Oblique-Length Contraction Factor in the Special Theory of Relativity

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In this paper one generalizes the Lorentz Contraction Factor for the case when the lengths are moving at an oblique angle with respect to the motion direction. One shows that the angles of the moving relativistic objects are distorted.

1 Introduction

According to the Special Theory of Relativity, the Lorentz Contraction Factor is referred to the lengths moving along the motion direction. The lengths which are perpendicular on the direction motion do not contract at all [1].

In this paper one investigates the lengths that are oblique to the motion direction and one finds their Oblique-Length Contraction Factor [3], which is a generalization of the Lorentz Contraction Factor (for $\theta = 0$) and of the perpendicular lengths (for $\theta = \pi/2$). We also calculate the distorted angles of lengths of the moving object.

2 Length-Contraction Factor

Length-Contraction Factor $C(v)$ is just Lorentz Factor:

$$C(v) = \sqrt{1 - \frac{v^2}{c^2}} \in [0, 1] \;\; for\; v \in [0, 1] \qquad (1)$$

$$L = L' \cdot C(v) \qquad (2)$$

where L = non-proper length (length contracted), L' = proper length. $C(0) = 1$, meaning no space contraction [as in Absolute Theory of Relativity (ATR)].

$C(c) = 0$, which means according to the Special Theory of Relativity (STR) that if the rocket moves at speed 'c' then the rocket length and laying down astronaut shrink to zero! This is unrealistic.

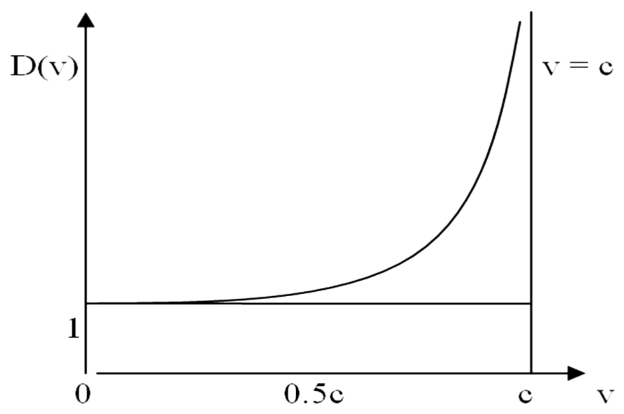

Fig. 1: The graph of the Time-Dilation Factor

3 Time-Dilation Factor

Time-Dilation Factor D(v) is the inverse of Lorentz Factor:

$$D(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \in [1, +\infty] \;\; for\; v \in [0, c] \qquad (3)$$

$$\Delta t = \Delta t' \cdot D(v) \qquad (4)$$

where Δt = non-proper time and, $\Delta t'$ = proper time. $D(0) = 1$, meaning no time dilation [as in Absolute Theory of Relativity (ATR)]; $D(c) = \lim_{v\to c} D(v) = +\infty$, which means according to the Special Theory of Relativity (STR) that if the rocket moves at speed 'c' then the observer on earth measures the elapsed non-proper time as infinite, which is unrealistic. $v = c$ is the equation of the vertical asymptote to the curve of $D(v)$.

4 Oblique-Length Contraction Factor

The Special Theory of Relativity asserts that all lengths in the direction of motion are contracted, while the lengths at right angles to the motion are unaffected. But it didn't say anything about lengths at oblique angle to the motion (i.e. neither perpendicular to, nor along the motion direction), how would they behave? This is a generalization of Galilean Relativity, i.e. we consider the oblique lengths. The length contraction factor in the motion direction is:

$$C(v) = \sqrt{1 - \frac{v^2}{c^2}}. \qquad (5)$$

Suppose we have a rectangular object with width W and length L that travels at a constant speed v with respect to an observer on Earth.

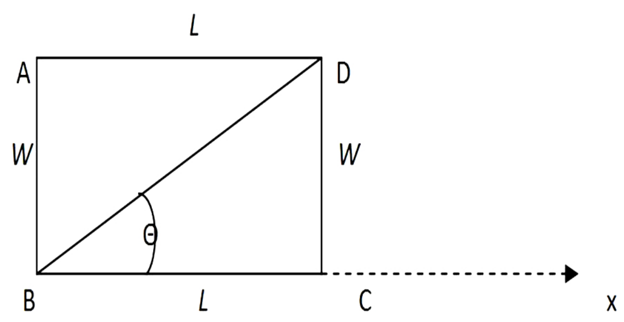

Fig. 2: A rectangular object moving along the x-axis

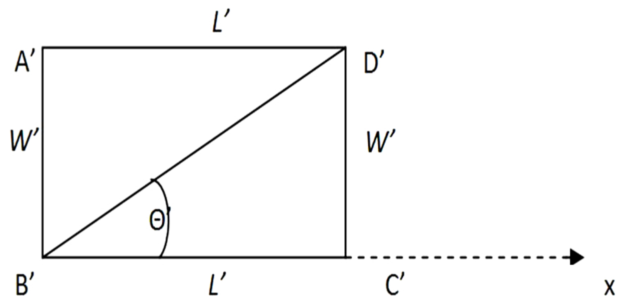

Fig. 3: Contracted lengths of the rectangular object moving along the x-axis

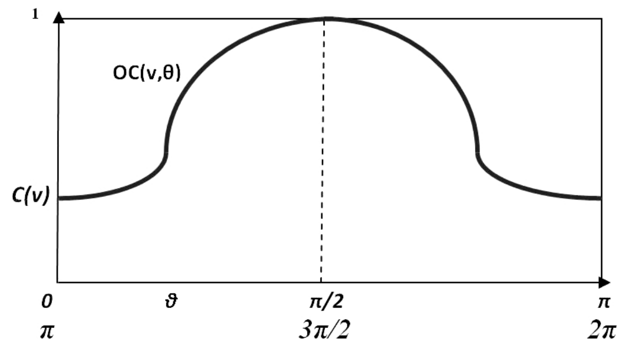

Fig. 4: The graph of the Oblique-Length Contraction Factor $OC(v, \theta)$

Then its lengths contract and its new dimensions will be L' and W': where $L' = L \cdot C(v)$ and $W' = W$. The initial diagonal of the rectangle ABCD is:

$$\begin{aligned} \delta &= |AC| = |BD| = \sqrt{L^2 + W^2} \\ &= \sqrt{L^2 + L^2 \tan^2\theta} = L\sqrt{1 + \tan^2\theta} \end{aligned} \tag{6}$$

while the contracted diagonal of the rectangle $A'B'C'D'$ is:

$$\begin{aligned} \delta' &= |A'C'| = |B'D'| \\ &= \sqrt{(L')^2 + (W')^2} = \sqrt{L^2 \cdot C(v)^2 + W^2} \\ &= \sqrt{L^2 C(v)^2 + L^2 \tan^2\theta} = L\sqrt{C(v)^2 + \tan^2\theta}. \end{aligned} \tag{7}$$

Therefore the lengths at oblique angle to the motion are contracted with the oblique factor

$$\begin{aligned} OC(v,\theta) &= \frac{\delta'}{\delta} = \frac{L\sqrt{C(v)^2 + \tan^2\theta}}{L\sqrt{1 + \tan^2\theta}} \\ &= \sqrt{\frac{C(v)^2 + \tan^2\theta}{1 + \tan^2\theta}} = \sqrt{C(v)^2 \cos^2\theta + \sin^2\theta} \end{aligned} \tag{8}$$

which is different from C(v).

$$\delta' = \delta \cdot OC(v,\theta) \tag{9}$$

where $0 \le OC(v,\theta) \le 1$.

For unchanged constant speed v, the greater is θ in $\left(0, \frac{\pi}{2}\right)$ the larger gets the oblique-length contradiction factor, and reciprocally. By oblique length contraction, the angle

$$\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \tag{10}$$

is not conserved.

In Fig. 4 the horizontal axis represents the angle θ, while the vertical axis represents the values of the Oblique-Length Contraction Factor $OC(v,\theta)$ for a fixed speed v. Hence $C(v)$ is thus a constant in this graph. The graph, for v fixed, is periodic of period π, since:

$$\begin{aligned} OC(v,\pi+\theta) &= \sqrt{C(v)^2 \cos^2(\pi+\theta) + \sin^2(\pi+\theta)} \\ &= \sqrt{C(v)^2[-\cos\theta]^2 + [-\sin\theta]^2} \\ &= \sqrt{C(v)^2 \cos^2\theta + \sin^2\theta} \\ &= OC(v,\theta). \end{aligned} \tag{11}$$

More exactly about the $OC(v,\theta)$ range:

$$OC(v,\theta) \in [C(v), 1] \tag{12}$$

but since $C(v) \in [0, 1]$, one has:

$$OC(v,\theta) \in [0, 1]. \tag{13}$$

The Oblique-Length Contractor

$$OC(v,\theta) = \sqrt{C(v)^2 \cos^2\theta + \sin^2\theta} \tag{14}$$

is a generalization of Lorentz Contractor $C(v)$, because: when $\theta = 0$ or the length is moving along the motion direction, then $OC(v, 0) = C(v)$. Similarly

$$OC(v,\pi) = OC(v, 2\pi) = C(v). \tag{15}$$

Also, if $\theta = \frac{\pi}{2}$, or the length is perpendicular on the motion direction, then $OC(v,\pi/2) = 1$, i.e. no contraction occurs. Similarly $OC(v, \frac{3\pi}{2}) = 1$.

5 Angle Distortion

Except for the right angles ($\pi/2, 3\pi/2$) and for the 0, π, and 2π, all other angles are distorted by the Lorentz transform.

Let's consider an object of triangular form moving in the direction of its bottom base (on the x-axis), with speed v, as in Fig. 5:

$$\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \tag{16}$$

is not conserved.

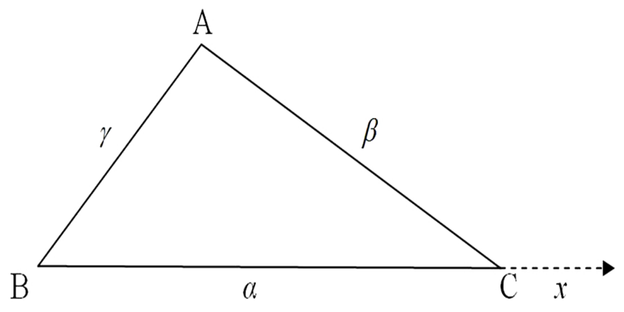

Fig. 5:

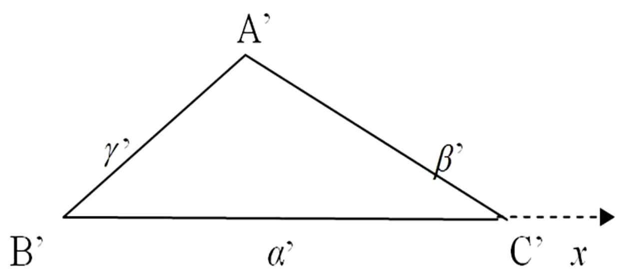

Fig. 6:

The side $|BC| = \alpha$ is contracted with the contraction factor $C(v)$ since BC is moving along the motion direction, therefore $|B'C'| = \alpha \cdot C(v)$. But the oblique sides AB and CA are contracted respectively with the oblique-contraction factors $OC(v, \measuredangle B)$ and $OC(v, \measuredangle\pi - C)$, where $\measuredangle B$ means angle B:

$$\left|A'B'\right| = \gamma \cdot OC(v, \measuredangle B) \tag{17}$$

and

$$\left|C'A'\right| = \beta \cdot OC(v, \measuredangle\pi - C) = \beta \cdot OC(v, \measuredangle A + B) \tag{18}$$

since

$$\measuredangle A + \measuredangle B + \measuredangle C = \pi. \tag{19}$$

Triangle ABC is shrunk and distorted to $A'B'C'$ as in Fig. 6.

Hence one gets:

$$\begin{aligned} \alpha' &= \alpha \cdot C(v) \\ \beta' &= \beta \cdot OC(v, \measuredangle A + B) \\ \gamma' &= \gamma \cdot OC(v, \measuredangle B) \end{aligned} \tag{20}$$

In the resulting triangle $A'B'C'$, since one knows all its side lengths, one applies the Law of Cosine in order to find each angle $\measuredangle A'$, $\measuredangle B'$, and $\measuredangle C'$. Therefore:

$$\measuredangle A' = \arccos \frac{-\alpha^2 \cdot C(v)^2 + \beta^2 \cdot OC(v, \measuredangle A + B)^2 + \gamma^2 \cdot OC(v, \measuredangle B)^2}{2\beta \cdot \gamma \cdot OC(v, \measuredangle B) \cdot OC(v, \measuredangle A + B)}$$

$$\measuredangle B' = \arccos \frac{\alpha^2 \cdot C(v)^2 - \beta^2 \cdot OC(v, \measuredangle A + B)^2 + \gamma^2 \cdot OC(v, \measuredangle B)^2}{2\alpha \cdot \gamma \cdot OC(v) \cdot OC(v, \measuredangle B)}$$

$$\measuredangle C' = \arccos \frac{\alpha^2 \cdot C(v)^2 + \beta^2 \cdot OC(v, \measuredangle A + B)^2 - \gamma^2 \cdot OC(v, \measuredangle B)^2}{2\alpha \cdot \beta \cdot OC(v) \cdot OC(v, \measuredangle A + B)}.$$

As we can see, the angles $\measuredangle A'$, $\measuredangle B'$, and $\measuredangle C'$ are, in general, different from the original angles A, B, and C respectively.

The distortion of an angle is, in general, different from the distortion of another angle.

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