

Neutrosophic vague soft expert set theory

Ashraf Al-Quran and Nasruddin Hassan*

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Malaysia

Abstract. In this paper, we first introduce the concept of neutrosophic vague soft expert sets (NVSESs for short) which combines neutrosophic vague sets and soft expert sets to be more effective and useful. We also define its basic operations, namely complement, union, intersection, AND and OR along with illustrative examples, and study some related properties with supporting proofs. Lastly, this concept is applied to a decision making problem and its effectiveness is demonstrated using a hypothetical example.

Keywords: Neutrosophic soft expert set, neutrosophic vague set, neutrosophic vague soft set, soft expert set

1. Introduction

In reality, the limitation of precise research is increasingly being recognized in many fields, such as economics, social science, and management science. In recent years, uncertain theories such as probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], vague set theory [3], rough set theory [4] and interval mathematics have been widely applied in uncertain and ambiguous environment. However, these theories do not handle the indeterminate and inconsistent information. Thus neutrosophic set (NS in short) is defined [5], as a new mathematical tool for dealing with problems involving incomplete, indeterminacy and inconsistent knowledge. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. Many research and applications based on neutrosophic set were undertaken such as aggregation operators of interval neutrosophic linguistic numbers [6], similarity measures between interval neutrosophic sets in multicriteria decision-making [7], aggregation operators for simplified neutrosophic sets [8] and improved

correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making [9]. Molodtsov [10] firstly proposed soft set theory as a general mathematical tool to cope with uncertainty and vagueness. Since then, soft set has been developed rapidly to possibility fuzzy soft set [11], soft multiset theory [12], multiparameterized soft set [13], soft intuitionistic fuzzy sets [14], Q-fuzzy soft sets [15–17], and multi Q-fuzzy sets [18–21], thereby opening avenues to many applications [22, 23]. Cagman et al. [24, 25] studied fuzzy soft set theory and fuzzy parameterized fuzzy soft set theory with its applications. Deli and Cagman [26] introduced the concept of intuitionistic fuzzy parameterized soft set and gave its application in decision making. Deli and Karatas [27] also introduced interval valued intuitionistic fuzzy parameterized soft set theory and its decision making. Vague soft set theory was provided by Xu [28], while Alhazaymeh and Hassan [29] introduced the concept of generalized vague soft set followed by possibility vague soft set [30], and interval-valued vague soft sets [31]. They also introduced the concept of possibility interval-valued vague soft set [32]. Maji [33] introduced neutrosophic soft set, which was extended to relations on interval valued neutrosophic soft sets [34], distance and similarity measures of interval neutrosophic soft sets [35], neutrosophic soft relations and some

*Corresponding author. Nasruddin Hassan, School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi Selangor DE, Malaysia. Tel.: +60 3 89213710; E-mail: nas@ukm.edu.my.

properties [36], neutrosophic soft matrices and NSM-decision making [37], interval-valued neutrosophic soft sets and its decision making [38] and interval valued neutrosophic parameterized soft set theory [39]. Alkhazaleh and Salleh then proceeded to introduce the notion of fuzzy soft expert sets [40], while Hassan and Alhazaymeh introduced the theory of vague soft expert sets [41], mapping on generalized vague soft expert set [42] and vague soft set relations [43].

In this paper we first introduce the concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set to improve the reasonability of decision making in reality, and then define its basic operation, namely complement, union, intersection, AND, and OR, and study their properties. Finally we present an application of this concept in solving a decision making problem to show its advantage compared to that of vague soft expert set as proposed by Hassan and Alhazaymeh [41].

2. Preliminaries

In this section, we recall some basic notions in neutrosophic vague set, neutrosophic vague soft set, soft expert set and neutrosophic soft expert set.

Definition 2.1. (see [44]) A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \{ \langle x; \widehat{T}_{A_{NV}}(x); \widehat{I}_{A_{NV}}(x); \widehat{F}_{A_{NV}}(x) \rangle; x \in X \}$ whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as $\widehat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\widehat{I}_{A_{NV}}(x) = [I^-, I^+]$ and $\widehat{F}_{A_{NV}}(x) = [F^-, F^+]$, where (1) $T^+ = 1 - F^-$, (2) $F^+ = 1 - T^-$ and (3) $-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.2. (see [44]) If Ψ_{NV} is a NVS of the universe U , where $\forall u_i \in U$, $\widehat{T}_{\Psi_{NV}}(x) = [1, 1]$, $\widehat{I}_{\Psi_{NV}}(x) = [0, 0]$, $\widehat{F}_{\Psi_{NV}}(x) = [0, 0]$, then Ψ_{NV} is called a unit NVS, where $1 \leq i \leq n$. If Φ_{NV} is a NVS of the universe U , where $\forall u_i \in U$, $\widehat{T}_{\Phi_{NV}}(x) = [0, 0]$, $\widehat{I}_{\Phi_{NV}}(x) = [1, 1]$, $\widehat{F}_{\Phi_{NV}}(x) = [1, 1]$, then Φ_{NV} is called a zero NVS, where $1 \leq i \leq n$.

Definition 2.3. (see [44]) Let A_{NV} and B_{NV} be two NVSs of the universe U . If $\forall u_i \in U$, (1) $\widehat{T}_{A_{NV}}(u_i) = \widehat{T}_{B_{NV}}(u_i)$, (2) $\widehat{I}_{A_{NV}}(u_i) = \widehat{I}_{B_{NV}}(u_i)$ and (3) $\widehat{F}_{A_{NV}}(u_i) = \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is equal to B_{NV} , denoted by $A_{NV} = B_{NV}$, where $1 \leq i \leq n$.

Definition 2.4. (see [44]) Let A_{NV} and B_{NV} be two NVSs of the universe U . If $\forall u_i \in U$, (1) $\widehat{T}_{A_{NV}}(u_i) \leq \widehat{T}_{B_{NV}}(u_i)$, (2) $\widehat{I}_{A_{NV}}(u_i) \geq \widehat{I}_{B_{NV}}(u_i)$ and (3) $\widehat{F}_{A_{NV}}(u_i) \geq \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV} , denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 2.5. (see [44]) The complement of a NVS A_{NV} is denoted by A^c and is defined by

$$\begin{aligned} \widehat{T}_{A_{NV}^c}(x) &= [1 - T^+, 1 - T^-], \\ \widehat{I}_{A_{NV}^c}(x) &= [1 - I^+, 1 - I^-] \quad \text{and} \\ \widehat{F}_{A_{NV}^c}(x) &= [1 - F^+, 1 - F^-]. \end{aligned}$$

Definition 2.6. (see [44]) The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned} \widehat{T}_{C_{NV}}(x) &= \left[\max \left(T_{A_{NV}x}^-, T_{B_{NV}x}^- \right), \max \left(T_{A_{NV}x}^+, T_{B_{NV}x}^+ \right) \right], \\ \widehat{I}_{C_{NV}}(x) &= \left[\min \left(I_{A_{NV}x}^-, I_{B_{NV}x}^- \right), \min \left(I_{A_{NV}x}^+, I_{B_{NV}x}^+ \right) \right] \end{aligned}$$

and

$$\widehat{F}_{C_{NV}}(x) = \left[\min \left(F_{A_{NV}x}^-, F_{B_{NV}x}^- \right), \min \left(F_{A_{NV}x}^+, F_{B_{NV}x}^+ \right) \right]$$

Definition 2.7. (see [44]) The intersection of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned} \widehat{T}_{H_{NV}}(x) &= \left[\min \left(T_{A_{NV}x}^-, T_{B_{NV}x}^- \right), \min \left(T_{A_{NV}x}^+, T_{B_{NV}x}^+ \right) \right], \\ \widehat{I}_{H_{NV}}(x) &= \left[\max \left(I_{A_{NV}x}^-, I_{B_{NV}x}^- \right), \max \left(I_{A_{NV}x}^+, I_{B_{NV}x}^+ \right) \right] \end{aligned}$$

and

$$\widehat{F}_{H_{NV}}(x) = \left[\max \left(F_{A_{NV}x}^-, F_{B_{NV}x}^- \right), \max \left(F_{A_{NV}x}^+, F_{B_{NV}x}^+ \right) \right]$$

Definition 2.8. (see [44]) Let U be an initial universal set and let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and let $A \subseteq E$. A collection of pairs (\widehat{F}, E) is called a *neutrosophic vague soft set* $\{NVSS\}$ over U , where \widehat{F} is a mapping given by $\widehat{F} : A \rightarrow NV(U)$.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.9. (see [45]) A pair (F, A) is called a soft expert set over U , where F is a mapping given by $F : A \rightarrow P(U)$, where $P(U)$ denotes the power set of U .

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.10. (see [46]) A pair (F, A) is called a neutrosophic soft expert set (NSES in short) over U , where F is a mapping given by $F : A \rightarrow P(U)$, where $P(U)$ denotes the power neutrosophic set of U .

Definition 2.11. (see [46]) Let (F, A) and (G, B) be two NSESs over the common universe U . (F, A) is said to be neutrosophic soft expert subset of (G, B) , if $A \subseteq B$ and $T_{F(e)}(X) \leq T_{G(e)}(X)$, $I_{F(e)}(X) \leq I_{G(e)}(X)$, $F_{F(e)}(X) \leq F_{G(e)}(X) \quad \forall e \in A, X \in U$. We denote it by $(F, A) \subseteq (G, B)$.

(F, A) is said to be neutrosophic soft expert superset of (G, B) if (G, B) is a neutrosophic soft expert subset of (F, A) . We denote it by $(F, A) \supseteq (G, B)$.

Definition 2.12. (see [46]) Two (NSESs) (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft expert subset of (G, B) and (G, B) is neutrosophic soft expert subset of (F, A) . We denote it by $(F, A) = (G, B)$.

Definition 2.13. (see [46]) Let $E = \{-e_1, e_2, \dots, e_n\}$ be a set of parameters. The NOT set of E is denoted by $\neg E = \{\neg e_1, \neg e_2, \dots, \neg e_n\}$, where $\neg e_i = \text{not } e_i, \forall i$.

Definition 2.14. (see [46]) The complement of a NSES (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \neg A)$, where $F^c : \neg A \rightarrow P(U)$ is given by $F^c(x) = \text{neutrosophic soft expert complement with } T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}, F_{F^c(x)} = T_{F(x)}$.

Definition 2.15. (see [46]) An agree-NSES $(F, A)_1$ over U is a neutrosophic soft expert subset of (F, A) defined as

$$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}.$$

Definition 2.16. (see [46]) A disagree-NSES $(F, A)_0$ over U is a neutrosophic soft expert subset of (F, A) defined as

$$(F, A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}.$$

Definition 2.17. (see [46]) Let (H, A) and (G, B) be two NSESs over the common universe U . Then the union of (H, A) and (G, B) is denoted by $(H, A) \widetilde{\cup} (G, B)$ and is defined by $(H, A) \widetilde{\cup} (G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$T_{H(e)}(m) = \begin{cases} T_{H(e)}(m), & \text{if } e \in A - B, \\ T_{G(e)}(m), & \text{if } e \in B - A, \\ \max(T_{H(e)}(m), T_{G(e)}(m)), & \text{if } e \in A \cap B, \end{cases}$$

$$I_{H(e)}(m) = \begin{cases} I_{H(e)}(m), & \text{if } e \in A - B, \\ I_{G(e)}(m), & \text{if } e \in B - A, \\ \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, & \text{if } e \in A \cap B, \end{cases}$$

$$F_{H(e)}(m) = \begin{cases} F_{H(e)}(m), & \text{if } e \in A - B, \\ F_{G(e)}(m), & \text{if } e \in B - A, \\ \min(F_{H(e)}(m), F_{G(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.18. (see [46]) Let (H, A) and (G, B) be two NSESs over the common universe U . The intersection of (H, A) and (G, B) is denoted by $(H, A) \widetilde{\cap} (G, B)$ and is defined by $(H, A) \widetilde{\cap} (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$T_{K(e)}(m) = \min(T_{H(e)}(m), T_{G(e)}(m)),$$

$$I_{K(e)}(m) = \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2},$$

$$F_{K(e)}(m) = \max(F_{H(e)}(m), F_{G(e)}(m)),$$

if $e \in A \cap B$.

Definition 2.19. (see [46]) Let (H, A) and (G, B) be two NSESs over the common universe U . The ‘‘AND’’ operation on them is denoted by $(H, A) \widetilde{\wedge} (G, B)$ and is defined by $(H, A) \widetilde{\wedge} (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership

and falsity-membership of $(K, A \times B)$ are as follows:

$$\begin{aligned}
 T_{K(\alpha,\beta)}(m) &= \min (T_{H(\alpha)}(m), T_{G(\beta)}(m)), \\
 I_{K(\alpha,\beta)}(m) &= \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}, \\
 F_{K(\alpha,\beta)}(m) &= \max (F_{H(\alpha)}(m), F_{G(\beta)}(m)), \\
 \forall \alpha \in A, \forall \beta \in B.
 \end{aligned}$$

Definition 2.20. (see [46]) Let (H, A) and (G, B) be two NSEs over the common universe U . The ‘‘OR’’ operation on them is denoted by $(H, A) \widetilde{\vee} (G, B)$ and is defined by $(H, A) \widetilde{\vee} (G, B) = (O, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(O, A \times B)$ are as follows:

$$\begin{aligned}
 T_{O(\alpha,\beta)}(m) &= \max (T_{H(\alpha)}(m), T_{G(\beta)}(m)), \\
 I_{O(\alpha,\beta)}(m) &= \frac{I_{H(\alpha)}(m) + I_{G(\beta)}(m)}{2}, \\
 F_{O(\alpha,\beta)}(m) &= \min (F_{H(\alpha)}(m), F_{G(\beta)}(m)), \\
 \forall \alpha \in A, \forall \beta \in B.
 \end{aligned}$$

3. Neutrosophic vague soft expert set

In this section, we introduce the definition of a neutrosophic vague soft expert set and give basic properties of this concept.

Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 3.1. A pair (F, A) is called a neutrosophic vague soft expert set over U , where F is a mapping given by $F : A \rightarrow NV^U$, where NV^U denotes the power neutrosophic vague set of U .

Suppose $F : A \rightarrow NV^U$ is a function defined as $F(a) = F(a)(u), \forall u \in U$. For each $a_i \in A, F(a_i) = F(a_i)(u)$, where $F(a_i)$ represents the degree of belongingness, degree of indeterminacy and non-belongingness of the elements of U in $F(a_i)$. Hence $F(a_i)$ can be written as:

$$F(a_i) = \left\{ \frac{u_i}{F(a_i)(u_i)} \right\}, \quad \text{for } i = 1, 2, 3, \dots,$$

where $F(a_i)(u_i) = \langle [T_{F(a_i)}^-(u_i), T_{F(a_i)}^+(u_i)],$

$[I_{F(a_i)}^-(u_i), I_{F(a_i)}^+(u_i)], [F_{F(a_i)}^-(u_i), F_{F(a_i)}^+(u_i)] \rangle$ and $T_{F(a_i)}^+(u_i) = 1 - F_{F(a_i)}^-(u_i), F_{F(a_i)}^+(u_i) = 1 - T_{F(a_i)}^-(u_i)$ with $[T_{F(a_i)}^-(u_i), T_{F(a_i)}^+(u_i)], [I_{F(a_i)}^-(u_i), I_{F(a_i)}^+(u_i)]$

and $[F_{F(a_i)}^-(u_i), F_{F(a_i)}^+(u_i)]$ representing the truth-membership function, indeterminacy-membership function and falsity-membership function of each of the elements $u_i \in U$, respectively.

Example 3.2. Suppose that a company produced new types of its products and wishes to take the opinion of some experts concerning these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products, $E = \{e_1, e_2\}$ a set of decision parameters where $e_i (i = 1, 2)$ denotes the decision ‘‘easy to use,’’ and ‘‘quality,’’ respectively, and let $X = \{p, q\}$ be a set of experts. Suppose that the company has distributed a questionnaire to the two experts to make decisions on the company’s products, and we get the following:

$$\begin{aligned}
 &F(e_1, p, 1) \\
 &= \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\} \\
 &F(e_1, q, 1) \\
 &= \left\{ \frac{u_1}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.4]; [0.2, 0.4]; [0.6, 0.8] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0, 0.5]; [0.5, 0.7]; [0.5, 1] \rangle}, \frac{u_4}{\langle [0.6, 0.7]; [0.2, 0.4]; [0.3, 0.4] \rangle} \right\} \\
 &F(e_2, p, 1) \\
 &= \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.3]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.2, 0.5]; [0.2, 0.5]; [0.5, 0.8] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.6, 0.9]; [0.1, 0.7]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.4]; [0.2, 0.2]; [0.6, 0.8] \rangle} \right\} \\
 &F(e_2, q, 1) \\
 &= \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.1, 0.4]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.2, 0.4]; [0.7, 0.9] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.1, 0.5]; [0.5, 0.7]; [0.5, 0.9] \rangle}, \frac{u_4}{\langle [0.2, 0.7]; [0.2, 0.4]; [0.3, 0.8] \rangle} \right\} \\
 &F(e_1, p, 0) \\
 &= \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.7, 0.9]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.9]; [0.5, 0.8]; [0.1, 0.7] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.4, 0.5]; [0.3, 0.7]; [0.5, 0.6] \rangle}, \frac{u_4}{\langle [0, 0.2]; [0.8, 0.9]; [0.8, 1] \rangle} \right\} \\
 &F(e_1, q, 0) \\
 &= \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.6, 0.8]; [0.6, 0.8]; [0.2, 0.4] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.5, 1]; [0.3, 0.5]; [0, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.4]; [0.6, 0.8]; [0.6, 0.7] \rangle} \right\} \\
 &F(e_2, p, 0) \\
 &= \left\{ \frac{u_1}{\langle [0.1, 0.7]; [0.7, 0.9]; [0.3, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.5, 0.8]; [0.2, 0.5] \rangle}, \right. \\
 &\quad \left. \frac{u_3}{\langle [0.1, 0.4]; [0.3, 0.9]; [0.6, 0.9] \rangle}, \frac{u_4}{\langle [0.6, 0.8]; [0.8, 0.8]; [0.2, 0.4] \rangle} \right\}
 \end{aligned}$$

$$F(e_2, q, 0) = \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.6, 0.9]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.7, 0.9]; [0.6, 0.8]; [0.1, 0.3] \rangle}, \frac{u_3}{\langle [0.5, 0.9]; [0.3, 0.5]; [0.1, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.8]; [0.6, 0.8]; [0.2, 0.7] \rangle} \right\}$$

The neutrosophic vague soft expert set (F, Z) is a parameterized family $\{F(e_i), i = 1, 2, 3, \dots\}$ of all neutrosophic vague sets of U and describes a collection of approximation of an object.

Definition 3.3. Let (F, A) and (G, B) be two neutrosophic vague soft expert sets over the common universe U . (F, A) is said to be neutrosophic vague soft expert subset of (G, B) if

1. $A \subseteq B$
2. $\forall \varepsilon \in A, F(\varepsilon)$ is a neutrosophic vague subset of $G(\varepsilon)$.

This relationship is denoted by $(F, A) \widetilde{\subseteq} (G, B)$. In this case (G, B) is called a neutrosophic vague soft expert superset of (F, A) .

Definition 3.4. Two neutrosophic vague soft expert sets (F, A) and (G, B) over U are said to be equal if (F, A) is a neutrosophic vague soft expert subset of (G, B) and (G, B) is a neutrosophic vague soft expert subset of (F, A) .

Example 3.5. Consider Example 3.2. Suppose that the company takes the opinion of the experts twice again over a period of time after using the products. Let $A = \{(e_1, p, 1), (e_1, q, 0)\}$, and $B = \{(e_1, p, 1), (e_1, q, 0), (e_2, p, 1)\}$

Clearly $A \subseteq B$. Let (F, A) and (G, B) be defined as follows:

$$(F, A) = \left\{ (e_1, p, 1), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \frac{u_3}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_4}{\langle [0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6] \rangle} \right\}, (e_1, q, 0), \left\{ \frac{u_1}{\langle [0.6, 0.9]; [0.2, 0.4]; [0.1, 0.4] \rangle}, \frac{u_2}{\langle [0.7, 0.8]; [0.3, 0.5]; [0.2, 0.3] \rangle}, \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right\}$$

$$(G, B) = \left\{ (e_1, p, 1), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.2]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.5, 0.7]; [0.2, 0.5]; [0.3, 0.5] \rangle} \right\} \right\}$$

$$\left. \left. \left. \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\} \right\}, (e_1, q, 0), \left\{ \frac{u_1}{\langle [0.7, 0.9]; [0.2, 0.3]; [0.1, 0.3] \rangle}, \frac{u_2}{\langle [0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2] \rangle}, \frac{u_3}{\langle [0.9, 0.9]; [0.1, 0.3]; [0.1, 0.1] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle} \right\}, (e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.2, 0.4]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.7, 0.8]; [0.3, 0.5]; [0.2, 0.3] \rangle}, \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right\}$$

Therefore $(F, A) \widetilde{\subseteq} (G, B)$.

Definition 3.6. An agree-neutrosophic vague soft expert set $(F, A)_1$ over U is a neutrosophic vague soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}$$

Example 3.7. Consider Example 3.2. Then the agree-neutrosophic vague soft expert set $(F, A)_1$ over U is

$$(F, A)_1 = \left\{ (e_1, p, 1), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\}, (e_1, q, 1), \left\{ \frac{u_1}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.4]; [0.2, 0.4]; [0.6, 0.8] \rangle}, \frac{u_3}{\langle [0, 0.5]; [0.5, 0.7]; [0.5, 1] \rangle}, \frac{u_4}{\langle [0.6, 0.7]; [0.2, 0.4]; [0.3, 0.4] \rangle} \right\}, (e_2, p, 1), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.3]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.2, 0.5]; [0.2, 0.5]; [0.5, 0.8] \rangle}, \frac{u_3}{\langle [0.6, 0.9]; [0.1, 0.7]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.4]; [0.2, 0.2]; [0.6, 0.8] \rangle} \right\}, (e_2, q, 1), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.1, 0.4]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.2, 0.4]; [0.7, 0.9] \rangle}, \frac{u_3}{\langle [0.1, 0.5]; [0.5, 0.7]; [0.5, 0.9] \rangle}, \frac{u_4}{\langle [0.2, 0.7]; [0.2, 0.4]; [0.3, 0.8] \rangle} \right\} \right\}$$

Definition 3.8. A disagree-neutrosophic vague soft expert set $(F, A)_0$ over U is a neutrosophic vague soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}$$

Example 3.9. Consider Example 3.2. The disagree-neutrosophic vague soft expert set $(F, A)_0$ over U is

$$(F, A)_0 = \left\{ \left((e_1, p, 0), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.7, 0.9]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.9]; [0.5, 0.8]; [0.1, 0.7] \rangle}, \frac{u_3}{\langle [0.4, 0.5]; [0.3, 0.7]; [0.5, 0.6] \rangle}, \frac{u_4}{\langle [0.2, 0.2]; [0.8, 0.9]; [0.8, 1] \rangle} \right\} \right), \\ \left((e_1, q, 0), \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.6, 0.8]; [0.6, 0.8]; [0.2, 0.4] \rangle}, \frac{u_3}{\langle [0.5, 1]; [0.3, 0.5]; [0, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.4]; [0.6, 0.8]; [0.6, 0.7] \rangle} \right\} \right), \\ \left((e_2, p, 0), \left\{ \frac{u_1}{\langle [0.1, 0.7]; [0.7, 0.9]; [0.3, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.5, 0.8]; [0.2, 0.5] \rangle}, \frac{u_3}{\langle [0.1, 0.4]; [0.3, 0.9]; [0.6, 0.9] \rangle}, \frac{u_4}{\langle [0.6, 0.8]; [0.8, 0.8]; [0.2, 0.4] \rangle} \right\} \right), \\ \left((e_2, q, 0), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.6, 0.9]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.7, 0.9]; [0.6, 0.8]; [0.1, 0.3] \rangle}, \frac{u_3}{\langle [0.5, 0.9]; [0.3, 0.5]; [0.1, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.8]; [0.6, 0.8]; [0.2, 0.7] \rangle} \right\} \right) \right\}.$$

4. Basic operations on neutrosophic vague soft expert sets

In this section, we introduce some basic operations on neutrosophic vague soft expert sets, namely the complement, union and intersection of neutrosophic vague soft expert sets, derive their properties and give some examples.

We define the complement operation for neutrosophic vague soft expert set and give an illustrative example and proved proposition.

Definition 4.1. The complement of a neutrosophic vague soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow NV^U$ is a mapping given by

$$F^c(\alpha) = \tilde{c}(F(\alpha)), \forall \alpha \in A$$

where \tilde{c} is a neutrosophic vague complement.

Example 4.2. Consider Example 3.2. By using the basic neutrosophic vague complement, we have

$$(F, Z)^c = \left\{ \left((e_1, p, 1), \right. \right.$$

$$\left. \left. \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.7, 0.9]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.9]; [0.5, 0.8]; [0.1, 0.7] \rangle}, \frac{u_3}{\langle [0.4, 0.5]; [0.3, 0.7]; [0.5, 0.6] \rangle}, \frac{u_4}{\langle [0, 0.2]; [0.8, 0.9]; [0.8, 1] \rangle} \right\} \right), \\ \left((e_1, q, 1), \left\{ \frac{u_1}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_2}{\langle [0.6, 0.8]; [0.6, 0.8]; [0.2, 0.4] \rangle}, \frac{u_3}{\langle [0.5, 1]; [0.3, 0.5]; [0, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.4]; [0.6, 0.8]; [0.6, 0.7] \rangle} \right\} \right), \\ \left((e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.7]; [0.7, 0.9]; [0.3, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.5, 0.8]; [0.2, 0.5] \rangle}, \frac{u_3}{\langle [0.1, 0.4]; [0.3, 0.9]; [0.6, 0.9] \rangle}, \frac{u_4}{\langle [0.6, 0.8]; [0.8, 0.8]; [0.2, 0.4] \rangle} \right\} \right), \\ \left((e_2, q, 1), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.6, 0.9]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.7, 0.9]; [0.6, 0.8]; [0.1, 0.3] \rangle}, \frac{u_3}{\langle [0.5, 0.9]; [0.3, 0.5]; [0.1, 0.5] \rangle}, \frac{u_4}{\langle [0.3, 0.8]; [0.6, 0.8]; [0.2, 0.7] \rangle} \right\} \right) \right), \\ \left((e_1, p, 0), \left\{ \frac{u_1}{\langle [0.2, 0.8]; [0.1, 0.3]; [0.2, 0.8] \rangle}, \frac{u_2}{\langle [0.1, 0.7]; [0.2, 0.5]; [0.3, 0.9] \rangle}, \frac{u_3}{\langle [0.5, 0.6]; [0.3, 0.7]; [0.4, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\} \right), \\ \left((e_1, q, 0), \left\{ \frac{u_1}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_2}{\langle [0.2, 0.4]; [0.2, 0.4]; [0.6, 0.8] \rangle}, \frac{u_3}{\langle [0, 0.5]; [0.5, 0.7]; [0.5, 1] \rangle}, \frac{u_4}{\langle [0.6, 0.7]; [0.2, 0.4]; [0.3, 0.4] \rangle} \right\} \right), \\ \left((e_2, p, 0), \left\{ \frac{u_1}{\langle [0.3, 0.9]; [0.1, 0.3]; [0.1, 0.7] \rangle}, \frac{u_2}{\langle [0.2, 0.5]; [0.2, 0.5]; [0.5, 0.8] \rangle}, \frac{u_3}{\langle [0.6, 0.9]; [0.1, 0.7]; [0.1, 0.4] \rangle}, \frac{u_4}{\langle [0.2, 0.4]; [0.2, 0.2]; [0.6, 0.8] \rangle} \right\} \right), \\ \left((e_2, q, 0), \left\{ \frac{u_1}{\langle [0.4, 0.6]; [0.1, 0.4]; [0.4, 0.6] \rangle}, \frac{u_2}{\langle [0.1, 0.3]; [0.2, 0.4]; [0.7, 0.9] \rangle}, \frac{u_3}{\langle [0.1, 0.5]; [0.5, 0.7]; [0.5, 0.9] \rangle}, \frac{u_4}{\langle [0.2, 0.7]; [0.2, 0.4]; [0.3, 0.8] \rangle} \right\} \right) \right\}.$$

Proposition 4.3. If (F, A) is a neutrosophic vague soft expert set over U , then $((F, A)^c)^c = (F, A)$

Proof. From Definition 4.1. We have $(F, A)^c = (F^c, A)$, where $F^c(\alpha) = \bar{1} - F(\alpha), \forall \alpha \in A$. Now, $((F, A)^c)^c = ((F^c)^c, A)$, where $(F^c)^c(\alpha) = \bar{1} - (\bar{1} - F(\alpha)), \forall \alpha \in A = F(\alpha), \forall \alpha \in A$.

We define the union of two neutrosophic vague soft expert sets and give an illustrative example.

Definition 4.4. The union of two neutrosophic vague soft expert sets (F, A) and (G, B) over U , denoted by $(F, A) \tilde{\cup} (G, B)$, is a neutrosophic vague soft expert set (H, C) , where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$(H, C) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B, \\ G(\varepsilon), & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \tilde{\cup} G(\varepsilon), & \text{if } \varepsilon \in A \cap B. \end{cases}$$

where $\tilde{\cup}$ denote the neutrosophic vague set union.

Example 4.5. Consider Example 3.2. Suppose that the company takes the opinion of the experts twice again over a period of time after using the products. Let $A = \{(e_1, p, 1), (e_1, q, 0), (e_1, p, 0)\}$ and $B = \{(e_1, p, 1), (e_1, q, 0), (e_2, p, 1)\}$.

Suppose (F, A) and (G, B) are two neutrosophic vague soft expert sets over U such that:

$$(F, A) = \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.7, 0.8]; [0.1, 0.3]; [0.2, 0.3] \rangle}, \frac{u_2}{\langle [0.4, 0.7]; [0.2, 0.5]; [0.3, 0.6] \rangle}, \frac{u_3}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_4}{\langle [0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6] \rangle} \right\} \right), \\ (e_1, q, 0), \left\{ \frac{u_1}{\langle [0.1, 0.9]; [0.2, 0.4]; [0.1, 0.9] \rangle}, \frac{u_2}{\langle [0.4, 0.8]; [0.3, 0.5]; [0.2, 0.6] \rangle}, \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right), \\ (e_1, p, 0), \left\{ \frac{u_1}{\langle [0.2, 0.3]; [0.7, 0.9]; [0.7, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.6]; [0.5, 0.8]; [0.4, 0.7] \rangle}, \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.8, 0.9]; [0.4, 0.5] \rangle} \right\} \right\}, \\ (G, B) = \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.3, 0.6]; [0.1, 0.4]; [0.4, 0.7] \rangle}, \frac{u_2}{\langle [0.5, 0.7]; [0.2, 0.5]; [0.3, 0.5] \rangle}, \frac{u_3}{\langle [0.5, 0.8]; [0.3, 0.7]; [0.2, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\} \right), \\ (e_1, q, 0), \left\{ \frac{u_1}{\langle [0.1, 0.9]; [0.2, 0.3]; [0.1, 0.9] \rangle}, \frac{u_2}{\langle [0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2] \rangle}, \frac{u_3}{\langle [0.9, 0.9]; [0.1, 0.3]; [0.1, 0.1] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle} \right\} \right), \\ (e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5] \rangle}, \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right\}.$$

$$\left\{ \frac{u_1}{\langle [0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5] \rangle}, \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\}.$$

By using basic neutrosophic vague union, we have $(F, A) \tilde{\cup} (G, B) = (H, C)$, where

$$(H, C) = \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{\langle [0.7, 0.8]; [0.1, 0.3]; [0.2, 0.3] \rangle}, \frac{u_2}{\langle [0.5, 0.7]; [0.2, 0.5]; [0.3, 0.5] \rangle}, \frac{u_3}{\langle [0.5, 0.8]; [0.3, 0.7]; [0.2, 0.5] \rangle}, \frac{u_4}{\langle [0.8, 1]; [0.1, 0.2]; [0, 0.2] \rangle} \right\} \right), \\ (e_1, q, 0), \left\{ \frac{u_1}{\langle [0.1, 0.9]; [0.2, 0.3]; [0.1, 0.9] \rangle}, \frac{u_2}{\langle [0.8, 0.9]; [0.2, 0.4]; [0.1, 0.2] \rangle}, \frac{u_3}{\langle [0.9, 0.9]; [0.1, 0.3]; [0.1, 0.1] \rangle}, \frac{u_4}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle} \right\} \right), \\ (e_1, p, 0), \left\{ \frac{u_1}{\langle [0.2, 0.3]; [0.7, 0.9]; [0.7, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.6]; [0.5, 0.8]; [0.4, 0.7] \rangle}, \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.8, 0.9]; [0.4, 0.5] \rangle} \right\} \right\}, \\ (e_2, p, 1), \left\{ \frac{u_1}{\langle [0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5] \rangle}, \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right\}.$$

We define the intersection of two neutrosophic vague soft expert sets and give an illustrative example.

Definition 4.6. The intersection of two neutrosophic vague soft expert sets (F, A) and (G, B) over a universe U , is a neutrosophic vague soft expert set (H, C) , denoted by $(F, A) \tilde{\cap} (G, B)$, such that $C = A \cup B$ and $\forall e \in c$

$$(H, C) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \tilde{\cap} G(e), & \text{if } e \in A \cap B. \end{cases}$$

where $\tilde{\cap}$ denoted the neutrosophic vague set intersection.

Example 4.7. Consider Example 4.5. By using basic neutrosophic vague intersection, we have $(F, A) \tilde{\cap} (G, B) = (H, C)$, where

$$\begin{aligned}
 (H, C) = & \left\{ (e_1, p, 1), \right. \\
 & \left. \left\{ \frac{u_1}{\langle [0.3, 0.6]; [0.1, 0.4]; [0.4, 0.7] \rangle}, \frac{u_2}{\langle [0.4, 0.7]; [0.2, 0.5]; [0.3, 0.6] \rangle}, \right. \right. \\
 & \left. \left. \frac{u_3}{\langle [0.1, 0.2]; [0.6, 0.7]; [0.8, 0.9] \rangle}, \frac{u_4}{\langle [0.4, 0.5]; [0.1, 0.2]; [0.5, 0.6] \rangle} \right\} \right\}, \\
 & (e_1, q, 0), \\
 & \left\{ \frac{u_1}{\langle [0.1, 0.9]; [0.2, 0.4]; [0.1, 0.9] \rangle}, \frac{u_2}{\langle [0.4, 0.8]; [0.3, 0.5]; [0.2, 0.6] \rangle}, \right. \\
 & \left. \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.7]; [0.4, 0.6]; [0.3, 0.5] \rangle} \right\} \right\}, \\
 & (e_1, p, 0), \\
 & \left\{ \frac{u_1}{\langle [0.2, 0.3]; [0.7, 0.9]; [0.7, 0.8] \rangle}, \frac{u_2}{\langle [0.3, 0.6]; [0.5, 0.8]; [0.4, 0.7] \rangle}, \right. \\
 & \left. \frac{u_3}{\langle [0.8, 0.9]; [0.3, 0.4]; [0.1, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.8, 0.9]; [0.4, 0.5] \rangle} \right\} \right\}, \\
 & (e_2, p, 1), \\
 & \left\{ \frac{u_1}{\langle [0.1, 0.4]; [0.2, 0.4]; [0.6, 0.9] \rangle}, \frac{u_2}{\langle [0.5, 0.8]; [0.3, 0.5]; [0.2, 0.5] \rangle}, \right. \\
 & \left. \frac{u_3}{\langle [0.8, 0.8]; [0.1, 0.4]; [0.2, 0.2] \rangle}, \frac{u_4}{\langle [0.5, 0.6]; [0.5, 0.6]; [0.4, 0.5] \rangle} \right\} \right\}.
 \end{aligned}$$

5. AND and OR operations

In this section, we introduce the definitions of AND and OR operations for neutrosophic vague soft expert set and derive their properties.

Definition 5.1. Let (F, A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z) .

Then “ (F, A) AND (G, B) ” denoted $(F, A) \tilde{\wedge} (G, B)$ is defined by $(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$, where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where \cap represents the basic intersection.

Definition 5.2. Let (F, A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z) .

Then “ (F, A) OR (G, B) ” denoted $(F, A) \tilde{\vee} (G, B)$ is defined by $(F, A) \tilde{\vee} (G, B) = (H, A \times B)$, where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where \cup represents the basic union.

Proposition 5.3. If (F, A) and (G, B) are two neutrosophic vague soft expert sets over a soft universe (U, Z) . Then,

1. $((F, A) \tilde{\wedge} (G, B))^c = (F, A)^c \tilde{\vee} (G, B)^c$
2. $((F, A) \tilde{\vee} (G, B))^c = (F, A)^c \tilde{\wedge} (G, B)^c$

Proof. (1) Suppose that (F, A) and (G, B) are two neutrosophic vague soft expert sets over a soft universe (U, Z) defined as:

$(F, A) = F(\alpha)$ for all $\alpha \in A \subseteq Z$ and $(G, B) = G(\beta)$ for all $\beta \in B \subseteq Z$. By definitions 4.8 and 4.9 it follows that:

$$\begin{aligned}
 ((F, A) \tilde{\wedge} (G, B))^c &= ((F(\alpha) \tilde{\wedge} G(\beta))^c \\
 &= ((F(\alpha) \cap G(\beta))^c \\
 &= (\tilde{c}(F(\alpha) \cap G(\beta))) \\
 &= (\tilde{c}(F(\alpha) \cup \tilde{c}G(\beta))) \\
 &= (F(\alpha))^c \tilde{\vee} (G(\beta))^c \\
 &= (F, A)^c \tilde{\vee} (G, B)^c.
 \end{aligned}$$

(2) The proof is similar to that in part(1) and therefore is omitted.

6. Application of NVSES in a decision making problem

In this section, we introduce a generalized algorithm which will be applied to the NVSES model introduced in Section 3 and used to solve a hypothetical decision making problem.

Example 6.1. Suppose that company Y is looking to hire a person to fill in the vacancy for a position in their company. Out of all the people who applied for the position, two candidates were shortlisted and these two candidates form the universe of elements, $U = \{u_1, u_2\}$. The hiring committee consists of the hiring manager and head of department and this committee is represented by the set $X = \{p, q\}$ (a set of experts) while the set $Q = \{1 = agree, 0 = disagree\}$ represents the set of opinions of the hiring committee members. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3\}$, where the parameters e_i ($i = 1, 2, 3$) represent the characteristics or qualities that the candidates are assessed on, namely, “relevant job experience”, “excellent academic qualifications in the relevant field” and “attitude and level of professionalism”, respectively. After interviewing the two candidates and going through their certificates and other supporting documents, the hiring committee constructs the following NVSES:

$$(F, A) = \left\{ (e_1, p, 1) = \left\{ \left(\frac{u_1}{\langle [0.3, 0.7]; [0.5, 0.9]; [0.3, 0.7] \rangle} \right) \right\} \right\},$$

$$\left(\frac{u_2}{([0.6, 0.9], [0.2, 0.5], [0.1, 0.4])} \right) \}} \},$$

$$\left\{ (e_2, p, 1) = \left\{ \left(\frac{u_1}{([0.5, 0.7], [0.1, 0.2], [0.3, 0.5])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.2, 0.3], [0.8, 0.9], [0.7, 0.8])} \right) \right\} \},$$

$$\left\{ (e_3, p, 1) = \left\{ \left(\frac{u_1}{([0.5, 0.7], [0.2, 0.2], [0.3, 0.5])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.4, 0.5], [0.7, 0.9], [0.5, 0.6])} \right) \right\} \},$$

$$\left\{ (e_1, q, 1) = \left\{ \left(\frac{u_1}{([0.4, 0.8], [0.5, 0.8], [0.2, 0.6])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.5, 0.6], [0.5, 0.5], [0.4, 0.5])} \right) \right\} \},$$

$$\left\{ (e_2, q, 1) = \left\{ \left(\frac{u_1}{([0.7, 0.8], [0.4, 0.6], [0.2, 0.3])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.2, 0.3], [0.4, 0.7], [0.7, 0.8])} \right) \right\} \},$$

$$\left\{ (e_3, q, 1) = \left\{ \left(\frac{u_1}{([0.4, 0.9], [0.5, 0.6], [0.1, 0.6])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.4, 0.7], [0.4, 0.4], [0.3, 0.6])} \right) \right\} \},$$

$$\left\{ (e_1, p, 0) = \left\{ \left(\frac{u_1}{([0.3, 0.7], [0.1, 0.5], [0.3, 0.7])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.1, 0.4], [0.5, 0.8], [0.6, 0.9])} \right) \right\} \},$$

$$\left\{ (e_2, p, 0) = \left\{ \left(\frac{u_1}{([0.3, 0.5], [0.8, 0.9], [0.5, 0.7])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.7, 0.8], [0.1, 0.2], [0.2, 0.3])} \right) \right\} \},$$

$$\left\{ (e_3, p, 0) = \left\{ \left(\frac{u_1}{([0.3, 0.5], [0.8, 0.8], [0.5, 0.7])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.5, 0.6], [0.1, 0.3], [0.4, 0.5])} \right) \right\} \},$$

$$\left\{ (e_1, q, 0) = \left\{ \left(\frac{u_1}{([0.2, 0.6], [0.2, 0.5], [0.4, 0.8])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.4, 0.5], [0.5, 0.5], [0.5, 0.6])} \right) \right\} \},$$

$$\left\{ (e_2, q, 0) = \left\{ \left(\frac{u_1}{([0.2, 0.3], [0.4, 0.6], [0.7, 0.8])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.7, 0.8], [0.3, 0.6], [0.2, 0.3])} \right) \right\} \},$$

$$\left\{ (e_3, q, 0) = \left\{ \left(\frac{u_1}{([0.1, 0.6], [0.4, 0.5], [0.4, 0.9])} \right), \right. \right.$$

$$\left. \left(\frac{u_2}{([0.3, 0.6], [0.6, 0.6], [0.4, 0.7])} \right) \right\} \}.$$

Next, the NVSES (F, A) is used together with a generalized algorithm to solve the decision making problem stated at the beginning of this section.

The algorithm given below is employed by the hiring committee to determine the best or most suitable candidate to be hired for the position. The generalized algorithm is as follows:

Algorithm

1. Input the NVSES (F, A) .
2. Find the values of $\alpha_{F(a_i)}(u_i) = T_{F(a_i)}^-(u_i) - F_{F(a_i)}^-(u_i)$ for interval truth-membership part $[T_{F(a_i)}^-(u_i), T_{F(a_i)}^+(u_i)]$, where $T_{F(a_i)}^+(u_i) = 1 - F_{F(a_i)}^-(u_i)$, for each element $u_i \in U$.
3. Take the arithmetic average $\beta_{F(a_i)}(u_i)$ of the end points of the interval indeterminacy-membership part $[I_{F(a_i)}^-(u_i), I_{F(a_i)}^+(u_i)]$, for each element $u_i \in U$.
4. Find the values of $\gamma_{F(a_i)}(u_i) = F_{F(a_i)}^-(u_i) - T_{F(a_i)}^-(u_i)$ for interval falsity-membership part $[F_{F(a_i)}^-(u_i), F_{F(a_i)}^+(u_i)]$, where $F_{F(a_i)}^+(u_i) = 1 - T_{F(a_i)}^-(u_i)$, for each element $u_i \in U$.
5. Find the values of $\alpha_{F(a_i)}(u_i) - \beta_{F(a_i)}(u_i) - \gamma_{F(a_i)}(u_i)$ for each element $u_i \in U$.
6. Find the highest numerical grade for the agree-NVSES and disagree-NVSES.
7. Compute the score of each element $u_i \in U$ by taking the sum of the products of the numerical grade of each element for the agree-NVSES and disagree-NVSES, denoted by A_i and D_i , respectively.
8. Find the values of the score $r_i = A_i - D_i$ for each element $u_i \in U$.
9. Determine the value of the highest score $s = \max_{u_i \in U} \{r_i\}$. The decision is to choose element u_i as the optimal or best solution to the problem. If there are more than one element with the highest r_i score, then any one of those elements can be chosen as the optimal solution.

Table 1 gives the values of $\alpha_{F(a_i)}(u_i)$, $\beta_{F(a_i)}(u_i)$ and $\gamma_{F(a_i)}(u_i)$ for each element $u_i \in U$ and gives the values of $\alpha_{F(a_i)}(u_i) - \beta_{F(a_i)}(u_i) - \gamma_{F(a_i)}(u_i)$ for each element $u_i \in U$.

It is to be noted that the upper and lower terms for each element in Table 1 represent the values of $\alpha_{F(a_i)}(u_i)$, $\beta_{F(a_i)}(u_i)$ and $\gamma_{F(a_i)}(u_i)$ for each element

Table 1
Values of $\alpha_{F(a_i)}(u_i)$, $\beta_{F(a_i)}(u_i)$ and $\gamma_{F(a_i)}(u_i)$

| | u_1 | u_2 | | u_1 | u_2 |
|---------------|------------------------------|------------------------------|---------------|------------------------------|-------------------------------|
| $(e_1, p, 1)$ | $(0, 0.7, 0)$ -0.7 | $(0.5, 0.35, -0.5)$ 0.65 | $(e_1, p, 0)$ | $(0, 0.3, 0)$ -0.3 | $(-0.5, 0.65, 0.5)$ -1.654 |
| $(e_2, p, 1)$ | $(0.2, 0.15, -0.2)$ 0.25 | $(-0.5, 0.85, 0.5)$ -1.85 | $(e_2, p, 0)$ | $(-0.2, 0.85, 0.2)$ -1.25 | $(0.5, 0.15, -0.5)$ 0.85 |
| $(e_3, p, 1)$ | $(0.2, 0.2, -0.2)$ 0.2 | $(-0.1, 0.8, 0.1)$ -1 | $(e_3, p, 0)$ | $(-0.2, 0.8, 0.2)$ -1.2 | $(0.1, 0.2, -0.1)$ 0 |
| $(e_1, q, 1)$ | $(0.2, 0.65, -0.2)$ -0.25 | $(0.1, 0.5, -0.1)$ -0.3 | $(e_1, q, 0)$ | $(-0.2, 0.35, 0.2)$ -0.75 | $(-0.1, 0.5, 0.1)$ -0.7 |
| $(e_2, q, 1)$ | $(0.5, 0.5, -0.5)$ 0.5 | $(-0.5, 0.55, 0.5)$ -1.55 | $(e_2, q, 0)$ | $(-0.5, 0.5, 0.5)$ -1.5 | $(0.5, 0.45, -0.5)$ 0.55 |
| $(e_3, q, 1)$ | $(0.3, 0.55, -0.3)$ 0.05 | $(0.1, 0.4, -0.1)$ -0.2 | $(e_3, q, 0)$ | $(-0.3, 0.45, 0.3)$ -1.05 | $(-0.1, 0.6, 0.1)$ -0.8 |

Table 2
Numerical grade for agree-NVSES

| | u_i | Highest numerical grade |
|---------------|-------|-------------------------|
| $(e_1, p, 1)$ | u_2 | 0.65 |
| $(e_2, p, 1)$ | u_1 | 0.25 |
| $(e_3, p, 1)$ | u_1 | 0.2 |
| $(e_1, q, 1)$ | u_1 | -0.25 |
| $(e_2, q, 1)$ | u_1 | 0.5 |
| $(e_3, q, 1)$ | u_1 | 0.05 |

Table 3
Numerical grade for disagree-NVSES

| | u_i | Highest numerical grade |
|---------------|-------|-------------------------|
| $(e_1, p, 0)$ | u_1 | -0.3 |
| $(e_2, p, 0)$ | u_2 | 0.85 |
| $(e_3, p, 0)$ | u_2 | 0 |
| $(e_1, q, 0)$ | u_2 | -0.7 |
| $(e_2, q, 0)$ | u_2 | 0.55 |
| $(e_3, q, 0)$ | u_2 | 0.8 |

Table 4
The score $r_i = A_i - D_i$

| A_i | D_i | r_i |
|------------------------|------------------------|-------|
| Score (u_1) = 0.75 | Score (u_1) = -0.3 | 1.05 |
| Score (u_2) = 0.65 | Score (u_2) = 1.5 | -0.85 |

$u_i \in U$ and the values of $\alpha_{F(a_i)}(u_i) - \beta_{F(a_i)}(u_i) - \gamma_{F(a_i)}(u_i)$ for each element $u_i \in U$, respectively.

Tables 2 and 3 give the highest numerical grade for the elements in the agree-NVSES and disagree-NVSES, respectively.

Let A_i and D_i , represent the score of each numerical grade for the agree-NVSES and disagree-NVSES, respectively. These values are given in Table 4.

Thus $s = \max_{u_i \in U} \{r_i\} = r_1$. Therefore, the hiring committee is advised to hire candidate u_1 to fill the vacant position.

To illustrate the advantages of our proposed method using NVSES as compared to that of vague soft expert set as proposed by Hassan and Alhazaymeh [41], let us consider Example 6.1 above. The vague soft expert set can describe this problem as follows.

$$(F_\mu, Z) = \{(e_1, p, 1) = \left\{ \left(\frac{u_1}{([0.3, 0.7])} \right), \left(\frac{u_2}{([0.6, 0.9],)} \right) \right\}, \dots$$

Note that the NVSES is a generalization of vague soft expert set. Thus as shown in Example 6.1 above, the NVSES can explain the universal U in more detail with three membership functions, especially when there are many parameters involved, whereas vague soft expert set can tell us a limited information about the universal U . It can only handle the incomplete information considering both the truth-membership and falsity-membership values, while NVSES can handle problems involving imprecise, indeterminacy and inconsistent data, which makes it more accurate and realistic than vague soft expert set.

7. Conclusion

In this paper, we reviewed the basic concepts of neutrosophic vague set and neutrosophic soft expert set, and gave some basic operations on both neutrosophic vague set and neutrosophic soft expert set, before establishing the concept of neutrosophic vague soft expert set. The basic operations on neutrosophic vague soft expert set, namely complement, union, intersection, AND, and OR operations, were defined. Subsequently, the basic properties of these operations such as De Morgan's laws and other relevant laws pertaining to the concept of neutrosophic vague

soft expert set are proved. Finally, a generalized algorithm is introduced and applied to the NVSES model to solve a hypothetical decision making problem. This new extension will provide a significant addition to existing theories for handling indeterminacy, and spurs more developments of further research and pertinent applications.

Acknowledgments

The authors would like to acknowledge the financial support received from Universiti Kebangsaan Malaysia under the research grant IP-2014-071.

References

- [1] L.A. Zadeh, Fuzzy set, *Information and Control* **8**(3) (1965), 338–353.
- [2] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets Syst* **20**(1) (1986), 87–96.
- [3] W.L. Gau and D.J. Buehrer, Vague sets, *IEEE Transactions on Systems, Man and Cybernetics* **23**(2) (1993), 610–614.
- [4] Z. Pawlak, Rough sets, *International Journal of Information and Computer Sciences* **11** (1982), 341–356.
- [5] F. Smarandache, Neutrosophic set - A generalisation of the intuitionistic fuzzy sets, *International Journal of Pure and Applied Mathematics* **24**(3) (2005), 287–297.
- [6] J. Ye, Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **27**(5) (2014), 2231–2241.
- [7] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, *Journal of Intelligent and Fuzzy Systems* **26**(1) (2014), 165–172.
- [8] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, *Journal of Intelligent and Fuzzy Systems* **26**(5) (2014), 2459–2466.
- [9] J. Ye, Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **27**(5) (2014), 2453–2462.
- [10] D. Molodtsov, Soft set theory—first results, *Computers and Mathematics with Applications* **37**(2) (1999), 19–31.
- [11] S. Alkhazaleh, A.R. Salleh and N. Hassan, Possibility fuzzy soft set, *Advances in Decision Sciences* **2011**, Article ID479756.
- [12] S. Alkhazaleh, A.R. Salleh and N. Hassan, Soft multi-sets theory, *Applied Mathematical Sciences* **5**(72) (2011), 3561–3573.
- [13] A.R. Salleh, S. Alkhazaleh, N. Hassan and A.G. Ahmad, Multiparameterized soft set, *Journal of Mathematics and Statistics* **8**(1) (2012), 92–97.
- [14] K. Alhazaymeh, S.A. Halim, A.R. Salleh and N. Hassan, Soft intuitionistic fuzzy sets, *Applied Mathematical Sciences* **6**(54) (2012), 2669–2680.
- [15] F. Adam and N. Hassan, Q-fuzzy soft matrix and its application, *AIP Conference Proceedings*, 1602, 2014, pp. 772–778.
- [16] F. Adam and N. Hassan, Q-fuzzy soft set, *Applied Mathematical Sciences* **8**(174) (2014), 8689–8695.
- [17] F. Adam and N. Hassan, Operations on Q-fuzzy soft set, *Applied Mathematical Sciences* **8**(175) (2014), 8697–8701.
- [18] F. Adam and N. Hassan, Multi Q-fuzzy parameterized soft set and its application, *Journal of Intelligent and Fuzzy Systems* **27**(1) (2014), 419–424.
- [19] F. Adam and N. Hassan, Properties on the multi Q-fuzzy soft matrix, *AIP Conference Proceedings* **1614** (2014), 834–839.
- [20] F. Adam and N. Hassan, Multi Q-fuzzy soft set and its application, *Far East Journal of Mathematical Sciences* **97**(7) (2015), 871–881.
- [21] F. Adam and N. Hassan, Multi Q-Fuzzy soft expert set and its applications, *Journal of Intelligent and Fuzzy Systems* **30**(2) (2016), 943–950.
- [22] M. Varnamkhashi and N. Hassan, A hybrid of adaptive neurofuzzy inference system and genetic algorithm, *Journal of Intelligent and Fuzzy Systems* **25**(3) (2013), 793–796.
- [23] M. Varnamkhashi and N. Hassan, Neurogenetic algorithm for solving combinatorial engineering problems, *Journal of Applied Mathematics* **2012**, Article ID 253714.
- [24] N. Cagman, S. Enginoglu and F. Citak, Fuzzy soft set theory and its applications, *Iranian Journal of Fuzzy Systems* **8**(3) (2011), 137–147.
- [25] N. Cagman, F. Citak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy Systems* **1** (2010), 21–35.
- [26] I. Deli and N. Cagman, Intuitionistic fuzzy parameterized soft set theory and its decision making, *Applied Soft Computing* **28** (2015), 109–113.
- [27] I. Deli and S. Karatas, Interval valued intuitionistic fuzzy parameterized soft set theory and its decision making, *Journal of Intelligent and Fuzzy Systems*. DOI:10.3233/IFS-151920
- [28] W. Xu, J. Ma, S. Wang and G. Hao, Vague soft sets and their properties, *Computers and Mathematics with Applications* **59**(2) (2010), 787–794.
- [29] K. Alhazaymeh and N. Hassan, Generalized vague soft set and its application, *International Journal of Pure and Applied Mathematics* **77**(3) (2012), 391–401.
- [30] K. Alhazaymeh and N. Hassan, Possibility vague soft set and its application in decision making, *International Journal of Pure and Applied Mathematics* **77**(4) (2012), 549–563.
- [31] K. Alhazaymeh and N. Hassan, Interval-valued vague soft sets and its application, *Advances in Fuzzy Systems* **2012**, Article ID 208489.
- [32] K. Alhazaymeh and N. Hassan, Possibility interval-valued vague soft set, *Applied Mathematical Sciences* **7**(140) (2013), 6989–6994.
- [33] P.K. Maji, Neutrosophic soft set, *Annals of Fuzzy Mathematics and Informatics* **5**(1) (2013), 157–168.
- [34] S. Broumi, I. Deli and F. Smarandache, Relations on interval valued neutrosophic soft sets, *Journal of New Results in Science* **5** (2014), 1–20.
- [35] S. Broumi, I. Deli and F. Smarandache, Distance and similarity measures of interval neutrosophic soft sets, *Critical Review* **8** (2014), 14–31.
- [36] I. Deli and S. Broumi, Neutrosophic soft relations and some properties, *Annals of Fuzzy Mathematics and Informatics* **9**(1) (2015), 169–182.
- [37] I. Deli and S. Broumi, Neutrosophic soft matrices and NSMdecision making, *Journal of Intelligent and Fuzzy Systems* **28**(5) (2015), 2233–2241.

- [38] I. Deli, Interval-valued neutrosophic soft sets and its decision making, *International Journal of Machine Learning and Cybernetics*. DOI: 10.1007/s13042-015-0461-3
- [39] S. Broumi, I. Deli and F. Smarandache, Interval valued neutrosophic parameterized soft set theory and its decision making, *Journal of New Results in Science* **7** (2014), 58–71.
- [40] S. Alkhazaleh and A.R. Salleh, Fuzzy soft expert set and its application, *Applied Mathematics* **5**(9) (2014), 1349–1368.
- [41] N. Hassan and K. Alhazaymeh, Vague soft expert set theory, *American Institute of Physics Conference Proceedings* **1522** (2013), 953–958.
- [42] K. Alhazaymeh and N. Hassan, Mapping on generalized vague soft expert set, *International Journal of Pure and Applied Mathematics* **93**(3) (2014), 369–376.
- [43] K. Alhazaymeh and N. Hassan, Vague soft set relations and functions, *Journal of Intelligent and Fuzzy Systems* **28**(3) (2015), 1205–1212.
- [44] S. Alkhazaleh, Neutrosophic vague set theory, *Critical Review* **X** (2015), 29–39.
- [45] S. Alkhazaleh and A.R. Salleh, Soft expert sets, *Advances in Decision Sciences* **2011**, Article ID 757868.
- [46] M. Sahin, S. Alkhazaleh and V. Ulucay, Neutrosophic soft expert sets, *Applied Mathematics* **6**(1) (2015), 116–127.