1965 Osmon published [29] pictures of observed time spectra of annihilation of positrons in inert gases (He, Ne, Ar, Kr, Xe). In his experiments he used ²²NaCl as a source of β^+ -decay positrons. Analyzing the results of the experiments, Levin noted that the spectrum in neon was peculiar compared to those in other one-atom gases: in neon points in the curve were so widely scattered, that presence of a "shoulder" was unsure. Repeated measurements of temporal spectra of annihilation of positrons in He, Ne, and Ar, later accomplished by Levin [31, 32], have proven existence of anomaly in neon. Specific feature of the experiments done by Osmon, Levin and some other researchers in the UK, Canada, and Japan is that the source of positrons was ²²Na, while the moment of birth of positron was registered according to γ_n -quantum of decay of excited ^{22*}Ne



from one of products of β^+ -decay of ^{22*}Na.

In his experiments [33, 34] Levin discovered that the peculiarity of annihilation spectrum in neon (abnormally wide scattered points) is linked to presence in natural neon of substantial quantity of its isotope ²²Ne (around 9%). Levin called this effect *isotope anomaly*. Temporal spectra were measured in neon environments of two isotopic compositions: (1) natural neon (90.88% of ²⁰Ne, 0.26% of ²¹Ne, and 8.86% of ²²Ne); (2) neon with reduced content of ²²Ne (94.83% of ²⁰Ne, 0.22% of ²¹Ne, and 4.91% of ²²Ne). Comparison of temporal spectra of positron decay revealed: in natural neon (the 1st composition) the shoulder is fuzzy, while in neon poor with ²²Ne (the 2nd composition) the shoulder is always clearly pronounced. In the part of spectrum, to which ^TPs-decay mostly contributes, the ratio between intensity of decay in poor neon and that in natural neon (with much isotope ²²Ne) is 1.85 ± 0.1 [34].

Another anomaly is substantially higher measured rate of annihilation of orthopositronium (the value reciprocal to its life span) compared to that predicted by QED.

Measurement of orthopositronium annihilation rate is among the main tests aimed to experimental verification of QED laws of conservation. In 1987 thanks to new precision technology a group of researchers based in the University of Michigan (Ann Arbor) made a breakthrough in this area. The obtained results showed substantial gap between experiment and theory. The anomaly that the Michigan group revealed

was that measured rates of annihilation at $\lambda_{T(\text{exp})} = 7.0514 \pm 0.0014 \mu\text{s}^{-1}$ and $\lambda_{T(\text{exp})} = 7.0482 \pm 0.0016 \mu\text{s}^{-1}$ (with unseen-before precision of 0.02% and 0.023% using vacuum and gas methods [35–38]) were much higher compared to $\lambda_{T(\text{theor})} = 7.00383 \pm 0.00005 \mu\text{s}^{-1}$ as predicted by QED [39–42]. The effect was later called λ_T -anomaly [43].

Theorists foresaw possible annihilation rate anomaly long before the first experiments were accomplished in Michigan. In 1986 Holdom [44] suggested that “mixed type” particles may exist, which being in the state of oscillation stay for some time in our world and for some time in the mirror Universe, possessing negative masses and energies. In the same year Glashow [45] gave further development to the idea and showed that in case of 3-photon annihilation ^TPs will “mix up” with its mirror twin thus producing two effects: (1) higher annihilation rate due to additional mode of decay $^T\text{Ps} \rightarrow \text{nothing}$, because products of decay passed into the mirror Universe can not be detected; (2) the ratio between orthopositronium and parapositronium numbers will decrease from $^T\text{Ps} : ^S\text{Ps} = 3:1$ to $1.5:1$. But at that time (in 1986) Glashow concluded that no interaction is possible between our-world and mirror-world particles.

On the other hand, by the early 1990’s these theoretic studies encouraged many researchers worldwide for experimental search of various “exotic” (unexplained in QED) modes of ^TPs -decay, which could lit some light on abnormally high rate of decay. These were, to name just a few, search for $^T\text{Ps} \rightarrow \text{nothing}$ mode [46], check of possible contribution from 2-photon mode [47–49] or from other exotic modes [50–52]. As a result it has been shown that no exotic modes can contribute to the anomaly, while contribution of $^T\text{Ps} \rightarrow \text{nothing}$ mode is limited to 5.8×10^{-4} of the regular decay.

The absence of theoretical explanation of λ_T -anomaly encouraged Adkins et al. [53] to suggest experiments made in Japan [54] in 1995 as an alternative to the basic Michigan experiments. No doubt, high statistical accuracy of the Japanese measurements puts them on the same level with the basic experiments [35–38]. But all Michigan measurements possessed the property of a “full experiment”, which in this particular case means no external influence could affect wave function of positronium. Such influence is inevitable due to electrodynamic nature of positronium and can be avoided only using special technique. In Japanese measurements [54] this was not taken into account and thus they do not possess property of “full experiment”. Latest experiments of the Michigans [55], so-called *Resolution of Orthopositronium-Lifetime Puzzle*, as well do not possess property of “full experiment”, because the qualitative another statement included external influence of electromagnetic field [56, 57].

As early as in 1993 Karshenboim [58] showed that QED had actually run out of any of its theoretical capabilities to explain orthopositronium anomaly.

Electric interactions and weak interactions were joined into a common electroweak interaction in the 1960’s by com-

monly Salam, Glashow, Weinberg, etc. Today’s physicists attempt to join electroweak interaction and strong interaction (unfinished yet). They follow an intuitive idea that forces, connecting electrons and a nucleus, and forces, connecting nucleons inside a nucleus, are particular cases of a common interaction. That is the basis of our claim. If that is true, our claim is that orthopositronium atoms born in neon of different isotope contents (^{22}Ne , ^{21}Ne , ^{20}Ne) should be different from each other. There should be an effect of “inner” structure of neon nuclei if built by the Brightsen scheme, because the different proton-neutron contents built by different compositions of nucleon pairs. As soon as a free positron drags an electron from a neon atom, the potential of electro-weak interactions have changed in the atom. Accordingly, there in the nucleus itself should be re-distribution of strong interactions, than could be once as the re-building of the Brightsen pairs of nucleons there. So, lost electron of ^{22}Ne should have a different “inner” structure than that of ^{21}Ne or ^{20}Ne . Then the life span of orthopositronium built on such electrons should be as well different.

Of course, we can only qualitatively predict that difference, because we have no exact picture of what really happens inside a “structurized” nucleus. Yet only principal predictions are possible there. However even in such case we vote for continuation of “isotope anomaly” experiments with orthopositronium in neon of different isotope contents. If further experiments will be positive, it could be considered as one more auxiliary proof that the Brightsen model is true.

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References

1. Smarandache F. Matter, antimatter, and unmatter. *Infinite Energy*, 2005, v. 11, issue 62, 50–51.
 2. Smarandache F. A new form of matter – unmatter, composed of particles and anti-particles. *Progress in Physics*, 2005, v. 1, 9–11.
 3. Smarandache F. Verifying unmatter by experiments, more types of unmatter, and a quantum chromodynamics formula. *Progress in Physics*, 2005, v. 2, 113–116.
- UNMATTER BASIS EXPERIMENTS
4. Gray L., Hagerty P., Kalogeropoulos T. E. *Phys. Rev. Lett.*, 1971, v. 26, 1491–1494.
 5. Carrol A. S., Chiang I.-H., Kucia T. F., Li K. K., Mazur P. O., Michael D. N., Mockett P., Rahm D. C., Rubinstein R. *Phys. Rev. Lett.*, 1974, v. 32, 247–250.
 6. Kalogeropoulos T. E., Vayaki A., Grammatikakis G., Tsilimigras T., Simopoulou E. *Phys. Rev. Lett.*, 1974, v. 33, 1635–1637.
 7. Chapiro I. S. *Physics-Uspexhi*, 1973, v. 109, 431.

8. Bogdanova L.N., Dalkarov O.D., Chapiro I.S. *Annals of Physics*, 1974, v. 84, 261–284.
 9. Cassidy D.B., Deng S.H.M., Greaves R.G., Maruo T., Nishiyama N., Snyder J.B., Tanaka H.K.M., Mills A.P. Jr. *Phys. Rev. Lett.*, 2005, v. 95, No. 19, 195006.
 10. Ball Ph. *News Nature*, 22 November 2005.
- THE BRIGHTSEN MODEL
11. Brightsen R.A. Nucleon cluster structures in beta-stable nuclides. *Infinite Energy*, 1995, v. 1, no. 4, 55–56.
 12. Brightsen R.A. Correspondence of the Nucleon Cluster Model with the Periodic Table of Elements. *Infinite Energy*, 1995/96, v. 1(5/6), 73–74.
 13. Brightsen R.A. Correspondence of the Nucleon Cluster Model with the Classical Periodic Table of Elements. *J. New Energy*, 1996, v. 1(1), 75–78.
 14. Brightsen R.A. The Nucleon Cluster Model and the Periodic Table of Beta-Stable Nuclides. 1996 (available online at <http://www.brightsenmodel.phoenixrising-web.net>).
 15. Brightsen R.A. The nucleon cluster model and thermal neutron fission. *Infinite Energy*, 2000, v. 6(31), 55–63.
 16. Brightsen R.A., Davis R. Appl. of the Nucleon Cluster Model to experimental results. *Infinite Energy*, 1995, v. 1(3), 13–15.
 17. Bass R.W. Experimental evidence favoring Brightsen's nucleon cluster model. *Infinite Energy*, 1996, v. 2(11), 78–79.
- SOME REVIEW PAPERS ON CLUSTER MODELS
18. Buck B., Merchant A.C., Perez S.M. *Phys. Rev. C*, 2005, v. 71(1), 014311–15.
 19. Akimune H., Yamagata T., Nakayama S., Fujiwara M., Fushimi K., Hara K., Hara K.Y., Ichihara K., Kawase K., Matsui K., Nakanishi K., Shiokawa A., Tanaka M., Utsunomiya H., and Yosoi M. *Physics of Atomic Nuclei*, 2004, v. 67(9), 1721–1725.
 20. *Clustering Aspects of Nuclear Structure and Dynamics, Cluster '99, Proc. of the 7th Intern. Conf.*, 1999, Rab (Croatia).
 21. Wuosmaa A.H., Betts R.R., Freer M., and Fulton B.R. *Annual Review of Nuclear and Particle Science*. 1995, v. 45, 89–131.
 22. Bromley D.A. *Clust. Aspects of Nucl. Structure, Proc. of the 4th Intern. Conf.*, Chester (UK), D. Reidel Publ., Dordrecht.
 23. Horiuchi H. and Ikeda K. *Cluster Models and Other Topics, Intern. Rev. of Nucl. Physics*, 1986, v. 4, World Scientific, Singapore, 1–259.
- CLUSTER MODELS THAT THE BRIGHTSEN MODEL BUILDS ON
24. Wheeler J.A. *Phys. Rev.*, 1937, v. 52, 1083.
 25. Wheeler J.A. *Phys. Rev.*, 1937, v. 52, 1107.
 26. Pauling L. *Proc. Natl. Acad. Sci. USA*, 1965, v. 54, no. 4, 989.
 27. Pauling L. *Science*, 1965, v. 150, no. 3694, 297.
 28. Pauling L. *Revue Roumain de Physique*, 1966, v. 11, no. 9/10, 825–833.
- ANOMALIES OF ORTHOPOSITRONIUM ANNIHILATION
29. Osmon P.E. *Physical Review B*, 1965, v. 138, 216.
 30. Tao S.J., Bell J., and Green J.H. *Proceedings of the Physical Society*, 1964, v. 83, 453.
 31. Levin B.M. and Shantarovich V.P. *High Energy Chemistry*, 1977, v. 11(4), 322–323.
 32. Levin B.M. *Soviet J. Nucl. Physics*, 1981, v. 34(6), 917–918.
 33. Levin B.M. and Shantarovich V.P. *Soviet J. Nucl. Physics*, 1984, v. 39(6), 855–856.
 34. Levin B.M., Kochenda L.M., Markov A.A., and Shantarovich V.P. *Soviet J. Nucl. Physics*, 1987, v. 45(6), 1119–1120.
 35. Gidley D.W., Rich A., Sweetman E., and West D. *Physical Review Letters*, 1982, v. 49, 525–528.
 36. Westbrook C.I., Gidley D.W., Conti R.S., and Rich A. *Physical Review Letters*, 1987, v. 58, 1328–1331.
 37. Westbrook C.I., Gidley D.W., Conti R.S., and Rich A. *Physical Review A*, 1989, v. 40, 5489–5499.
 38. Nico J.S., Gidley D.W., Rich A., and Zitzewitz P.W. *Physical Review Letters*, 1990, v. 65, 1344–1347.
 39. Caswell W.E. and Lepage G.P. *Phys. Rev. A*, 1979, v. 20, 36.
 40. Adkins G.S. *Ann. Phys. (N.Y.)*, 1983, v. 146, 78.
 41. Adkins G.S., Salahuddin A.A., and Schalm K.E. *Physical Review A*, 1992, v. 45, 3333–3335.
 42. Adkins G.S., Salahuddin A.A., and Schalm K.E. *Physical Review A*, 1992, v. 45, 7774–7780.
 43. Levin B.M. *Physics of Atomic Nuclei*, 1995, v. 58(2), 332–334.
 44. Holdom B. *Physics Letters B*, 1986, v. 166, 196–198.
 45. Glashow S.L. *Physics Letters B*, 1986, v. 167, 35–36.
 46. Atoyan G.S., Gninenko S.N., Razin V.I., and Ryabov Yu. V. *Physics Letters B*, 1989, v. 220, 317–320.
 47. Asai S., Orito S., Sanuki T., Yasuda M., and Yokoi T. *Physical Review Letters*, 1991, v. 66, 1298–1301.
 48. Gidley D.W., Nico J.S., and Skalsey M. *Physical Review Letters*, 1991, v. 66, 1302–1305.
 49. Al-Ramadhan A.H. and Gidley D.W. *Physical Review Letters*, 1994, v. 72, 1632–1635.
 50. Orito S., Yoshimura K., Haga T., Minowa M., and Tsuchiaki M. *Physical Review Letters*, 1989, v. 63, 597–600.
 51. Mitsui T., Fujimoto R., Ishisaki Y., Ueda Y., Yamazaki Y., Asai S., and Orito S. *Phys. Rev. Lett.*, 1993, v. 70, 2265–2268.
 52. Skalsey M. and Conti R.S. *Phys. Rev. A*, 1997, v. 55(2), 984.
 53. Adkins G.S., Melnikov K., and Yelkhovsky A. *Phys. Rev. A*, 1999, v. 60(4), 3306–3307.
 54. Asai S., Orito S., and Shinohara N. *Physics Letters B*, 1995, v. 357, 475–480.
 55. Vallery R.S., Zitzewitz P.W., and Gidley D.W. *Phys. Rev. Lett.*, 2003, v. 90, 203402.
 56. Levin B.M. arXiv: quant-ph/0303166.
 57. Levin B.M. CERN EXT-2004-016.
 58. Karshenboim S.G. *Yadern. Fizika*, 1993, v. 56(12), 155–171.

Fermions as Topological Objects

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A preon-based composite model of the fundamental fermions is discussed, in which the fermions are bound states of smaller entities — primitive charges (preons). The preon is regarded as a dislocation in a dual 3-dimensional manifold — a topological object with no properties, save its unit mass and unit charge. It is shown that the dualism of this manifold gives rise to a hierarchy of complex structures resembling by their properties three families of the fundamental fermions. Although just a scheme for building a model of elementary particles, this description yields a quantitative explanation of many observable particle properties, including their masses. PACS numbers: 12.60.Rc, 12.15.Ff, 12.10.Dm

1 Introduction

The hierarchical pattern observed in the properties of the fundamental fermions (quarks and leptons) points to their composite nature [1], which goes beyond the scope of the Standard Model of particle physics. The particles are grouped into three generations (families), each containing two quarks and two leptons with their electric charges, spins and other properties repeating from generation to generation: the electron and its neutrino, e^- , ν_e , the muon and its neutrino, μ^- , ν_μ , the tau and its neutrino, τ^- , ν_τ , the up and down quarks, $u^{+2/3}$, $d^{-1/3}$, charm and strange, $c^{+2/3}$, $s^{-1/3}$, top and bottom, $t^{+2/3}$, $b^{-1/3}$ (here the charges of quarks are indicated by superscripts). The composite models of quarks and leptons [2] are based on fewer fundamental particles than the Standard Model (usually two or three) and are able to reproduce the above pattern as to the electric and colour charges, spins and, in some cases, the variety of species. However, the masses of the fundamental fermions are distributed in a rather odd way [3]. They cannot be predicted from any application of first principles of the Standard Model; nor has any analysis of the observed data [4] or development of new mathematical ideas [5] yielded an explanation as to why they should have strictly the observed values instead of any others. Even there exist claims of randomness of this pattern [6]. However, the history of science shows that, whenever a regular pattern was observed in the properties of matter (e.g., the periodical table of elements or eight-fold pattern of mesons and baryons), this pattern could be explained by invoking some underlying structures. In this paper we shall follow this lead by assuming that quarks and leptons are bound states of smaller particles, which are usually called “pre-quarks” or “preons” [7]. Firstly, we shall guess at the basic symmetries of space, suggesting that space, as any other physical entity, is dual. We propose that it is this property that is responsible for the emergence

of different types of interactions from a unique fundamental interaction. To be absolutely clear, we have to emphasise that our approach will be based on classical (deterministic) fields, which is opposed to the commonly-held view that quarks and leptons are quantum objects. But we shall see that by using classical fields on small scales we can avoid the problems related to the short-range energy divergences and anomalies, which is the main problem of all quantum field theories.

2 The universe

Let us begin from a few conjectures (postulates) about the basic properties of space:

- P1 Matter is structured, and the number of its structural levels is finite;
- P2 The simplest (and, at the same time, the most complex) structure in the universe is the universe itself;
- P3 The universe is self-contained (by definition);
- P4 All objects in the universe spin (including the universe itself).

The postulate P1 is based on the above mentioned historical experience with the patterns and structures behind them. These patterns are known to be simpler on lower structural levels, which suggests that matter could be structured down to the simplest possible entity with almost no properties. We shall relate this entity to the structure of the entire universe (postulate P2). This is not, of course, a novelty, since considering the universe as a simple uniform object lies in the heart of modern cosmology. The shape (topology) of this object is not derivable from Einstein’s equations, but for simplicity it is usually considered as a hyper-sphere (S^3) of positive, negative or zero curvature. However, taking into account the definition of the universe as a self-contained object (postulate P3), the spherical shape becomes inappropriate, because any sphere has at least two unrelated

hyper-surfaces, which is incompatible with the definition of the uniqueness and self-containedness of the universe. More convenient would be a manifold with a unique hyper-surface, such as the Klein-bottle, K^3 [8]. Similarly to S^3 , it can be of positive, negative or zero curvature. An important feature of K^3 is the unification of its inner and outer surfaces (Fig. 1). In the case of the universe, the unification might well occur on the sub-quark level, giving rise to the structures of elementary particles and, supposedly, resulting in the identification of the global cosmological scale with the local microscopic scale of elementary particles. In Fig. 1b the unification region is marked as Π (primitive particle).

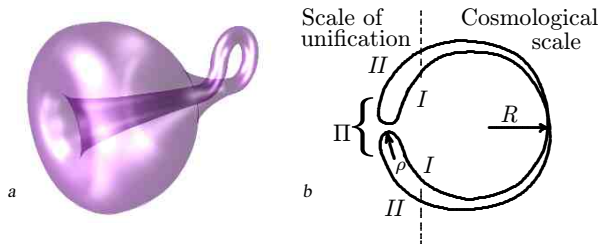


Fig. 1: (a) Klein-bottle and (b) its one-dimensional representation; the “inner” (I) and “outer” (II) hyper-surfaces are unified through the region Π (primitive particle); R and ρ are, respectively, the global and local radii of curvature.

3 The primitive particle

Let us assume that space is smooth and continuous, i.e., that its local curvature cannot exceed some finite value ε : $|\rho|^{-1} < \varepsilon$. Then, within the region Π (Fig. 1b) space will be locally curved “inside-out”. In these terms, the primitive particle can be seen as a dislocation (topological defect) of the medium and, thus, cannot exist independently of this medium. Then, the postulate P4 about the spinning universe gives us an insight into the possible origin of the particle mass. This postulate is not obvious, although the idea of spinning universe was proposed many years ago by A. Zelmanov [9] and K. Gödel [10]. It comes from the common fact that so far non-rotating objects have never been observed.

The universe spinning with its angular velocity ω (of course, if considered from the embedding space) would result in the linear velocity $\pm\omega R$ of the medium in the vicinity of the primitive particle, where R is the global radius of curvature of the universe; and the sign depends on the choice of the referent direction (either inflow or outflow from the inversion region).

Due to the local curvature, ρ^{-1} , in the vicinity of the primitive particle, the spinning universe must give rise to a local acceleration, a_g , of the medium moving through the region Π , which is equivalent to the acceleration of the particle itself. According to Newton’s second law, this acceleration can be described in terms of a force, $F_g =$

$= m_g a_g$, proportional to this acceleration. The coefficient of proportionality between the acceleration and the force corresponds to the inertial mass of the particle. However, for an observer in the coordinate frame of the primitive particle this mass will be perceived as gravitational (m_g) because the primitive particle is at rest in this coordinate frame. Thus, the spinning universe implies the accelerated motion of the primitive particle along its world line (time-axis). If now the particle is forced to move along the spatial coordinates with an additional acceleration a_i , it will resist this acceleration in exactly the same way as it does when accelerating along the time-axis. A force $F_i = m_i a_i$, which is required in order to accelerate the particle, is proportional to a_i with the coefficient of proportionality m_i (inertial mass). But, actually, we can see that within our framework the inertial, m_i , and gravitational, m_g , masses are generated by the same mechanism of acceleration. That is, mass in this framework is a purely inertial phenomenon ($m_i \equiv m_g$).

It is seen that changing the sign of ωR does not change the sign of the second derivative $a_g = \frac{\partial^2(ict)}{\partial t^2}$, i.e., of the “gravitational” force $F_g = m_g a_g$. This is obvious, because the local curvature, ρ^{-1} , is the property of the manifold and does not depend on the direction of motion. By contrast, the first derivative $\frac{\partial(ict)}{\partial t}$ can be either positive or negative, depending on the choice of the referent direction. It would be natural here to identify the corresponding force as electrostatic. For simplicity, in this paper we shall use unit values for the mass and electric charge of the primitive particle, denoting them as m_o and q_o .

In fact, the above mass acquisition scheme has to be modified because, besides the local curvature, one must account for torsion of the manifold (corresponding to the Weyl tensor). In the three-dimensional case, torsion has three degrees of freedom, and the corresponding field can be resolved into three components (six — when both manifestations of space, I and II , are taken into account). It is reasonable to relate these three components to three polarities (colours) of the strong interaction.

Given two manifestations of space, we can resolve the field of the particle into two components, ϕ_s and ϕ_e . To avoid singularities we shall assume that infinite energies are not accessible in nature. Then, since it is an experimental fact that energy usually increases as distance decreases, we can hypothesise that the energy of both ϕ_e and ϕ_s , after reaching a maximum, decays to zero at the origin. The simplest form for the split field that incorporates the requirements above is the following:

$$\begin{aligned} F &= \phi_s + \phi_e, \\ \phi_s &= s \exp(-\rho^{-1}), \quad \phi_e = -\phi_s'(\rho). \end{aligned} \quad (1)$$

Here the signature $s = \pm 1$ indicates the sense of the interaction (attraction or repulsion); the derivative of ϕ_s is taken with respect to the radial coordinate ρ . Far from the

source, the second component of the split field F mimics the Coulomb gauge, whereas the first component extends to infinity being almost constant (similarly to the strong field).

In order to formalise the use of tripolar fields we have to introduce a set of auxiliary 3×3 singular matrices Π^i with the following elements:

$$\pm \pi_{jk}^i = \pm \delta_j^i (-1)^{\delta_j^k}, \quad (2)$$

where δ_j^i is the Kronecker delta-function; the (\pm) -signs correspond to the sign of the charge; and the index i stands for the colour ($i = 1, 2, 3$ or red, green and blue). The diverging components of the field can be represented by reciprocal elements: $\tilde{\pi}_{jk} = \pi_{jk}^{-1}$. Then we can define the (unit) charges and masses of the primitive particles by summation of these matrix elements:

$$\begin{aligned} q_{\Pi} &= \mathbf{u}^T \Pi \mathbf{u}, & \tilde{q}_{\Pi} &= \mathbf{u}^T \tilde{\Pi} \mathbf{u} \\ m_{\Pi} &= |\mathbf{u}^T \Pi \mathbf{u}|, & \tilde{m}_{\Pi} &= |\mathbf{u}^T \tilde{\Pi} \mathbf{u}| \end{aligned} \quad (3)$$

(\mathbf{u} is the diagonal of a unit matrix; \tilde{q}_{Π} and \tilde{m}_{Π} diverge). Assuming that the strong and electric interactions are manifestations of the same entity and taking into account the known pattern [11] of the colour-interaction (two like-charged but unlike-coloured particles are attracted, otherwise they repel), we can write the signature s_{ij} of the chromoelectric interaction between two primitive particles, say of the colours i and j , as:

$$s_{ij} = -\mathbf{u}^T \Pi^i \Pi^j \mathbf{u}. \quad (4)$$

4 Colour dipoles

Obviously, the simplest structures allowed by the tripolar field are the monopoles, dipoles and tripoles, unlike the conventional bipolar (electric) field, which allows only the monopoles and dipoles. Let us first consider the colour-dipole configuration. It follows from (4) that two like-charged particles with unlike-colours will combine and form a charged colour-dipole, g^{\pm} . Similarly, a neutral colour-dipole, g^0 , can also be formed – when the constituents of the dipole have unlike-charges.

The dipoles g^{\pm} and g^0 are classical oscillators with the double-well potential $V(\rho)$, Fig. 2, derived from the split field (1). The oscillations take place within the region $\rho \in (0, \rho_{\max})$, with the maximal distance between the components $\rho_{\max} \approx 1.894 \rho_0$ (assuming the initial condition $E_0 = V(0)$ and setting this energy to zero).

Let us assume that the field $F(\rho)$ does not act instantaneously at a distance. Then, we can define the mass of a system with, say, N primitive particles as proportional to the number of these particles, wherever the field flow rate is not cancelled. For this purpose we shall regard the total field flow rate, v_N , of such a system as a superposition of the individual volume flow rates of its N constituents. Then

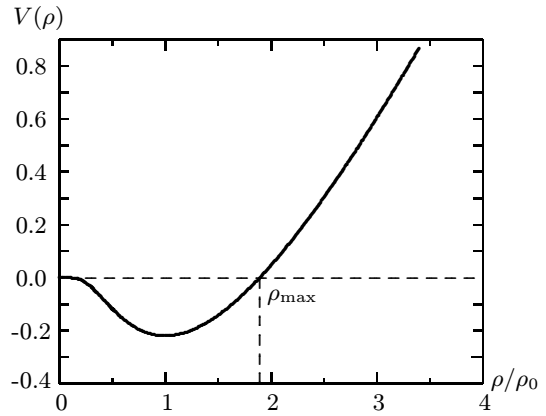


Fig. 2: Equilibrium potential based on the split field (1)

the net mass of the system can be calculated (to a first-order of approximation) as the number of particles, N , times the normalised to unity (Lorentz-additive) field flow rate v_N :

$$m_N = |N| v_N. \quad (5)$$

Here v_N is calculated recursively from

$$v_i = \frac{q_i + v_{i-1}}{1 + |q| |v_{i-1}|}, \quad (6)$$

with $i = 2, \dots, N$ and putting $v_1 = q_1$. Then, when two unlike-charged particles combine (say red and antigreen), the magnitudes of their oppositely directed flow rates cancel each other (resulting in a neutral system). The corresponding, acceleration also vanishes, which is implicit in (5), formalising the fact that the mass of a neutral system is nullified. This formula implies the complete cancellation of masses in the systems with vanishing electric fields, but this is only an approximation because in our case the primitive particles are separated by the average distance ρ_0 , whereas the complete cancellation of flows is possible only when the flow source centres coincide.

In the matrix notation, the positively charged dipole, g_{12}^+ , is represented as a sum of two matrices, Π^1 and Π^2 :

$$g_{12}^+ = \Pi^1 + \Pi^2 = \begin{pmatrix} -1 & +1 & +1 \\ +1 & -1 & +1 \\ 0 & 0 & 0 \end{pmatrix}, \quad (7)$$

with the charge $q_{g_{12}^+} = +2$ and mass $m_{g_{12}^+} \approx 2$ and $\tilde{m}_{g_{12}^+} = \infty$, according to (3). If two components of the dipole are oppositely charged, say, $g_{12}^0 = \Pi^1 + \bar{\Pi}^2$ (of whatever colour combination), then their electric fields and masses are nullified: $q_{g^0} = 0$, $m_{g^0} \approx 0$ (but still $\tilde{m}_{g^0} = \infty$ due to the null-elements in the matrix g^0). The infinities in the expressions for the reciprocal masses of the dipoles imply that neither g^{\pm} nor g^0 can exist in free states (because of their infinite energies). However, in a large ensemble of neutral colour-dipoles g^0 , not only electric but all the chromatic components of the field can be cancelled (statistically). Then, the mass of the neutral

dipole g_{ik}^0 with an extra charged particle Π^l belonging this ensemble but coupled to the dipole, will be derived from the unit mass of Π^l :

$$\begin{aligned} m(\Pi^i, \bar{\Pi}^k, \Pi^l) &= 1, \\ \text{but still } \tilde{m}(\Pi^i, \bar{\Pi}^k, \Pi^l) &= \infty. \end{aligned} \quad (8)$$

The charge of this system will also be derived from the charge of the extra charged particle Π^l .

5 Colour triplets

Three primitive particles with complementary colour-charges will tend to cohere and form a Y-shaped structure (tripole). For instance, by completing the set of colour-charges in the charged dipole [adding the blue-charged component to the system (7)] one would obtain a colour-neutral but electrically charged tripole:

$$Y = \Pi^1 + \Pi^2 + \Pi^3 = \begin{pmatrix} -1 & +1 & +1 \\ +1 & -1 & +1 \\ +1 & +1 & -1 \end{pmatrix},$$

which is colour-neutral at infinity but colour-polarised nearby (because the centres of its constituents do not coincide). Both m and \tilde{m} of the tripole are finite, $m_Y = \tilde{m}_Y = 3 [m_o]$, since all the diverging components of its chromofield are mutually cancelled (converted into the binding energy of the tripole).

6 Doublets of triplets



Fig. 3: The triplets (Y-particles) can combine pairwise, rotated by 180° (a) or 120° (b) with respect to each other.

One can show [12] that two like-charged Y-tripoles can combine pole-to-pole with each other and form a charged doublet $\delta^+ = Y\Lambda$ (Fig. 3a). Here the rotated symbol Λ is used to indicate the rotation of the triplets through 180° with respect to each other, which corresponds to their equilibrium position angle. The marked arm of the symbol Y indicates one of the colours, say, red, in order to visualise mutual orientations of colour-charges in the neighbouring triplets. The charge of the doublet, $q_\delta = +6 [q_o]$, is derived from the charges of its two constituent triplets; the same is applied to its mass: $m_\delta = \tilde{m}_\delta = 6 [m_o]$. Similarly, if two unlike-charged Y-particles are combined, they will form a neutral doublet, $\gamma = Y\bar{\Lambda}$ (Fig. 3b) with $q_\gamma = 0$ and $m_\gamma = \tilde{m}_\gamma = 0$. The shape of the potential well in the vicinity of the doublet allows a certain degree of freedom for its components to rotate oscillating within $\pm 120^\circ$ with respect to their equilibrium position angle (see [12] for details). We shall use the symbols \odot and \ominus to denote the clockwise and anticlockwise rotations.

7 Triplets of triplets

The $\frac{2}{3}\pi$ -symmetry of the tripole allows up to three of them to combine if they are like-charged. Necessarily, they will combine into a loop, denoted hereafter with the symbol e . It is seen that this loop can be found in one of two possible configurations corresponding to two possible directions of rotation of the neighbouring triplets: clockwise, $e_\odot^+ = YYY$, and anticlockwise, $e_\ominus^+ = \bar{Y}\bar{Y}\bar{Y}$. The vertices of the triplets can be directed towards the centre of the structure (Fig. 4a) or outwards (Fig. 4b), but it is seen that these two orientations

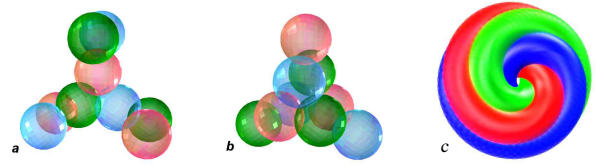


Fig. 4: Three like-charged triplets joined with their vertices directed towards (a) and outwards (b) of the centre of the structure; (c): trajectories of colour charges in this structure.

correspond to different phases of the same structure, with its colour charges spinning around its ring-closed axis. These spinning charges will generate a toroidal (ring-closed) magnetic field which will force them to move along the torus. Their circular motion will generate a secondary (poloidal) magnetic field, contributing to their spin around the ring-axis, and so forth. The corresponding trajectories of colour-charges (currents) are shown in Fig. 4c. This mechanism, known as dynamo, is responsible for generating a self-consistent magnetic field of the triplet e .

To a first order of approximation, we shall derive the mass of the triplet from its nine constituents, suggesting that this mass is proportional to the density of the currents, neglecting the contribution to the mass of the binding and oscillatory energies of the triplets. That is, we put $m_e = 9 [m_o]$ (bearing in mind that the diverging components, \tilde{m}_o , are almost nullified). The charge of the triplet is also derived from the number of its constituents: $q_e = \pm 9 [q_o]$.

8 Hexaplets

Unlike-charged triplets, combined pairwise, can form chains with the following patterns:

$$\begin{aligned} \nu_{e\odot} &= Y\bar{Y} + \bar{\Lambda}\Lambda + Y\bar{Y} + \bar{\Lambda}\Lambda + Y\bar{Y} + \bar{\Lambda}\Lambda + \dots \\ \nu_{e\ominus} &= Y\bar{Y} + \bar{\Lambda}\Lambda + Y\bar{Y} + \bar{\Lambda}\Lambda + Y\bar{Y} + \bar{\Lambda}\Lambda + \dots \end{aligned} \quad (9)$$

corresponding to two possible directions of rotation of the neighbouring triplets with respect to each other. The cycle of rotations repeats after each six consecutive links, making the orientation of the sixth link compatible with (attractive to) the first link by the configuration of their colour-charges.

This allows the closure of the chain in a loop (which we shall call hexaplet and denote as ν_e). The pattern (9) is visualised in Fig. 5a where the antipreons are coded with lighter colours. The corresponding trajectories of charges (currents) are shown in Fig. 5b. They are clockwise or anticlockwise helices, similar to those of the triplet e^- . The hexaplet consists of $n_{\nu_e} = 36$ preons (twelve tripoles); it is electrically neutral and, therefore, almost massless, according to Eq. (3).

Some properties of the simple preon-based structures are summarised in Table 1.

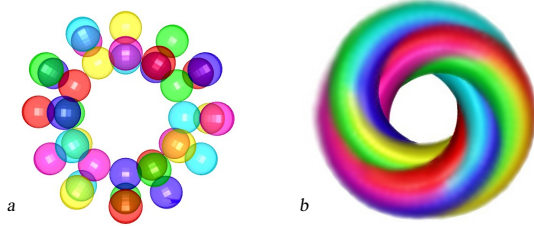


Fig. 5: (a) Structure of the hexaplet $\nu_e = 6\bar{Y}$ and (b) the corresponding helical trajectories (currents) formed by the motions of the hexaplet's colour-charges.

9 Combinations of triplets and hexaplets

The looped structures $e = 3Y$ and $\nu_e = 6\bar{Y}$ can combine with each other, as well as with the simple tripole Y , because of their $\frac{2}{3}\pi$ -symmetry and residual chromaticism. That is, separated from other particles, the structure ν_e will behave like a neutral particle. But, if two such particles approach one another, they will be either attracted or repulsed from each other because of van der Waals forces caused by their residual chromaticism and polarisation. The sign of this interaction depends on the twisting directions of the particles' currents. One can show [12] that the configuration of colour charges in the hexaplet ν_e matches (is attractive to) that of the triplet e if both particles have like-helicities (topological charges). On the contrary, the force between the particles of the same kind is attractive for the opposite helicities ($2e_{\odot\odot}^+$ or $e_{\odot}^+e_{\ominus}^-$) and repulsive for like-helicities ($2e_{\ominus\ominus}^+$ or $e_{\ominus}^+e_{\ominus}^-$). So, the combined effective potential of the system $2e$ with unlike-helicities, will have an attractive inner and repulsive outer region, allowing an equilibrium configuration of the two particles. In the case of like-helicities, both inner and outer regions of the potential are repulsive and the particles e with like-helicities will never combine. This coheres with (and probably explains) the Pauli exclusion principle, suggesting that the helicity (topological charge) of a particle can straightforwardly be related to the quantum notion of spin. This conjecture is also supported by the fact that quantum spin is measured in units of angular momentum (\hbar), and so too — the topological charge in question, which is derived from the rotational motion of the tripoles Y around the ring-closed axis of the triplet e or hexaplet ν_e .

Relying upon the geometrical resemblance between the tripoles Y , triplets e , and hexaplets ν_e and following the pattern replicated on different complexity levels we can deduce how these structures will combine with each other. Obviously, the hexaplet ν_e , formed of twelve tripoles, is geometrically larger than a single tripole. Thus, these two structures can combine only when the former enfolds the latter. The combined structure, which we shall denote as $Y_1 = \nu_e + Y$, will have a mass derived from its 39 constituents: $m_{Y_1} = n_{\nu_e} + m_Y = 36 + 3 = 39 [m_o]$. Its charge will be derived from the charge of its central tripole: $q_{Y_1} = \pm 3 [q_o]$. By their properties, the tripole, Y , and the “helical tripole”, Y_1 , are alike, except for the helicity property of the latter derived from the helicity of its constituent hexaplet.

When considering the combination of the hexaplet, ν_e , with the triplet, e , we can observe that the hexaplet must be stiffer than the triplet because of stronger bonds between the unlike-charged components of the former, while the repulsion between the like-charged components of the latter makes the bonds between them weaker. Then, the amplitude of the fluctuations of the triplet's radius will be larger than that of the hexaplet. Thus, in the combined structure, which we shall denote as $W = 6Y\bar{Y}3Y$ (or $\nu_e e$), it is the triplet that would enfold the hexaplet. The charge of this structure will correspond to the charge of its charged component, e : $q_W = \pm 9 [q_o]$; its mass can also be derived from the masses of its constituents if oscillations are dampened:

$$m_W = m_e + n_{\nu_e} = 9 + 36 = 45 [m_o].$$

Like the simple Y -tripoles, the “helical” ones, Y_1 , can form bound states with each other (doublets, strings, loops, etc.). Two hexaplets, if both enfold like-charged tripoles, will *always* have like-topological charges (helicities), which means that the force between them due to their topological charges will be repulsive (in addition to the usual repulsive force between like-charges). Thus, two like-charged helical tripoles Y_1 will never combine, unless there exists an intermediate hexaplet (ν_e) between them, with the topological charge opposite to that of the components of the pair. This would neutralise the repulsive force between these components and allow the formation of the following positively charged bound state (“helical” doublet):

$$u^+ = Y_{1\odot}\nu_{e\odot}Y_{1\odot} \quad \text{or} \quad Y_1 \check{\text{Y}} Y_1. \quad (10)$$

For brevity we have denoted the intermediate hexaplet with the symbol $\check{\text{Y}}$, implying that it creates a bond force between the otherwise repulsive components on its sides. By its properties, the helical doublet can be identified with the u -quark. Its net charge, $q_u = +6 [q_o]$, is derived from the charges of its two charged components (Y_1 -tripoles). Its mass is also derived from the number of particles that constitute these charged components: $m_u = 2 \times 39 = 78 [m_o]$.

Table 1: Simple preon-based structures

Structure	Constituents of the structure	Number of colour charges in the structure	Charge (q_0 units)	Mass (m_0 -units)
The primitive particle (preon Π)				
Π	1Π	1	+1	1
First-order structures (combinations of preons)				
ϱ	2Π	2	+2	2
g^0	$1\bar{\Pi} + 1\Pi$	2	$-1 + 1 = 0$	~ 0
Y	3Π	3	+3	3
Second-order structures (combinations of tripoles Y)				
δ	$2Y$	6	+6	6
γ	$1\bar{Y} + 1Y$	6	$-3 + 3 = 0$	~ 0
e^-	$3\bar{Y}$	9	-9	9
Third-order structures				
$2e^-$	$3\bar{Y} + 3\bar{Y}$	$9 + 9 = 18$	-18	18
e^-e^+	$3\bar{Y} + 3Y$	$9 + 9 = 18$	$-9 + 9 = 0$	$\sim 16^\dagger$
ν_e	$6\bar{Y}Y$	$6 \times (3 + 3) = 36$	$6 \times (-3 + 3) = 0$	$7.9 \times 10^{-8}^\dagger$
Y_1	$\nu_e + Y$	$36 + 3 = 39$	$0 - 3 = -3$	$36 + 3 = 39$
W^-	$\nu_e + e^-$	$36 + 9 = 45$	$0 - 9 = -9$	$36 + 9 = 45$
u	$Y_1 \diamond \bar{Y}_1$	$39 + 36 + 39 = 114$	$+3 + 0 + 3 = +6$	$39 + 39 = 78$
ν_μ	$Y_1 \vdots \bar{Y}_1$	$39 + 36 + 39 = 114$	$-3 + 0 + 3 = 0$	$1.4 \times 10^{-7}^\dagger$
d	$u + W^-$	$114 + 45 = 159$	$+6 - 9 = -3$	$78 + 45 = 123$
μ	$\nu_\mu + W^-$	$114 + 45 = 159$	$0 - 9 = -9$	$\overline{48 + 39}^\ddagger$
and so on . .				

[†]quantities estimated in [13]

[‡]system with two oscillating components (see further)

The positively charged u -quark can combine with the negatively charged structure $W^- = \bar{\nu}_e e^-$ (of 45-units mass), forming the d -quark:

$$d^- = u^+ + \bar{\nu}_e e^- \tag{11}$$

of a 123-units mass ($m_d = m_u + m_W = 78 + 45$). The charge of this structure will correspond to the charge of a single triplet: $q_d = q_u + q_e = +6 - 9 = -3 [q_0]$ (see Fig. 6 that below).

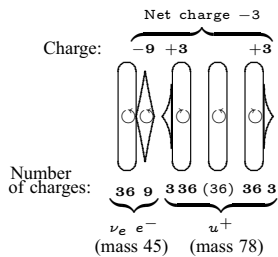


Fig. 6: Scheme of the d -quark. The symbol \diamond is used for the triplet (e), the symbols $\{$ and $\}$ denote the tripoles (Y -particles), and the symbols \square denote the hexaplets (ν_e).

10 The second and third generations of the fundamental fermions

When two unlike-charged helical tripoles combine, their polarisation modes and helicity signs will *always* be opposite (simply because their central tripoles have opposite charges). This would cause an attractive force between these two particles, in addition to the usual attractive force corresponding to the opposite electric charges of Y_1 and \bar{Y}_1 . Since all the forces here are attractive, the components of this system will coalesce and then disintegrate into neutral doublets γ . However, this coalescence can be prevented by an additional hexaplet ν_e with oscillating polarisation, which would create a repulsive stabilising force (barrier) between the combining particles:

$$\nu_\mu = Y_{1\circ} \nu_{e\circ\circ} \bar{Y}_{1\circ} \tag{12}$$

It is natural to identify this structure with the muon-neutrino — a neutral lepton belonging to the second family of the fundamental fermions. The intermediate hexaplet oscillates between the tripoles $Y_{1\circ}$ and $\bar{Y}_{1\circ}$, changing synchronously its polarisation state: $\nu_{e\circ} \leftrightarrow \nu_{e\circ}$. For brevity, we shall use vertical dots separating the components of ν_μ to

denote this barrier-hexaplet:

$$\nu_\mu = Y_1 \cdot \bar{Y}_1. \quad (13)$$

By analogy, we can derive the tau-neutrino structure:

$$\nu_\tau = Y_1 \cdot \bar{Y}_1 \cdot Y_1 \cdot \bar{Y}_1, \quad (14)$$

as well as the structures of the muon (Fig. 7):

$$\mu^- = \nu_\mu \bar{\nu}_e e^- \quad (15)$$

and tau-lepton (Fig. 8):

$$\tau^- = \nu_\tau \bar{\nu}_\mu \mu^-. \quad (16)$$

Drawing also an analogy with molecular equilibrium configurations, where the rigidity of a system depends on the number of local minima of its combined effective potential [14], we can consider the second and third generation fermions as non-rigid structures with oscillating components (clusters) rather than stiff entities with dampened oscillations. In Fig. 7 and Fig. 8 we mark the supposedly clustered components of the μ^- - and τ^- -leptons with braces. Obtaining the ground-state energies (masses) of these complex structures is not a straightforward task because they may have a great variety of oscillatory modes contributing to the mass. However, in principle, these masses are computable, as can be shown by using the following empirical formula:

$$m_{\text{clust}} = \overline{m_1 + m_2 + \dots + m_N} = m\tilde{m}, \quad (17)$$

where N is the number of oscillating clusters, each with the mass m_i ($i = 1, \dots, N$); m is the sum of these masses:

$$m = m_1 + m_2 + \dots + m_N,$$

and \tilde{m} is the reduced mass based on the components (3):

$$\tilde{m}^{-1} = \tilde{m}_1^{-1} + \tilde{m}_2^{-1} + \dots + \tilde{m}_N^{-1}.$$

For simplicity, we assume that unit conversion coefficients in this formula are set to unity. Each substructure here contains a well-defined number of constituents (preons) corresponding to the configuration with the lowest energy. Therefore, the number of these constituents is fixed by the basic symmetry of the potential, implying that the input quantities in (17) are not free parameters. The fermion masses computed with the use of this formula are summarised in Table 2.

As an example, let us compute the muon's mass. The masses of the muon's substructures, according to Fig. 7, are: $m_1 = \tilde{m}_1 = 48$, $m_2 = \tilde{m}_2 = 39$ (in units of m_o). And the muon's mass will be: $m_\mu = \overline{48+39} = \frac{48+39}{1/48+1/39} = 1872[m_o]$. For the τ^- -lepton, the constituent masses are $m_1 = \tilde{m}_1 = 201$, $m_2 = \tilde{m}_2 = 156$ (Fig. 8), and its mass is $m_\tau = \overline{201+156} =$

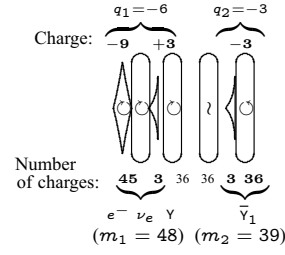


Fig. 7: Scheme of the muon.

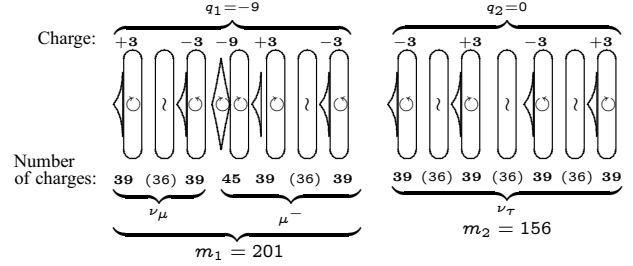


Fig. 8: Scheme of the tau-lepton.

$= 31356 [m_o]$. For the proton, the positively charged fermion consisting of two up ($N_u = 2$), one down ($N_d = 1$) quarks and submerged into a cloud of gluons g^0 , the masses of its components are $m_u = \tilde{m}_u = 78$, $m_d = \tilde{m}_d = 123$. The total number of primitive charges comprising the proton's structure is $N_p = 2m_u + m_d = 2 \times 78 + 123 = 279$, which would correspond to the number of gluons (N_g) interacting with each of these charges ($N_g = N_p = 279$). The masses of these gluons, according to (8), are $m_{g^0} = 1$, $\tilde{m}_{g^0} = \infty$, and the resulting proton mass is

$$m_p = \overline{N_u m_u + N_d m_d + N_g m_g} = 16523 [m_o], \quad (18)$$

which also reproduces the well-known but not yet explained proton-to-electron mass ratio, since $\frac{m_p}{m_e} = \frac{16523}{9} \approx 1836$.

With the value (18) one can convert m_e , m_μ , m_τ , and the masses of all other particles from units m_o into proton mass units, m_p , thus enabling these masses to be compared with the experimental data. The computed fermion masses are listed in Table 2 where the symbols Y_1 , Y_2 and Y_3 denote complex "helical" triplets that replicate the properties of the simple triplet Y on higher levels of the hierarchy. These helical triplets can be regarded as the combinations of "heavy neutrinos" with simple triplets. Like ν_e , the heavy neutrino consists of six pairs of helical triplets: $\nu_h = 6Y_1\bar{Y}_1$. They can further combine and form "ultra-heavy" neutrinos $\nu_{uh} = 3(\bar{Y}_1\nu_h u)e^-$ and so on. The components Y_2 and Y_3 of the c and t quarks have the following structures: $Y_2 = \nu\nu_e u\nu_e e^-$, consisting of 165 primitive particles, and $Y_3 = \nu_{uh}Y$, consisting of 1767 primitive particles.

Table 2: Computed masses of quarks and leptons. The values in the 4th column taken in units of m_o are converted into proton mass units (5th column) $m_p=16523$, Eq.(18). The overlined ones are shorthands for Eq. (17). The masses of ν_e , ν_μ and ν_τ are estimated in [13].

Particle and its structure (components)		Number of charges in the non-cancelled mass components	Computed masses in units of $[m_p]$	Masses converted into m_p	Experimental masses [3] in units of $[m_p]$
First family					
ν_e	$6\overline{Y\overline{Y}}$	≈ 0	7.864×10^{-8}	4.759×10^{-12}	$< 3 \times 10^{-9}$
e^-	$3\overline{Y}$	9	9	0.0005447	0.0005446170232
u	$Y_1 \overline{Y} Y_1$	78	78	0.004720	0.0021 to 0.0058
d	$u + \overline{\nu_e}e^-$	123	123	0.007443	0.0058 to 0.0115
Second family					
ν_μ	$Y_1 \overline{Y}_1$	≈ 0	1.4×10^{-7}	8.5×10^{-12}	$< 2 \times 10^{-4}$
μ^-	$\nu_\mu + \overline{\nu_e}e^-$	$\overline{48 + 39}$	1872	0.1133	0.1126095173
c	$Y_2 \overline{Y} Y_2$	$\overline{165 + 165}$	27225	1.6477	1.57 to 1.95
s	$c + e^-$	$\overline{165 + 165 + 9}$	2751	0.1665	0.11 to 0.19
Third family					
ν_τ	$Y_1 \overline{Y}_1 : Y_1 \overline{Y}_1$	≈ 0	1.5896×10^{-7}	9.6192×10^{-12}	$< 2 \times 10^{-2}$
τ^-	$\nu_\tau + \overline{\nu_\mu}\mu^-$	$\overline{156 + 201}$	31356	1.8977	1.8939 ± 0.0003
t	$Y_3 \overline{Y} Y_3$	$\overline{1767 + 1767}$	3122289	188.94	189.7 ± 4.5
b	$t + \mu^-$	$\overline{1767 + 1767 + 48 + 39}$	76061.5	4.603	4.3 to 4.7

11 Conclusions

The results presented in Table 2 show that our model agree with experiment to an accuracy better then 0.5%. The discrepancies should be attributed to the simplifications we have assumed here (e.g., neglecting the binding and oscillatory energies, as well as the neutrino residual masses, which contribute to the masses of many structures in our model).

By matching the pattern of properties of the fundamental particles our results confirm that our conjecture about the dualism of space and the symmetry of the basic field corresponds, by a grand degree of confidence, to the actual situation. Thus, our model seems to unravel a new layer of physical reality, which bears the causal mechanisms underlying quantum phenomena. This sets a foundation from which one can explain many otherwise inexplicable observational facts that plague modern physics.

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References

1. Kalman C. S. *Nucl. Phys. Proc. Suppl.*, 2005, v.142, 235–237.
2. Dugne J.-J. Higgs pain? Take a preon! arXiv: hep-ph/9709227. Peccei R.D. The mystery of flavor. arXiv: hep-ph/9712422. Weinberg S. *Phys. Rev. D*, 1976, v.13, 974.
3. Eidelman S., et al. (Particle Data Group), *Phys. Lett. B*, 2004, v. 592, 1–33 (online <http://pdg.lbl.gov>).
4. Pesteil P. *Apeiron*, 1991, v. 1, 15–29.
5. Golfand Yu. A. and Likhtman E. P. *JETP Lett.*, 1971, v. 13, 323. Randall L. and Sundrum R. *Phys. Rev. Lett.*, 1999, v. 83, 3370. Libanov M. and Nougayev E. *JHEP*, 2002, v. 204, 55. Spaans M. On the topological nature of fundamental interactions. arXiv: gr-qc/9901025.
6. Donoghue J. F. *Phys. Rev. D*, 1998, v. 57, 5499–5508.
7. D'Souza I. A. and Kalman C. S. Preons. World Scientific Publ., Singapore, 1992.
8. Kastrup H. A. in *Symmetries in physics (1600–1980)* (ed. by Doncel M. G.) World Sci. Publ., Singapore, 1988, 113–163.
9. Zelmanov A. L., Chronometric invariants (dissertation), 1944. Publ. CERN EXT-2004-117, 236 p.
10. Gödel K. in *Albert Einstein: philosopher, scientist*, (ed. by Schilpp P. A.) Harper Tordis Books, N.Y., 1959, v.II, 555–562.
11. Suisso E. F., de Melo J. P. B. C., Frederico T. *Phys. Rev. D*, 2002, v. 65, 094009.
12. Yershov V. N. *Few-Body Systems*, 2005, v. 37, 79–106.
13. Yershov V. N. Neutrino masses and the structure of the weak-gauge boson. arXiv: physics/0301034.
14. Burenin A. V. *Physics-Uspeski*, 2002, v. 45(7), 753–776.

Dynamical Fractal 3-Space and the Generalised Schrödinger Equation: Equivalence Principle and Vorticity Effects

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The new dynamical “quantum foam” theory of 3-space is described at the classical level by a velocity field. This has been repeatedly detected and for which the dynamical equations are now established. These equations predict 3-space “gravitational wave” effects, and these have been observed, and the 1991 DeWitte data is analysed to reveal the fractal structure of these “gravitational waves”. This velocity field describes the differential motion of 3-space, and the various equations of physics must be generalised to incorporate this 3-space dynamics. Here a new generalised Schrödinger equation is given and analysed. It is shown that from this equation the equivalence principle may be derived as a quantum effect, and that as well this generalised Schrödinger equation determines the effects of vorticity of the 3-space flow, or “frame-dragging”, on matter, and which is being studied by the Gravity Probe B (GP-B) satellite gyroscope experiment.

1 Introduction

Extensive experimental evidence [1, 2, 3] has shown that a complex dynamical 3-space underlies reality. The evidence involves the repeated detection of the motion of the Earth relative to that 3-space using Michelson interferometers operating in gas mode [3], particularly the experiment by Miller [4] in 1925/26 at Mt. Wilson, and the coaxial cable RF travel time measurements by Torr and Kolen in Utah, and the DeWitte experiment in 1991 in Brussels [3]. All such 7 experiments are consistent with respect to speed and direction. It has been shown that effects caused by motion relative to this 3-space can mimic the formalism of spacetime, but that it is the 3-space that is “real”, simply because it is directly observable [1].

The 3-space is in differential motion, that is one part has a velocity relative to other parts, and so involves a velocity field $\mathbf{v}(\mathbf{r}, t)$ description. To be specific this velocity field must be described relative to a frame of observers, but the formalism is such that the dynamical equations for this velocity field must transform covariantly under a change of observer. As shown herein the experimental data from the DeWitte experiment shows that $\mathbf{v}(\mathbf{r}, t)$ has a fractal structure. This arises because, in the absence of matter, the dynamical equations for $\mathbf{v}(\mathbf{r}, t)$ have no scale. This implies that the differential motion of 3-space manifests at all scales. This fractal differential motion of 3-space is missing from all the fundamental equations of physics, and so these equations require a generalisation. Here we report on the necessary generalisation of the Schrödinger equation, and which results in some remarkable results: (i) the equivalence principle emerges, as well as (ii) the effects of vorticity of this velocity

field. These two effects are thus seen to be quantum-theoretic effects, i. e. consequences of the wave nature of matter. The equivalence principle, as originally formulated by Galileo and then Newton, asserts that the gravitational acceleration of an object is independent of its composition and speed. However we shall see that via the vorticity effect, the velocity of the object does affect the acceleration by causing rotations.

It has been shown [1, 5] that the phenomenon of gravity is a consequence of the time-dependence and inhomogeneities of $\mathbf{v}(\mathbf{r}, t)$. So the dynamical equations for $\mathbf{v}(\mathbf{r}, t)$ give rise to a new theory of gravity, when combined with the generalised Schrödinger equation, and the generalised Maxwell and Dirac equations. The equations for $\mathbf{v}(\mathbf{r}, t)$ involve the Newtonian gravitational constant G and a dimensionless constant that determines the strength of a new spatial self-interaction effect, which is missing from both Newtonian Gravity and General Relativity. Experimental data has revealed [1, 5] the remarkable discovery that this constant is the fine structure constant $\alpha \approx 1/137$. This dynamics then explains numerous gravitational anomalies, such as the bore hole g anomaly, the so-called “dark matter” anomaly in the rotation speeds of spiral galaxies, and that the effective mass of the necessary black holes at the centre of spherical matter systems, such as globular clusters and spherical galaxies, is $\alpha/2$ times the total mass of these systems. This prediction has been confirmed by astronomical observations [6].

The occurrence of α suggests that space is itself a quantum system undergoing on-going classicalisation. Just such a proposal has arisen in *Process Physics* [1] which is an information-theoretic modelling of reality. There quantum space and matter arise in terms of the Quantum Homotopic Field Theory (QHFT) which, in turn, may be related to the

standard model of matter. In the QHFT space at this quantum level is best described as a “quantum foam”. So we interpret the observed fractal 3-space as a classical approximation to this “quantum foam”.

While here we investigate the properties of the generalised Schrödinger equation, analogous generalisations of the Maxwell and Dirac equations, and in turn the corresponding generalisations to the quantum field theories for such systems, may also be made. In the case of the Maxwell equations we obtain the light bending effects, including in particular gravitational lensing, caused by the 3-space differential and time-dependent flow.

2 The physics of 3-space

Because of the dominance of the spacetime ontology, which has been the foundation of physics over the last century, the existence of a 3-space as an observable phenomenon has been overlooked, despite extensive experimental detection over that period, and earlier. This spacetime ontology is distinct from the role of spacetime as a mathematical formalism implicitly incorporating some real dynamical effects, though this distinction is rarely made. Consequently the existence of 3-space has been denied, and so there has never been a dynamical theory for 3-space. In recent years this situation has dramatically changed. We briefly summarise the key aspects to the dynamics of 3-space.

Relative to some observer 3-space is described by a velocity field $\mathbf{v}(\mathbf{r}, t)$. It is important to note that the coordinate \mathbf{r} is not itself 3-space, rather it is merely a label for an element of 3-space that has velocity \mathbf{v} , relative to some observer. This will become more evident when we consider the necessary generalisation of the Schrödinger equation. Also it is important to appreciate that this “moving” 3-space is not itself embedded in a “space”; the 3-space is all there is, although as noted above its deeper structure is that of a “quantum foam”.

In the case of zero vorticity $\nabla \times \mathbf{v} = 0$ the 3-space dynamics is given by, in the non-relativistic limit,

$$\nabla \cdot \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \frac{\alpha}{8} ((\text{tr} D)^2 - \text{tr}(D^2)) = -4\pi G \rho, \quad (1)$$

where ρ is the matter density, and where

$$D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \quad (2)$$

The acceleration of an element of space is given by the Euler form

$$\mathbf{g}(\mathbf{r}, t) \equiv \lim_{\Delta t \rightarrow 0} \frac{\mathbf{v}(\mathbf{r} + \mathbf{v}(\mathbf{r}, t)\Delta t, t + \Delta t) - \mathbf{v}(\mathbf{r}, t)}{\Delta t} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v}. \quad (3)$$

These forms are mandated by Galilean covariance under change of observer*. This non-relativistic modelling of the dynamics for the velocity field gives a direct account of the various phenomena noted above. A generalisation to include vorticity and relativistic effects of the motion of matter through this 3-space is given in [1]. From (1) and (2) we obtain that

$$\nabla \cdot \mathbf{g} = -4\pi G \rho - 4\pi G \rho_{DM}, \quad (4)$$

where

$$\rho_{DM}(\mathbf{r}) = \frac{\alpha}{32\pi G} ((\text{tr} D)^2 - \text{tr}(D^2)). \quad (5)$$

In this form we see that if $\alpha \rightarrow 0$, then the acceleration of the 3-space elements is given by Newton’s Law of Gravitation, in differential form. But for a non-zero α we see that the 3-space acceleration has an additional effect, the ρ_{DM} term, which is an effective “matter density” that mimics the new self-interaction dynamics. This has been shown to be the origin of the so-called “dark matter” effect in spiral galaxies. It is important to note that (4) does not determine \mathbf{g} directly; rather the velocity dynamics in (1) must be solved, and then with \mathbf{g} subsequently determined from (3). Eqn. (4) merely indicates that the resultant non-Newtonian aspects to \mathbf{g} could be mistaken as being the result of a new form of matter, whose density is given by ρ_{DM} . Of course the saga of “dark matter” shows that this actually happened, and that there has been a misguided and fruitless search for such “matter”.

The numerous experimental confirmations of (1) imply that Newtonian gravity is not universal at all. Rather a key aspect to gravity was missed by Newton because it so happens that the 3-space self-interaction dynamics does not necessarily explicitly manifest outside of spherical matter systems, such as the Sun. To see this it is only necessary to see that the velocity field

$$\mathbf{v}(\mathbf{r}) = -\sqrt{\frac{2GM'}{r}} \hat{\mathbf{r}}, \quad (6)$$

is a solution to (1) external to a spherical mass M , where $M' = (1 + \frac{\alpha}{2})M + \dots$. Then (6) gives, using (3), the resultant external “inverse square law” acceleration

$$\mathbf{g}(\mathbf{r}) = -\frac{GM'}{r^2} \hat{\mathbf{r}}. \quad (7)$$

Hence in this special case the 3-space dynamics predicts an inverse square law form for \mathbf{g} , as confirmed in the non-relativistic regime by Kepler’s laws for planetary motion, with only a modified value for the effective mass M' . So for this reason we see how easy it was for Newton to have overlooked a velocity formalism for gravity, and so missed the self-interaction dynamics in (1). Inside a spherical matter

*However this does not exclude so-called relativistic effects, such as the length contraction of moving rods or the time dilations of moving clocks.

system Newtonian gravity and the new gravity theory differ, and it was this difference that explained the bore hole g anomaly data [5], namely that g does not decrease down a bore hole as rapidly as Newtonian gravity predicts. It was this anomaly that led to the discovery that α was in fact the fine structure constant, up to experimental errors. As well the 3-space dynamics in (1) has “gravitational wave” solutions [7]. Then there are regions where the velocity differs slightly from the enveloping region. In the absence of matter these waves will be in general fractal because there is no dimensioned constant, and so no natural scale. These waves were seen by Miller, Torr and Kolen, and by DeWitte [1, 7] as shown in Fig. 2.

However an assumption made in previous analyses was that the acceleration of the 3-space itself, in (3), was also the acceleration of matter located in that 3-space. The key result herein is to derive this result by using the generalised Schrödinger equation. In doing so we discover the additional effect that vorticity in the velocity field causes quantum states to be rotated, as discussed in sect. 7.

3 Newtonian gravity and the Schrödinger equation

Let us consider what might be regarded as the conventional “Newtonian” approach to including gravity in the Schrödinger equation [8]. There gravity is described by the Newtonian potential energy field $\Phi(\mathbf{r}, t)$, such that $\mathbf{g} = -\nabla\Phi$, and we have for a “free-falling” quantum system, with mass m ,

$$i\hbar \frac{\partial\psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + m\Phi(\mathbf{r}, t) \psi(\mathbf{r}, t) \equiv H(t)\psi, \quad (8)$$

where the hamiltonian is in general now time dependent, because the masses producing the gravitational acceleration may be moving. Then the classical-limit trajectory is obtained via the usual Ehrenfest method [9]: we first compute the time rate of change of the so-called position “expectation value”

$$\begin{aligned} \frac{d\langle \mathbf{r} \rangle}{dt} &\equiv \frac{d}{dt} \langle \psi, \mathbf{r} \psi \rangle = \frac{i}{\hbar} \langle H\psi, \mathbf{r} \psi \rangle - \frac{i}{\hbar} \langle \psi, \mathbf{r} H\psi \rangle \\ &= \frac{i}{\hbar} \langle \psi, [H, \mathbf{r}] \psi \rangle, \end{aligned} \quad (9)$$

which is valid for a normalised state ψ . The norm is time invariant when H is hermitian ($H^\dagger = H$) even if H itself is time dependent,

$$\begin{aligned} \frac{d}{dt} \langle \psi, \psi \rangle &= \frac{i}{\hbar} \langle H\psi, \psi \rangle - \frac{i}{\hbar} \langle \psi, H\psi \rangle \\ &= \frac{i}{\hbar} \langle \psi, H^\dagger \psi \rangle - \frac{i}{\hbar} \langle \psi, H\psi \rangle = 0. \end{aligned} \quad (10)$$

Next we compute the matter “acceleration” from (9)

$$\frac{d^2 \langle \mathbf{r} \rangle}{dt^2} = \frac{i}{\hbar} \frac{d}{dt} \langle \psi, [H, \mathbf{r}] \psi \rangle =$$

$$\begin{aligned} &= \left(\frac{i}{\hbar}\right)^2 \langle \psi, [H, [H, \mathbf{r}]] \psi \rangle + \frac{i}{\hbar} \left\langle \psi, \left[\frac{\partial H(t)}{\partial t}, \mathbf{r} \right] \psi \right\rangle \\ &= -\langle \psi, \nabla \Phi \psi \rangle = \langle \psi, \mathbf{g}(\mathbf{r}, t) \psi \rangle = \langle \mathbf{g}(\mathbf{r}, t) \rangle, \end{aligned} \quad (11)$$

where for the commutator

$$\left[\frac{\partial H(t)}{\partial t}, \mathbf{r} \right] = \left[m \frac{\partial \Phi(\mathbf{r}, t)}{\partial t}, \mathbf{r} \right] = 0. \quad (12)$$

In the classical limit ψ has the form of a wavepacket where the spatial extent of ψ is much smaller than the spatial region over which $\mathbf{g}(\mathbf{r}, t)$ varies appreciably. Then we have the approximation $\langle \mathbf{g}(\mathbf{r}, t) \rangle \approx \mathbf{g}(\langle \mathbf{r} \rangle, t)$, and finally we arrive at the Newtonian 2nd-law equation of motion for the wavepacket,

$$\frac{d^2 \langle \mathbf{r} \rangle}{dt^2} \approx \mathbf{g}(\langle \mathbf{r} \rangle, t). \quad (13)$$

In this classical limit we obtain the equivalence principle, namely that the acceleration is independent of the mass m and of the velocity of that mass. But of course that followed by construction, as the equivalence principle is built into (8) by having m as the coefficient of Φ . In Newtonian gravity there is no explanation for the origin of Φ or \mathbf{g} . In the new theory gravity is explained in terms of a velocity field, which in turn has a deeper explanation within *Process Physics*.

4 Dynamical 3-space and the generalised Schrödinger equation

The key insight is that conventional physics has neglected the interaction of various systems with the dynamical 3-space. Here we generalise the Schrödinger equation to take account of this new physics. Now gravity is a dynamical effect arising from the time-dependence and spatial inhomogeneities of the 3-space velocity field $\mathbf{v}(\mathbf{r}, t)$, and for a “free-falling” quantum system with mass m the Schrödinger equation now has the generalised form

$$i\hbar \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} \right) \psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t), \quad (14)$$

which we write as

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = H(t) \psi(\mathbf{r}, t), \quad (15)$$

where now

$$H(t) = -i\hbar \left(\mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} \right) - \frac{\hbar^2}{2m} \nabla^2. \quad (16)$$

This form for H specifies how the quantum system must couple to the velocity field, and it uniquely follows from two considerations: (i) the generalised Schrödinger equation must remain form invariant under a change of observer, i. e. with $t \rightarrow t$, and $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{v}t$, where \mathbf{v} is the relative velocity of the two observers. Then we compute that $\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} \rightarrow$

→ $\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v}$, i. e. that it is an invariant operator, and (ii) requiring that $H(t)$ be hermitian, so that the wavefunction norm is an invariant of the time evolution. This implies that the $\frac{1}{2} \nabla \cdot \mathbf{v}$ term must be included, as $\mathbf{v} \cdot \nabla$ by itself is not hermitian for an inhomogeneous $\mathbf{v}(\mathbf{r}, t)$. Then the consequences for the motion of wavepackets is uniquely determined; they are fixed by these two quantum-theoretic requirements.

Then again the classical-limit trajectory is obtained via the position “expectation value”, first with

$$\begin{aligned} \mathbf{v}_O &\equiv \frac{d\langle \mathbf{r} \rangle}{dt} = \frac{d}{dt} (\psi, \mathbf{r} \psi) = \frac{i}{\hbar} (\psi, [H, \mathbf{r}] \psi) = \\ &= \left(\psi, \left(\mathbf{v}(\mathbf{r}, t) - \frac{i\hbar}{m} \nabla \right) \psi \right) = \langle \mathbf{v}(\mathbf{r}, t) \rangle - \frac{i\hbar}{m} \langle \nabla \rangle, \end{aligned} \quad (17)$$

on evaluating the commutator using $H(t)$ in (16), and which is again valid for a normalised state ψ .

Then for the “acceleration” we obtain from (17) that*

$$\begin{aligned} \frac{d^2 \langle \mathbf{r} \rangle}{dt^2} &= \frac{d}{dt} \left(\psi, \left(\mathbf{v} - \frac{i\hbar}{m} \nabla \right) \psi \right) = \\ &= \left(\psi, \left(\frac{\partial \mathbf{v}(\mathbf{r}, t)}{\partial t} + \frac{i}{\hbar} \left[H, \left(\mathbf{v} - \frac{i\hbar}{m} \nabla \right) \right] \right) \psi \right) = \\ &= \left(\psi, \frac{\partial \mathbf{v}(\mathbf{r}, t)}{\partial t} \psi \right) + \\ &+ \left(\psi, \left(\mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} - \frac{i\hbar}{2m} \nabla^2 \right) \left(\mathbf{v} - \frac{i\hbar}{m} \nabla \right) \psi \right) - \\ &- \left(\psi, \left(\mathbf{v} - \frac{i\hbar}{m} \nabla \right) \left(\mathbf{v} \cdot \nabla + \frac{1}{2} \nabla \cdot \mathbf{v} - \frac{i\hbar}{2m} \nabla^2 \right) \psi \right) = \\ &= \left(\psi, \left(\frac{\partial \mathbf{v}(\mathbf{r}, t)}{\partial t} + ((\mathbf{v} \cdot \nabla) \mathbf{v}) - \frac{i\hbar}{m} (\nabla \times \mathbf{v}) \times \nabla \right) \psi \right) + \\ &+ \left(\psi, \frac{i\hbar}{2m} (\nabla \times (\nabla \times \mathbf{v})) \psi \right) \approx \\ &\approx \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + (\nabla \times \mathbf{v}) \times \left(\frac{d\langle \mathbf{r} \rangle}{dt} - \mathbf{v} \right) + \\ &+ \frac{i\hbar}{2m} (\nabla \times (\nabla \times \mathbf{v})) = \\ &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + (\nabla \times \mathbf{v}) \times \left(\frac{d\langle \mathbf{r} \rangle}{dt} - \mathbf{v} \right) = \\ &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + (\nabla \times \mathbf{v}) \times \mathbf{v}_R, \end{aligned} \quad (18)$$

where in arriving at the 3rd last line we have invoked the small-wavepacket approximation, and used (17) to identify

$$\mathbf{v}_R \equiv -\frac{i\hbar}{m} \langle \nabla \rangle = \mathbf{v}_O - \mathbf{v}, \quad (19)$$

where \mathbf{v}_O is the velocity of the wavepacket or object “O” relative to the observer, so then \mathbf{v}_R is the velocity of the

wavepacket relative to the local 3-space. Then all velocity field terms are now evaluated at the location of the wavepacket. Note that the operator

$$-\frac{i\hbar}{m} (\nabla \times \mathbf{v}) \times \nabla + \frac{i\hbar}{2m} (\nabla \times (\nabla \times \mathbf{v})) \quad (20)$$

is hermitian, but that separately neither of these two operators is hermitian. Then in general the scalar product in (18) is real. But then in arriving at the last line in (18) by means of the small-wavepacket approximation, we must then self-consistently use that $\nabla \times (\nabla \times \mathbf{v}) = 0$, otherwise the acceleration acquires a spurious imaginary part. This is consistent with (27) outside of any matter which contributes to the generation of the velocity field, for there $\rho = 0$. These observations point to a deep connection between quantum theory and the velocity field dynamics, as already argued in [1].

We see that the test “particle” acquires the acceleration of the velocity field, as in (3), and as well an additional vorticity induced acceleration which is the analogue of the Helmholtz acceleration in fluid mechanics. Hence we find that the equivalence principle arises from the unique generalised Schrödinger equation and with the additional vorticity effect. This vorticity effect depends on the absolute velocity \mathbf{v}_R of the object relative to the local space, and so requires a change in the Galilean or Newtonian form of the equivalence principle.

The vorticity acceleration effect is the origin of the Lense-Thirring so-called “frame-dragging” effect[†] [10] discussed in sect. 7. While the generation of the vorticity is a relativistic effect, as in (27), the response of the test particle to that vorticity is a non-relativistic effect, and follows from the generalised Schrödinger equation, and which is not present in the standard Schrödinger equation with coupling to the Newtonian gravitational potential, as in (8). Hence the generalised Schrödinger equation with the new coupling to the velocity field is more fundamental. The Helmholtz term in (18) is being explored by the Gravity Probe B gyroscope precession experiment, however the vorticity caused by the motion of the Earth is extremely small, as discussed in sect. 7.

An important insight emerges from the form of (15) and (16): here the generalised Schrödinger equation involves two fields $\mathbf{v}(\mathbf{r}, t)$ and $\psi(\mathbf{r}, t)$, where the coordinate \mathbf{r} is merely a label to relate the two fields, and is not itself the 3-space. In particular while \mathbf{r} may have the form of a Euclidean 3-geometry, the space itself has time-dependence and inhomogeneities, and as well in the more general case will exhibit vorticity $\omega = \nabla \times \mathbf{v}$. Only in the unphysical case does the description of the 3-space become identified with the coordinate system \mathbf{r} , and that is when the velocity field $\mathbf{v}(\mathbf{r}, t)$ becomes uniform and time independent. Then by a suitable choice of observer we may put $\mathbf{v}(\mathbf{r}, t) = 0$, and the generalised Schrödinger equation reduces to the usual “free”

*Care is needed to indicate the range of the various ∇ 's. Extra parentheses (...) are used to limit the range when required.

[†]In the spacetime formalism it is mistakenly argued that it is “spacetime” that is “dragged”.

Schrödinger equation. As we discuss later the experimental evidence is that $\mathbf{v}(\mathbf{r}, t)$ is fractal and so cannot be removed by a change to a preferred observer. Hence the generalised Schrödinger equation in (15)–(16) is a major development for fundamental physics. Of course in general other non-3-space potential energy terms may be added to the RHS of (16). A prediction of this new quantum theory, which also extends to a generalised Dirac equation, is that the fractal structure to space implies that even at the scale of atoms etc there will be time-dependencies and inhomogeneities, and that these will affect transition rates of quantum systems. These effects are probably those known as the Shnoll effects [11].

5 Free-fall minimum proper-time trajectories

The acceleration in (18) also arises from the following argument, which is the analogue of the Fermat least-time formalism. Consider the elapsed time for a comoving clock travelling with the test particle. Then taking account of the Lamour time-dilation effect that time is given by

$$\tau[\mathbf{r}_0] = \int dt \left(1 - \frac{\mathbf{v}_R^2}{c^2}\right)^{1/2} \quad (21)$$

with \mathbf{v}_R given by (19) in terms of \mathbf{v}_O and \mathbf{v} . Then this time effect relates to the speed of the clock relative to the local 3-space, and that c is the speed of light relative to that local 3-space. We are using a relativistic treatment in (21) to demonstrate the generality of the results*. Under a deformation of the trajectory

$$\mathbf{r}_0(t) \rightarrow \mathbf{r}_0(t) + \delta\mathbf{r}_0(t), \quad \mathbf{v}_0(t) \rightarrow \mathbf{v}_0(t) + \frac{d\delta\mathbf{r}_0(t)}{dt}, \quad (22)$$

and then

$$\begin{aligned} \mathbf{v}(\mathbf{r}_0(t) + \delta\mathbf{r}_0(t), t) &= \\ &= \mathbf{v}(\mathbf{r}_0(t), t) + (\delta\mathbf{r}_0(t) \cdot \nabla) \mathbf{v}(\mathbf{r}_0(t), t) + \dots \end{aligned} \quad (23)$$

Evaluating the change in proper travel time to lowest order

$$\begin{aligned} \delta\tau &= \tau[\mathbf{r}_0 + \delta\mathbf{r}_0] - \tau[\mathbf{r}_0] = \\ &= - \int dt \frac{1}{c^2} \mathbf{v}_R \cdot \delta\mathbf{v}_R \left(1 - \frac{\mathbf{v}_R^2}{c^2}\right)^{-1/2} + \dots = \\ &= \int dt \frac{1}{c^2} \frac{\mathbf{v}_R \cdot (\delta\mathbf{r}_0 \cdot \nabla) \mathbf{v} - \mathbf{v}_R \cdot \frac{d(\delta\mathbf{r}_0)}{dt}}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} = \\ &= \int dt \frac{1}{c^2} \left(\frac{\mathbf{v}_R \cdot (\delta\mathbf{r}_0 \cdot \nabla) \mathbf{v}}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} + \delta\mathbf{r}_0 \cdot \frac{d}{dt} \frac{\mathbf{v}_R}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} \right) = \end{aligned}$$

*A non-relativistic analysis may be alternatively pursued by first expanding (21) in powers of $1/c^2$.

$$= \int dt \frac{1}{c^2} \delta\mathbf{r}_0 \cdot \left(\frac{(\mathbf{v}_R \cdot \nabla) \mathbf{v} + \mathbf{v}_R \times (\nabla \times \mathbf{v})}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} + \frac{d}{dt} \frac{\mathbf{v}_R}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} \right).$$

Hence a trajectory $\mathbf{r}_0(t)$ determined by $\delta\tau = 0$ to $O(\delta\mathbf{r}_0(t)^2)$ satisfies

$$\frac{d}{dt} \frac{\mathbf{v}_R}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}} = - \frac{(\mathbf{v}_R \cdot \nabla) \mathbf{v} + \mathbf{v}_R \times (\nabla \times \mathbf{v})}{\sqrt{1 - \frac{\mathbf{v}_R^2}{c^2}}}. \quad (24)$$

Substituting $\mathbf{v}_R(t) = \mathbf{v}_0(t) - \mathbf{v}(\mathbf{r}_0(t), t)$ and using

$$\frac{d\mathbf{v}(\mathbf{r}_0(t), t)}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}_0 \cdot \nabla) \mathbf{v}, \quad (25)$$

we obtain

$$\begin{aligned} \frac{d\mathbf{v}_0}{dt} &= \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + (\nabla \times \mathbf{v}) \times \mathbf{v}_R - \\ &\quad - \frac{\mathbf{v}_R}{1 - \frac{\mathbf{v}_R^2}{c^2}} \frac{1}{2} \frac{d}{dt} \left(\frac{\mathbf{v}_R^2}{c^2} \right). \end{aligned} \quad (26)$$

Then in the low speed limit $v_R \ll c$ we may neglect the last term, and we obtain (18). Hence we see a close relationship between the geodesic equation, known first from General Relativity, and the 3-space generalisation of the Schrödinger equation, at least in the non-relativistic limit. So in the classical limit, i.e when the wavepacket approximation is valid, the wavepacket trajectory is specified by the least proper-time geodesic.

The relativistic term in (26) is responsible for the precession of elliptical orbits and also for the event horizon effect. Hence the trajectory in (18) is a non-relativistic minimum travel-time trajectory, which is Fermat's Principle. The relativistic term in (26) will arise from a generalised Dirac equation which would then include the dynamics of 3-space.

6 Fractal 3-space and the DeWitte experimental data

In 1991 Roland DeWitte working within Belgacom, the Belgium telecommunications company, accidentally made yet another detection of absolute motion, and one which was 1st-order in v/c . 5 MHz radio frequency (RF) signals were sent in both directions through two buried coaxial cables linking the two clusters of cesium atomic clocks.

Changes in propagation times were observed and eventually observations over 178 days were recorded. A sample of the data, plotted against sidereal time for just three days, is shown in Fig. 1. The DeWitte data was clear evidence of absolute motion with the Right Ascension for minimum/maximum propagation time agreeing almost exactly with

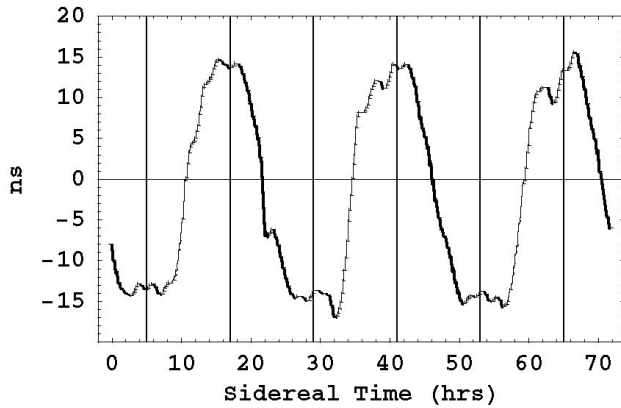


Fig. 1: Variations in twice the one-way travel time, in ns, for an RF signal to travel 1.5 km through a buried coaxial cable between Rue du Marais and Rue de la Paille, Brussels. An offset has been used such that the average is zero. The cable has a North-South orientation, and the data is \pm difference of the travel times for NS and SN propagation. The sidereal time for maximum effect of ~ 5 hr (or ~ 17 hr) (indicated by vertical lines) agrees with the direction found by Miller[4]. Plot shows data over 3 sidereal days and is plotted against sidereal time. The main effect is caused by the rotation of the Earth. The superimposed fluctuations are evidence of turbulence i.e gravitational waves. Removing the Earth induced rotation effect we obtain the first experimental data of the fractal structure of space, and is shown in Fig. 2. DeWitte performed this experiment over 178 days, and demonstrated that the effect tracked sidereal time and not solar time[1].

Miller's direction* ($\alpha = 5.2^{\text{hr}}$, $\delta = -67^\circ$)[†], and with speed 420 ± 30 km/s. This local absolute motion is different from the CMB motion, in the direction ($\alpha = 11.20^{\text{hr}}$, $\delta = -7.22^\circ$) with speed of 369 km/s, for that would have given the data a totally different sidereal time signature, namely the times for maximum/ minimum would have been shifted by 6hrs. The CMB velocity is motion relative to the distant early universe, whereas the velocity measured in the DeWitte and related experiments is the velocity relative to the local space. The declination of the velocity observed in this DeWitte experiment cannot be determined from the data as only three days of data are available. However assuming exactly the same declination as Miller the speed observed by DeWitte appears to be also in excellent agreement with the Miller speed. The dominant effect in Fig. 1 is caused by the rotation of the Earth, namely that the orientation of the coaxial cable

*This velocity arises after removing the effects of the Earth's orbital speed about the Sun, 30 km/s, and the gravitational in-flow past the Earth towards the Sun, 42 km/s, as in (6).

[†]The opposite direction is not easily excluded due to errors within the data, and so should also be considered as possible. A new experiment will be capable of more accurately determining the speed and direction, as well as the fractal structure of 3-space. The author is constructing a more compact version of the Torr-Kolen - DeWitte coaxial cable RF travel-time experiment. New experimental techniques have been developed to increase atomic-clock based timing accuracy and stability, so that shorter cables can be used, which will permit 3-arm devices.

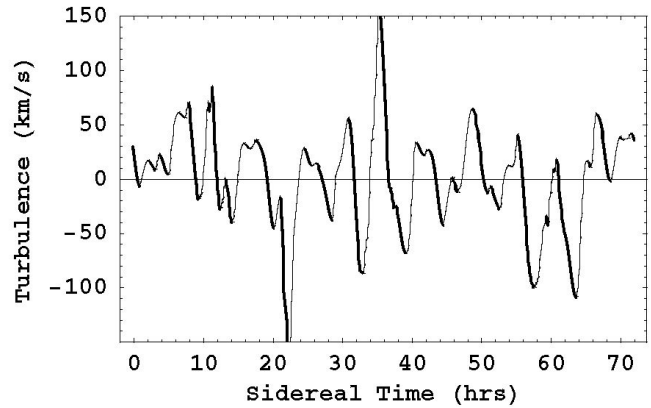


Fig. 2: Shows the velocity fluctuations, essentially "gravitational waves" observed by DeWitte in 1991 from the measurement of variations in the RF coaxial-cable travel times. This data is obtained from that in Fig. 1 after removal of the dominant effect caused by the rotation of the Earth. Ideally the velocity fluctuations are three-dimensional, but the DeWitte experiment had only one arm. This plot is suggestive of a fractal structure to the velocity field. This is confirmed by the power law analysis shown in Fig. 3.

with respect to the direction of the flow past the Earth changes as the Earth rotates. This effect may be approximately unfolded from the data, leaving the gravitational waves shown in Fig. 2. This is the first evidence that the velocity field describing 3-space has a complex structure, and is indeed fractal.

The fractal structure, i. e. that there is an intrinsic lack of scale, to these speed fluctuations is demonstrated by binning the absolute speeds $|v|$ and counting the number of speeds $p(|v|)$ within each bin. A least squares fit of the log-log plot to a straightline was then made. Plotting $\log[p(|v|)]$ vs $\log|v|$, as shown in Fig. 3 we see that the fit gives $p(v) \propto |v|^{-2.6}$. With the new experiment considerably more data will become available.

7 Observing 3-space vorticity

The vorticity effect in (18) can be studied experimentally in the Gravity Probe B (GP-B) gyroscope satellite experiment in which the precession of four on-board gyroscopes has been measured to unprecedented accuracy [12, 13]. In a generalisation of (1) [1] the vorticity $\nabla \times \mathbf{v}$ is generated by matter in motion through the 3-space, where here \mathbf{v}_R is the absolute velocity of the matter relative to the local 3-space.

$$\nabla \times (\nabla \times \mathbf{v}) = \frac{8\pi G\rho}{c^2} \mathbf{v}_R. \quad (27)$$

We then obtain from (27) the vorticity (ignoring homogeneous vortex solutions)

$$\vec{\omega}(\mathbf{r}, t) = \frac{2G}{c^2} \int d^3 r' \frac{\rho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|^3} \mathbf{v}_R(\mathbf{r}', t) \times (\mathbf{r} - \mathbf{r}'). \quad (28)$$

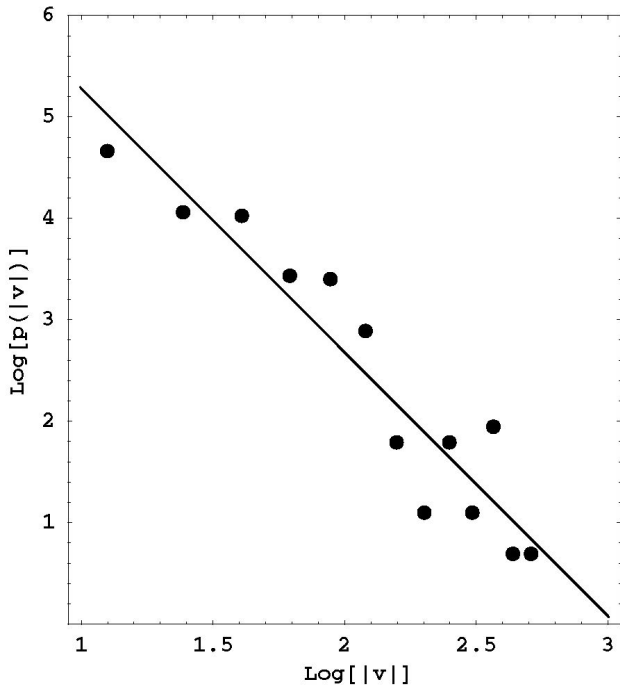


Fig. 3: Shows that the velocity fluctuations in Fig. 2 are scale free, as the probability distribution from binning the speeds has the form $p(v) \propto |v|^{-2.6}$. This plot shows $\log[p(v)]$ vs $\log|v|$. This shows that the velocity field has a fractal structure, and so requiring the generalisation of the Schrödinger equation, as discussed herein, and also the Maxwell and Dirac equations (to be discussed elsewhere).

For the smaller Earth-rotation induced vorticity effect $\mathbf{v}_R(\mathbf{r}) = \mathbf{w} \times \mathbf{r}$ in (28), where \mathbf{w} is the angular velocity of the Earth, giving

$$\vec{\omega}(\mathbf{r})_{\text{rot}} = 4 \frac{G}{c^2} \frac{3(\mathbf{r} \cdot \mathbf{L})\mathbf{r} - r^2\mathbf{L}}{2r^5}, \quad (29)$$

where \mathbf{L} is the angular momentum of the Earth, and \mathbf{r} is the distance from the centre.

In general the vorticity term in (18) leads to a apparent “torque”, according to a distant observer, acting on the angular momentum \mathbf{S} of the gyroscope,

$$\vec{\tau} = \int d^3r \rho(\mathbf{r}) \mathbf{r} \times (\vec{\omega}(\mathbf{r}) \times \mathbf{v}_R(\mathbf{r})), \quad (30)$$

where ρ is its density, and where now \mathbf{v}_R is used here to describe the motion of the matter forming the gyroscope relative to the local 3-space. Then $d\mathbf{S} = \vec{\tau}dt$ is the change in \mathbf{S} over the time interval dt . For a gyroscope $\mathbf{v}_R(\mathbf{r}) = \mathbf{s} \times \mathbf{r}$, where \mathbf{s} is the angular velocity of the gyroscope. This gives

$$\vec{\tau} = \frac{1}{2} \vec{\omega} \times \mathbf{S} \quad (31)$$

and so $\vec{\omega}/2$ is the instantaneous angular velocity of precession of the gyroscope. The component of the vorticity in (29) has

been determined from the laser-ranged satellites LAGEOS (NASA) and LAGEOS 2 (NASA-ASI) [14], and the data implies the indicated coefficient on the RHS of (27) to $\pm 10\%$. For GP-B the direction of \mathbf{S} has been chosen so that this precession is cumulative and, on averaging over an orbit, corresponds to some 7.7×10^{-6} arcsec per orbit, or 0.042 arcsec per year. GP-B has been superbly engineered so that measurements to a precision of 0.0005 arcsec are possible.

However for the Earth-translation induced precession if we use $v_R = 430$ km/s (in the direction RA = 5.2^{hr} , Dec = -67°), (28) gives

$$\vec{\omega}(\mathbf{r})_{\text{trans}} = \frac{2GM}{c^2} \frac{\mathbf{v}_R \times \mathbf{r}}{r^3}, \quad (32)$$

and then the total vorticity is $\vec{\omega} = \vec{\omega}_{\text{rot}} + \vec{\omega}_{\text{trans}}$. The maximum magnitude of the speed of this precession component is $\omega_{\text{trans}}/2 = gv_C/c^2 = 8 \times 10^{-6}$ arcsec/s, where here g is the usual gravitational acceleration at the altitude of the satellite. This precession has a different signature: it is not cumulative, and is detectable by its variation over each single orbit, as its orbital average is zero, to first approximation.

Essentially then these spin precessions are caused by the rotation of the “wavepackets” describing the matter forming the gyroscopes, and caused in turn by the vorticity of 3-space. The above analysis shows that the rotation is exactly the same as the rotation of the 3-space itself, just as the acceleration of “matter” was exactly the same as the acceleration of the 3-space. We thus obtain a much clearer insight into the nature of motion, and which was not possible in the spacetime formalism.

8 Conclusions

We have seen herein that the new theory of 3-space has resulted in a number of fundamental developments, namely that a complex “quantum foam” dynamical 3-space exists and has a fractal “flow” structure, as revealed most clearly by the extraordinary DeWitte coaxial-cable experiment. This fractal structure requires that the fundamental equations of physics be generalised to take account of, for the first time, the physics of this 3-space and, in particular, here the inclusion of that dynamics within the dynamics of quantum systems. We saw that the generalisation of the Schrödinger equation is unique, and that from an Ehrenfest wavepacket analysis we obtained the equivalence principle, with the acceleration of “matter” being shown to be identical to the acceleration of the 3-space; which while not unexpected, is derived here for the first time. This result shows that the equivalence principle is really a quantum-theoretic effect. As well we obtained by that same analysis that any vorticity in the 3-space velocity field will result in a corresponding rotation of wavepackets, and just such an effect is being studied in the GP-B gyroscope experiment. So for the first time we see that the original Schrödinger equation actually lacked a key

dynamical ingredient. We saw that self-consistency within the small-wavepacket approximation imposed restrictions on the dynamical equations that determine the vorticity, giving yet another indication of the close connection between quantum theory and the phenomena of 3-space and gravity. As well because the 3-space is fractal the generalised Schrödinger equation now contains a genuine element of stochasticity.

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References

1. Cahill R. T. Process physics: from information theory to quantum space and matter. Nova Science Pub., N.Y., 2005.
2. Cahill R. T. Absolute motion and gravitational effects. *Apeiron*, 2004, v. 11, no.1, 53–111.
3. Cahill R. T. The Michelson and Morley 1887 experiment and the discovery of absolute motion. *Progress in Physics*, 2005, v. 3, 25–29; Cahill R. T. and Kitto K. *Apeiron*, 2003, v. 10, no. 2, 104–117.
4. Miller D. C. *Rev. Mod. Phys.*, 1933, v. 5, 203–242.
5. Cahill R. T. “Dark matter” as a quantum foam in-flow effect. *Trends in Dark Matter Research* (ed. by J. Val Blain), Nova Science Pub., N.Y., 2005; Cahill R. T. Gravitation, the “dark matter” effect, and the fine structure constant. *Apeiron*, 2005, v. 12, no.2, 155–177.
6. Cahill R. T. Black holes in elliptical and spiral galaxies and in globular clusters. *Progress in Physics*, 2005, v. 3, 51–56.
7. Cahill R. T. Quantum foam, gravity and gravitational waves. *Relativity, Gravitation, Cosmology*, Nova Science Pub., N.Y., 2004, 168–226.
8. Schrödinger E. *Ann. d. Physik*, 1926, v. 79, 361–376, 489–527, 734–756; *Die Naturwissenschaften*, 1926, v. 14, 664; *Phys. Rev.*, 1926, v. 28, 1049.
9. Ehrenfest P. *Z. Physik*, 1927, v. 45, 455.
10. Lense J. and Thirring H. *Phys. Z.*, 1918, v. 29, 156.
11. Shnoll S. E. *et al.* Experiments with radioactive decay of ^{239}Pu evidence sharp anisotropy of space. *Progress in Physics*, 2005, v. 1, 81–84, and references therein.
12. Cahill R. T. Novel gravity probe B frame-dragging effect. *Progress in Physics*, 2005, v. 3, 30–33.
13. Schiff L. I. *Phys. Rev. Lett.*, 1960, v. 4, 215.
14. Ciufolini I. and Pavlis E. *Nature*, 2004, v. 431, 958–960.

Zelmanov's Anthropic Principle and the Infinite Relativity Principle

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Zelmanov's Anthropic Principle, although introduced in the 1940's, has been published only recently: "The Universe has the interior we observe because we observe the Universe in this way. It is impossible to divorce the Universe from the observer. If the observer is changed, then the observed world will present in some other way, so the Universe observed will also be changed. If no observers exist then the observable Universe as well does not exist." Zelmanov's mathematical apparatus of physical observable quantities employs the Principle to the General Theory of Relativity. Using this apparatus he developed the Infinite Relativity Principle: "In homogeneous isotropic cosmological models spatial infinity of the Universe and infinity of its evolution span depend on our choice of the observer's reference frame."

Abraham Zelmanov (1913–1987), a prominent cosmologist, introduced his Anthropic Principle in the 1940's, but it has been published only recently. It is probable that he reached his ideas not only from his pure mathematical studies on the General Theory of Relativity and relativistic cosmology — besides these he had an excellent knowledge of religious considerations on world-genesis and the origin of humanity. We can now only guess at the way in which he came to his idea of the Anthropic Principle. The fact is that for more than 60 years his Anthropic Principle remained known only a close circle of several of his pupils. His book containing his main fundamental studies on the General Theory of Relativity and relativistic cosmology was written in 1944 and had survived only in manuscript until it has been published in 2004 [1].

Zelmanov stated his Anthropic Principle in two versions. The first version sets forth the law of human evolution dependent upon fundamental physical constants:

Humanity exists at the present day and we observe world constants completely because the constants bear their specific numerical values at this time. When the world constants bore other values humanity did not exist. When the constants change to other values humanity will disappear. That is, humanity can exist only with the specific scale of the numerical values of the cosmological constants. Humanity is only an episode in the life of the Universe. At the present time cosmological conditions are such that humanity develops.

In the second form he argues that any observer depends on the Universe he observes in the same way that the Universe depends on him:

The Universe has the interior we observe, because we observe the Universe in this way. It is impossible to divorce the Universe from the observer. The observable Universe depends on the observer and the observer depends on the Universe. If the contemporary

physical conditions in the Universe change then the observer is changed. And vice versa, if the observer is changed then he will observe the world in another way. So the Universe he observes will be also changed. If no observers exist then the observable Universe as well does not exist.

It is probable that by proceeding from his Anthropic Principle, in 1941–1944 Zelmanov solved the well-known problem of physical observable quantities in the General Theory of Relativity [1, 2]. It should be noted, many researchers were working on the theory of observable quantities in the 1940's. For example, Landau and Lifshitz in their famous *The Classical Theory of Fields* [3] introduced observable time and the observable three-dimensional interval similar to those introduced by Zelmanov. But they limited themselves only to this particular case and did not arrive at general mathematical methods to define physical observable quantities in pseudo-Riemannian spaces. It was only Cattaneo, an Italian mathematician, who developed his own approach to the problem, not far removed from Zelmanov's solution. Cattaneo published his results on the theme in 1958 and later [4, 5].

In 1944 Zelmanov completed a complete mathematical apparatus [1, 2] to calculate physical observable quantities in four-dimensional pseudo-Riemannian space, that is the strict solution of that problem. He called the apparatus the *theory of chronometric invariants*. The essence of his theory is that if an observer accompanies his physical reference body, his observable quantities are projections of four-dimensional quantities on his time line and the spatial section — *chronometrically invariant quantities*, made by projecting operators $b^\alpha = \frac{dx^\alpha}{ds}$ and $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$ which fully define his real reference space (here b^α is his velocity with respect to his real references). Thus, the chr.inv.-projections of a world-vector Q^α are $b_\alpha Q^\alpha = \frac{Q_0}{\sqrt{g_{00}}}$ and $h_\alpha^i Q^\alpha = Q^i$, while chr.inv.-projections of a world-tensor of the 2nd rank $Q^{\alpha\beta}$

are $b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}$, $h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q_0^i}{\sqrt{g_{00}}}$, $h^\alpha_i h^\beta_k Q^{\alpha\beta} = Q^{ik}$. Physically observable properties of the space are derived from the fact that chr.inv.-differential operators $\frac{\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{\partial}{\partial t}$ are non-commutative $\frac{\partial}{\partial t} \frac{\partial}{\partial x^i} - \frac{\partial}{\partial x^i} \frac{\partial}{\partial t} = \frac{1}{c^2} F_i \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x^i} \frac{\partial}{\partial x^k} - \frac{\partial}{\partial x^k} \frac{\partial}{\partial x^i} = \frac{2}{c^2} A_{ik} \frac{\partial}{\partial t}$, and also from the fact that the chr.inv.-metric tensor h_{ik} may not be stationary. The observable characteristics are the chr.inv.-vector of gravitational inertial force F_i , the chr.inv.-tensor of angular velocities of the space rotation A_{ik} , and the chr.inv.-tensor of rates of the space deformations D_{ik} , namely

$$F_i = \frac{1}{\sqrt{g_{00}}} \left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right), \quad \sqrt{g_{00}} = 1 - \frac{w}{c^2}$$

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i),$$

$$D_{ik} = \frac{1}{2} \frac{\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2} \frac{\partial h^{ik}}{\partial t}, \quad D_k^k = \frac{\partial \ln \sqrt{h}}{\partial t},$$

where w is gravitational potential, $v_i = -c \frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ is the chr.inv.-metric tensor, and also $h = \det \|h_{ik}\|$, $h_{g00} = -g$, $g = \det \|g_{\alpha\beta}\|$. Observable inhomogeneity of the space is set up by the chr.inv.-Christoffel symbols $\Delta_{jk}^i = h^{im} \Delta_{jk,m}$, which are built just like Christoffel's usual symbols $\Gamma_{\mu\nu}^\alpha = g^{\alpha\sigma} \Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$.

A four-dimensional generalization of the main chr.inv.-quantities F_i , A_{ik} , and D_{ik} (by Zelmanov, the 1960's [10]) is: $F_\alpha = -2c^2 b^\beta a_{\beta\alpha}$, $A_{\alpha\beta} = ch_\alpha^\mu h_\beta^\nu a_{\mu\nu}$, $D_{\alpha\beta} = ch_\alpha^\mu h_\beta^\nu d_{\mu\nu}$, where $a_{\alpha\beta} = \frac{1}{2} (\nabla_\alpha b_\beta - \nabla_\beta b_\alpha)$, $d_{\alpha\beta} = \frac{1}{2} (\nabla_\alpha b_\beta + \nabla_\beta b_\alpha)$.

In this way, for any equations obtained using general covariant methods, we can calculate their physically observable projections on the time line and the spatial section of any particular reference body and formulate the projections in terms of their real physically observable properties, from which we obtain equations containing only quantities measurable in practice.

Zelmanov deduced chr.inv.-formulae for the space curvature [1]. He followed that procedure by which the Riemann-Christoffel tensor was built: proceeding from the non-commutativity of the second derivatives of an arbitrary vector ${}^* \nabla_i {}^* \nabla_k Q_l - {}^* \nabla_k {}^* \nabla_i Q_l = \frac{2A_{ik}}{c^2} \frac{\partial Q_l}{\partial t} + H_{lki}^{\dots j} Q_j$, he obtained the chr.inv.-tensor $H_{lki}^{\dots j} = \frac{\partial \Delta_{il}^j}{\partial x^k} - \frac{\partial \Delta_{kl}^j}{\partial x^i} + \Delta_{il}^m \Delta_{km}^j - \Delta_{kl}^m \Delta_{im}^j$, which is similar to Schouten's tensor from the theory of non-holonomic manifolds. The tensor $H_{lki}^{\dots j}$ differs algebraically from the Riemann-Christoffel tensor because of the presence of the space rotation A_{ik} in the formula.

Nevertheless its generalization gives the chr.inv.-tensor

$$C_{lkij} = \frac{1}{4} (H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}),$$

which possesses all the algebraic properties of the Riemann-Christoffel tensor in this three-dimensional space and, at the same time, the property of chronometric invariance. Therefore Zelmanov called C_{iklj} the *chr.inv.-curvature tensor* as the tensor of the observable curvature of the observer's spatial section. Its contraction step-by-step

$$C_{kj} = C_{kij}^{\dots i} = h^{im} C_{kimj}, \quad C = C_j^j = h^{lj} C_{lj}$$

gives the chr.inv.-scalar C , which is the *observable three-dimensional curvature* of this space.

Chr.inv.-projections of the Riemann-Christoffel tensor are [1]: $X^{ik} = -c^2 \frac{R_{00}^{ik}}{g_{00}}$, $Y^{ijk} = -c \frac{R_{0\dots}^{ijk}}{\sqrt{g_{00}}}$, $Z^{ijkl} = c^2 R^{ijkl}$.

Solving Einstein's equations with this mathematical apparatus, Zelmanov obtained the total system of all cosmological models (senarios of the Universe's evolution) which could be possible as derived from the equations [1, 6]. In particular, he had arrived at the possibility that infinitude may be relative. Later, in the 1950's, he enunciated the *Infinite Relativity Principle*:

In homogeneous isotropic cosmological models spatial infinity of the Universe depends on our choice of that reference frame from which we observe the Universe (the observer's reference frame). If the three-dimensional space of the Universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. The same is just as well true for the time during which the Universe evolves.

In other words, using purely mathematical methods of the General Theory of Relativity, Zelmanov showed that any observer forms his world-picture from a comparison between his observation results and some standards he has in his laboratory — the standards of different objects and their physical properties. So the "world" we see as a result of our observations depends directly on that set of physical standards we have, so the "visible world" depends directly on our considerations about some objects and phenomena.

The mathematical apparatus of physical observable quantities and those results it gave in relativistic cosmology were the first results of Zelmanov's application of his Anthropic Principle to the General Theory of Relativity. To obtain the results with general covariant methods (standard in the General Theory of Relativity), where observation results do not depend on the observer's reference properties, would be impossible.

Now, according to the wishes of those who knew Zelmanov closely, I would like to say a few good words in memory of him.

Abraham Leonidovich Zelmanov was born in May 15, 1913 in Poltava Gubernya of the Russian Empire. His father

was a Judaic religious scientist, a specialist in comments on Torah and Kabbalah. In 1937 Zelmanov completed his education at the Mechanical Mathematical Department of Moscow University. After 1937 he was a research-student at the Sternberg Astronomical Institute in Moscow, where he presented his dissertation in 1944. In 1953 he was arrested for “cosmopolitanism” within the framework of Stalin’s campaign against Jews, however as soon as Stalin had died Zelmanov was set free after some months of in gaol. For several decades Zelmanov and his paralyzed parents lived in a room in a shared flat with neighbours. He took everyday care of his parents, so they lived into old age. Only in the 1970’s did he obtain a personal municipal flat. He was married three times. Zelmanov worked on the academic staff of the Sternberg Astronomical Institute all life, until his death on the winter’s day, the 2nd of February, 1987.



Abraham Zelmanov, 1940's

He was very thin in physique, like an Indian yogi, rather shorter than average, and a very delicate man. From his appearance it was possible to think that his life and thoughts were rather ordinary or uninteresting. However, in acquaintance with him and his scientific discussions in friendly company one formed another opinion about him. Those were discussions with a great scientist and humanist who reasoned in a very unorthodox way. Some-

times we thought that we were not speaking with a contemporary scientist of the 20th century, but some famous philosopher from Classical Greece or the Middle Ages. So the themes of those discussions are eternal – the interior of the Universe, what is the place of a human in the Universe, what are space and time.

Zelmanov liked to remark that he preferred to make mathematical “instruments” more than to use them in practice. Perhaps thereby his main goal in science was the mathematical apparatus of physical observable quantities in the General Theory of Relativity known as the *theory of chronometric invariants* [1, 2]. In developing the apparatus he also created other mathematical methods, namely – *kinematic invariants* [9] and *monad formalism* [10]. Being very demanding of himself, Zelmanov published less than a dozen scientific articles during his life (see References), so every publication is a concentrate of his fundamental scientific ideas.

Most of his time was spent in scientific work, but he sometimes gave lectures on the General Theory of Relativity and relativistic cosmology as a science for the geometrical structure of the Universe. Stephen Hawking, a young scientist in the 1960’s, attended Zelmanov’s seminars on cosmology at the Sternberg Astronomical Institute in Moscow. Zelmanov presented him as a “promising young cosmologist”. Hawking read a brief report at one of those seminars.

Because Zelmanov made scientific creation the main goal of his life writing articles was a waste of time to him. However he never regreted time spent on long discussions in friendly company, where he set forth his philosophic concepts on the geometrical structure of the Universe and the ways of human evolution. In those discussions he formulated his famous Anthropic Principle and the Infinite Relativity Principle.

Now everyone may read it. I hope that Zelmanov’s classical works on the General Theory of Relativity and cosmology, in particular his Anthropic Principle and the Infinite Relativity Principle known to a very close circle of his pupils, will become more widely known and appreciated.

References

1. Zelmanov A.L. Chronometric invariants. Dissertation, 1944. First published: CERN, EXT-2004-117, 236 pages.
2. Zelmanov A.L. Chronometric invariants and co-moving coordinates in the general relativity theory. *Doklady Acad. Nauk USSR*, 1956, v. 107 (6), 815–818.
3. Landau L.D. and Lifshitz E.M. The classical theory of fields. GITTL, Moscow, 1939 (ref. with the 4th final exp. edition, Butterworth–Heinemann, 1980).
4. Cattaneo C. General Relativity: Relative standard mass, momentum, energy, and gravitational field in a general system of reference. *Il Nuovo Cimento*, 1958, v. 10 (2), 318–337.
5. Cattaneo C. Problèmes d’interprétation en Relativité Générale. *Colloques Internationaux du Centre National de la Recherche Scientifique*, No. 170 “Fluides et champ gravitationnel en Relativité Générale”, Éditions du Centre National de la Recherche Scientifique, Paris, 1969, 227–235.
6. Zelmanov A.L. To relativistic theory of anisotropic inhomogeneous Universe. *Proceedings of the 6th Soviet Conference on Cosmogony*, Nauka, Moscow, 1959, 144–174 (*in Russian*).
7. Zelmanov A.L. To statement of the problem of the infinity of space in the general relativity theory. *Doklady Acad. Nauk USSR*, 1959, v. 124 (5), 1030–1034.
8. Zelmanov A.L. To problem of the deformation of the co-moving space in Einstein’s theory of gravitation. *Doklady Acad. Nauk USSR*, 1960, v. 135 (6), 1367–1370.
9. Zelmanov A.L. Kinematic invariants and their relation to chronometric invariants in Einstein’s theory of gravitation. *Doklady Acad. Nauk USSR*, 1973, v. 209 (4), 822–825.
10. Zelmanov A.L. Orthometric form of monad formalism and its relations to chronometric and kinematic invariants. *Doklady Acad. Nauk USSR*, 1976, v. 227 (1), 78–81.
11. Zelmanov A.L. and Khabibov Z.R. Chronometrically invariant variations in Einstein’s gravitation theory. *Doklady Acad. Nauk USSR*, 1982, v. 268 (6), 1378–1381.
12. Zelmanov A.L. and Agakov V.G. Elements of the General Theory of Relativity. Nauka, Moscow, 1989 (*in Russian*).

On Nonlinear Quantum Mechanics, Brownian Motion, Weyl Geometry and Fisher Information

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A new nonlinear Schrödinger equation is obtained explicitly from the (fractal) Brownian motion of a massive particle with a complex-valued diffusion constant. Real-valued energy plane-wave solutions and solitons exist in the free particle case. One remarkable feature of this nonlinear Schrödinger equation based on a (fractal) Brownian motion model, over all the other nonlinear QM models, is that the quantum-mechanical energy functional coincides precisely with the field theory one. We finalize by showing why a *complex* momentum is essential to fully understand the physical implications of Weyl's geometry in QM, along with the interplay between Bohm's Quantum potential and Fisher Information which has been overlooked by several authors in the past.

1 Introduction

Over the years there has been a considerable debate as to whether linear QM can fully describe Quantum Chaos. Despite that the quantum counterparts of classical chaotic systems have been studied via the techniques of linear QM, it is our opinion that Quantum Chaos is truly a new paradigm in physics which is associated with non-unitary and nonlinear QM processes based on non-Hermitian operators (implementing time symmetry breaking). This Quantum Chaotic behavior should be linked more directly to the Nonlinear Schrödinger equation without any reference to the nonlinear behavior of the classical limit. For this reason, we will analyze in detail the fractal geometrical features underlying our Nonlinear Schrödinger equation obtained in [6].

Nonlinear QM has a practical importance in different fields, like condensed matter, quantum optics and atomic and molecular physics; even quantum gravity may involve nonlinear QM. Another important example is in the modern field of quantum computing. If quantum states exhibit small nonlinearities during their temporal evolution, then quantum computers can be used to solve NP-complete (non polynomial) and #P problems in polynomial time. Abrams and Lloyd [19] proposed logical gates based on non linear Schrödinger equations and suggested that a further step in quantum computing consists in finding physical systems whose evolution is amenable to be described by a NLSE.

On other hand, we consider that Nottale and Ord's formulation of quantum mechanics [1], [2] from first principles based on the combination of scale relativity and fractal space-time is a very promising field of future research. In this work we extend Nottale and Ord's ideas to derive the nonlinear Schrödinger equation. This could shed some light on the physical systems which could be appropriately described by

the nonlinear Schrödinger equation derived in what follows.

The contents of this work are the following: In section 2 we derive the nonlinear Schrödinger equation by extending Nottale-Ord's approach to the case of a fractal Brownian motion with a complex diffusion constant. We present a thorough analysis of such nonlinear Schrödinger equation and show why it cannot linearized by a naive complex scaling of the wavefunction $\psi \rightarrow \psi^\lambda$.

Afterwards we will describe the explicit interplay between Fisher Information, Weyl geometry and the Bohm's potential by introducing an action based on a *complex* momentum. The connection between Fisher Information and Bohm's potential has been studied by several authors [24], however the importance of introducing a *complex* momentum $P_k = p_k + iA_k$ (where A_k is the Weyl gauge field of dilatations) in order to fully understand the physical implications of Weyl's geometry in QM, along with the interplay between Bohm's quantum potential and Fisher Information, has been overlooked by several authors in the past [24], [25]. For this reason we shall review in section 3 the relationship between Bohm's Quantum Potential and the Weyl curvature scalar of the Statistical ensemble of particle-paths (an Abelian fluid) associated to a single particle that was initially developed by [22]. A Weyl geometric formulation of the Dirac equation and the nonlinear Klein-Gordon wave equation was provided by one of us [23]. In the final section 4, we summarize our conclusions and include some additional comments.

2 Nonlinear QM as a fractal Brownian motion with a complex diffusion constant

We will be following very closely Nottale's derivation of the ordinary Schrödinger equation [1]. Recently Nottale and

Celerier [1] following similar methods were able to derive the Dirac equation using bi-quaternions and after breaking the parity symmetry $dx^\mu \leftrightarrow -dx^\mu$, see references for details. Also see the Ord's paper [2] and the Adlers's book on quaternionic QM [16]. For simplicity the one-particle case is investigated, but the derivation can be extended to many-particle systems. In this approach particles do not follow smooth trajectories but fractal ones, that can be described by a continuous but non-differentiable fractal function $\vec{r}(t)$. The time variable is divided into infinitesimal intervals dt which can be taken as a given scale of the resolution.

Then, following the definitions given by Nelson in his stochastic QM approach (Lemos in [12] p. 615; see also [13, 14]), Nottale define mean backward and forward derivatives

$$\frac{d_{\pm}\vec{r}(t)}{dt} = \lim_{\Delta t \rightarrow \pm 0} \left\langle \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \right\rangle, \quad (1)$$

from which the forward and backward mean velocities are obtained,

$$\frac{d_{\pm}\vec{r}(t)}{dt} = \vec{b}_{\pm}. \quad (2)$$

For his deduction of Schrödinger equation from this fractal space-time classical mechanics, Nottale starts by defining the complex-time derivative operator

$$\frac{\delta}{dt} = \frac{1}{2} \left(\frac{d_+}{dt} + \frac{d_-}{dt} \right) - i \frac{1}{2} \left(\frac{d_+}{dt} - \frac{d_-}{dt} \right), \quad (3)$$

which after some straightforward definitions and transformations takes the following form,

$$\frac{\delta}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} - iD\nabla^2, \quad (4)$$

D is a real-valued diffusion constant to be related to the Planck constant.

The D comes from considering that the scale dependent part of the velocity is a Gaussian stochastic variable with zero mean, (see de la Peña at [12] p. 428)

$$\langle d\xi_{\pm i} d\xi_{\pm j} \rangle = \pm 2D\delta_{ij}dt. \quad (5)$$

In other words, the fractal part of the velocity $\vec{\xi}$, proportional to the $\vec{\zeta}$, amount to a Wiener process when the fractal dimension is 2.

Afterwards, Nottale defines a set of complex quantities which are generalization of well known classical quantities (Lagrange action, velocity, momentum, etc), in order to be coherent with the introduction of the complex-time derivative operator. The complex time dependent wave function ψ is expressed in terms of a Lagrange action S by $\psi = e^{iS/(2mD)}$. S is a complex-valued action but D is real-valued. The velocity is related to the momentum, which can be expressed as the gradient of S , $\vec{p} = \vec{\nabla}S$. Then the following known relation is found,

$$\vec{V} = -2iD\vec{\nabla} \ln \psi. \quad (6)$$

The Schrödinger equation is obtained from the Newton's equation (force = mass times acceleration) by using the expression of \vec{V} in terms of the wave function ψ ,

$$-\vec{\nabla}U = m \frac{\delta}{dt} \vec{V} = -2imD \frac{\delta}{dt} \vec{\nabla} \ln \psi. \quad (7)$$

Replacing the complex-time derivation (4) in the Newton's equation gives us

$$-\vec{\nabla}U = -2im \left(D \frac{\partial}{\partial t} \vec{\nabla} \ln \psi \right) - 2D\vec{\nabla} \left(D \frac{\nabla^2 \psi}{\psi} \right). \quad (8)$$

Simple identities involving the $\vec{\nabla}$ operator were used by Nottale. Integrating this equation with respect to the position variables finally yields

$$D^2 \nabla^2 \psi + iD \frac{\partial \psi}{\partial t} - \frac{U}{2m} \psi = 0, \quad (9)$$

up to an arbitrary phase factor which may set to zero. Now replacing D by $\hbar/(2m)$, we get the Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi = U\psi. \quad (10)$$

The Hamiltonian operator is Hermitian, this equation is linear and clearly is homogeneous of degree one under the substitution $\psi \rightarrow \lambda\psi$.

Having reviewed Nottale's work [1] we can generalize it by relaxing the assumption that the diffusion constant is real; we will be working with a complex-valued diffusion constant; i. e. with a complex-valued \hbar . This is our new contribution. The reader may be immediately biased against such approach because the Hamiltonian ceases to be Hermitian and the energy becomes complex-valued. However this is not always the case. We will explicitly find plane wave solutions and soliton solutions to the nonlinear and non-Hermitian wave equations with real energies and momenta. For a detailed discussion on complex-valued spectral representations in the formulation of quantum chaos and time-symmetry breaking see [10]. Nottale's derivation of the Schrödinger equation in the previous section required a complex-valued action S stemming from the complex-valued velocities due to the breakdown of symmetry between the forwards and backwards velocities in the fractal zigzagging. If the action S was complex then it is not farfetched to have a complex diffusion constant and consequently a complex-valued \hbar (with same units as the complex-valued action).

Complex energy is not alien in ordinary linear QM. They appear in optical potentials (complex) usually invoked to model the absorption in scattering processes [8] and decay of unstable particles. Complex potentials have also been used to describe decoherence. The accepted way to describe resonant states in atomic and molecular physics is based on the complex scaling approach, which in a natural way deals with complex energies [17]. Before, Nottale wrote,

$$\langle d\xi_{\pm} d\xi_{\pm} \rangle = \pm 2Ddt, \quad (11)$$

with D and $2mD = \hbar$ real. Now we set

$$\langle d\zeta_{\pm} d\zeta_{\pm} \rangle = \pm(D + D^*) dt, \quad (12)$$

with D and $2mD = \hbar = \alpha + i\beta$ complex. The complex-time derivative operator becomes now

$$\frac{\delta}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} - \frac{i}{2}(D + D^*) \nabla^2. \quad (13)$$

In the real case $D = D^*$. It reduces to the complex-time-derivative operator described previously by Nottale. Writing again the ψ in terms of the complex action S ,

$$\psi = e^{iS/(2mD)} = e^{iS/\hbar}, \quad (14)$$

where S , D and \hbar are complex-valued, the complex velocity is obtained from the complex momentum $\vec{p} = \vec{\nabla} S$ as

$$\vec{V} = -2iD\vec{\nabla} \ln \psi. \quad (15)$$

The NLSE (non-linear Schrödinger equation) is obtained after we use the generalized Newton's equation (force = mass times acceleration) in terms of the ψ variable,

$$-\vec{\nabla} U = m \frac{\delta}{dt} \vec{V} = -2imD \frac{\delta}{dt} \vec{\nabla} \ln \psi. \quad (16)$$

Replacing the complex-time derivation (13) in the generalized Newton's equation gives us

$$\begin{aligned} \vec{\nabla} U = 2im \left[D \frac{\partial}{\partial t} \vec{\nabla} \ln \psi - 2iD^2 (\vec{\nabla} \ln \psi \cdot \vec{\nabla}) \times \right. \\ \left. \times (\vec{\nabla} \ln \psi) - \frac{i}{2} (D + D^*) D \nabla^2 (\vec{\nabla} \ln \psi) \right]. \end{aligned} \quad (17)$$

Now, using the next three identities: (i) $\vec{\nabla} \nabla^2 = \nabla^2 \vec{\nabla}$; (ii) $2(\vec{\nabla} \ln \psi \cdot \vec{\nabla})(\vec{\nabla} \ln \psi) = \vec{\nabla}(\vec{\nabla} \ln \psi)^2$; and (iii) $\nabla^2 \ln \psi = \nabla^2 \psi / \psi - (\vec{\nabla} \ln \psi)^2$ allows us to integrate such equation above yielding, after some straightforward algebra, the NLSE

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\alpha}{\hbar} \nabla^2 \psi + U\psi - i \frac{\hbar^2}{2m} \frac{\beta}{\hbar} (\vec{\nabla} \ln \psi)^2 \psi. \quad (18)$$

Note the crucial minus sign in front of the kinematic pressure term and that $\hbar = \alpha + i\beta = 2mD$ is complex. When $\beta = 0$ we recover the linear Schrödinger equation.

The nonlinear potential is now complex-valued in general. Defining

$$W = W(\psi) = -\frac{\hbar^2}{2m} \frac{\beta}{\hbar} (\vec{\nabla} \ln \psi)^2, \quad (19)$$

and U the ordinary potential, we rewrite the NLSE as

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \frac{\alpha}{\hbar} \nabla^2 + U + iW \right) \psi. \quad (20)$$

This is the fundamental nonlinear wave equation of this work. It has the form of the ordinary Schrödinger equation

with the complex potential $U + iW$ and the complex \hbar . The Hamiltonian is no longer Hermitian and the potential $V = U + iW(\psi)$ itself depends on ψ . Nevertheless one could have meaningful physical solutions with real valued energies and momenta, like the plane-wave and soliton solutions studied in the next section. Here are some important remarks.

• Notice that the NLSE above *cannot* be obtained by a naive scaling of the wavefunction

$$\begin{aligned} \psi = e^{iS/\hbar_0} \rightarrow \psi' = e^{iS/\hbar} = e^{(iS/\hbar_0)(\hbar_0/\hbar)} = \\ = \psi^\lambda = \psi^{\hbar_0/\hbar}, \quad \hbar = \text{real} \end{aligned} \quad (21)$$

related to a scaling of the diffusion constant $\hbar_0 = 2mD_0 \rightarrow \hbar = 2mD$. Upon performing such scaling, the ordinary linear Schrödinger equation in the variable ψ will appear to be *nonlinear* in the new scaled wavefunction ψ'

$$\begin{aligned} i\hbar \frac{\partial \psi'}{\partial t} = -\frac{\hbar^2}{2m} \frac{\hbar_0}{\hbar} \nabla^2 \psi' + U\psi' - \\ - \frac{\hbar^2}{2m} \left(1 - \frac{\hbar_0}{\hbar}\right) (\vec{\nabla} \ln \psi')^2 \psi', \end{aligned} \quad (22)$$

but this apparent nonlinearity is only an *artifact* of the change of variables (the scaling of ψ).

Notice that the latter (apparent) nonlinear equation, despite having the same form as the NLSE, obtained from a complex-diffusion constant, differs *crucially* in the actual values of the coefficients multiplying each of the terms. The NLSE has the complex coefficients α/\hbar (in the kinetic terms), and $-i\beta/\hbar$ (in the nonlinear logarithmic terms) with $\hbar = \alpha + i\beta = \text{complex}$. However, the nonlinear equation obtained from a naive scaling involves *real* and *different* numerical coefficients than those present in the NLSE. Therefore, the genuine NLSE *cannot* be obtained by a naive scaling (redefinition) of the ψ and the diffusion constant.

Notice also that even if one scaled ψ by a complex exponent $\psi \rightarrow \psi^\lambda$ with $\lambda = \hbar_0/\hbar$ and $\hbar = \text{complex}$, the actual numerical values in the apparent nonlinear equation, in general, would have still been different than those present in the NLSE. However, there is an actual equivalence, if, and only if, the scaling exponent $\lambda = \hbar_0/\hbar$ obeyed the condition:

$$\alpha = \hbar_0 \Rightarrow 1 - \frac{\hbar_0}{\hbar} = 1 - \frac{\alpha}{\hbar} = 1 - \frac{\hbar - i\beta}{\hbar} = i \frac{\beta}{\hbar} \quad (23)$$

in this very special case, the NLSE would be obtained from a linear Schrödinger equation after scaling the wavefunction $\psi \rightarrow \psi^\lambda$ with a complex exponent $\lambda = \hbar_0/\hbar = \alpha/\hbar$. In this very special and restricted case, the NLSE could be *linearized* by a scaling of the wavefunction with complex exponent.

From this analysis one infers, immediately, that if one defines the norm of the complex \hbar : $\|\hbar\| = \sqrt{\alpha^2 + \beta^2} = \hbar_0$ to coincide precisely with the observed value \hbar_0 of Planck's constant, then $\alpha \neq \hbar_0$, $i\beta \neq \hbar - \hbar_0$ and, consequently, the

NLSE *cannot* be obtained from the ordinary (linear) Schrödinger equations after a naive scaling, with a complex exponent, $\psi \rightarrow \psi^\lambda = \psi^{\hbar_0/\hbar}$. Therefore, a complex diffusion constant $2mD = \hbar = \alpha + i\beta$, with the condition $2m\|D\| = \|\hbar\| = \sqrt{\alpha^2 + \beta^2} = \hbar_0$ (observed value of Planck's constant) ensures that the NLSE is not a mere artifact of the scaling of the wavefunction $\psi \rightarrow \psi^\lambda = \psi^{\hbar_0/\hbar}$ in the ordinary linear Schrödinger equation.

It is important to emphasize that the diffusion constant is always chosen to be related to Planck constant as follows: $2m\|D\| = \|\hbar\| = \hbar_0$ which is just the transition length from a fractal to a scale-independence non-fractal regime discussed by Nottale in numerous occasions. In the relativistic scale it is the Compton wavelength of the particle (say an electron): $\lambda_c = \hbar_0/(mc)$. In the nonrelativistic case it is the de Broglie wavelength of the electron.

Therefore, the NLSE based on a fractal Brownian motion with a complex valued diffusion constant $2mD = \hbar = \alpha + i\beta$ represents truly a new physical phenomenon and a hallmark of nonlinearity in QM. For other generalizations of QM see experimental tests of quaternionic QM (in the book by Adler [16]). Equation (18) is the fundamental NLSE of this work.

- A Fractal Scale Calculus description of our NLSE was developed later on by Cresson [20] who obtained, on a rigorous mathematical footing, the same functional form of our NLSE equation above (although with different complex numerical coefficients) by using Nottale's fractal scale-calculus that obeyed a quantum bialgebra. A review of our NLSE was also given later on by [25]. Our nonlinear wave equation originated from a complex-valued diffusion constant that is related to a complex-valued extension of Planck's constant. Hence, a fractal spacetime is deeply ingrained with *nonlinear* wave equations as we have shown and it was later corroborated by Cresson [20].

- Complex-valued viscosity solutions to the Navier-Stokes equations were also analyzed by Nottale leading to the Fokker-Planck equation. Clifford-valued extensions of QM were studied in [21] C-spaces (Clifford-spaces whose enlarged coordinates are polyvectors, i.e. antisymmetric tensors) that involved a Clifford-valued number extension of Planck's constant; i.e. the Planck constant was a hyper-complex number. Modified dispersion relations were derived from the underlying QM in Clifford-spaces that lead to faster than light propagation in ordinary spacetime but without violating causality in the more fundamental Clifford spaces. Therefore, one should not exclude the possibility of having complex-extensions of the Planck constant leading to nonlinear wave equations associated with the Brownian motion of a particle in fractal spacetimes.

- Notice that the NLSE (34) obeys the homogeneity condition $\psi \rightarrow \lambda\psi$ for any constant λ . All the terms in the NLSE are scaled respectively by a factor λ . Moreover, our two parameters α, β are intrinsically connected to a complex Planck constant $\hbar = \alpha + i\beta$ such that $\|\hbar\| = \sqrt{\alpha^2 + \beta^2} = \hbar_0$

(observed Planck's constant) rather than being *ad-hoc* constants to be determined experimentally. Thus, the nonlinear QM equation derived from the fractal Brownian motion with complex-valued diffusion coefficient is intrinsically tied up with a non-Hermitian Hamiltonian and with complex-valued energy spectra [10].

- Despite having a non-Hermitian Hamiltonian we still could have eigenfunctions with real valued energies and momenta. Non-Hermitian Hamiltonians (pseudo-Hermitian) have captured a lot of interest lately in the so-called *PT* symmetric *complex* extensions of QM and QFT [27]. Therefore these ideas cannot be ruled out and they are the subject of active investigation nowadays.

3 Complex momenta, Weyl geometry, Bohm's potential and Fisher information

Despite that the interplay between Fisher Information and Bohm's potential has been studied by several authors [24] the importance of introducing a *complex* momentum $P_k = p_k + iA_k$ in order to fully understand the physical implications of Weyl's geometry in QM has been overlooked by several authors [24], [25]. We shall begin by reviewing the relationship between the Bohm's Quantum Potential and the Weyl curvature scalar of the Statistical ensemble of particle-paths (a fluid) associated to a single particle and that was developed by [22]. A Weyl geometric formulation of the Dirac equation and the nonlinear Klein-Gordon wave equation was provided by one of us [23]. Afterwards we will describe the interplay between Fisher Information and the Bohm's potential by introducing an action based on a *complex* momentum $P_k = p_k + iA_k$.

In the description of [22] one deals with a geometric derivation of the nonrelativistic Schrödinger Equation by relating the Bohm's quantum potential Q to the Ricci-Weyl scalar curvature of an ensemble of particle-paths associated to *one* particle. A quantum mechanical description of many particles is far more complex. This ensemble of particle paths resemble an Abelian *fluid* that permeates spacetime and whose ensemble density ρ affects the Weyl curvature of spacetime, which in turn, determines the geodesics of spacetime in guiding the particle trajectories. See [22], [23] for details.

Again a relation between the *relativistic* version of Bohm's potential Q and the Weyl-Ricci curvature exists but without the ordinary nonrelativistic probabilistic connections. In relativistic QM one does not speak of probability density to find a particle in a given spacetime point but instead one refers to the particle number current $J^\mu = \rho dx^\mu/d\tau$. In [22], [23] one begins with an ordinary Lagrangian associated with a point particle and whose statistical ensemble average over all particle-paths is performed only over the random initial data (configurations). Once the initial data is specified the trajectories (or rays) are completely determined by the

Hamilton-Jacobi equations. The statistical average over the random initial Cauchy data is performed by means of the ensemble density ρ . It is then shown that the Schrödinger equation can be derived after using the Hamilton-Jacobi equation in conjunction with the continuity equation and where the “quantum force” arising from Bohm’s quantum potential Q can be related to (or described by) the Weyl geometric properties of space. To achieve this one defines the Lagrangian

$$L(q, \dot{q}, t) = L_C(q, \dot{q}, t) + \gamma(\hbar^2/m) R(q, t), \quad (24)$$

where $\gamma = (1/6)(d-2)/(d-1)$ is a dimension-dependent numerical coefficient and R is the Weyl scalar curvature of the corresponding d -dimensional Weyl spacetime M where the particle lives.

Covariant derivatives are defined for contravariant vectors V^k : $V^k_{;\beta} = \partial_i V^k - \Gamma^k_{im} V^m$ where the Weyl connection coefficients are composed of the ordinary Christoffel connection plus terms involving the Weyl gauge field of dilatations A_i . The curvature tensor R^i_{mkn} obeys the same symmetry relations as the curvature tensor of Riemann geometry as well as the Bianchi identity. The Ricci symmetric tensor R_{ik} and the scalar curvature R are defined by the same formulas also, viz. $R_{ik} = R^n_{ink}$ and $R = g^{ik} R_{ik}$

$$R_{\text{Weyl}} = R_{\text{Riemann}} + (d-1) \left[(d-2) A_i A^i - \frac{2}{\sqrt{g}} \partial_i (\sqrt{g} A^i) \right], \quad (25)$$

where R_{Riemann} is the ordinary Riemannian curvature defined in terms of the Christoffel symbols without the Weyl-gauge field contribution.

In the special case that the space is flat from the Riemannian point of view, after some algebra one can show that the Weyl scalar curvature contains only the Weyl gauge field of dilatations

$$R_{\text{Weyl}} = (d-1)(d-2)(A_k A^k) - 2(d-1)(\partial_k A^k). \quad (26)$$

Now the Weyl geometrical properties are to be derived from physical principles so the A_i cannot be arbitrary but must be related to the distribution of matter encoded by the ensemble density of particle-paths ρ and can be obtained by the same (averaged) least action principle giving the motion of the particle. The minimum is to be evaluated now with respect to the class of all Weyl geometries having arbitrarily Weyl-gauge fields but with fixed metric tensor.

A variational procedure [22] yields a minimum for

$$A_i(q, t) = -\frac{1}{d-2} \partial_k (\log \rho) \Rightarrow F_{ij} = \partial_i A_j - \partial_j A_i = 0, \quad (27)$$

which means that the ensemble density ρ is Weyl-covariantly constant

$$\begin{aligned} \mathcal{D}_i \rho = 0 &= \partial_i \rho + \omega(\rho) \rho A_i = 0 \Rightarrow \\ \Rightarrow A_i(q, t) &= -\frac{1}{d-2} \partial_i (\log \rho), \end{aligned} \quad (28)$$

where $\omega(\rho)$ is the Weyl weight of the density ρ . Since A_i is a *total* derivative the length of a vector transported from A to B along *different* paths changes by the *same amount*. Therefore, a vector after being transported along a *closed* path does *not* change its overall length. This is of fundamental importance to be able to solve in a satisfactory manner Einstein’s objections to Weyl’s geometry. If the lengths were to change in a path-dependent manner as one transports vectors from point A to point B , two atomic clocks which followed different paths from A to B will tick at *different* rates upon arrival at point B .

The continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}} \partial_i (\sqrt{g} \rho v^i) = 0. \quad (29)$$

In this spirit one goes next to a geometrical derivation of the Schrödinger equation. By inserting

$$A_k = -\frac{1}{d-2} \frac{\partial \log \rho}{\partial x^k} \quad (30)$$

into

$$R_{\text{Weyl}} = (d-1)(d-2)(A_k A^k) - 2(d-1) \partial_k A^k \quad (31)$$

one gets for the Weyl scalar curvature, in the special case that the space is flat from the Riemannian point of view, the following expression

$$R_{\text{Weyl}} = \frac{1}{2\gamma\sqrt{\rho}} (\partial_i \partial^i \sqrt{\rho}), \quad (32)$$

which is precisely equal to the Bohm’s Quantum potential up to numerical factors.

The Hamilton-Jacobi equation can be written as

$$\frac{\partial S}{\partial t} + H_C(q, S, t) - \gamma \left(\frac{\hbar^2}{2m} \right) R = 0, \quad (33)$$

where the effective Hamiltonian is

$$\begin{aligned} H_C - \gamma \left(\frac{\hbar^2}{m} \right) R &= \frac{1}{2m} g^{jk} p_j p_k + V - \gamma \frac{\hbar^2}{m} R = \\ &= \frac{1}{2m} g^{jk} \frac{\partial S}{\partial x^j} \frac{\partial S}{\partial x^k} + V - \gamma \frac{\hbar^2}{m} R. \end{aligned} \quad (34)$$

When the above expression for the Weyl scalar curvature (Bohm’s quantum potential given in terms of the ensemble density) is inserted into the Hamilton-Jacobi equation, in conjunction with the continuity equation, for a momentum given by $p_k = \partial_k S$, one has then a set of two nonlinear coupled partial differential equations. After some straightforward algebra, one can verify that these two coupled differential equations will lead to the Schrödinger equation after the substitution $\Psi = \sqrt{\rho} e^{iS/\hbar}$ is made.

For example, when $d=3$, $\gamma=1/12$ and consequently, Bohm’s quantum potential $Q = -(\hbar^2/12m)R$ (when R_{Riemann} is zero) becomes

$$R = \frac{1}{2\gamma\sqrt{\rho}} \partial_i g^{ik} \partial_k \sqrt{\rho} \sim \frac{1}{2\gamma} \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} \Rightarrow Q = -\frac{\hbar^2}{2m} \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} \quad (35)$$

as is should be and from the two coupled differential equations, the Hamilton-Jacobi and the continuity equation, they both reduce to the standard Schrödinger equation in flat space

$$i\hbar \frac{\partial \Psi(\vec{x}, t)}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi(\vec{x}, t) + V\Psi(\vec{x}, t) \quad (36)$$

after, and only after, one defines $\Psi = \sqrt{\rho} e^{iS/\hbar}$.

If one had a *curved* spacetime with a nontrivial metric one would obtain the Schrödinger equation in a curved spacetime manifold by replacing the Laplace operator by the Laplace-Beltrami operator. This requires, of course, to write the continuity and Hamilton Jacobi equations in a explicit covariant manner by using the covariant form of the divergence and Laplace operator [22], [23]. In this way, the geometric properties of space are indeed affected by the presence of the particle and in turn the alteration of geometry acts on the particle through the quantum force $f_i = \gamma(\hbar^2/m)\partial_i R$ which depends on the Weyl gauge potential A_i and its derivatives. It is this peculiar feedback between the Weyl geometry of space and the motion of the particle which recapture the effects of Bohm's quantum potential.

The formulation above from [22] was also developed for a derivation of the Klein-Gordon (KG) equation. The Dirac equation and Nonlinear Relativistic QM equations were found by [23] via an average action principle. The *relativistic* version of the Bohm potential (for signature $- , + , + , +$) can be written

$$Q \sim \frac{1}{m^2} \frac{(\partial_\mu \partial^\mu \sqrt{\rho})}{\sqrt{\rho}} \quad (37)$$

in terms of the D'Alambertian operator.

To finalize this section we will explain why the Bohm-potential/Weyl scalar curvature relationship in a flat spacetime

$$Q = -\frac{\hbar^2}{2m} \frac{1}{\sqrt{\rho}} g^{ik} \partial_i \partial_k \sqrt{\rho} = \frac{\hbar^2 g^{ik}}{8m} \left(\frac{2\partial_i \partial_k \rho}{\rho} - \frac{\partial_i \rho \partial_k \rho}{\rho^2} \right) \quad (38)$$

encodes already the explicit connection between Fisher Information and the Weyl-Ricci scalar curvature R_{Weyl} (for Riemann flat spaces) after one realizes the importance of the *complex* momentum $P_k = p_k + iA_k$. This is typical of Electromagnetism after a minimal coupling of a charged particle (of charge e) to the $U(1)$ gauge field A_k is introduced as follows $\Pi_k = p_k + ieA_k$. Weyl's initial goal was to unify Electromagnetism with Gravity. It was later realized that the gauge field of Weyl's dilatations A was *not* the same as the $U(1)$ gauge field of Electromagnetism \mathcal{A} .

Since we have reviewed the relationship between the Weyl scalar curvature and Bohm's Quantum potential, it is not surprising to find automatically a connection between Fisher information and Weyl Geometry after a *complex* momentum $P_k = p_k + iA_k$ is introduced. A complex momentum has already been discussed in previous sections within the context of fractal trajectories moving forwards and backwards in time by Nottale and Ord.

If ρ is defined over an d -dimensional manifold with metric g^{ik} one obtains a natural definition of the Fisher information associated with the ensemble density ρ

$$I = g^{ik} I_{ik} = \frac{g^{ik}}{2} \int \frac{1}{\rho} \frac{\partial \rho}{\partial y^i} \frac{\partial \rho}{\partial y^k} d^n y. \quad (39)$$

In the Hamilton-Jacobi formulation of classical mechanics the equation of motion takes the form

$$\frac{\partial S}{\partial t} + \frac{1}{2m} g^{jk} \frac{\partial S}{\partial x^j} \frac{\partial S}{\partial x^k} + V = 0. \quad (40)$$

The momentum field p^j is given by $p^j = g^{jk}(\partial S/\partial x^k)$. The ensemble probability density of particle-paths $\rho(t, x^\mu)$ obeys the normalization condition $\int d^n x \rho = 1$. The continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{1}{m} \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(\sqrt{g} \rho g^{jk} \frac{\partial S}{\partial x^k} \right) = 0. \quad (41)$$

These equations completely describe the motion and can be derived from the action

$$S = \int \rho \left(\frac{\partial S}{\partial t} + \frac{1}{2m} g^{jk} \frac{\partial S}{\partial x^j} \frac{\partial S}{\partial x^k} + V \right) dt d^n x \quad (42)$$

using fixed endpoint variation in S and ρ .

The Quantization via the Weyl geometry procedure is obtained by defining the *complex* momentum in terms of the Weyl gauge field of dilatations A_k as $P_k = p_k + ieA_k$ and constructing the modified Hamiltonian in terms of the norm-squared of the *complex* momentum $P^k P_k^*$ as follows

$$H_{\text{Weyl}} = \frac{g^{jk}}{2m} [(p_j + ieA_j)(p_k - ieA_k)] + V. \quad (43)$$

The *modified* action is now:

$$S_{\text{Weyl}} = \int dt d^n x \left[\frac{\partial S}{\partial t} + \frac{g^{jk}}{2m} (p_j + ieA_j)(p_k - ieA_k) + V \right]. \quad (44)$$

The relationship between the Weyl gauge potential and the ensemble density ρ was

$$A_k \sim \frac{\partial \log(\rho)}{\partial x^k} \quad (45)$$

the proportionality factors can be re-absorbed into the coupling constant e as follows $P_k = p_k + ieA_k = p_k + i\partial_k(\log \rho)$. Hence, when the spacetime metric is flat (diagonal) $g^{jk} = \delta^{jk}$, S_{Weyl} becomes

$$\begin{aligned} S_{\text{Weyl}} &= \int dt d^n x \frac{\partial S}{\partial t} + \frac{g^{jk}}{2m} \left[\left(\frac{\partial S}{\partial x^j} + i \frac{\partial \log(\rho)}{\partial x^j} \right) \times \right. \\ &\times \left. \left(\frac{\partial S}{\partial x^k} - i \frac{\partial \log(\rho)}{\partial x^k} \right) \right] + V = \int dt d^n x \left[\frac{\partial S}{\partial t} + V + \right. \\ &\left. + \frac{g^{jk}}{2m} \left(\frac{\partial S}{\partial x^j} \right) \left(\frac{\partial S}{\partial x^k} \right) \right] + \frac{1}{2m} \int dt d^n x \left[\frac{1}{\rho} \frac{\partial \rho}{\partial x^k} \right]^2. \end{aligned} \quad (46)$$

The expectation value of S_{Weyl} is

$$\langle S_{\text{Weyl}} \rangle = \langle S_C \rangle + S_{\text{Fisher}} = \int dt d^n x \rho \left[\frac{\partial S}{\partial t} + \frac{g^{jk}}{2m} \left(\frac{\partial S}{\partial x^j} \right) \left(\frac{\partial S}{\partial x^k} \right) + V \right] + \frac{1}{2m} \int dt d^n x \rho \left[\frac{1}{\rho} \frac{\partial \rho}{\partial x^k} \right]^2. \quad (47)$$

This is how we have reproduced the Fisher Information expression directly from the last term of $\langle S_{\text{Weyl}} \rangle$:

$$S_{\text{Fisher}} \equiv \frac{1}{2m} \int dt d^n x \rho \left[\frac{1}{\rho} \frac{\partial \rho}{\partial x^k} \right]^2. \quad (48)$$

An Euler variation of the expectation value of the action $\langle S_{\text{Weyl}} \rangle$ with respect to the ρ yields:

$$\frac{\partial S}{\partial t} + \frac{\delta \langle S_{\text{Weyl}} \rangle}{\delta \rho} - \partial_j \left(\frac{\delta \langle S_{\text{Weyl}} \rangle}{\delta (\partial_j \rho)} \right) = 0 \Rightarrow \quad (49)$$

$$\frac{\partial S}{\partial t} + V + \frac{1}{2m} g^{jk} \left[\frac{\partial S}{\partial x^j} \frac{\partial S}{\partial x^k} + \left(\frac{1}{\rho^2} \frac{\partial \rho}{\partial x^j} \frac{\partial \rho}{\partial x^k} - \frac{2}{\rho} \frac{\partial^2 \rho}{\partial x^j \partial x^k} \right) \right] = 0. \quad (50)$$

Notice that the last term of the Euler variation

$$\frac{1}{2m} g^{jk} \left[\left(\frac{1}{\rho^2} \frac{\partial \rho}{\partial x^j} \frac{\partial \rho}{\partial x^k} - \frac{2}{\rho} \frac{\partial^2 \rho}{\partial x^j \partial x^k} \right) \right] \quad (51)$$

is precisely the same as the Bohm's quantum potential, which in turn, is proportional to the Weyl scalar curvature. If the continuity equation is implemented at this point one can verify once again that the last equation is equivalent to the Schrödinger equation after the replacement $\Psi = \sqrt{\rho} e^{iS/\hbar}$ is made.

Notice that in the Euler variation variation of $\langle S_{\text{Weyl}} \rangle$ w.r.t the ρ one must include those terms involving the derivatives of ρ as follows

$$-\partial_j \left(\frac{\delta [\rho (\partial_k \rho / \rho)^2]}{\delta (\partial_j \rho)} \right) = -\frac{1}{\rho} \partial_j \left(\frac{\delta (\partial_k \rho)^2}{\delta (\partial_j \rho)} \right) = -\frac{2}{\rho} \partial_j \partial^j \rho. \quad (52)$$

This explains the origins of all the terms in the Euler variation that yield Bohm's quantum potential.

Hence, to conclude, we have shown how the *last* term of the Euler variation of the averaged action $\langle S_{\text{Weyl}} \rangle$, that automatically incorporates the Fisher Information expression after a *complex* momentum $P_k = p_k + i\partial_k(\log \rho)$ is introduced via the Weyl gauge field of dilations $A_k \sim -\partial_k \log \rho$, generates once again Bohm's potential:

$$Q \sim \left(\frac{1}{\rho^2} \frac{\partial \rho}{\partial x^j} \frac{\partial \rho}{\partial x^k} - \frac{2}{\rho} \frac{\partial^2 \rho}{\partial x^j \partial x^k} \right). \quad (53)$$

To conclude, the Quantization of a particle whose Statistical ensemble of particle-paths permeate a spacetime background endowed with a Weyl geometry allows to construct a

complex momentum $P_k = \partial_k S + i\partial_k(\log \rho)$ that yields automatically the Fisher Information S_{Fisher} term. The latter Fisher Information term is crucial in generating Bohm's quantum potential Q after an Euler variation of the expectation value of the $\langle S_{\text{Weyl}} \rangle$ with respect to the ρ is performed. Once the Bohm's quantum potential is obtained one recovers the Schrödinger equation after implementing the continuity equation and performing the replacement $\Psi = \sqrt{\rho} e^{iS/\hbar}$. This completes the relationship among Bohm's potential, the Weyl scalar curvature *and* Fisher Information *after* introducing a *complex* momentum.

4 Concluding remarks

Based on Nottale and Ord's formulation of QM from first principles; i. e. from the fractal Brownian motion of a massive particle we have derived explicitly a nonlinear Schrödinger equation. Despite the fact that the Hamiltonian is not Hermitian, real-valued energy solutions exist like the plane wave and soliton solutions found in the free particle case. The remarkable feature of the fractal approach versus *all* the Nonlinear QM equation considered so far is that the Quantum Mechanical energy functional coincides precisely with the field theory one.

It has been known for some time, see Puskarz [8], that the expression for the energy functional in nonlinear QM does *not* coincide with the QM energy functional, nor it is unique. The classic Gross-Pitaveskii NLSE (of the 1960's), based on a quartic interaction potential energy, relevant to Bose-Einstein condensation, contains the nonlinear cubic terms in the Schrödinger equation, after differentiation, $(\psi^* \psi) \psi$. This equation does not satisfy the Weinberg homogeneity condition [9] and also the energy functional differs from the E_{QM} by factors of two.

However, in the fractal-based NLSE there is no discrepancy between the quantum-mechanical energy functional and the field theory energy functional. Both are given by

$$H_{\text{fractal}}^{\text{NLSE}} = -\frac{\hbar^2}{2m} \frac{\alpha}{\hbar} \psi^* \nabla^2 \psi + U \psi^* \psi - i \frac{\hbar^2}{2m} \frac{\beta}{\hbar} \psi^* (\vec{\nabla} \ln \psi)^2 \psi. \quad (54)$$

This is why we push forward the NLSE derived from the fractal Brownian motion with a complex-valued diffusion coefficient. Such equation does admit plane-wave solutions with the dispersion relation $E = \vec{p}^2 / (2m)$. It is not hard to see that after inserting the plane wave solution into the fractal-based NLSE we get (after setting $U = 0$),

$$E = \frac{\hbar^2}{2m} \frac{\alpha}{\hbar} \frac{\vec{p}^2}{\hbar^2} + i \frac{\beta}{\hbar} \frac{\vec{p}^2}{2m} = \frac{\vec{p}^2}{2m} \frac{\alpha + i\beta}{\hbar} = \frac{\vec{p}^2}{2m}, \quad (55)$$

since $\hbar = \alpha + i\beta$. Hence, the plane-wave *is* a solution to our fractal-based NLSE (when $U = 0$) with a real-valued energy and has the correct energy-momentum dispersion relation.

Soliton solutions, with real-valued energy (momentum) are of the form

$$\psi \sim [F(x - vt) + iG(x - vt)] e^{ipx/\hbar - iEt/\hbar}, \quad (56)$$

with F , G two functions of the argument $x - vt$ obeying a coupled set of two nonlinear differential equations.

It is warranted to study solutions when one turns-on an external potential $U \neq 0$ and to generalize this construction to the Quaternionic Schrödinger equation [16] based on the Hydrodynamical Nonabelian-fluid Madelung's formulation of QM proposed by [26]. And, in particular, to explore further the consequences of the Non-Hermitian Hamiltonian (pseudo-Hermitian) associated with our NLSE (34) within the context of the so-called PT symmetric *complex* extensions of QM and QFT [27]. Arguments why a quantum theory of gravity should be nonlinear have been presented by [28] where a *different* non-linear Schrödinger equation, but with a similar logarithmic dependence, was found. This equation [28] is also similar to the one proposed by Doebner and Goldin [29] from considerations of unitary representations of the diffeomorphism group.

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References

1. Nottale L. *Chaos, Solitons and Fractals*, 1994, v.4(3), 361; Celerier M. and Nottale L. Dirac equation in scale relativity. arXiv: hep-th/0112213.
2. Ord G. N. *Journal of Physics A: Math. Gen.*, 1983, v. 16, 1869.
3. Bialynicki-Birula I., Mycielsky J. *Annal of Physics*, 1976, v. 100, 62.
4. Pardy M. To the nonlinear QM. arXiv: quant-ph/0111105.
5. Bohm D. and Vigier J. *Phys. Rev.*, 1954, v. 96, 208.
6. Castro C., Mahecha J., and Rodriguez B. Nonlinear QM as a Fractal Brownian motion with complex diffusion constant. arXiv: quant-ph/0202026.
7. Madelung E. *Z. Physik*, 1926, v. 40, 322.
8. Staruszkiewicz A. *Acta Physica Polonica*, 1983, v. 14, 907; Puzscharz W. On the Staruszkiewicz modification of the Schrödinger equation. arXiv: quant-ph/9912006.
9. Weinberg S. *Ann. Phys.*, 1989, v. 194, 336.
10. Petrosky T., Prigogine I. *Chaos, Solitons and Fractals*, 1994, v. 4(3), 311.
11. Castro C. *Chaos, Solitons and Fractals*, 2001, v. 12, 101.
12. Gómez B., Moore S., Rodríguez A., and Rueda A. (eds). *Stochastic processes applied to physics and other related fields*. World Scientific, Singapore, 1983.
13. Lemos N. A. *Phys. Lett. A*, 1980, v. 78, 237; 1980, v. 78, 239.
14. Ghirardi G. C., Omero C., Rimini A., and Weber T. *Rivista del Nuovo Cimento*, 1978, v. 1, 1.
15. Kamesberger J., Zeilinger J. *Physica B*, 1988, v. 151, 193.
16. Adler S. L. *Quaternionic quantum mechanics and quantum fields*. Oxford University Press, Oxford, 1995.
17. Yaris R. and Winkler P. *J. Phys. B: Atom. Molec Phys.*, 1978, v. 11, 1475; 1978, v. 11, 1481; Moiseyev N. *Phys. Rep.*, 1998, v. 302, 211.
18. Mensky M. B. *Phys. Lett. A*, 1995, v. 196, 159.
19. Abrams D. S. and Lloyd S. *Phys. Rev. Lett.*, 1998, v. 81, 3992.
20. Cresson J. *Scale Calculus and the Schrodinger Equation*. arXiv: math.GM/0211071.
21. Castro C. *Foundations of Physics*, 2000, v. 8, 1301; *Chaos, Solitons and Fractals*, 2000, v. 11(11), 1663–1670; On the noncommutative Yang's spacetime algebra, holography, area quantization and C-space relativity. (*Submitted to European Journal of Physics C*); Castro C., Pavsic M. The Extended Relativity Theory in Clifford-spaces. *Progress in Physics*, 2005, v. 1, 31–64.
22. Santamato E. *Phys. Rev. D*, 1984, v. 29, 216; 1985, v. 32, 2615; *Jour. Math. Phys.*, 1984, v. 25, 2477.
23. Castro C. *Foundations of Physics*, 1992, v. 22, 569; *Foundations of Physics Letters*, 1991, v. 4, 81; *Jour. Math. Phys.*, 1990, v. 31(11), 2633.
24. Frieden B. *Physics from Fisher Information*. Cambridge University Press, Cambridge, 1998; Hall M., Reginatto M. *Jour. Phys. A*, 2002, v. 35, 3829; Frieden B., Plastino A., Plastino A. R., and Soffer B. A Schrödinger link between non-equilibrium thermodynamics and Fisher information. arXiv: cond-mat/0206107.
25. Carroll R. Fisher, Kahler, Weyl and the Quantum Potential. arXiv: quant-ph/0406203; Remarks on the Schrödinger Equation. arXiv: quant-ph/0401082.
26. Love P., Boghosian B. Quaternionic madelung transformation and Nonabelian fluid dynamics. arXiv: hep-th/0210242.
27. Bender C. Introduction to PT-Symmetric Quantum Theory. arXiv: quant-ph/0501052; Bender C., Cervero-Pelaez I., Milton K. A., Shajesh K. V. PT-Symmetric Quantum Electrodynamics. arXiv: hep-th/0501180.
28. Singh T., Gutti S., and Tibrewala R. Why Quantum Gravity should be Nonlinear. arXiv: gr-qc/0503116.
29. Doebner H. and Goldin G. *Phys. Lett. A*, 1992, v. 162, 397.

The Gravity of Photons and the Necessary Rectification of Einstein Equation

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It is pointed out that Special Relativity together with the principle of causality implies that the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed. Since a gravitational wave carries energy-momentum, this accompanying wave would make the energy-stress tensor of the light to be different from the electromagnetic energy-stress tensor, and thus can produce a geodesic equation for the photons. Moreover, it is found that the appropriate Einstein equation must additionally have the photonic energy-stress tensor with the antigravity coupling in the source term. This would correct that, in disagreement with the calculations for the bending of light, existing solutions of gravity for an electromagnetic wave, is unbounded. This rectification is confirmed by calculating the gravity of electromagnetic plane-waves. The gravity of an electromagnetic wave is indeed an accompanying gravitational wave. Moreover, these calculations show the first time that Special Relativity and General Relativity are compatible because the physical meaning of coordinates has been clarified. The success of this rectification makes General Relativity standing out further among theories of gravity.

1 Introduction

The physical basis of Special Relativity is constancy of the light speed, which is also the velocity of an electromagnetic wave [1]. On the other hand, the physical basis of quantum mechanics is that light can be considered as consisting of the photons [2]. Currently, it seems, there is no theoretical connection between constancy of light speed and photons, except that both are proposed by Einstein. However, since constancy of the light speed and the notion of photon are two aspects of the same physical phenomenon, from the viewpoint of physics, a theoretical connection of these notions must exist. Moreover, such a connection would be a key to understand the relationship between these two theories.

To this end, General Relativity seems to hold a special position because of the bending of light. The fact that a photon follows the geodesic of a massless particle [3, 4] manifests that there is a connection between the light speed and the photon. This suggests that General Relativity may provide some insight on the existence of the photons. In other words, the existence of the photons, though an observed fact, may be theoretically necessary because the light speed is the maximum.

On the other hand, since electromagnetism is a source for gravity [5], an electromagnetic wave would generate gravity. Thus, it is natural to ask whether its gravity is related to the existence of the photon. In other words, would the existence of the photon be an integral part of the theory of General Relativity? It will be shown here that the answer is affirmative. In fact, this is also a consequence of Special

Relativity provided that the theoretical framework of General Relativity is valid.

2 Special Relativity and the accompanying gravity of an electromagnetic wave

In a light ray, the massless light energy is propagating in vacuum with the maximum speed c . Thus, the gravity due to the light energy should be distinct from that generated by massive matter [6–7]. Since a field emitted from an energy density unit means a non-zero velocity relative to that unit, it is instructive to study the velocity addition. According to Special Relativity, the addition of velocities is as follows [1]:

$$u_x = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_z v/c^2} u'_x, \quad u_y = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_z v/c^2} u'_y, \quad (1)$$

$$\text{and } u_z = \frac{u'_z + v}{1 + u'_z v/c^2},$$

where velocity \vec{v} is in the z -direction, (u'_x, u'_y, u'_z) is a velocity in a system moving with velocity v , c is the light speed, $u_x = dx/dt$, $u_y = dy/dt$, and $u_z = dz/dt$. When $v = c$, independent of (u'_x, u'_y, u'_z) one has

$$u_x = 0, \quad u_y = 0, \quad \text{and } u_z = c. \quad (2)$$

Thus, neither the direction nor the magnitude of the velocity \vec{v} ($= \vec{c}$) have been changed.

This implies that nothing can be emitted from a light ray, and therefore no field can be generated outside the light ray. To be more specific, from a light ray, no gravitational field

can be generated outside the ray although, accompanying the light ray, a gravitational field g_{ab} ($\neq \eta_{ab}$ the flat metric) is allowed within the ray.

According to the principle of causality [7], this accompanying gravity g_{ab} should be a gravitational wave since an electromagnetic wave is the physical cause. This would put General Relativity into a severe test for theoretical consistency. However, this examination would also have the benefit of knowing that electrodynamics is completely compatible with General Relativity.

3 The accompanying gravitational wave and the photonic energy-stress tensor

Observations confirm that photons follow a geodesic. One may expect that the light energy-stress tensor $T(L)_{ab}$ would generate the photonic geodesic since the massive tensor $T(m)_{ab}$ generates the geodesic through $\nabla^c T(m)_{cb} = 0$ [5]. This means that $T(L)_{ab}$ is different from the electromagnetic energy-stress tensor $T(E)_{ab}$ since $\nabla^c T(E)_{cb}$ is the Lorentz force [7, 8].

Nevertheless, this can be resolved since a gravitational wave carries an additional energy-stress tensor $T(g)_{ab}$, i. e., one should have

$$T(L)_{ab} = T(E)_{ab} + T(g)_{ab} \quad (3)$$

since there is no other type of energy. Then, one may expect that Eq. (3) allows $\nabla^c T(L)_{cb} = 0$ to generate the necessary geodesic equation for photons.

If the light is emitted and absorbed in terms of photons, physically the photons contain all the energy of the light, i. e., the photonic energy-stress tensor,

$$T(P)_{ab} = T(L)_{ab}. \quad (4)$$

One might object on the ground that, in quantum theory, $T(E)_{ab}$ is considered as identical to the photonic energy-stress tensor $T(P)_{ab}$. However, one should note also that gravity is ignored in quantum electrodynamics.

4 The Einstein equation for an electromagnetic wave

Einstein [9] suggested the field equation for the gravity of an electromagnetic wave was

$$G_{ab} = -KT(E)_{ab}, \quad (5)$$

where G_{ab} is the Einstein tensor, and K is the coupling constant. However, to generate the photonic geodesic, the source term must include the photonic energy-stress $T(P)_{ab}$. The need of a modified equation is supported by the fact that all existing solutions, in disagreement with light bending calculation, are unbounded [7].

Moreover, if the gravity of an electromagnetic wave is a gravitational wave, validity of Eq. (5) is questionable. It

has been known from the binary pulsar experiments, that when radiation is included, the anti-gravity coupling must be included in the Einstein equation [10],

$$G_{ab} = -K[T(m)_{ab} - t(g)_{ab}], \quad (6)$$

where $T(m)_{ab}$ and $t(g)_{ab}$ are respectively the energy-stress tensors for massive matter and gravity. The need of $t(g)_{ab}$ was first conjectured by Hogarth [12]. The possibility of such an coupling was suggested by Pauli [13]. Moreover, if a space-time singularity is not a reality, the existence of an antigravity coupling is implicitly given by the singularity theorems which assume the coupling constants are of the same sign [14].

There are theories such as the Brans-Dicke's [15] and the Yilmaz's [16] that provide an extra source term in vacuum. However, it is not clear that they can provide the right formula for the gravity of an electromagnetic wave since their connection with the notion of photon was never mentioned. Besides, it is more appropriate to consider a fundamental problem from the basics.

The above analysis suggests that, to obtain an appropriate Einstein equation, one may start from considering the gravitational radiation with Einstein's radiation formula as follows:

(a) For the gravitational wave generated by massive matter, the gravitational energy-stress $t(g)_{ab}$ of Einstein's radiation formula is approximately [11].

$$t(g)_{ab} = \frac{G_{ab}^{(2)}}{K}, \quad \text{where } G_{ab}^{(2)} = G_{ab} - G_{ab}^{(1)}, \quad (7)$$

where $G_{ab}^{(1)}$ consists of all first order terms of G_{ab} . Moreover, if the gravitational energy is the same as the gravitational wave energy, one has

$$t(g)_{ab} = T(g)_{ab}. \quad (8)$$

(b) Since g_{ab} is a wave propagating with the electromagnetic wave, one may have the linear terms, $G_{ab}^{(1)} = 0$ on a time average. This suggests $G_{ab} = KT(g)_{ab}$. Thus, it follows from Eqs. (3) and (4) that

$$G_{ab} = KT(g)_{ab} = -K[T(E)_{ab} - T(P)_{ab}] \quad (9)$$

would be the appropriate Einstein equation. Comparing with Eq. (5), there is an additional term $T(P)_{ab}$.

(c) Since the Lorentz force $\nabla^c T(E)_{cb} = 0$ and $\nabla^c G_{cb} = 0$, as expected, one has the necessary formula

$$\nabla^c T(P)_{cb} = 0 \quad (10)$$

generate the photonic geodesic equation. However, to verify Eq. (9), one must first show that Eq. (5) cannot be valid for at least one example and then find the photonic energy-stress tensor $T(P)_{ab}$ for Eq. (9).

Alternatively, Eq. (9) can be derived from the principle of causality [7, 8] since the electromagnetic plane-wave as a

spatial local idealization has been justified in electrodynamics. In general, without an idealization, to solve the gravity of an electromagnetic wave is very difficult [4].

5 The reduced Einstein equation for plane-waves

Due to the speed of light is the maximum, the influence of an electromagnetic wave to its accompanying gravity is spatially local. Thus, an electromagnetic plane-wave is also a valid modeling for the problem of gravity.

Now, let us consider the electromagnetic potential $A_k(t-z)$ which represents the photons moving in the z -direction. Then, Eq. (5) is reduced to a differential equation of $u (= t-z)$ [6] as follows:

$$\begin{aligned} G'' - g'_{xx}g'_{yy} + (g'_{xy})^2 - G' \frac{g'}{2g} &= 2GR_{tt} = \\ &= 2K (F_{xt}^2 g_{yy} + F_{yt}^2 g_{xx} - 2F_{xt}F_{yt} g_{xy}), \end{aligned} \quad (11)$$

where

$$G = g_{xx}g_{yy} - g_{xy}^2, \quad g = |g_{ab}|$$

is the determinant of the metric, $F_{ab} = \partial_a A_b - \partial_b A_a$ is the electromagnetic field tensor, and R_{ab} is the Ricci tensor. The metric elements are connected as follows:

$$g = Gg_t^2, \quad \text{where } g_t \equiv g_{tt} + g_{tz}. \quad (12)$$

Moreover, the massless of photons implies that

$$g_{tt} + 2g_{tz} + g_{zz} = 0, \quad \text{and } g^{tt} - 2g^{tz} + g^{zz} = 0.$$

Note that Eq. (35.31) and Eq. (35.44) in reference [4] and Eq. (2.8) in reference [17] are special cases of Eq. (5). They believed that bounded wave solutions can be obtained [7].

It has been shown that $A_t, g_{xt}, g_{yt},$ and g_{zt} are allowed to be zero. Although there are four remaining metric elements ($g_{xx}, g_{xy}, g_{yy},$ and g_{tt}) to be determined, based on Einstein's notion of weak gravity and Eq. (5), it will be shown that there is no physical solution [6]. In other words, in contrast to Einstein's belief [9], the difficulty of his equation is not limited to mathematics.

6 Verification of the rectified Einstein equation

Now, consider an electromagnetic plane-waves of circular polarization, propagating to the z -direction

$$A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u, \quad \text{and } A_y = \frac{1}{\sqrt{2}} A_0 \sin \omega u, \quad (13)$$

The rotational invariants with respect to the z -axis are constants. These invariants are: $G_{tt}, R_{tt}, T(E)_{tt}, G, (g_{xx} + g_{yy}), g_{tz}, g_{tt}, g,$ and etc. It follows that [6, 7]

$$\begin{aligned} g_{xx} &= -1 - C + B_\alpha \cos(\omega_1 u + \alpha), \\ g_{yy} &= -1 - C - B_\alpha \cos(\omega_1 u + \alpha), \\ g_{xy} &= \pm B_\alpha \sin(\omega_1 u + \alpha), \end{aligned} \quad (14)$$

where C and B_α are small constants, and $\omega_1 = 2\omega$. Thus, metric (14) is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (13). On the other hand, one also has $G = (1 + C)^2 - B_\alpha^2 \geq 0$,

$$G_{tt} = \frac{2\omega^2 B_\alpha^2}{G} \geq 0, \quad (15)$$

$$T(E)_{tt} = \frac{g_{yy}}{G} \omega^2 A_0^2 (1 + C - B_\alpha \cos \alpha) > 0.$$

Thus, it is not possible to satisfy Einstein equation (5) because $T(E)_{tt}$ and G_{tt} have the same sign [6]. Thus, it is necessary to have a photonic energy-stress tensor.

Given that a geodesic equation must be produced, for a monochromatic wave, the form of a photonic energy tensor should be similar to that of massive matter. Observationally, there is very little interaction, if any, among photons of the same ray. Theoretically, since photons travel in the velocity of light, there should not be any interaction among them.

Therefore, the photonic energy tensor should be dust-like with the momentum of the photon P_a as follows:

$$T_{ab}(P) = \rho P_a P_b, \quad (16)$$

where ρ is a scalar and is a function of u . In the units $c = \hbar = 1, P_t = \omega$. It has been obtained [6] that

$$\rho(u) = -A_m g^{mn} A_n \geq 0. \quad (17)$$

Here, $\rho(u)$ is related to gravity through g^{mn} . Since light intensity is proportional to the square of the wave amplitude, ρ which is Lorentz gauge invariant, can be considered as the density function of photons. Then

$$\begin{aligned} T_{ab} &= -T(g)_{ab} = T(E)_{ab} - T(P)_{ab} = \\ &= T(E)_{ab} + A_m g^{mn} A_n P_a P_b. \end{aligned} \quad (18)$$

Thus, $T_{ab}(P)$ has been derived completely from the electromagnetic wave A_k and g_{ab} .

Physically, such a tensor should be unique. It remains to see whether all the severe physical requirements can be satisfied. In particular, validity of the light bending calculation requires compatibility with the notion of weak gravity [3]. Also, the photonic energy tensor of Misner et al. [4], is an approximation of the time average of $T_{ab}(P)$.

As expected, this tensor $T_{ab}(P)$ enables a gravity solution for wave (13). According to Eq. (8),

$$T_{tt} = -\frac{1}{G} \omega^2 A_0^2 B_\alpha \cos \alpha \leq 0, \quad (19)$$

since $B_\alpha = \frac{K}{2} A_0^2 \cos \alpha$. the energy density of the photonic energy tensor is indeed larger than that of the electromagnetic wave. $T(g)_{tt}$ is of order K . Note that, pure electromagnetic waves can exist since $\cos \alpha = 0$ is also possible. To confirm the general validity of (16), consider a wave linearly polarized in the x -direction,

$$A_x = A_0 \cos \omega(t - z). \quad (20)$$

Then, one has

$$T_{tt} = \frac{g_{yy}}{G} \omega^2 A_0^2 \cos 2\omega(t - z), \quad (21)$$

since the gravitational component is not an independent wave, $T(g)_{tt}$ is allowed to be negative. Eq. (21) implies that its polarization has to be different.

It turns out that the solution is a linearly polarized gravitational wave and that the time-average of $T(g)_{tt}$ is positive of order K [7]. From the viewpoint of physics, for an x -directional polarization, gravitational components related to the y -direction, remains the same. In other words,

$$g_{xy} = 0, \quad \text{and} \quad g_{yy} = -1. \quad (22)$$

It follows that the general solution of wave (20) is:

$$-g_{xx} = 1 + C_1 - \frac{K}{2} A_0^2 \cos 2\omega(t - z), \quad (23)$$

and $g_{tt} = -g_{zz} = \sqrt{\frac{g}{g_{xx}}}$,

where C_1 is a constant. Note that the frequency ratio is the same as that of a circular polarization, but there is no phase difference to control the amplitude of the gravitational wave.

For a polarization in the diagonal direction of the $x - y$ plane, the solution is:

$$g_{xx} = g_{yy} = -1 - \frac{C_1}{2} + \frac{K}{4} A_0^2 \cos 2\omega(t - z), \quad (24)$$

$$g_{xy} = -\frac{C_1}{2} + \frac{K}{4} A_0^2 \cos 2\omega(t - z), \quad (25)$$

$$g_{tt} = -g_{zz} = \sqrt{\frac{-g}{1 - 2g_{xy}}}. \quad (26)$$

Note that for a perpendicular polarization, the metric element g_{xy} changes sign. The time averages of their T_{tt} are also negative as required. If $g = -1$, relativistic causality requires $C_1 \geq K A_0^2 / 2$.

7 Compatibility between Special Relativity and General Relativity

We implicitly use the same coordinate system whether the calculation is done in terms of Special Relativity or General Relativity. However, according to Einstein's "covariance principle" [1], coordinates have no physical meaning whereas the coordinates in Special Relativity have very clear meaning [18]. Thus, all the above calculations could have no meaning. Recently, it has been proven that a physical coordinate system for General Relativity necessarily has a frame of reference⁽¹⁾ with the Euclidean-like structure [19–21]. Moreover, the time

coordinate will be the same as in Special Relativity if the metric is asymptotically flat.

Many theorists, including Einstein, overlooked that the metric of a Riemannian space actually is compatible with the space coordinates with the Euclidean-like structure. Let us illustrate this with the Schwarzschild solution in quasi-Minkowskian coordinates [11],

$$-ds^2 = -\left(1 - \frac{2M\kappa}{r}\right) c^2 dt^2 + \left(1 - \frac{2M\kappa}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (27)$$

where (r, θ, φ) transforms to (x, y, z) by,

$$x = r \sin \theta \cos \varphi, \quad y = r \sin \theta \sin \varphi, \quad (28)$$

and $z = r \cos \theta$.

Coordinate transformation (28) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. Metric (27) and the Euclidean-like structure (28) are complementary to each other in the Riemannian space. Then, a light speed ($ds^2 = 0$) is defined in terms of dx/dt , dy/dt , and dz/dt [1]. This is necessary though insufficient for a physical space [19–21].

Einstein's oversight made his theory inconsistent, and thus rejected by Whitehead [22] for being not a theory in physics. For instance, his theory of measurement is incorrect because it is modeled after⁽²⁾ measurements for a Riemannian space embedded in a higher dimensional space [19–21]. In General Relativity, the local distance ($\sqrt{-ds^2}$, where $dt = 0$) represents the space contraction, which is measured in a free fall local space [1, 3]. Thus, this is a dynamic measurement since the measuring instrument is in a free fall state.

Einstein's error is that he overlooked the free fall state, and thus has mistaken this dynamic local measurement as a static measurement. Moreover, having different states at different points, this makes such a measurement for an extended object not executable.

The Euclidean-like structure determines the distance between two points in a frame of reference, and the observed light bending supports this physical meaning. This is why the interpretation of Hubble's law as a consequence of receding velocity⁽³⁾ is invalid [23]. Because the measurement theory of Einstein is invalid, the miles long arms of the laser interferometer in LIGO would not change their length under the influence of gravitational waves [24]. In other words, LIGO would inadvertently further confirm that Einstein's theory of measurement is invalid.

It has been solved that the coordinate system of General Relativity and that of Special Relativity are actually the same for this problem. We must show also that the plane waves

would satisfy the Maxwell equation in General Relativity, see [11; p. 125],

$$\frac{\partial}{\partial x^a} \sqrt{g} F^{ab} = -\sqrt{g} J^b, \quad (29)$$

$$\frac{\partial}{\partial x^a} F^{bc} + \frac{\partial}{\partial x^b} F^{ca} + \frac{\partial}{\partial x^c} F^{ab} = 0. \quad (30)$$

Since equation (30) is the same as in Special Relativity, it remains to show that (29) is satisfied for $J^a = 0$. To show this, we can use the facts that g^{ab} and F^{ab} are function of u , and that $g^{tt} + g^{zz} = 0$. It follows that

$$\begin{aligned} \frac{\partial}{\partial x^a} \sqrt{g} F^{ab} &= \frac{\partial \sqrt{g}}{\partial t} (F^{tb} - F^{zb}) = \\ &= \frac{\partial \sqrt{g}}{\partial t} g^{tt} (\partial_t A_c + \partial_z A_c) g^{cb} = 0. \end{aligned} \quad (31)$$

We thus complete the compatibility proof.

8 Conclusions and Discussions

A crucial argument for this case is that both Special Relativity and General Relativity use the same coordinate system. This is impossible, according to Einstein's theory of measurement. A major problem of Einstein's theory is that the physical meaning of coordinates is not only ambiguous, but also confusing⁽⁴⁾ since the physical meaning of the coordinates depends on the metric. Moreover, Einstein's equivalence principle actually contradicts the so-called "covariance principle". P. Morrison of MIT [21, 25] remarked that the "covariance principle" is physically invalid because it disrupts the necessary physical continuity from Special Relativity to General Relativity.

Now, a photonic energy-stress tensor has been obtained as physics requires. The energy and momentum of a photon is proportional to its frequency although, as expected from a classical theory, their relationship with the Planck constant \hbar is not yet clear; and the photonic energy-stress tensor is a source term in the Einstein equation. As predicted by Special Relativity, the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed. Moreover, the gravity of light is proven to be compatible with the notion of weak gravity.

In the literature [4, 26–29], however, solutions of Eq. (5) are unbounded.⁽⁵⁾ Thus, they are incompatible with the approximate validity of electrodynamics and violate physical principles including the equivalence principle and the principle of causality [7, 30]. (The existence of local Minkowski spaces is only a necessary condition⁽⁶⁾ for Einstein's equivalence principle [31].) Naturally, one may question whether the gravity due to the light is negligible. Now, the claim that the bending of light experiment confirms General Relativity, is no longer inflated.

In addition, the calculation answers a long-standing question on the propagation of gravity in General Relativity. Since an electromagnetic wave has an accompanying gravitational wave, gravity should propagate in the same speed as electromagnetism. It is interesting to note that Rabounski [32] reached the same conclusion on the propagation of gravity with a completely different method, which is independent of the Einstein equation.

One might argue that since $E = mc^2$ and the gravitational effect of the wave energy density should be outside a light ray. However, this is a misinterpretation [33, 34]. One should not, as Tolman [35] did, ignore Special Relativity and the fact that the light energy density is propagating with the maximum velocity possible. There are intrinsically different characteristics in such an energy form according to Special Relativity. This calculation confirms a comment of Einstein [23] that $E = mc^2$ must be understood in the context of energy conservation.

To illustrate this, consider the case of a linear polarization, for which Eq. (5) still has a solution [6]

$$-g_{xx} = 1 - \frac{K}{4} A_0^2 [2\omega^2(t-z)^2 + \cos 2\omega(t-z)]. \quad (32)$$

However, solution (32) is invalid since $(t-z)^2$ grows very large as time goes by. This would "represent" the effects that the wave energy were equivalent to mass. This illustrates also that Einstein's notion of weak gravity may not be compatible with an inadequate source.

The theoretical consistency between Special Relativity and General Relativity is further established. This is a very strong confirming evidence for General Relativity beyond the requirements of the equivalence principle. Moreover, this rectification makes General Relativity standing out among all theories of gravity. Moreover, since light has a gravitational wave component, it would be questionable to quantize gravity independently as in the current approach.

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Endnotes

- ⁽¹⁾ In a Riemannian geometry, a frame of reference may not exist since the coordinates can be arbitrary. However, for a physical space, a frame of reference with the Euclidean-like structure must exist because of physical requirements [19–21]. Note that the Euclidean-like structure is independent of the metric.

- (2) In the initial development of Riemannian geometry, the metric was identified formally with the notion of distance in analogy as the case of the Euclidean space. Such geometry is often illustrated with the surface of a sphere, a subspace embedded in a flat space [4, 36]. Then, the distance is determined by the flat space and can be measured with a static method. For a general case, however, the issue of measurement was not addressed before Einstein's theory.
- (3) Einstein's theory of measurements is not supported by observation, which requires [21, 37] that the light speed must be defined in terms of the Euclidean-like structure as in Einstein's own papers [1, 3].
- (4) If the "covariance principle" was valid, it has been shown that the "event of horizon" for a black hole could be just any arbitrary constant [38].
- (5) In fact, all existing solutions involving waves are unbounded because the term to accommodate gravitational wave energy-stress is missing. It is interesting that Einstein and Rosen are the first to discover the non-existence of wave solutions [39]. However, their arguments that led to their correct conclusion was incorrect. Robertson as a referee of Physical Review pointed out that the singularities mentioned are actually removable [39]. However, there are other reasons for a wave solution to be invalid. It has been found that a wave solution necessarily violates Einstein's equivalence principle and the principle of causality [10, 19].
- (6) Many theorists do not understand Einstein's equivalence principle because they failed in understanding the Einstein-Minkowski condition that the local space of a particle under gravity must be locally Minkowskian [1, 3]. This condition is crucial to obtain the time dilation and space contractions [21].

References

1. Einstein A., Lorentz H. A., Minkowski H., and Weyl H. The Principle of Relativity. Dover, New York, 1923.
2. Einstein A. *Annalen der Physik*, 1905, Bd. 17, 132–148.
3. Einstein A. The Meaning of Relativity (1921). Princeton Univ. Press, 1954.
4. Misner C. W., Thorne K. S., and Wheeler J. A. Gravitation. Freeman, San Francisco, 1973.
5. Lo C. Y. *Phys. Essays*, 1997, v.10(4), 540–545.
6. Lo C. Y. *Proc. Sixth Marcel Grossmann Meeting On Gen. Relativity, Kyoto 1991*, ed. H. Sato, and T. Nakamura, World Sci., Singapore, 1992, 1496.
7. Lo C. Y. *Phys. Essays*, 1997, v.10(3), 424–436.
8. Lo C. Y. *Phys. Essays*, 1999, v.12(2), 226–241.
9. Einstein A. Physics and Reality (1936) in Ideas and Opinions. Crown, New York, 1954, p. 311.
10. Lo C. Y. *Astrophys. J.*, 1995, v. 455, 421–428.
11. Weinberg S. Gravitation and Cosmology. John Wiley Inc., New York, 1972.
12. Hogarth J. E., Ph.D. Thesis, 1953, Dept. of Math. Royal Holloway College, University of London, p. 6.
13. Pauli W. Theory of Relativity. Pergamon Press, London, 1958, p. 163.
14. Hawking S. W. and Penrose R., *Proc. Roy. Soc. London A*, 1970, v. 314, 529–548.
15. Brans C. and Dicke R. H. *Phys. Rev.*, 1961, v. 124, 925.
16. Yilmaz H. *Hadronic J.*, 1979, v. 2, 997–1020.
17. Bondi H., Pirani F. A. E., and Robinson I. *Proc. R. Soc. London A*, 1959, v. 251, 519–533.
18. Einstein A. The Problem of Space, Ether, and the Field in Physics (1934) in Ideas and Opinions Crown, New York, 1954.
19. Lo C. Y. *Phys. Essays*, 2002, v. 15(3), 303–321.
20. Lo C. Y. *Chinese J. of Phys.*, 2003, v. 41(4), 1–11.
21. Lo C. Y. *Phys. Essays*, 2005, v. 18(4).
22. Whitehead A. N. The Principle of Relativity. Cambridge Univ. Press, Cambridge, 1962.
23. Lo C. Y. *Progress in Phys.*, 2006, v. 1, 10–13.
24. Lo C. Y. The Detection of Gravitational Wave with the Laser Interferometer and Einstein's Theoretical Errors on Measurements. (*In preparation*).
25. Lo C. Y. *Phys. Essays*, 2005, v. 18(1).
26. Peres A. *Phys. Rev.*, 1960, v. 118, 1105.
27. Penrose R. *Rev. Mod. Phys.*, 1965, v. 37(1), 215–220.
28. Bonnor W. B. *Commun. Math. Phys.*, 1969, v. 13, 173.
29. Kramer D., Stephani H., Herlt E., and MacCallum M. *Exact Solution of Einstein's Field Equations*, ed. E. Schmutzer, Cambridge Univ. Press, Cambridge, 1980.
30. Lo C. Y. *Phys. Essays*, 1994, v. 7(4), 453–458.
31. Lo C. Y. *Phys. Essays*, 1998, v. 11(2), 274–272.
32. Rabounski D. *Progress in Physics*, 2005, v. 1, 3–6.
33. Einstein A. $E = Mc^2$ (1946) Ideas and Opinions. Crown, New York, 1954, p. 337
34. Lo C. Y. *Astrophys. J.*, 1997, v. 477, 700–704.
35. Tolman R. C. Relativity, Thermodynamics, and Cosmology. Dover, New York, 1987, p. 273.
36. Dirac P. A. M. General Theory of Relativity. John Wiley, New York, 1975.
37. Lo C. Y. *Phys. Essays*, 2003, v. 16(1), 84–100.
38. Lo C. Y. *Chin. Phys.*, 2004, v. 13(2), 159.
39. Kennefick D. Einstein versus the Physical Review. *Physics Today*, September 2005.

On the Theory and Physics of the Aether

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Physical Space is identified as the universal Aether Space. An Aether Equation is deduced, predicting the Temperature of the Cosmic Background Radiation T_{CMBR} , and indicating that G and c are universal dependent variables. The strong nuclear force is found to be a strong gravitational force at extreme energy densities of the neutron, indicating a Grand Unified Theory, when gravity is a process of enduring exchange of radiant energy between all astrophysical objects. The big bang hypothesis is refuted by interpretation of the Hubble redshift as evidence of gravitational work. Conditions for application of STR and GTR in the scientific cosmological research are deduced.

Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers.

Newton. Letter to Bentley, 1693.

We assume to find in every point of space a flow in all directions of radiant energy from all astrophysical objects, meaning that space everywhere has a specific energy U [erg] and an energy density $u = U/V$ [erg/cm³], which of course is a local variable depending on the position in space.

The radiant energy will we name the "Aether", and since it is present throughout the Universe, we will call space the "Aether-Space". Presuming the aether the medium sustaining all physical fields and forces, the aether-space is the universal physical space.

A set of equations can be found for this situation [1] from which may be derived the aether equation with the minimum energy U at the temperature T_{Aether} , which has been confirmed by the COBE observations of $T_{\text{CMBR}} = 2.735 \pm 0.06$ Kelvin⁽¹⁾

$$\kappa UV = Ghc^2,$$

$$U = 3.973637 \times 10^{-13} \text{ erg at } T_{\text{Aether}} = 2.692064 \text{ Kelvin},$$

$$K = Gc/\kappa L^2 = UL/hc = 2.000343 \times 10^3.$$

Defining $\kappa \equiv 1 \text{ erg}/(\text{sec} \times \text{g}^2)$ and $V = 1 \text{ cm}^3$, it is seen that if U is a variable, then the Newtonian G and the velocity of light c are dependent variables if Planck's h is a universal constant.

At higher energy densities of the aether, such as in the galaxies, G and c would have other and higher values than $G = 6.672426 \times 10^{-8} \text{ cm}^3/(\text{g} \times \text{sec}^2)$ and $c = 2.99792458 \times 10^{10} \text{ cm/sec}$ of the aether equation and will need some coefficient ρ to G , while the maximum value of c is supposed from a possible coefficient function to be $c_{\text{max}} = \sqrt{2} c$.

To have an idea of the extreme energy densities and their corresponding ρ -values, we will have a look at the Schwarzschild solution for the electron, from which to derive G :⁽²⁾

$$Gm_e/r_e c^2 = 1/\rho_e = Gm_e^2/e^2,$$

$$\rho_e G e^2 = c^4 r_e^2,$$

$$m_e = 9.109535 \times 10^{-28} \text{ g},$$

$$e = 4.803242 \times 10^{-10} \text{ esu},$$

$$r_e = 2.817937 \times 10^{-13} \text{ cm},$$

$$\rho_e = 4.166705 \times 10^{42}.$$

Considering the composite neutron, the proton⁺, and the neutron-meson⁻ we find that the meson must be the mass difference between the neutron and the proton, and that the meson must be a special heavy neutron-electron, since the free neutron in relatively short time disintegrates into a proton, an electron, and some neutrino energy depending on the velocities and directions of the parting massive particles. We therefore have with α , the fine structure constant:

$$m_n = 1.674954 \times 10^{-24} \text{ g},$$

$$m_p = 1.672648 \times 10^{-24} \text{ g},$$

$$m_m = 2.305589 \times 10^{-27} \text{ g},$$

$$m_p m_m / m_e^2 = \alpha K^2 / 2\pi = K^3 e^2 / UL =$$

$$= \rho_e / \rho_{p,m} = 4.64723 \times 10^3,$$

$$\alpha = 7.297349 \times 10^{-3},$$

$$\rho_{p,m} = 8.965996 \times 10^{38}.$$

As an analogon to the Schwarzschild electron solution we find:

$$\rho_{p,m} G m_n / r_n c^2 = \rho_{p,m} G m_p m_m / e^2 = 1,$$

$r_n = 1.11492 \times 10^{-13} \text{ cm}$ would then be the radius of the neutron, and if the proton is calculated with the same coefficient $\rho_{p,m}$,

$$\rho_{p,m} G m_p / r_p c^2 = 1,$$

$$r_p = 1.113386 \times 10^{-13} \text{ cm}.$$

If the neutron-meson should in fact be a heavy electron, and $m_m/m_e \sim 2.53$, it would make sense if the mass-difference $m_m - m_e$ was the virtual gravitational mass of the neutron's intrinsic proton-electron pair, whence we find from a first calculation m_{vir} :

$$\rho_{p,m} G m_p m_e / r_n c^2 = 9.096998 \times 10^{-28} \text{ g},$$

$$\rho_{p,m} G m_p m_e / r_p c^2 = 9.109531 \times 10^{-28} \text{ g}.$$

We have hereby accounted for a neutron-meson of twice the electron's mass, while we need an explanation for the extra mass of $\frac{1}{2}$ electron-mass in the neutron-meson. We will abstain from further calculations here and for the moment consider it sufficient to have shown a double electron-mass in the meson, pointing to the self-gravitation also of the virtual mass as a probable solution to the deficiency of $\sim 4.83 \times 10^{-28}$ g meson-mass.

Regarding the self-gravitation of the neutron, it may be shown from a normalization of the neutron's gravitational potential P , that the potential with respect of the central proton, when the self-gravitation means an increment of the meson-mass from $\sim 2m_e$ to m_m , would result in a slightly greater value of ρ by a factor of $r_n/r_p = m_n/m_p = 1.001378$ ⁽³⁾ from $\rho_{p,m}$ to ρ_n , so that $\rho_n = 1.001378 \rho_{p,m} = 8.978353 \times 10^{38}$. We then find from considering the gravitational potential of the neutron, as if produced by the central proton alone in the distance r_n , that it leads to the resulting potential

$$P = \rho_n G m_p / r_n = \rho_{p,m} G m_n / r_n = c^2,$$

$$\rho_n G m_p m_m / r_n = E_m.$$

$E_m = m_m c^2$ is the total energy of the heavily augmented neutron-electron to the full mass of the neutron-meson, $m_m = m_n - m_p$. That the virtual gravitational mass of the free neutron equals one electron-mass may be seen from the following equation, which interestingly shows the ratio between radii r_p and r_e . It appears then that all the relational conditions of the free neutron are completely deduced:

$$m_{vir} = \rho_{p,m} G m_p m_e / r_p c^2 = m_m r_p / r_e = e^2 / r_e c^2 = m_e,$$

$$m_m = e^2 / r_p c^2.$$

Having demonstrated that the Newtonian G must be a variable of very great values at extreme energy densities, such as in the composite neutron ($\rho_{p,m} G \sim 6 \times 10^{31}$), it seems reasonable to believe that the strong nuclear force is caused by such extreme values of the Newtonian gravitational factor.

We therefore assume that the neutron-meson would be able to bind two protons in the atomic nucleus by orbiting in such a way that it shifts constantly between the two protons, of which the one may be considered a neutron, when the other is a proton and vice versa in constant shifts of constitution in the neutron-proton pair of a nucleus.

The binding orbit may hence be thought of in a most simple theoretical illustration as the meson following an Oval of Cassini around the two heavy electrically positive charged particles, forcing them to the constant shifts of neutron-proton phase. And as will be known, the Lemniscate is the extreme curve of the Cassini Oval, with the parameters $a = b$, where the strong particle-binding would break in a proton and a free neutron that may possibly leave the nucleus. ⁽⁴⁾

Of course, the real conditions of an "orbiting neutron-meson" cannot be made really lucid, since we know that the interaction is rather a question of probability of distribution of charges and masses, when we observe the weak magnetic moment of the electrically neutral neutron.

However, it seems that the strong nuclear force may be accounted for as a very strong gravitational force at extreme energy densities, to which it is remarked that in the galaxies, with their very intense radiation from stars and gasses, we may also expect special dynamics due to the variability of the factor G , which would therefore account for the observed galactic differential velocities and probably would explain also the so-called "problem of missing mass in the Universe".

As in fact gravitational action according to the aether physics is an electromagnetic phenomenon of energy exchange in Planck quanta leaving an enduring train of impulses unto the gravitating masses, it seems that a unification of the four fundamental forces in nature may be expected from consideration of the physics of the aether.

From the aether equation we have found the constant K . Considering the composite neutron, $m_p + m_m = m_n$, we have the mass relation and the energy-charge relation:

$$(m_p m_m) / m_e^2 = K^3 (e^2 / UL) = K^2 (e^2 / hc),$$

$$E_e r_e = E_m r_p = E_p r_m = e^2.$$

It further follows that $K \Phi / c = G m_x m_y / L^2$ for any pair of gravitating masses in mutual distance $L = 1$ cm, when the radiant flux Φ [erg/sec] is $\Phi_{x,y} = \kappa m_x m_y$.

We will therefore show that a radiant aether flux Φ is the common cause of the Coulomb force and the extremely strong force of gravity in the neutron, manifest as the strong nuclear force

$$e^2 / r_n^2 = \rho_{p,m} [(K \Phi_{p,m}) / c] \times [L^2 / r_n^2] = \rho_{p,m} G m_p m_m / r_n^2 \text{ dynes},$$

$$e^2 c / r_n^2 = \rho_{p,m} K \Phi_{p,m} L^2 / r_n^2 = \rho_{p,m} G m_p m_m c / r_n^2 \text{ erg/sec},$$

$$e^2 / m_p m_m = \rho_{p,m} K \kappa L^2 / c = \rho_{p,m} \kappa UV / hc^2 = \rho_{p,m} G,$$

$$e^2 / G = \rho_{p,m} m_p m_m = M_{js}^2.$$

For any pair of fundamental particles of unit charge $\pm e$ there seems to exist a dimensionless factor of proportionality $\rho_{1,2}$, which, if made a coefficient of G , will balance the electrostatic Coulomb force and the Newtonian force of

gravity at any distance between the charged particles. For any charged pair of $mass_1$ and $mass_2$ the factor of proportionality will be ρ with the Johnstone-Stoney mass squared e^2/G as a constant: $\rho_{1,2} = M_{js}^2/(m_1 m_2)$.

Demonstrating the validity of the foregoing derivations, it may be shown that, with the magnitude found for the coefficient $\rho_{p,m}$, the dimensions r_n , r_p , and r_m of the neutron masses m_n , m_p , and m_m are most easily given by the following simple relations:

$$\rho K \kappa m L^2 / c^3 = \rho G m / c^2 = r;$$

as is with ρ_e and m_e the Schwarzschild radius of the electron r_e .

Generally, provided a local value of ρ can be found or estimated, the local gravitational potential P at any distance R from the center of a gravitating mass M will be:

$$P = \rho G M / R \text{ (cm/sec)}^2.$$

When, however, all ponderable matter is constituted as a sum of charged particles, and the force of gravity as shown is an electromagnetic phenomenon by energy exchange in the aether space between any pair of masses via a radiant flux Φ [erg/sec], which is proportional to the product of the two masses, we generally have with some local value of ρ the Newtonian force between M_1 and M_2 :

$$\begin{aligned} F &= \rho G M_1 M_2 / R^2 = \rho \kappa M_1 M_2 UV / hc^2 R^2 = \\ &= \rho K [\Phi / c] \times [L^2 / R^2] \text{ dynes.} \end{aligned}$$

The radiant flux $K\Phi$ may be thought of as aether energy at the velocity of light, which is bound in the line of distance R between the gravitating masses, representing the gravitational energy $K\Phi L^2 / Rc$ and the equivalent virtual gravitational mass $K\Phi L^2 / Rc^3$ that belongs to the binary system. It should therefore be added to the sum of gravitating masses for calculations of total potential and force including the self-gravitation of the aether energy in Φ .

In the composite neutron, however, only two elementary charges are acting, the proton's $+e$ esu and the meson-electron's $-e$ esu. The latter is an ordinary electron, when the neutron disintegrates, and we have no idea whatsoever of a variation in the elementary charge $e = 4.803242 \times 10^{-10}$ esu. We conclude from the neutron equation, as from Schwarzschild's electron solution:

$$\begin{aligned} e^2 / r_n^2 &= \rho_{p,m} G m_p m_m / r_n^2, \\ e^2 / m_p m_m &= \rho_{p,m} G, \end{aligned}$$

that gravity is an electromagnetic phenomenon, and that it is the relation shown herein between charges and masses which governs the gravitational force between the neutron's proton and electron at the extreme energy density of the free neutron.

Presumably, it is the gravitational interaction between the free neutron and all other masses in the aether space, by enduring energy exchange with the radiant energy of the aether, that makes the neutron unstable by emitting more energy to the aether field than is absorbed in the same interval of time. This loss of energy is by radiation at the cost of the meson-mass, which diminishes, meaning a loss of mass and of the neutron's energy density, thereby a reduction of the coefficient ρ , of G . That means an increase in r_n , the radius of the free neutron, to a considerably greater dimension as a so-called "cold neutron" until the proton and the neutron-electron part with a random measure of the electron-meson's binding energy as a massless supply of neutrino-energy to the aether.

The aether energy represented in the radiant flux Φ is, according to the theory, present in the aether space of infinite energy as random radiation at all wavelengths and in all directions to and from the gravitating systems. Therefore the action of gravity is immediate, say if one of the gravitating masses is suddenly increased, while any change in the gravitating system will result in a signal which propagates in the aether space as a gravitational wave with the velocity of light. Such a signal may therefore be thought of as a modulation of the present radiant aether energy. The flux Φ is not a flow of energy from mass 1 to mass 2 and back again. It is a result of the energy exchange in all directions between the aether and the complete system and its single gravitating masses. According to the aether theory we have:

$$\alpha (K m_e)^2 = 2\pi m_p m_m g^2,$$

\Downarrow

$$[e^2 / h\nu] \times [G m_e^2 / \lambda^2] = [L \Phi_{p,m} / U] \times [h / \lambda] \text{ erg.}$$

Aether energy which is absorbed by a mass is immediately re-emitted randomly to the aether, and in all directions. The action of gravity means work by impulses $h\nu/c = h/\lambda$ both at absorption and radiation of energy, while reflection means a double-pulse [2]. The gravitational work done by the aether causes an increasing loss of aether energy, shown in the Hubble-effect of increasing redshift with distance of all light from distant sources. The universal redshift thus is evidence of gravitational work, and not of any universal expansion interpreted as a Doppler-effect. The redshift is in complete accordance with the gravitational effect described by Einstein's theory of relativity, where we have to discriminate between two types of gravitational effects: (1) the local redshift of a single mass also deflecting passing rays of light; (2) the redshift of distance called the Hubble redshift.

The speculative big-bang hypothesis therefore seems absurd and way beyond rational science, since General Relativity has meaning only in application to a finite physical space of known and observable contents of masses and energy, while the Universe is for all reasons of an infinite mightiness beyond some apparent limit of observation, and

when the idea of the Newtonian gravitational factor G as an universal constant cannot be upheld. The multitude of individual “galactic worlds” of very different types and ages in some general ongoing process of creation and decay by age should, on the other hand, be an obvious goal for scientific cosmological research.

The loss of energy to gravitational work is replenished by the stars and all the energy producing astrophysical objects by irradiation of new energy into the aether space at the cost of their masses. It seems clear that there ought to exist a feedback effect working to keep the aether at a constant energy level, which, however, may be left to future research.

The replenishment of free radiant energy to the aether-space by irradiation of Planck quanta at the velocity of light is, as seen from the aether equation, regardless of any local coefficient ρ :

$$h\nu/c = \kappa UV/(Gc^2\lambda) = (U/Kc) \times L/\lambda;$$

$$Kh\nu = U \times L/\lambda.$$

By the foregoing presentation of the theory and physics of the aether we have shown that gravitation is an electromagnetic phenomenon, and that the force of gravity is the result of an enduring exchange of radiant energy between mass and aether, by which the energy of the fundamental particles fluctuates consistently with the QED-findings regarding the fundamental charge/mass proportion of the electron.

The theory of the aether thereby seems to confirm also Einstein’s finding 1928 [3] that “The separation of the gravitational and the electromagnetic field appears artificial”, — when, of course, the aether-space is the seat of all physical fields and forces.

In modern 5-dimensional Kaluza-Klein Theory the specific space energy of the aether, or some identical local aether-parameter, such as for instance T_{Aether} Kelvin, would apparently represent the 5th dimension.

Provided the speculative unphysical STR is defined with a local energy density u of the aether space, and with the condition that u shall be constant all over the actual physical space, ensuring a constant light-velocity c , the Special Theory of Relativity is a valid physical theory confirmed by observations.

Provided in any application of GTR the λ -term is defined with the parameters of a black body radiation of energy density u at the temperature T_{Aether} in the actual finite physical aether space, and provided the local coefficient ρ to the Einsteinian gravitational factor χ is estimated correct, the General Theory of Relativity may be applied to a first approximation.

STR and GTR thereby should be useful sub-theories in the Theory and Physics of the Aether which, as here described, appears as a natural continuation and extension of Drude’s famous *Physik des Aethers* [4]. In thermodynamics it should be noted that gravitational energy exchange by

radiation is a reversible process in open systems, therefore in no matter of the 2nd law.

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Endnotes

- (1) Since the aether is a perfect boson-gas, we have with $a = 8\pi V/c^3$ and $b = kT/h$, when $\zeta(x)$ is the Riemann ζ -function, $L = 1$ cm, $V = 1$ cm³, $m = 1$ g the following solutions:

$$pV/kT = a2b^3\zeta(4), \quad U = ah6b^4\zeta(4);$$

$$p = u/3, \quad pV = U/3;$$

$$R/N = R_G/N_A = k = 1.380662 \times 10^{-16} \text{ erg/Kelvin};$$

$$RT = ah2b^4\zeta(3) = kTN;$$

$$R_{\text{Aether}} = 5.464489 \times 10^{-14} \text{ erg/Kelvin};$$

$$N_{\text{Aether}} = a2b^3\zeta(3) = 3.957876 \times 10^2;$$

$$S = 4U/3T = R\zeta(4)/\zeta(3) = 4Ghc^2/3kVT;$$

$$S_{\text{Aether}} = 1.968074 \times 10^{-13} \text{ erg/Kelvin};$$

$$\kappa = \Phi/m^2 = \chi hc^4/8\pi UV = 4Ghc^2/3SVT;$$

$$\chi = \text{Einstein's gravitational factor};$$

$$\Phi/L^2 = (Gm^2/L^2) \times (4hc^2/3SVT) = \kappa m^2/L^2.$$

- (2) When for every mass m it holds that $E = mc^2$, and the de Broglie wavelength $\lambda_B = h/mv$, we have for $v = c$ that $E_m\lambda_{B=c} = hc$. When further the fine structure constant is $\alpha = 2\pi e^2/hc$, a precise theoretical value of the Newtonian G may be derived from iterations on the shown Schwarzschild solution for the electron and the very well known value of α .

This theoretical value of G , which of all physical magnitudes is the most difficult to measure experimentally, is the universal Newtonian constant $G = 6.672426 \times 10^{-8}$ cm³/(g×sec²) at the minimum specific energy of the aether at the defined universal minimum temperature $T_{\text{Aether}} = 2.692064$ Kelvin = T_{CMBR} according to the theory of the aether. At any higher aether temperature $T_{\text{Aether}} > T_{\text{CMBR}}$, thus at a proportionally greater local energy density u erg/cm³, $u_{\text{Aether}} > u_{\text{CMBR}}$, the Newtonian constant becomes a variable: $\rho G > G_{\text{CMBR}}$ by a dimensionless coefficient of proportionality.

According to the aether equation we furthermore find $KE_m\lambda_c = UL$, confirming the derived magnitudes of U and G with utmost precision; thereby also the predicted temperature T_{Aether} comparable with the experimental value from measurements of T_{CMBR} .

The relation $K = UL/E_m \lambda_c = UL/hc$ may be of interest in particle physics as in wave mechanics, since according to Planck the fundamental particles may be regarded as oscillating electromagnetic energy in standing waves, with the oscillator parameters L [cm] and C [Farad], in which case we have for the elementary charged particles of energy E_m , and besides for the electron of energy $E_e = m_e c^2$ especially: $E_m = h\nu = mc^2 = hc/\lambda_{B=C} = mL C \omega^2$; $E_e = E_{m(e)} = e^2/r_e$.

(3) One finds from the small factor $1.001378 = r_n/r_p = m_n/m_p$,

$$\begin{aligned} r_n m_p / r_p m_n &= r_e m_e / r_p m_m = 1, \\ r_e m_e &= e^2 m_e / m_e c^2 = e^2 / c^2 = r_p m_m, \\ r_n m_p / r_e m_e &= r_n / r_p' = r_n / (r_n - r_p), \\ r_n - r_p &= r_p' = e^2 / m_p c^2, \\ r_e m_e m_n / m_m m_p r_n &= 1, \\ [e^2 / m_m m_p] \times m_n / r_n c^2 &= 1, \\ e^2 / (m_m m_p) &= \rho_{p,m} G, \\ e^2 / G &= \rho_{p,m} m_m m_p = M_{JS}^2, \\ \rho_{p,m} G m_n / r_n &= c^2, \end{aligned}$$

that both $\rho_{p,m} G$ and the Johnstone-Stoney mass M_{JS}^2 can be derived with extreme precision alone from the found dimensions r_e and masses m_e , when at the same time showing correctly that the meson-mass m_m and the proton-mass m_p are both charged with e esu, whereas no electric charge occurs at the neutron m_n . It is such an overwhelming demonstration of the valid derivation of all the found dimensions, that no doubt seems possible.

The small extension $r_p' = 1.534 \times 10^{-16}$ cm of space the proton-radius up to the neutron-radius, which in fact would be the radius r_p' of the proton, if calculated strictly like the radius of the electron according to the Schwarzschild solution, is the thickness of an outer spherical shell surrounding the central proton of the free neutron, is why we may say that the volume of this spherical shell of extremely narrow depth r_p' is the location of the bound heavy neutron-meson.

Calculation of $r_n' = 1.532 \times 10^{-16}$ cm $= e^2 / m_n c^2$ retains the ratio $1.001378 = r_p' / r_n'$ and the exceedingly small difference $r_p' - r_n' = 2.113 \times 10^{-19}$ cm < 0.002 pro mille of the neutron radius r_n . If of any relevance at all, it will have to await the results and precision of future research.

(4) From two protons in a torus of radii r_p and r_e may be generated the family of Cassini Ovals in planes parallel with the torus axis. The Lemniscate may be seen in a section cut in a plane parallel to the axis through a point on the inside of the torus, i. e. in the distance $(r_e - r_p)$ from the axis.

The mutual distance of the protons in the Lemniscate is $2\sqrt{(r_e)^2 - (r_e - r_p)^2} = 4.488 \times 10^{-13}$ cm, or $4.031 \times r_p$ cm apart (according to Pythagorean calculation).

In case of a change of radii, $r_e \rightarrow r_p$, or contrary $r_p \rightarrow r_e$, the torus will degenerate into a non-Riemannian surface with one singularity in the axis.

References

1. Yourgrau W., van der Merwe A., Raw G. Treatise on irreversible and statistical thermodynamics. The Macmillan Company, N.Y., 1966, Chap. 3–9, 10, Statistics of a photon gas. Blackbody radiation and the laws of Planck and Stefan. (The Dover edition 1982 is an unabridged and extensively corrected republication).
2. Einstein A. Zur Quantentheorie der Strahlung. *Physikalische Zeitschrift*, XVIII, Leipzig 1917, 121–128.
3. Einstein A. Neue Möglichkeit für eine einheitliche Feldtheorie von Gravitation und Elektrizität. *Sitzungber. K. Preuss. Akad. Wissenschaften*, Phys.-Math. Klasse, Berlin 1928, 224–227.
4. Drude P. Physik des Aethers auf Elektromagnetischer Grundlage. Stuttgart, Verlag von Ferdinand Enke, 1894.

Open Letter by the Editor-in-Chief

Declaration of Academic Freedom (Scientific Human Rights)

Article 1: Preamble

The beginning of the 21st century reflects more than at any other time in the history of Mankind, the depth and significance of the role of science and technology in human affairs.

The powerfully pervasive nature of modern science and technology has given rise to a commonplace perception that further key discoveries can be made principally or solely by large government or corporation funded research groups with access to enormously expensive instrumentation and hordes of support personnel.

The common perception is however, mythical, and belies the true nature of how scientific discoveries are really made. Large and expensive technological projects, howsoever complex, are but the result of the application of the profound scientific insights of small groups of dedicated researchers or lone scientists, often working in isolation. A scientist working alone is now and in the future, just as in the past, able to make a discovery that can substantially influence the fate of humanity and change the face of the whole planet upon which we so insignificantly dwell.

Groundbreaking discoveries are generally made by individuals working in subordinate positions within government agencies, research and teaching institutions, or commercial enterprises. Consequently, the researcher is all too often constrained or suppressed by institution and corporation directors who, working to a different agenda, seek to control and apply scientific discovery and research for personal or organizational profit, or self-aggrandisement.

The historical record of scientific discovery is replete with instances of suppression and ridicule by establishment, yet in later years revealed and vindicated by the inexorable march of practical necessity and intellectual enlightenment. So too is the record blighted and besmirched by plagiarism and deliberate misrepresentation, perpetrated by the unscrupulous, motivated by envy and cupidity. And so it is today.

The aim of this Declaration is to uphold and further the fundamental doctrine that scientific research must be free of the latent and overt repressive influence of bureaucratic, political, religious and pecuniary directives, and that scientific creation is a human right no less than other such rights and forlorn hopes as propounded in international covenants and international law.

All supporting scientists shall abide by this Declaration, as an indication of solidarity with the concerned international scientific community, and to vouchsafe the rights of the citizenry of the world to unfettered scientific creation ac-

ording to their individual skills and disposition, for the advancement of science and, to their utmost ability as decent citizens in an indecent world, the benefit of Mankind. Science and technology have been far too long the handmaidens of oppression.

Article 2: Who is a scientist

A scientist is any person who does science. Any person who collaborates with a scientist in developing and propounding ideas and data in research or application is also a scientist. The holding of a formal qualification is not a prerequisite for a person to be a scientist.

Article 3: Where is science produced

Scientific research can be carried out anywhere at all, for example, at a place of work, during a formal course of education, during a sponsored academic programme, in groups, or as an individual at home conducting independent inquiry.

Article 4: Freedom of choice of research theme

Many scientists working for higher research degrees or in other research programmes at academic institutions such as universities and colleges of advanced study, are prevented from working upon a research theme of their own choice by senior academic and/or administrative officials, not for lack of support facilities but instead because the academic hierarchy and/or other officials simply do not approve of the line of inquiry owing to its potential to upset mainstream dogma, favoured theories, or the funding of other projects that might be discredited by the proposed research. The authority of the orthodox majority is quite often evoked to scuttle a research project so that authority and budgets are not upset. This commonplace practice is a deliberate obstruction to free scientific thought, is unscientific in the extreme, and is criminal. It cannot be tolerated.

A scientist working for any academic institution, authority or agency, is to be completely free as to choice of a research theme, limited only by the material support and intellectual skills able to be offered by the educational institution, agency or authority. If a scientist carries out research as a member of a collaborative group, the research directors and team leaders shall be limited to advisory and consulting roles in relation to choice of a relevant research theme by a scientist in the group.

Article 5: Freedom of choice of research methods

It is frequently the case that pressure is brought to bear upon a scientist by administrative personnel or senior academics in relation to a research programme conducted within an academic environment, to force a scientist to adopt research methods other than those the scientist has chosen, for no reason other than personal preference, bias, institutional policy, editorial dictates, or collective authority. This practice, which is quite widespread, is a deliberate denial of freedom of thought and cannot be permitted.

A non-commercial or academic scientist has the right to develop a research theme in any reasonable way and by any reasonable means he considers to be most effective. The final decision on how the research will be conducted is to be made by the scientist alone.

If a non-commercial or academic scientist works as a member of a collaborative non-commercial or academic team of scientists the project leaders and research directors shall have only advisory and consulting rights and shall not otherwise influence, mitigate or constrain the research methods or research theme of a scientist within the group.

Article 6: Freedom of participation and collaboration in research

There is a significant element of institutional rivalry in the practice of modern science, concomitant with elements of personal envy and the preservation of reputation at all costs, irrespective of the scientific realities. This has often led to scientists being prevented from enlisting the assistance of competent colleagues located at rival institutions or others without any academic affiliation. This practice is too a deliberate obstruction to scientific progress.

If a non-commercial scientist requires the assistance of another person and that other person is so agreed, the scientist is at liberty to invite that person to lend any and all assistance, provided the assistance is within an associated research budget. If the assistance is independent of budget considerations the scientist is at liberty to engage the assisting person at his sole discretion, free of any interference whatsoever by any other person whomsoever.

Article 7: Freedom of disagreement in scientific discussion

Owing to furtive jealousy and vested interest, modern science abhors open discussion and wilfully banishes those scientists who question the orthodox views. Very often, scientists of outstanding ability, who point out deficiencies in current theory or interpretation of data, are labelled as crackpots, so that their views can be conveniently ignored. They are derided publicly and privately and are systematically barred from scientific conventions, seminars and colloquia so that their ideas cannot find an audience. Deliberate falsification

of data and misrepresentation of theory are now frequent tools of the unscrupulous in the suppression of facts, both technical and historical. International committees of scientific miscreants have been formed and these committees host and direct international conventions at which only their acolytes are permitted to present papers, irrespective of the quality of the content. These committees extract large sums of money from the public purse to fund their sponsored projects, by resort to deception and lie. Any objection to their proposals on scientific grounds is silenced by any means at their disposal, so that money can continue to flow into their project accounts, and guarantee them well-paid jobs. Opposing scientists have been sacked at their behest; others have been prevented from securing academic appointments by a network of corrupt accomplices. In other situations some have been expelled from candidature in higher degree programmes such as the PhD, for expressing ideas that undermine a fashionable theory, however longstanding that orthodox theory might be. The fundamental fact that no scientific theory is definite and inviolable, and is therefore open to discussion and re-examination, they thoroughly ignore. So too do they ignore the fact that a phenomenon may have a number of plausible explanations, and maliciously discredit any explanation that does not accord with orthodox opinion, resorting without demur to the use of unscientific arguments to justify their biased opinions.

All scientists shall be free to discuss their research and the research of others without fear of public or private materially groundless ridicule, or be accused, disparaged, impugned or otherwise discredited by unsubstantiated allegations. No scientist shall be put in a position by which livelihood or reputation will be at risk owing to expression of a scientific opinion. Freedom of scientific expression shall be paramount. The use of authority in rebuttal of a scientific argument is not scientific and shall not be used to gag, suppress, intimidate, ostracise, or otherwise coerce or bar a scientist. Deliberate suppression of scientific facts or arguments either by act or omission, and the deliberate doctoring of data to support an argument or to discredit an opposing view is scientific fraud, amounting to a scientific crime. Principles of evidence shall guide all scientific discussion, be that evidence physical or theoretical or a combination thereof.

Article 8: Freedom to publish scientific results

A deplorable censorship of scientific papers has now become the standard practice of the editorial boards of major journals and electronic archives, and their bands of alleged expert referees. The referees are for the most part protected by anonymity so that an author cannot verify their alleged expertise. Papers are now routinely rejected if the author disagrees with or contradicts preferred theory and the mainstream orthodoxy. Many papers are now rejected automatically by virtue of the appearance in the author list of a

particular scientist who has not found favour with the editors, the referees, or other expert censors, without any regard whatsoever for the contents of the paper. There is a black-listing of dissenting scientists and this list is communicated between participating editorial boards. This all amounts to gross bias and a culpable suppression of free thinking, and are to be condemned by the international scientific community.

All scientists shall have the right to present their scientific research results, in whole or in part, at relevant scientific conferences, and to publish the same in printed scientific journals, electronic archives, and any other media. No scientist shall have their papers or reports rejected when submitted for publication in scientific journals, electronic archives, or other media, simply because their work questions current majority opinion, conflicts with the views of an editorial board, undermines the bases of other current or planned research projects by other scientists, is in conflict with any political dogma or religious creed, or the personal opinion of another, and no scientist shall be blacklisted or otherwise censured and prevented from publication by any other person whomsoever. No scientist shall block, modify, or otherwise interfere with the publication of a scientist's work in the promise of any presents or other bribes whatsoever.

Article 9: Co-authoring of scientific papers

It is a poorly kept secret in scientific circles that many co-authors of research papers actually have little or nothing to do with the research reported therein. Many supervisors of graduate students, for instance, are not averse to putting their names to papers written by those persons who are but nominally working under their supervision. In many such cases, the person who actually writes the paper has an intellect superior to the nominal supervisor. In other situations, again for the purposes of notoriety, reputation, money, prestige, and the like, non-participating persons are included in a paper as co-author. The actual authors of such papers can only object at risk of being subsequently penalised in some way, or even being expelled from candidature for their higher research degree or from the research team, as the case may be. Many have actually been expelled under such circumstances. This appalling practice cannot be tolerated. Only those persons responsible for the research should be accredited authorship.

No scientist shall invite another person to be included and no scientist shall allow their name to be included as a co-author of a scientific paper if they did not significantly contribute to the research reported in the paper. No scientist shall allow himself or herself to be coerced by any representative of an academic institution, corporation, government agency, or any other person, to include their name as a co-author concerning research they did not significantly contribute to, and no scientist shall allow their name to be used as co-author in exchange for any presents or other bribes.

No person shall induce or attempt to induce a scientist in howsoever a way to allow that scientist's name to be included as a co-author of a scientific paper concerning matters to which they did not significantly contribute.

Article 10: Independence of affiliation

Many scientists are now employed under short-term contracts. With the termination of the employment contract, so too is the academic affiliation. It is often the policy of editorial boards that persons without an academic or commercial affiliation will not be published. In the absence of affiliation many resources are not available to the scientist, and opportunities to present talks and papers at conferences are reduced. This is a vicious practice that must be stopped. Science does not recognise affiliation.

No scientist shall be prevented from presenting papers at conferences, colloquia or seminars, from publication in any media, from access to academic libraries or scientific publications, from attending scientific meetings, or from giving lectures, for want of an affiliation with an academic institution, scientific institute, government or commercial laboratory, or any other organisation.

Article 11: Open access to scientific information

Most specialised books on scientific matters and many scientific journals render little or no profit so that commercial publishers are unwilling to publish them without a contribution of money from academic institutions, government agencies, philanthropic foundations, and the like. Under such circumstances commercial publishers should allow free access to electronic versions of the publications, and strive to keep the cost of the printed materials to a minimum.

All scientists shall strive to ensure that their research papers are available to the international scientific community free of charge, or in the alternative, if it cannot be avoided, at minimum cost. All scientists should take active measures to make their technical books available at the lowest possible cost so that scientific information can be available to the wider international scientific community.

Article 12: Ethical responsibility of scientists

History testifies that scientific discoveries are used for ends both good and evil, for the benefit of some and the destruction of others. Since the progress of science and technology cannot stop, some means for the containment of malevolent application should be established. Only a democratically elected government, free of religious, racial and other bias, can safeguard civilisation. Only democratically elected government, tribunals and committees can safeguard the right of free scientific creation. Today, various undemocratic states and totalitarian regimes conduct active research into nuclear physics, chemistry, virology, genetic engineering, etc in order

to produce nuclear, chemical and biological weapons. No scientist should willingly collaborate with undemocratic states or totalitarian regimes. Any scientist coerced into work on the development of weapons for such states should find ways and means to slow the progress of research programmes and to reduce scientific output so that civilisation and democracy can ultimately prevail.

All scientists bear a moral responsibility for their scientific creations and discoveries. No scientist shall willingly engage in the design or construction of weapons of any sort whatsoever for undemocratic states or totalitarian regimes or allow his or her scientific skills and knowledge to be applied to the development of anything whatsoever injurious to Mankind. A scientist shall live by the dictum that all undemocratic government and the violation of human rights is crime.

November 22, 2005

Dmitri Rabounski

*Editor-in-Chief,
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