The Neutrosophic Logic View to Schrödinger's Cat Paradox, Revisited

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The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot. To make this proposition clear, we revisit our previous paper by offering an illustration using modified coin tossing problem, known as Parrondo's game.

1 Introduction

The present article discusses Neutrosophic logic view to Schrödinger's cat paradox. In this article we argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic logic can take into consideration, whereas other methods including Fuzzy logic cannot.

In the preceding article we have discussed how Neutrosophic logic view can offer an alternative method to solve the well-known problem in Quantum Mechanics, i.e. the Schrödinger's cat paradox [1, 2], by introducing indeterminacy of the outcome of the observation.

In other article we also discuss possible re-interpretation of quantum measurement using Unification of Fusion Theories as generalization of Information Fusion [3, 4, 5], which results in proposition that one can expect to neglect the principle of "excluded middle"; therefore Bell's theorem can be considered as merely tautological. [6] This alternative view of Quantum mechanics as Information Fusion has also been proposed by G. Chapline [7]. Furthermore this Information Fusion interpretation is quite consistent with measurement theory of Quantum Mechanics, where the action of measurement implies information exchange [8].

In the first section we will discuss basic propositions of Neutrosophic probability and Neutrosophic logic. Then we discuss solution to Schrödinger's cat paradox. In subsequent section we discuss an illustration using modified coin tossing problem, and discuss its plausible link to quantum game.

While it is known that derivation of Schrödinger's equation is heuristic in the sense that we know the answer to which the algebra and logic leads, but it is interesting that Schrödinger's equation follows logically from de Broglie's *grande loi de la Nature* [9, p.14]. The simplest method to derive Schrödinger's equation is by using simple wave as [9]:

$$\frac{\partial^2}{\partial x^2}\,\mathtt{exp}(ikx) = -k^2 \cdot \mathtt{exp}(ikx)\,. \tag{1}$$

By deriving twice the wave and defining:

$$k = \frac{2\pi m v}{h} = \frac{mv}{\hbar} = \frac{p_x}{\hbar}\,, \tag{2}$$

where p_x, $\hbar$ represents momentum at x direction, and rationalised Planck constants respectively.

By introducing kinetic energy of the moving particle, T, and wavefunction, as follows [9]:

$$T = \frac{mv^2}{2} = \frac{p_x^2}{2m} = \frac{\hbar^2}{2m}\,k^2\,, \tag{3}$$

and

$$\psi(x) = \mathtt{exp}(ikx)\,. \tag{4}$$

Then one has the time-independent Schrödinger equation from [1, 3, 4]:

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\,\psi(x) = T \cdot \psi(x)\,. \tag{5}$$

It is interesting to remark here that by convention physicists assert that "the wavefunction is simply the *mathematical function* that describes the wave" [9]. Therefore, unlike the wave equation in electromagnetic fields, one should not consider that equation [5] has any physical meaning. Born suggested that the square of wavefunction represents the probability to observe the electron at given location [9, p.56]. Although Heisenberg rejected this interpretation, apparently Born's interpretation prevails until today.

Nonetheless the founding fathers of Quantum Mechanics (Einstein, De Broglie, Schrödinger himself) were dissatisfied with the theory until the end of their lives. We can summarize the situation by quoting as follows [9, p.13]:

> "The interpretation of Schrödinger's wave function (and of quantum theory generally) remains a matter of continuing concern and controversy among scientists who cling to philosophical belief that the natural world is basically logical and deterministic."

Furthermore, the "pragmatic" view of Bohr asserts that for a given quantum measurement [9, p.42]:

> "A system does not possess objective values of its physical properties until a measurement of one of them is made; the act of measurement is asserted to force the system into an eigenstate of the quantity being measured."

In 1935, Einstein-Podolsky-Rosen argued that the axiomatic basis of Quantum Mechanics is incomplete, and subsequently Schrödinger was inspired to write his well-known cat paradox. We will discuss solution of his cat paradox in subsequent section.

2 Cat paradox and imposition of boundary conditions

As we know, Schrödinger's deep disagreement with the Born interpretation of Quantum Mechanics is represented by his cat paradox, which essentially questioning the "statistical" interpretation of the wavefunction (and by doing so, denying the physical meaning of the wavefunction). The cat paradox has been written elsewhere [1, 2], but the essence seems quite similar to coin tossing problem:

> "Given $p = 0.5$ for each side of coin to pop up, we will never know the state of coin before we open our palm from it; unless we know beforehand the "state" of the coin (under our palm) using ESP-like phenomena. Prop. (1)."

The only difference here is that Schrödinger asserts that the state of the cat is half alive and half dead, whereas in the coin problem above, we can only say that we don't know the state of coin until we open our palm; i.e. the state of coin is *indeterminate* until we open our palm. We will discuss the solution of this problem in subsequent section, but first of all we shall remark here a basic principle in Quantum Mechanics, i.e. [9, p.45]:

> "Quantum Concept: The first derivative of the wavefunction Ψ of Schrödinger's wave equation must be single-valued everywhere. As a consequence, the wavefunction itself must be single-valued everywhere."

The above assertion corresponds to quantum logic, which can be defined as follows [10, p.30; 11]:

$$P \vee Q = P + Q - PQ\,. \tag{6}$$

As we will see, it is easier to resolve this cat paradox by releasing the aforementioned constraint of "single-valuedness" of the wavefunction and its first derivative. In fact, nonlinear fluid interpretation of Schrödinger's equation (using the level set function) also indicates that the physical meaning of wavefunction includes the notion of multivaluedness [12]. In other words, one can say that observation of spin-half electron at location x does not exclude its possibility to pop up somewhere else. This counter-intuitive proposition will be described in subsequent section.

3 Neutrosophic solution of the Schrödinger cat paradox

In the context of physical theory of information [8], Barrett has noted that "there ought to be a set theoretic language which applies directly to all quantum interactions". This is because the idea of a bit is itself straight out of *classical set theory*, the definitive and unambiguous assignment of an element of the set {0,1}, and so the assignment of an information content of the photon itself is fraught with the same difficulties [8]. Similarly, the problem becomes more adverse because the fundamental basis of conventional statistical theories is the same classical set {0,1}.

For example the Schrödinger's cat paradox says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory. In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function, which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Therefore the question can be summarized as follows [1]:

> "How to describe a particle ζ in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\zeta \in P_1$ and $\zeta \in \neg P_1$ is a true contradiction, with respect to Quantum Concept described above."

Now we will discuss some basic propositions in Neutrosophic logic [1].

3a Non-standard real number and subsets

Let T,I,F be standard or non-standard real subsets $\subseteq]^-0, 1^+[$,

with sup T = t_sup, inf T= t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup + i_sup + f_sup,
n_inf = t_inf + i_inf + f_inf.

Obviously, t_sup, i_sup, f_sup $\leqslant 1^+$; and t_inf, i_inf, f_inf $\geqslant {}^-0$, whereas n_sup $\leqslant 3^+$ and n_inf $\geqslant {}^-0$. The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single element; finite or infinite; union or intersection of various subsets etc. They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case where T, I, F are reduced to points).

For interpretation of this proposition, we can use modal logic [10]. We can use the notion of "world" in modal logic, which is semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is "true in all possible worlds." (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth*, as opposed to possible truth), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a "specific" world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each "world" is dynamic, depending on an ensemble of parameters, we introduce the sub-category "conjuncture" within it to reflect a particular state of the world.

In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the Neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the Neutrosophic indeterminacy value $NL_i(A) = 1$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

3b Neutrosophic probability definition

Neutrosophic probability is defined as: "Is a generalization of the classical probability in which the chance that an event A occurs is t% true — where t varies in the subset T, i% indeterminate — where i varies in the subset I, and f% false — where f varies in the subset F. One notes that NP(A) = (T, I, F)". It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

The universal set, endowed with a Neutrosophic probability defined for each of its subset, forms a Neutrosophic probability space.

3c Solution of the Schrödinger's cat paradox

Let's consider a neutrosophic set a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^-0$ and 1^+. For example: x (0.5, 0.2, 0.3) belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is *undecidable*); or y (0, 0, 1) belongs to A (which normally means y is not for sure in A); or z (0, 1, 0) belongs to A (which means one does know absolutely nothing about z's affiliation with A).

More general, x ((0.2–0.3), (0.40–0.45) ∪ [0.50–0.51], {0.2, 0.24, 0.28}) belongs to the set A, which means:

- with a probability in between 20-30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $\text{n\_sup} = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [1, 2], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i.e. we don't know if it is dead or alive), then herein the Neutrosophic logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We have described Neutrosophic solution of the Schrödinger's cat paradox. Alternatively, one may hypothesize four-valued logic to describe Schrödinger's cat paradox, see Rauscher *et al.* [13, 14].

In the subsequent section we will discuss how this Neutrosophic solution involving "possible truth" and "indeterminacy" can be interpreted in terms of coin tossing problem (albeit in modified form), known as Parrondo's game. This approach seems quite consistent with new mathematical formulation of game theory [20].

4 An alternative interpretation using coin toss problem

Apart from the aforementioned pure mathematics-logical approach to Schrödinger's cat paradox, one can use a well-known neat link between Schrödinger's equation and Fokker-Planck equation [18]:

$$D\,\frac{\partial^2 p}{\partial z^2} - \frac{\partial \alpha}{\partial z}\,p - \alpha\,\frac{\partial p}{\partial z} - \frac{\partial p}{\partial t} = 0\,. \qquad (7)$$

A quite similar link can be found between relativistic classical field equation and non-relativistic equation, for it is known that the time-independent Helmholtz equation and Schrödinger equation is formally identical [15]. From this reasoning one can argue that it is possible to explain Aharonov effect from pure electromagnetic field theory; and therefore it seems also possible to describe quantum mechanical phenomena without postulating the decisive role of "observer" as Bohr asserted. [16, 17]. In idiomatic form, one can expect that quantum mechanics does not have to mean that "the Moon is not there when nobody looks at".

With respect to the aforementioned neat link between Schrödinger's equation and Fokker-Planck equation, it is interesting to note here that one can introduce "finite difference" approach to Fokker-Planck equation as follows. First, we can define local coordinates, expanded locally about a point (z_0, t_0) we can map points between a real space (z, t) and an integer or discrete space (i, j). Therefore we can sample the space using linear relationship [19]:

$$(z, t) = (z_0 + i\lambda, t_0 + j\tau)\,, \tag{8}$$

where λ is the sampling length and τ is the sampling time. Using a set of finite difference approximations for the Fokker-Planck PDE:

$$\frac{\partial p}{\partial z} = A_1 = \frac{p\,(z_0 + \lambda, t_0 - \tau) - p\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,, \tag{9}$$

$$\frac{\partial^2 p}{\partial z^2} = 2A_2 = \\ = \frac{p\,(z_0 - \lambda, t_0 - \tau) - 2p\,(z_0, t_0 - \tau) + p\,(z_0 + \lambda, t_0 - \tau)}{\lambda^2}\,, \tag{10}$$

and

$$\frac{\partial p}{\partial t} = B_1 = \frac{p\,(z_0, t_0) - p\,(z_0, t_0 - \tau)}{\tau}\,. \tag{11}$$

We can apply the same procedure to obtain:

$$\frac{\partial \alpha}{\partial z} = A_1 = \frac{\alpha\,(z_0 + \lambda, t_0 - \tau) - \alpha\,(z_0 - \lambda, t_0 - \tau)}{2\lambda}\,. \tag{12}$$

Equations (9–12) can be substituted into equation (7) to yield the required finite partial differential equation [19]:

$$p\,(z_0, t_0) = a_{-1} \cdot p\,(z_0 - \lambda, t_0 - \tau) - a_0 \cdot p\,(z_0, t_0 - \tau) + \\ + a_{+1} \cdot p\,(z_0 + \lambda, t_0 - \tau)\,. \tag{13}$$

This equation can be written in terms of discrete space by using [8], so we have:

$$p_{i,j} = a_{-1} \cdot p_{i-1,j-1} + a_0 \cdot p_{i,j-1} + a_{+1} \cdot p_{i+1,j-1}\,. \tag{14}$$

Equation (14) is precisely the form required for Parrondo's game. The meaning of Parrondo's game can be described in simplest way as follows [19]. Consider a coin tossing problem with a biased coin:

$$p_{head} = \frac{1}{2} - \varepsilon\,, \tag{15}$$

where ε is an external bias that the game has to "overcome". This bias is typically a small number, for instance 1/200. Now we can express equation (15) in finite difference equation (14) as follows:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon\right) \cdot p_{i-1,j-1} + 0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon\right) \cdot p_{i+1,j-1}\,. \tag{16}$$

Furthermore, the bias parameter can be related to an applied external field.

With respect to the aforementioned Neutrosophic solution to Schrödinger's cat paradox, one can introduce a new "indeterminacy" parameter to represent conditions where the outcome may be affected by other issues (let say, apparatus setting of Geiger counter). Therefore equation (14) can be written as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ + a_0 \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{17}$$

where unlike the bias parameter (∼1/200), the indeterminacy parameter can be quite large depending on the system in question. For instance in the Neutrosophic example given above, we can write that:

$$\eta \sim 0.2 - 0.3 = k\left(\frac{d}{t}\right)^{-1} = k\left(\frac{t}{d}\right) \leqslant 0.50. \tag{18}$$

The only problem here is that in original coin tossing, one cannot assert an "intermediate" outcome (where the outcome is neither A nor B). Therefore one shall introduce modal logic definition of "possibility" into this model. Fortunately, we can introduce this possibility of intermediate outcome into Parrondo's game, so equation (17) shall be rewritten as:

$$p_{i,j} = \left(\frac{1}{2} - \varepsilon - \eta\right) \cdot p_{i-1,j-1} + \\ + (2\eta) \cdot p_{i,j-1} + \left(\frac{1}{2} + \varepsilon - \eta\right) \cdot p_{i+1,j-1}\,, \tag{19}$$

For instance, by setting $\eta \sim 0.25$, then one gets the finite difference equation:

$$p_{i,j} = (0.25 - \varepsilon) \cdot p_{i-1,j-1} + (0.5) \cdot p_{i,j-1} + \\ + (0.25 + \varepsilon) \cdot p_{i+1,j-1}\,, \tag{20}$$

which will yield more or less the same result compared with Neutrosophic method described in the preceding section.

For this reason, we propose to call this equation (19): *Neutrosophic-modified Parrondo's game.* A generalized expression of equation [19] is:

$$p_{i,j} = (p_0 - \varepsilon - \eta) \cdot p_{i-1,j-1} + (z\eta) \cdot p_{i,j-1} + \\ + (p_0 + \varepsilon - \eta) \cdot p_{i+1,j-1}\,, \tag{21}$$

where p_0, z represents the probable outcome in standard coin tossing, and a real number, respectively. For the practical meaning of η, one can think (by analogy) of this indeterminacy parameter as a variable that is inversely proportional to the "thickness ratio" (d/t) of the coin in question. Therefore using equation (18), by assuming $k = 0.2$, coin thickness = 1.0 mm, and coin diameter $d = 50$ mm, then we get $d/t = 50$, or $\eta = 0.2\,(50)^{-1} = 0.004$, which is negligible. But if we use a thick coin (for instance by gluing 100 coins altogether), then by assuming $k = 0.2$, coin thickness = 100 mm, and coin diameter $d = 50$ mm, we get $d/t = 0.5$, or $\eta = 0.2(0.5)^{-1} = 0.4$, which indicates that chance to get outcome neither A nor B is quite large. And so forth.

It is worth noting here that in the language of "modal logic" [10, p.54], the "intermediate" outcome described here is given name 'possible true', written $\Diamond A$, meaning that "it is not necessarily true that not-A is true". In other word, given that the cat cannot be found in location x, does not have to mean that it shall be in y.

Using this result (21), we can say that our proposition in the beginning of this paper (Prop. 1) has sufficient reasoning; i.e. it is possible to establish link from Schrödinger wave equation to simple coin toss problem, albeit in modified form. Furthermore, this alternative interpretation, differs appreciably from conventional Copenhagen interpretation.

It is perhaps more interesting to remark here that Heisenberg himself apparently has proposed similar thought on this problem, by introducing "potentia", which means "*a world devoid of single-valued actuality but teeming with unrealized possibility*" [4, p.52]. In Heisenberg's view an atom is certainly real, but its attributes dwell in an existential limbo "halfway between an idea and a fact", a quivering state of attenuated existence. Interestingly, experiments carried out by J. Hutchison seem to support this view, that a piece of metal can come in and out from existence [23].

In this section we discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game. Further observation and theoretical study is recommended to explore more implications of this plausible link.

5 Concluding remarks

In the present paper we revisit the Neutrosophic logic view of Schrödinger's cat paradox. We also discuss a plausible way to represent the Neutrosophic solution of cat paradox in terms of Parrondo's game.

It is recommended to conduct further experiments in order to verify and explore various implications of this new proposition, including perhaps for the quantum computation theory.

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References

1. Smarandache F. An introduction to the Neutrosophic probability applied in quantum physics. *Bull. Pure and Appl. Sci., Physics*, 2003, v. 22D, no. 1, 13–25.
2. Smarandache F. and Christianto V. The Neutrosophic logic view to Schrödinger's cat paradox. *Progr. in Phys.*, 2006, no. 2.
3. Smarandache F. and Christianto V. A note on geometric and information fusion interpretation of Bell's theorem and quantum measurement. *Progress in Physics*, 2006, no. 4.
4. Smarandache F. and Christianto V. Multivalued logic, neutrosophy and Schrödinger equation. Hexis, Phoenix (AZ), 2006, p.52–54.
5. Smarandache F. and Dezert J. Advances and applications of DSmT for information fusion. American Research Press, Rehoboth (NM), 2004.
6. Kracklauer A. La theorie de Bell, est-elle la plus grande meprise de l'histoire de la physique? *Ann. Fond. Louis de Broglie*, 2000, v. 25, 193.
7. Chapline G. arXiv: adap-org/9906002; quant-ph/9912019; Granik A. and Chapline G. arXiv: quant-ph/0302013.
8. Zurek W. (ed.) Complexity, entropy and the physics of information. Addison-Wesley Publ., 1990, p.378.
9. Hunter G. Quantum chemistry: wave mechanics applied to atoms and molecules. Lecture notes. Chapter 1: Genesis of Quantum Mechanics. 2001, p.14, 26, 42.
10. deVries A. Algebraic hierarchy of logics unifying fuzzy logic and quantum logic. arXiv: math.LO/0707.2161, p.30, 54.
11. Aerts D. Description of many separated physical entities without the paradoxes encountered in Quantum Mechanics. *Found. Phys.*, 1982, v. 12, no. 12, p.1142, 1149–1155.
12. Jin S., Liu H., Osher S., and Tsai R. Computing multivalued physical observables for the semiclassical limits of the Schrödinger equation. *J. Comp. Phys.*, 2005, v. 205, 222–241.
13. Rauscher E.A. and Targ R. The speed of thought: investigation of complex spacetime metric to describe psychic phenomena. *J. Scientific Exploration*, 2001, v. 15, no. 3, 344–354.
14. Rauscher E.A. and Amoroso R. The physical implications of multidimensional geometries and measurement. *Intern. J. Comp. Anticipatory Systems*, D. Dubois (ed.), 2006.
15. Lu J., Greenleaf J., and Recami E. Limited diffraction solutions to Maxwell and Schrödinger equation. arXiv: physics/9610012.
16. Aharonov Y., *et al.* arXiv: quant-ph/0311155.
17. Goldstein S. Quantum theory without observers — part one. *Physics Today*, March 1998, 42–46.
18. Ho C.-L. and Sasaki R. Deformed Fokker Planck equations. arXiv: cond-mat/0612318.
19. Allison A., *et al.* State space visualization and fractal properties of Parrondo's game. arXiv: cond-mat/0205536; cond-mat/0208470.
20. Wu J. A new mathematical representation of game theory. arXiv: quant-ph/0404159.
21. Smarandache F. Unification of fusion theories (UFT). *Intern. J. Appl. Math. and Stat.*, 2004, v. 2, 1–14.
22. Smarandache F. An in-depth look at information fusion rules and unification of fusion theories. Invited speech at NASA Langley Research Center, Hampton, VA, USA, November 5, 2004.
23. Smarandache F., Christianto V., Khrapko R., Yuhua F., Hutchison J. Unfolding labyrinth: open problems in physics, mathematics, astrophysics, and other areas of science. Hexis, Phoenix (AZ), 2006.