On Neutrosophic Implications

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Abstract: In this paper, we firstly review the neutrosophic set, and then construct new concepts called neutrosophic implication of type 1 and of type 2 for neutrosophic sets.

Keywords: Neutrosophic Implication, Neutrosophic Set, N-norm, N-conorm.

1 Introduction
Neutrosophic set (NS) was introduced by Florentin Smarandache in 1995 [1], as a generalization of the fuzzy set proposed by Zadeh [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on. This concept represents uncertain, imprecise, incomplete and inconsistent information existing in the real world. A NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and with lies in] 0, 1] [ the non-standard unit interval.

NS has been studied and applied in different fields including decision making problems [6, 7, 8], Databases [10], Medical diagnosis problem [11], topology [12], control theory [13], image processing [14, 15, 16] and so on.

In this paper, motivated by fuzzy implication [17] and intuitionistic fuzzy implication [18], we will introduce the definitions of two new concepts called neutrosophic implication for neutrosophic set.

This paper is organized as follow: In section 2 some basic definitions of neutrosophic sets are presented. In section 3, we propose some sets operations on neutrosophic sets. Then, two kind of neutrosophic implication are proposed. Finally, we conclude the paper.

2 Preliminaries
This section gives a brief overview of concepts of neutrosophic sets, single valued neutrosophic sets, neutrosophic norm and neutrosophic conorm which will be utilized in the rest of the paper.

Definition 1 (Neutrosophic set) [1]
Let X be a universe of discourse then, the neutrosophic set A is an object having the form:
A = { x: T(x), I(x), F(x) ∈ X}, where the functions T, I, F : X→ ]0, 1[ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ X to the set A with the condition.

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$ (1)

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of ]0, 1[. So instead of ]0, 1[, we need to take the interval [0, 1] for technical applications, because ]0, 1[ will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 (Single-valued Neutrosophic sets) [20]
Let X be an universe of discourse with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A$, an indeterminacy-membership function $I_A$, and a falsity-membership function $F_A$, for each point $x$ in $X$, $T_A(x)$, $I_A(x)$, $F_A(x)$, $x \in [0, 1]$.

When X is continuous, an SVNS A can be written as

$$A = \{ (x, T_A(x), I_A(x), F_A(x)) | x \in X \}.$$ (2)

When X is discrete, an SVNS A can be written as

$$A = \{ (x_i, T_A(x_i), I_A(x_i), F_A(x_i)) | x_i \in X \}.$$ (3)

Definition 3 (Neutrosophic norm, n-norm) [19]
Mapping $N_n$: [0, 1]×[0, 1]→[0, 1] such that $N_n(0,0)=1$, $N_n(1,1)=1$,

$$N_n(a,b) = \| a \times b \times \|.$$ (4)

are the truth/membership, indeterminacy, and respectively falsehood/ nonmembership components.
$N_a$ have to satisfy, for any $x, y, z$ in the neutrosophic logic/set $M$ of the universe of discourse $X$, the following axioms

a) Boundary Conditions: $N_a(x, 0) = 0$, $N_a(x, 1) = x$.

b) Commutativity: $N_a(x, y) = N_a(y, x)$.

c) Monotonicity: If $x \leq y$, then $N_a(x, z) \leq N_a(y, z)$.

d) Associativity: $N_a(N_a(x, y), z) = N_a(x, N_a(y, z))$.

$N_a$ represents the intersection operator in neutrosophic set theory.

Let $I \in \{T, I, F\}$ be a component.

Most known N-norms, as in fuzzy logic and set the T-norms, are:

- The Algebraic Product $N_{A}$-norm: $N_{A}(x, y) = xy$.
- The Bounded $N_{B}$-norm: $N_{B}(x, y) = \max\{0, x + y - 1\}$.
- The Default (min) $N_{D}$-norm: $N_{D}(x, y) = \min\{x, y\}$.
- The Algebraic Product $N_{A}$-norm: $N_{A}(x, y) = \max\{0, x + y\}$.

A general example of $N$-norm would be this. Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set $M$. Then:

$N_{A}(x, y) = (T_1 \land T_2, I_1 \lor I_2, F_1 \lor F_2)$ (4)

where the “$\land$” operator is a Neutrosophic Conorm, while the “$\lor$” operator is a Neutrosophic Norm.

Remark

For the sake of simplicity we have denoted:

$\alpha_1 = \min\{x, y\}$, $\alpha_2 = \max\{x, y\}$

where $\alpha_1$ and $\alpha_2$ represent the intersection and union operations between two neutrosophic sets.

3 Neutrosophic Implications

In this subsection, we introduce the set operations on neutrosophic set, which we will work with. Then, two neutrosophic implication are constructed on the basis of single valued neutrosophic set. The two neutrosophic implications are denoted by and . Also, important properties of and are demonstrated and proved.

Definition 6 (Set Operations on Neutrosophic sets)

Let $A$ and $B$ two neutrosophic sets, we propose the following operations on NSs as follows:

$A \oplus B = \{ (T_A + T_B, I_A + I_B, F_A + F_B) \}$, where

$< T_A, I_A, F_A > \in A$, $< T_B, I_B, F_B > \in B$.

$A \ominus B = \{ (T_A - T_B, I_A - I_B, F_A - F_B) \}$, where

$< T_A, I_A, F_A > \in A$, $< T_B, I_B, F_B > \in B$.

$A \odot B = \{ 2T_AT_B, 2I_AT_B, 2F_AT_B \}$, where

$< T_A, I_A, F_A > \in A$, $< T_B, I_B, F_B > \in B$.

$A \triangledown B = \{ T_A + T_B, I_A + I_B, F_A + F_B \}$, where

$< T_A, I_A, F_A > \in A$, $< T_B, I_B, F_B > \in B$.

$A \Box B = \{ T_A - T_B, I_A - I_B, F_A - F_B \}$, where

$< T_A, I_A, F_A > \in A$, $< T_B, I_B, F_B > \in B$.

where the “$\oplus$” operator is a N-union (verifying the above N-norms axioms); while the “$\ominus$” operator, is a N-norm.

For example, $\ominus$ can be the Algebraic Product T-norm/N-norm, so $T_1T_2= T_1T_2$ and $\ominus$ can be the Algebraic Product T-norm/N-norm, so $T_1T2= T1+T2-T1T2$.

Or $\ominus$ can be any T-norm/N-norm, and $\ominus$ any T-norm/N-norm from the above.

In 2013, A. Salama [21] introduced beside the intersection and union operations between two neutrosophic set $A$ and $B$, another operations defined as follows:

Definition 5

Let $A, B$ two neutrosophic sets

$\alpha_1 = \min\{x, y\}$, $\alpha_2 = \max\{x, y\}$

where $\alpha_1, \alpha_2$ represent the intersection and union set proposed by Florentin Smarandache and A. Salama.
Obviously, for every two $A$ and $B$, $(A \oplus B)$, $(A \oslash B)$, $(A \# B)$, $A \otimes B$ and $A \oslash B$ are also NSs. Based on definition of standard implication denoted by “$A \rightarrow B$”, which is equivalent to “non $A$ or $B$”. We extended it for neutrosophic set as follows:

**Definition 7**

Let $A(x) = \{ <x, T_A(x), I_A(x), F_A(x)> \}$ and $B(x) = \{ <x, T_B(x), I_B(x), F_B(x)> \}$. 
$A, B \in$ NS(X). So, depending on how we handle the indeterminacy, we can defined two types of neutrosophic implication, then is the neutrosophic type 1 defined as

$$A_{NS1} \circ B = \{ <x, F_A(x) \lor T_B(x), I_A(x) \land I_B(x), T_A(x)> \}$$

and

$$A_{NS2} \circ B = \{ <x, F_A(x) \lor T_B(x), I_A(x) \lor I_B(x), T_A(x)> \}$$

by $\forall$ and we denote a neutrosophic norm ($N$-norm) and neutrosophic conorm ($N$-conorm).

Note: The neutrosophic implications are not unique, as this depends on the type of functions used in $N$-norm and $N$-conorm.

Throughout this paper, we used the function (dual) $\min/\max$.

**Theorem 1**

For $A, B$ and $C \in$ NS(X),

- $\forall A \cup_1 B_{NS1} C = (A_{NS1} C) \bigcup_1 (B_{NS1} C)$
- $\forall A \cap_1 B_{NS1} C = (A_{NS1} C) \bigcap_1 (B_{NS1} C)$
- $\forall A \cap_1 B_{NS1} C = (A_{NS1} C) \bigcup_1 (B_{NS1} C) \bigcup_1 (A_{NS1} B_{NS1} C)$
- $\forall A \cap_1 B_{NS1} C = (A_{NS1} B_{NS1} C)$

**Proof**

(i) From definition in (5), we have

$$A \cup_1 B_{NS1} C = \{ <x, \min(\max(F_A, F_B), T_C), \min(\max(I_A, I_B)), \min(\max(T_A, T_B), T_C)> \}$$

and

$$(A_{NS1} C) \cap_1 (B_{NS1} C) = \{ <x, \min(\max(F_A, T_C), \max(F_B, T_C)), \min(\max(I_A, I_B), \min(\max(I_B, I_C)), \max(\min(T_A, T_B), T_C))> \}$$

Comparing the result of (8) and (9), we get

$$(\min(\max(F_A, F_B), T_C), \min(\max(I_A, I_B)), \min(\max(T_A, T_B), T_C)) \rightarrow x \in X$$

(ii) From definition in (5), we have

$$A_{NS1} B \cap_1 C = \{ (\min(F_A, \min(T_B, T_C)), \min(I_A, \max(I_B, I_C)), \min(T_A, \max(F_B, T_C)> \}$$

and

$$(A_{NS1} B \cap_1 C) = \{ <x, \min(\max(F_A, T_B), T_C)), \min(\max(I_A, I_B), \min(I_B, I_C)), \max(\min(T_A, T_B), T_C))> \}$$

Hence, $A \cup_1 B_{NS1} C = (A_{NS1} C) \cap_1 (B_{NS1} C)$

(iii) From definition in (5), we have

$$A \cap_1 B_{NS1} C = (A_{NS1} B_{NS1} C) \cup_1 (B_{NS1} C)$$

and

$$(A_{NS1} C) \cup_1 (B_{NS1} C) = \{ <x, \max(\min(F_A, F_B), T_C), \max(F_A, T_C)), \max(\min(I_A, I_B), \max(I_B, I_C)), \min(\min(T_A, T_B), T_C))> \}$$

Comparing the result of (10) and (11), we get

$$\min(\max(T_A, T_B), T_C)) \rightarrow x \in X$$

and

$$\min(\max(T_A, T_B), T_C)) \rightarrow x \in X$$

Hence, $A \cup_1 B_{NS1} C = (A_{NS1} C) \cap_1 (B_{NS1} C)$

**Theorem 2**

For $A, B$ and $C \in$ NS(X),

- $\forall A \cup_2 B_{NS1} C = (A_{NS1} C) \cap_1 (B_{NS1} C)$
ii. $A_{NS1} \cap B_{NS2} C = (A_{NS1} \cap B_{NS2}) C = (A_{NS1} B_{NS2}) C$

iii. $A_{NS1} \cap B_{NS2} C = (A_{NS1} \cap B_{NS2}) \cup_1 (A_{NS1} C)$

iv. $A_{NS1} \cup_2 B_{NS2} C = (A_{NS1} \cup_2 B_{NS2}) \cup_2 (A_{NS1} C)$

**Proof**
The proof is straightforward.

In view of $A_{NS2} B = \{x < F_A \lor T_B \cdot I_B \lor T_A \lor F_B > x \in X \}$, we have the following theorem:

**Theorem 3**

For $A$, $B$ and $C \in NS(X)$,

i. $A \cup_1 B_{NS2} C = A_{NS2} (A_{NS2} C) \cap_1 (B_{NS2} C)$

ii. $A_{NS2} \cap_1 C = A_{NS2} (A_{NS2} C) \cap_1 (B_{NS2} C)$

iii. $A_{NS2} \cap_1 B_{NS2} C = A_{NS2} (B_{NS2} C)$

iv. $A_{NS2} \cup_1 B_{NS2} C = A_{NS2} (A_{NS2} B) \cup_1 (A_{NS2} C)$

**Proof**

(i) From definition in (5), we have

$$A \cup_1 B_{NS2} C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Max}(\text{max}(I_A, I_B), I_C)\}$$

and $A_{NS2} C \cap_1 (B_{NS2} C) = \{x, \text{Min}(\text{max}(F_A, T_C), \text{Max}(\text{min}(I_A, I_C), \text{Max}(F_B, T_C)), \text{Min}(\text{min}(T_A, F_C), \text{min}(T_B, F_C))\} \text{ for } x \in X$ (16)

Comparing the result of (16) and (17), we get

$$\text{Max}(\text{min}(F_A, F_B), T_C) = \text{Min}(\text{max}(F_A, T_C), \text{Max}(\text{min}(I_A, I_C), \text{Max}(F_B, T_C)))$$

and (18)

(ii) From definition in (5), we have

$$A_{NS2} B \cap_1 C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Max}(\text{max}(I_A, I_B), I_C)\}$$

and $A_{NS2} C \cap_1 (B_{NS2} C) = \{x, \text{Min}(\text{max}(F_A, T_C), \text{Max}(\text{min}(I_A, I_C), \text{Max}(F_B, T_C)), \text{Min}(\text{min}(T_A, F_C), \text{min}(T_B, F_C))\} \text{ for } x \in X$ (19)

Comparing the result of (18) and (19), we get

$$\text{Max}(\text{min}(T_A, T_C)) = \text{Min}(\text{max}(F_A, T_B), \text{Max}(T_A, F_C))$$

Hence, $A_{NS2} B \cap_1 C = A_{NS2} (B_{NS2} C)$

(iii) From definition in (5), we have

$$A_{NS2} C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Max}(\text{max}(I_A, I_B), I_C)\}$$

and $A_{NS2} C \cap_1 (B_{NS2} C) = \{x, \text{Min}(\text{max}(F_A, T_C), \text{Max}(\text{min}(I_A, I_C), \text{Max}(F_B, T_C)), \text{Min}(\text{min}(T_A, F_C), \text{min}(T_B, F_C))\} \text{ for } x \in X$ (20)

Comparing the result of (20) and (21), we get

$$\text{Max}(\text{max}(F_A, T_C)) = \text{Min}(\text{max}(F_A, T_C), \text{Max}(\text{min}(I_A, I_C), \text{Max}(F_B, T_C)))$$

Hence, $A_{NS2} B_{NS2} C = (A_{NS2} C) \cup_1 (B_{NS2} C)$

(iv) From definition in (5), we have

$$A_{NS2} B_{NS2} C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Max}(\text{min}(I_A, I_B), \text{Min}(T_A, F_B, F_C))\} \text{ for } x \in X$ (22)

and (A_{NS2} B_{NS2} C) \cap_1 (A_{NS2} C) = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Max}(\text{min}(I_A, I_B), \text{Min}(T_A, F_B, F_C))\} \text{ for } x \in X$ (23)

Comparing the result of (22) and (23), we get

$$\text{Max}(\text{max}(F_A, T_B), \text{Max}(F_A, T_C)) = \text{Max}(\text{max}(F_A, T_B), \text{Max}(F_A, T_C))$$

Hence, $A_{NS2} B_{NS2} C = (A_{NS2} C) \cup_1 (B_{NS2} C)$

Using the two operators $\cap_1$ and $\cup_1$, we have

**Theorem 4**

For $A$, $B$ and $C \in NS(X)$,

i. $A \cup_1 B_{NS2} C = A_{NS2} (A_{NS2} C) \cap_1 (B_{NS2} C)$

ii. $A_{NS2} \cap_1 B_{NS2} C = A_{NS2} (A_{NS2} C) \cap_1 (B_{NS2} C)$

iii. $A_{NS2} \cap_1 B_{NS2} C = A_{NS2} (B_{NS2} C)$

iv. $A_{NS2} \cup_1 B_{NS2} C = A_{NS2} (A_{NS2} B) \cup_1 (A_{NS2} C)$

**Proof**

The proof is straightforward.

**Theorem 5**

For $A$, $B \in NS(X)$,

i. $A_{NS2} B_{NS2} C = A_{NS2} (A_{NS2} C) \cap_1 (B_{NS2} C)$

ii. $(A_{NS2} B_{NS2} C)^c = (A_{NS2} C)^c \cup_1 (B_{NS2} C)^c$ = $A_{NS2} (B_{NS2} C)^c$

iii. $(A_{NS2} B_{NS2} C)^c = A_{NS2} (B_{NS2} C)^c$

iv. $A_{NS2} B_{NS2} C = A_{NS2} (A_{NS2} B) \cup_1 (A_{NS2} C)$

**Proof**

(i) From definition in (5), we have

$$A_{NS2} B_{NS2} C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Min}(\text{min}(I_A, I_B), \text{Min}(T_A, F_B, F_C))\} \text{ for } x \in X$ (24)

and $A_{NS2} B_{NS2} C = \{x, \text{Max}(\text{min}(F_A, F_B), T_C), \text{Min}(\text{min}(I_A, I_B), \text{Min}(T_A, F_B, F_C))\} \text{ for } x \in X$ (25)
From (24) and (25), we get $A^{N_{2}} (B^{C})^{c} = A^{C} \cup_{1} B^{C}$

(ii) From definition in (5), we have

$$A^{C} \cup_{1} B^{C} = \{<x, \max (F_{A}, F_{B}), \min (I_{A} - I_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

(26)

and

$$\left((A^{C} \cup_{1} B^{C})^{c}\right) = \{<x, \min (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \min (F_{A} - F_{B}) > | x \in X \}$$

(27)

From (26) and (27), we get

$$\left((A^{C} \cup_{1} B^{C})^{c}\right) = A \cap_{1} B$$

(iii) From definition in (5), we have

$$\left(\left((A^{C} \cup_{1} B^{C})^{c}\right)^{c}\right) = \{<x, \min (T_{A} - T_{B}), \min (I_{A} - I_{B}) \} \max (F_{A} - F_{B}) > | x \in X \}$$

\[= A \cap_{1} B \]

(28)

A $\cap_{2} B = \{\min (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \}$

(29)

From (28) and (29), we get

$$\left(\left((A^{C} \cup_{1} B^{C})^{c}\right)^{c}\right) = A \cap_{2} B$$

(iv) $A^{C} \cup_{2} B = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \min (F_{A} - F_{B}) > | x \in X \}$$

(30)

and

$$\left((A \cap_{2} B)^{c}\right) = \{<x, \max (F_{A}, F_{B}), \min (I_{A} - I_{B}) \} \max (T_{A} - T_{B}) > | x \in X \}$$

(31)

From (30) and (31), we get

$$A^{N_{2}} B^{C} = (A \cap_{2} B)^{c}$$

Theorem 6

For $A, B \in NS(X)$,

i. $(A \cap_{1} B)^{N_{1}} (A \cap_{1} B) = (A \cap_{1} B)^{c}$

\[= \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \} \]

$$\left((A \cap_{1} B)^{c}\right) = \{<x, \max (F_{A}, F_{B}), \min (I_{A} - I_{B}) \} \max (T_{A} - T_{B}) > | x \in X \}$$

From (30) and (31), we get

$$A^{N_{2}} B^{C} = (A \cap_{2} B)^{c}$$

(ii) From definition in (6), we have

$$(A \cap_{1} B)^{N_{1}} (A \cap_{1} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

$$\left((A \cap_{1} B)^{c}\right) = \{<x, \max (F_{A}, F_{B}), \min (I_{A} - I_{B}) \} \max (T_{A} - T_{B}) > | x \in X \}$$

From (32) and (33), we get the result (i)

(iii) From definition in (6), we have

$$\left((A \cap_{1} B)^{c}\right)^{N_{1}} (A \cap_{1} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

$$\left((A \cap_{1} B)^{c}\right)^{c} = \{<x, \min (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \min (F_{A} - F_{B}) > | x \in X \}$$

From (30) and (31), we get

$$A^{N_{2}} B^{C} = (A \cap_{2} B)^{c}$$

(iv) $A \cap_{2} B = \{<x, \min (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \max (F_{A} - F_{B}) > | x \in X \}$$

(34)

$$(A \cap_{2} B)^{N_{1}} (A \cap_{2} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

$$(A \cap_{2} B)^{c} = \{<x, \min (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \min (F_{A} - F_{B}) > | x \in X \}$$

From (32) and (33), we get the result (i)

(v) From definition in (6), we have

$$(A \cap_{1} B)^{N_{1}} (A \cap_{1} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

$$(A \cap_{1} B)^{c} = \{<x, \min (T_{A}, T_{B}), \min (I_{A} - I_{B}) \} \min (F_{A} - F_{B}) > | x \in X \}$$

From (30) and (31), we get

$$A^{N_{2}} B^{C} = (A \cap_{2} B)^{c}$$

Proof

Let us recall following simple fact for any two real numbers $a$ and $b$.

$Max(a, b) + Min(a, b) = a + b$.

Max(a, b) x Min(a, b) = a x b.

(i) From definition in (6), we have

$$(A \cap_{2} B)^{N_{1}} (A \cap_{2} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

(35)

and

$$(A \cap_{1} B)^{N_{1}} (A \cap_{1} B) = \{<x, \max (T_{A}, T_{B}), \min (I_{A} - I_{B}), \max (F_{A} - F_{B}) \} \min (T_{A}, T_{B}) > | x \in X \}$$

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From (36) and (37), we get the result (iii).

From definition in (6), we have

\[(A \otimes B)^c = (A \otimes B)\]

From (38) and (39), we get the result (iv).

From definition in (6), we have

\[(A \otimes B)^c = (A \otimes B)\]

From (40) and (41), we get the result (v).

From definition in (6), we have

\[(A \otimes B)^c = (A \otimes B)\]

From (42) and (43), we get the result (vi). The following theorem is not valid.

**Theorem 7**

For \(A, B \in \text{NS}(X)\),

i. \((A \otimes B)^c = (A \otimes B)\)

ii. \((A \otimes B)^c = (A \otimes B)\)

iii. \((A \otimes B)^c = (A \otimes B)\)

iv. \((A \otimes B)^c = (A \otimes B)\)

v. \((A \otimes B)^c = (A \otimes B)\)

vi. \((A \otimes B)^c = (A \otimes B)\)

**Proof**

The proof is straightforward.

**Theorem 8**

For \(A, B \in \text{NS}(X)\),

i. \((A \otimes B)^c = (A \otimes B)\)

ii. \((A \otimes B)^c = (A \otimes B)\)

iii. \((A \otimes B)^c = (A \otimes B)\)

iv. \((A \otimes B)^c = (A \otimes B)\)

v. \((A \otimes B)^c = (A \otimes B)\)

vi. \((A \otimes B)^c = (A \otimes B)\)

**Proof**

The proof is straightforward.
From definition in (6), we have

$$\begin{align*}
(A \ominus B)^c & = (T_A + T_B - T_A T_B - I_A I_B, F_A) \\
F_B & = (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, F_A) \\
& = \{<x, y, z> | x \in X \} \\
& = F_A F_B, I_A I_B, T_A + T_B - T_A T_B, \frac{T_A + T_B}{2} \\
& = \{<x, y, z, F_A F_B > | x \in X \}
\end{align*}$$

and

$$\begin{align*}
(A \otimes B)^c & = (\frac{T_A + T_B}{2}, \frac{I_A + I_B}{2}, F_A) \\
& = \{<x, y, z, F_A F_B > | x \in X \}
\end{align*}$$

From (44) and (45), we get the result (i).

(ii) From definition in (6), we have

$$\begin{align*}
(A \ominus B)^c & = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, min(T_A + T_B, T_A T_B) > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \}
\end{align*}$$

and

$$\begin{align*}
(A \otimes B)^c & = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, min(T_A + T_B, T_A T_B) > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \}
\end{align*}$$

From (46) and (47), we get the result (ii).

(iii) From definition in (6), we have

$$\begin{align*}
(A \ominus B)^c & = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, min(T_A + T_B, T_A T_B) > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \}
\end{align*}$$

From definition in (6), we have

$$\begin{align*}
(A \otimes B)^c & = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, min(T_A + T_B, T_A T_B) > | x \in X \} \\
& = \{<T_A + T_B, I_A + I_B - I_A I_B, F_A F_B > | x \in X \}
\end{align*}$$
From (52) and (53), we get the result (v).

(vi) From definition in (2), we have

\[(A \text{ OR } B)^{c} = \max \{F_{A} + F_{B} - F_{A}F_{B}, F_{A}F_{B} \}, \]

Max \( I_{A} + I_{B} - I_{A}I_{B} \), Min \( T_{A}T_{B} \)

= \( F_{A} + F_{B} - F_{A}F_{B}, \min (I_{A}I_{B} + I_{A}I_{B} + I_{A}I_{B}) \)

\(= T_{B}T_{A} + I_{A}I_{B} + I_{B}I_{A}I_{B} + F_{A} + F_{B} - F_{A}F_{B} = (A \text{ OR } B) \)

\] (54)

From (54) and (55), we get the result (v).

The following are not valid.

\[
\begin{array}{|c|c|c|c|c|}
\hline
<T_{A}, F_{A}> & <T_{B}, F_{B}> & A_{NS1}B & A_{NS1}B & V(A \rightarrow B) \\
\hline
<0,1> & <0,1> & <1,0> & <1,0> & <1,0> \\
<0,1> & <1,0> & <1,0> & <1,0> & <1,0> \\
<1,0> & <0,1> & <0,1> & <0,1> & <0,1> \\
<1,0> & <1,0> & <1,0> & <1,0> & <1,0> \\
\hline
\end{array}
\]

Theorem 9

1. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

2. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

3. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

4. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

5. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

6. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

7. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

8. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

9. \((A \text{ OR } B)^{c} = (A \text{ OR } B)^{c} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} & \text{A \text{ OR } B} \\
\hline
\text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} & \text{NS1} \\
\hline
\end{array}
\]

Remark

We remark that if the indeterminacy values are restricted to 0, and the membership /non-membership are restricted to 0 and 1. The results of the two neutrosophic implications and collapse to the fuzzy /intuitionistic fuzzy implications defined (V(A \rightarrow B) ) in [17]

Table

Comparison of three kind of implications

From the table, we conclude that fuzzy /intuitionistic fuzzy implications are special case of neutrosophic implication.

Conclusion

In this paper, the neutrosophic implication is studied. The basic knowledge of the neutrosophic set is firstly reviewed, a two kind of neutrosophic implications are constructed, and its properties. These implications may be the subject of further research, both in terms of their properties or comparison with other neutrosophic implication, and possible applications.
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References


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