Neutrosophic Soft Sets with Applications in Decision Making

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Received: 23 January 2015; Accepted: 04 March 2015

Abstract

In this study, we firstly present definitions and properties in study of Maji [10] on neutrosophic soft sets. We then give a few notes on his study. Next, based on Çağman [5], we redefine the notion of neutrosophic soft set and neutrosophic soft set operations to make more functional. By using these new definitions we construct a decision making method and a group decision making method which selects a set of optimum elements from the alternatives. We finally present examples which shows that the methods can be successfully applied to many problems that contain uncertainties.

Keywords: Neutrosophic Set; Soft Set; Neutrosophic Soft Set; Decision Making.

1. Introduction

Many problems including uncertainties are a major issue in many fields of real life such as economics, engineering, environment, social sciences, medical sciences and business management. Uncertain data in these fields could be caused by complexities and difficulties in classical mathematical modeling. To avoid difficulties in dealing with uncertainties, many tools have been studied by researchers. Some of these tools are fuzzy sets [16], rough sets [14] and intuitionistic fuzzy sets [1]. Fuzzy sets and intuitionistic fuzzy sets are characterized by membership functions, membership and non-membership functions, respectively. In some real

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life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the indeterminate and incomplete information. But fuzzy sets and intuitionistic fuzzy sets don't handle the indeterminant and inconsistent information. Smarandache [13] defined the notion of neutrosophic set which is a mathematical tool for dealing with problems involving imprecise and indeterminant data.

Molodtsov introduced concept of soft sets [8] to solve complicated problems and various types of uncertainties. In [9], Maji et al. introduced several operators for soft set theory: equality of two soft sets, subsets and superset of soft sets, complement of soft set, null soft sets and absolute soft sets. But some of these definitions and their properties have few gaps, which have been pointed out by Ali et al. [11] and Yang [15]. In 2010, Çağman and Enginoğlu [4] made some modifications the operations of soft sets and filled in these gap. In 2014, Çağman [5] redefined soft sets using the single parameter set and compared definitions with those defined before.

Maji [10] combined the concept of soft set and neutrosophic set together by introducing a new concept called neutrosophic soft set and gave an application of neutrosophic soft set in decision making problem. Recently, the properties and applications on the neutrosophic sets have been studied increasingly [2, 3, 6, 7]. The propose of this paper is to fill the gaps of the Maji's neutrosophic soft set [11] definition and operations redefining concept of neutrosophic soft set and operations between neutrosophic soft sets. First, we present Maji's definitions and operations and we verify that some propositions are incorrect by a counterexample. Then based on Çağman 's [5] study we redefine neutrosophic soft sets and their operations. Also, we investigate properties of neutrosophic soft sets operations. Finally we present an application of a neutrosophic soft set in decision making.

2. Preliminary

In this section, we will recall the notions of neutrosophic sets [13] and soft sets [8]. Then, we will give some properties of soft sets and neutrosophic soft sets [10]. Throughout this paper X, E and $P(X)$ denote initial universe, set of parameters and power set of X, respectively.

Defintion 1:[13] A neutrosophic set A on the universe of discourse X is defined as

$$A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle : x\in X\}$$

where $T_A,I_A,F_A : X\rightarrow]^-0,1^+[$ and $^-0\leq T_A(x)+I_A(x)+F_A(x)\leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0,1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition 2:[8] Let consider a nonempty set A, $A\subseteq E$. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A\rightarrow P(X)$.

From now on, we will use f_A instead of (F,A).

Example 1: Let $X=\{x_l,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ be the universe which are eight houses and $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ be the set of parameters. Here, e_i $(i=1,2,3,4,5,6)$ stand for the parameters "*modern*", "*with parking*", "*expensive*", "*cheap*", "*large*" and "*near to city*" respectively. Then, following soft sets are described respectively Mr. A and Mr. B who are going to buy

$$f_A=\{(e_1,\{x_1,x_3,x_4\}),(e_2,\{x_1,x_4,x_7,x_8\}),(e_3,\{x_1,x_2,x_3,x_8\})\}$$

$$f_B=\{(e_2\{x_1,x_3,x_6\}),(e_3,X),(e_5,\{x_2,x_4,x_4,x_6\})\}.$$

From now on, we will use definitions and operations of soft sets which are more suitable for pure mathematics based on study of Çağman [5].

Definition 3:[5] A soft set f over X is a set valued function from E to $P(X)$. It can be written a set of ordered pairs

$$f=\{(e,f(e)):e\in E\}.$$

Note that if $f(e)=\emptyset$, then the element $(e,f(e))$ is not appeared in f. Set of all soft sets over X is denoted by $\mathbb{S}$.

Definition 4:[5] Let $f,g\in\mathbb{S}$. Then,

1. If $f(e)=\emptyset$ for all $e\in E$, f is said to be a null soft set, denoted by Φ.

2. If $f(e)=X$ for all $e\in E$, f is said to be absolute soft set, denoted by $\hat{X}$.

3. f is soft subset of g, denoted by $f\tilde{\subseteq}g$, if $f(e)\subseteq g(e)$ for all $e\in E$.

4. $f=g$, if $f\tilde{\subseteq}g$ and $g\tilde{\subseteq}f$.

5. Soft union of soft sets f and g, denoted by $f\tilde{\cup}g$, is a soft set over X and defined by $f\tilde{\cup}g:E\to P(X)$ such that $(f\tilde{\cup}g)(e)=f(e)\cup g(e)$ for all $e\in E$.

6. Soft intersection of soft sets f and g, denoted by $f\tilde{\cap}g$, is a soft set over X and defined by $f\tilde{\cap}g:E\to P(X)$ such that $(f\tilde{\cap}g)(e)=f(e)\cap g(e)$ for all $e\in E$.

7. Soft complement of f is denoted by $f^{\tilde{c}}$ and defined by $f^{\tilde{c}}:E\to P(X)$ such that $f^{\tilde{c}}(e)=X\setminus f(e)$ for all $e\in E$.

Example 2: Let us consider soft sets f,g in the Example 2.3. Then, we have

$$f \tilde{\cup} g = \{(e_1, \{x_1, x_3, x_4\}), (e_2, \{x_1, x_3, x_4, x_6, x_7, x_8\}), (e_3, X), (e_5, \{x_2, x_4, x_4, x_6\})\}$$

$$f \tilde{\cap} g = \{(e_2 \{x_1\}), (e_3, \{x_1, x_2, x_3, x_8\})\}$$

$$f^{\tilde{c}} = \{(e_1, \{x_2, x_5, x_6, x_7, x_8\}), (e_2, \{x_2, x_3, x_5, x_6\}), (e_3, \{x_4, x_5, x_6, x_7\}), (e_4, X), (e_5, X), (e_6, X)\}.$$

Definition 5:[10] Let X be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(X)$ denotes the set of all neutrosophic sets of X. The collection f_A is termed to be the neutrosophic soft set over X, where F is a mapping given by $F: A \rightarrow P(X)$.

For illustration we consider an example.

Example 3:**[10]** Let X be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E={beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe X given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'. Suppose that,

$f(beautiful) = \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle,$
$\langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\},$

$f(wooden) = \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle,$
$\langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\},$

$f(costly) = \{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\},$

$f(moderate) = \{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}.$

The neutrosophic soft set (NSS) f_E is a parameterized family $\{f_E(e_i); i = 1,2,...,10\}$ of all neutrosophic sets of X and describes a collection of approximation of an object.

Thus we can view the neutrosophic soft set (NSS) f_A as a collection of approximation as below:

$f_A = \{beautiful\ houses = \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle,$
$\langle h_5, 0.8, 0.2, 0.3\rangle\}, wooden\ houses = \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle,$
$\langle h_5, 0.8, 0.3, 0.6\rangle\}, costly\ houses = \{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle,$

$\langle h_5,0.7,0.3,0.4\rangle\}$, $moderate\ houses = \{\langle h_1,0.8,0.6,0.4\rangle,\langle h_2,0.7,0.9,0.6\rangle,$ $\langle h_3,0.7,0.6,0.4\rangle,$ $\langle h_4,0.7,0.8,0.6\rangle,\langle h_5,0.9,0.5,0.7\rangle\}\}$.

Definition 6:[10] Let f_A and g_B be two neutrosophic sets over the common universe X . f_A is said to be neutrosophic soft subset of g_B is $A \subset B$, and $T_{f(e)}(x) \le T_{g(e)}(x)$, $I_{f(e)}(x) \le I_{g(e)}(x)$ $F_{f(e)}(x) \ge F_{g(e)}(x)$, $\forall e \in A$, $\forall x \in U$. We denote it by $f_A \subseteq g_B$. f_A is said to be neutrosophic soft super set of g_B if g_B is a neutrosophic soft subset of f_A. We denote it by $f_A \supseteq g_B$.

If f_A is neutrosophic soft subset of g_B and g_B is neutrosophic soft subset of f_A. We denote it $f_A = g_B$.

Definition 7:[10] NOT set of a parameters. Let $E = \{e_1, e_2, ...e_n\}$ be a set of parameters. The NOT set of E, denoted by $\rceil E$ is defined by $\rceil E = \{\neg e_1, \neg e_2, ...\neg e_n\}$, where $\neg e_i =$ not e_i $\forall i$ (it may be noted that $\rceil$ and $\neg$ are different operators).

Definition 8:[10] Complement of a neutrosophic soft set f_A denoted by f_A^c and is defined as $f_A^c = (f^c, \rceil A)$, where $f^c : \rceil A \to P(X)$ is mapping given by $f^c(\alpha) =$ neutrosophic soft complement with $T_{f^c(x)} = F_{f(x)}$, $I_{f^c(x)} = I_{f(x)}$ and $F_{f^c(x)} = T_{f(x)}$.

Definition 9:[10] (Empty or null neutrosophic soft set with respect to a parameter.) A neutrosophic soft set h_A over the universe X is termed to be empty or null neutrosophic soft set with respect to the parameter e if $T_{h(e)}(m) = 0, F_{h(e)} = 0$ and $I_{h(e)}(m) = 0$ $\forall m \in X$, $\forall e \in A$

In this case the null neutrosophic soft set $(NNSS)$ is denoted by Φ_A.

Definition 10:[10] (Union of two neutrosophic soft sets.) Let h_A and g_B be two $NSSs$ over the common universe X . Then the union of h_A and g_B is defined by $h_A \cup g_B = k_C$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of k_C are as follow.

$$
\begin{aligned}
T_{k(e)}(m) &= T_{h(e)}(m), \quad if \quad e \in A - B \\
&= T_{g(e)}(m), \quad if \quad e \in B - A \\
&= max(T_{h(e)}(m), T_{g(e)}(m)), \quad if \quad e \in A \cap B
\end{aligned}
$$

$$
\begin{aligned}
I_{k(e)}(m) &= I_{h(e)}(m), \quad if \quad e \in A - B \\
&= I_{g(e)}(m), \quad if \quad e \in B - A \\
&= \frac{I_{h(e)}(m) + I_{g(e)}(m)}{2}, \quad if \quad e \in A \cap B.
\end{aligned}
$$

$$F_{k(e)}(m) = F_{h(e)}(m), \ if \quad e \in A - B$$
$$= F_{g(e)}(m), if e \in B - A$$
$$= min(F_{h(e)}(m), F_{g(e)}(m)), \ if \quad e \in A \cap B$$

Definition 11:[10] Let h_A and g_B be two $NSSs$ over the common universe X. Then, intersection of h_A and g_B is defined by $h_A \cap g_B = k_C$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of k_C are as follow.

$$T_{k(e)}(m) = min(T_{h(e)}(m), T_{g(e)}(m)), \ if \ e \in A \cap B$$
$$I_{k(e)}(m) = \frac{I_{h(e)}(m) + I_{g(e)}(m)}{2}, \ if \quad e \in A \cap B.$$
$$F_{k(e)}(m) = max(F_{h(e)}(m), F_{g(e)}(m)), \ if \quad e \in A \cap B$$

Proposition 1:[10] Let h_A and g_B be two $NSSs$ over the common universe X. Then,

1. $h_A \cup h_A = h_A$
2. $h_A \cup g_B = g_B \cup h_A$
3. $h_A \cap h_A = h_A$
4. $h_A \cap g_B = g_B \cap h_A$
5. $h_A \cup \Phi = h_A$
6. $h_A \cap \Phi = \Phi$
7. $[h_A^c]^c = h_A$

3. Notes on Neutrosophic Soft Sets [10]

Now, we verify that some propositions in the study of Maji [10] are incorrect by counterexamples.

1. According to the definitions of null neutrosophic soft set and neutrosophic soft subset, null neutrosophic soft set is not subset of every neutrosophic soft sets.
2. Proposition 1 -5 and 6, $f_A \cap \Phi = \Phi$ and $f_A \cup \Phi = f_A$ are incorrect.

We verify these notes by counter examples.

Example 4: Let us consider neutrosophic soft set f_A in Example 3 and null neutrosophic soft set Φ. According to the definitions of null neutrosophic soft set and neutrosophic soft subset it must be $\Phi \subseteq f_A$. Whereas $\Phi \not\subseteq f_A$, since $T_{\Phi(beautiful)}(h_1) \leq T_{F(beautiful)}(h_1)$ and $I_{\Phi(beautiful)}(h_1) \leq I_{F(beautiful)}(h_1)$ but $F_{\Phi(beautiful)}(h_1) \not\geq F_{F(beautiful)}(h_1)$, $\Phi \not\subseteq f_A$.

Example 5: Let us consider neutrosophic soft set f_A in Example 3 and null neutrosophic soft set Φ. Then,

$$\begin{aligned} f_A \cap \Phi = \{ & e_1 = \{\langle h_1,0,0.3,0.3\rangle,\langle h_2,0,0.35,0.6\rangle,\langle h_3,0,0.1,0.3\rangle,\langle h_4,0,0.15,0.2\rangle,\langle h_5,0,0.1,0.3\rangle\}, \\ & e_2 = \{\langle h_1,0,0.15,0.5\rangle,\langle h_2,0,0.2,0.3\rangle,\langle h_3,0,0.05,0.2\rangle,\langle h_4,0,0.05,0.3\rangle,\langle h_5,0,0.15,0.6\rangle\}, \\ & e_3 = \{\langle h_1,0,0.2,0.3\rangle,\langle h_2,0,0.35,0.2\rangle,\langle h_3,0,0.1,0.5\rangle,\langle h_4,0,0.1,0.6\rangle,\langle h_5,0,0.15,0.4\rangle\}, \\ & e_5 = \{\langle h_1,0,0.3,0.4\rangle,\langle h_2,0,0.45,0.6\rangle,\langle h_3,0,0.3,0.4\rangle,\langle h_4,0,0.4,0.6\rangle,\langle h_5,0,0.25,0.7\rangle\}\}. \\ & \neq \Phi \end{aligned}$$

and

$$\begin{aligned} f_A \cup \Phi = \{ & e_1 = \{\langle h_1,0.5,0.3,0\rangle,\langle h_2,0.40.35,0\rangle,\langle h_3,0.6,0.1,0\rangle,\langle h_4,0.7,0.15,0\rangle,\langle h_5,0.8,0.1,0\rangle\}, \\ & e_2 = \{\langle h_1,0.6,0.15,0\rangle,\langle h_2,0.7,0.2,0\rangle,\langle h_3,0.8,0.05,0\rangle,\langle h_4,0.7,0.05,0\rangle,\langle h_5,0.8,0.15,0\rangle\}, \\ & e_3 = \{\langle h_1,0.7,0.2,0\rangle,\langle h_2,0.6,0.35,0\rangle,\langle h_3,0.7,0.1,0\rangle,\langle h_4,0.5,0.1,0\rangle,\langle h_5,0.7,0.15,0\rangle\}, \\ & e_5 = \{\langle h_1,0.8,0.3,0\rangle,\langle h_2,0.7,0.45,0\rangle,\langle h_3,0.7,0.3,0\rangle,\langle h_4,0.7,0.4,0\rangle,\langle h_5,0.9,0.25,0\rangle\}\}. \\ & \neq f_A \end{aligned}$$

4. Neutrosophic Soft Sets

In this section, we will redefine the neutrosophic soft set based on paper of Çağman [5].

Definition 12: A neutrosophic soft set (or namely ns-set) f over X is a neutrosophic set valued function from E to $N(X)$. It can be written as

$$f = \{(e, \{\langle x, T_{f(e)}(x), I_{f(e)}(x), F_{f(e)}(x)\rangle : x \in X\}) : e \in E\}$$

where, $N(X)$ denotes all neutrosophic sets over X. Note that if $f(e) = \{\langle x,0,1,1\rangle : x \in X\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set f. Set of all *ns*-sets over X is denoted by NS.

Definition 13: Let $f, g \in \mathsf{NS}$. f is said to be neutrosophic soft subset of g, if $T_{f(e)}(x) \le T_{g(e)}(x)$, $I_{f(e)}(x) \ge I_{g(e)}(x)$ $F_{f(e)}(x) \ge F_{g(e)}(x)$, $\forall e \in E$, $\forall x \in U$. We denote it by $f \; \pi \; g$. f is said to be neutrosophic soft super set of g if g is a neutrosophic soft subset of f. We denote it by $f \; \phi \; g$.

Definition 14: If f is neutrosophic soft subset of g and g is neutrosophic soft subset of f. We denote it $f = g$ Let $f \in \mathsf{NS}$. If $T_{f(e)}(x) = 0$ and $I_{f(e)}(x) = F_{f(e)}(x) = 1$ for all $e \in E$ and for all $x \in X$, then f is called null *ns*-set and denoted by $\widetilde{\Phi}$.

Definition 15: Let $f \in \mathsf{NS}$. If $T_{f(e)}(x) = 1$ and $I_{f(e)}(x) = F_{f(e)}(x) = 0$ for all $e \in E$ and for all $x \in X$, then f is called universal *ns*-set and denoted by $\widetilde{X}$.

Definition 16: Let $f, g \in \text{NS}$. Then union and intersection of ns-sets f and g denoted by $f \oplus g$ and $f \otimes g$ respectively, are defined by as follow,

$$f \oplus g = \{(e, \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{g(e)}(x),\ F_{f(e)}(x) \wedge F_{g(e)}(x) \rangle : x \in X\}) : e \in E\}$$

and *ns*-intersection of f and g is defined as

$$f \otimes g = \{(e, \{\langle x, T_{f(e)}(x) \wedge T_{g(e)}(x), I_{f(e)}(x) \vee I_{g(e)}(x),\ F_{f(e)}(x) \vee F_{g(e)}(x) \rangle : x \in X\}) : e \in E\}.$$

Definition 17: Let $f, g \in \text{NS}$. Then complement of ns-set f, denoted by $f^{\tilde{c}}$, is defined as follow

$$f^{\tilde{c}} = \{(e, \{\langle x, F_{f(e)}(x), 1 - I_{f(e)}(x), T_{f(e)}(x) \rangle : x \in X\}) : e \in E\}.$$

Proposition 2: Let $f, g, h \in \text{NS}$. Then,

1. $\tilde{\Phi}\ \pi\ f$
2. $f\ \pi\ \tilde{X}$
3. $f\ \pi\ f$
4. $f\ \pi\ g$ and $g\ \pi\ h \Rightarrow f\ \pi\ h$

Proof : The proof is obvious from Definition 13, 14 and Definition 15.

Proposition 3: Let $f \in \text{NS}$. Then,

1. $\tilde{\Phi}^{\tilde{c}} = \tilde{X}$
2. $\tilde{X}^{\tilde{c}} = \tilde{\Phi}$
3. $(f^{\tilde{c}})^{\tilde{c}} = f$.

Proof: The proof is clear from Definition 14, 15 and 16.

Proposition 4: Let $f, g, h \in \text{NS}$. Then,

1. $f \otimes f = f$ and $f \oplus f = f$
2. $f \otimes g = g \otimes f$ and $f \oplus g = g \oplus f$
3. $f \otimes \tilde{\Phi} = \tilde{\Phi}$ and $f \otimes \tilde{X} = f$
4. $f \oplus \tilde{\Phi} = f$ and $f \oplus \tilde{X} = \tilde{X}$
5. $f \otimes (g \otimes h) = (f \otimes g) \otimes h$ and $f \oplus (g \oplus h) = (f \oplus g) \oplus h$
6. $f \otimes (g \oplus h) = (f \otimes g) \oplus (f \otimes h)$ and $f \oplus (g \otimes h) = (f \oplus g) \otimes (f \oplus h)$.

Proof. The proof is clear from definition and operations of neutrosophic soft sets.

Theorem 1: Let $f, g \in \text{NS}$. Then, De Morgan's law is valid.

1. $(f \oplus g)^{\tilde{c}} = f^{\tilde{c}} \otimes g^{\tilde{c}}$
2. $(f \otimes g)^{\tilde{c}} = f^{\tilde{c}} \oplus g^{\tilde{c}}$

Proof: $f, g \in \mathsf{NS}$ is given

1. From Definition 3, we have

$$(f \oplus g)^{\tilde{c}} = \{(e, \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{f(e)}(x), F_{f(e)}(x) \wedge F_{f(e)}(x)\rangle : x \in X\}) : e \in E\}^{\tilde{c}}$$

$$= \{(e, \{\langle x, F_{f(e)}(x) \wedge F_{f(e)}(x), 1-(I_{f(e)}(x) \wedge I_{f(e)}(x)), T_{f(e)}(x) \vee T_{g(e)}(x)\rangle : x \in X\}) : e \in E\}$$

$$= \{(e, \{\langle x, F_{f(e)}(x), 1-I_{f(e)}(x), T_{f(e)}(x)\rangle : x \in X)\} \otimes \{(e, \{\langle x, F_{g(e)}(x), 1-I_{g(e)}(x), T_{g(e)}(x)\rangle : x \in X\}$$
$$= f^{\tilde{c}} \otimes g^{\tilde{c}}.$$

2. It can be proved similar way (*1*)

Definition 18: Let $f, g \in \mathsf{NS}$. Then, difference of f and g, denoted by $f \setminus g$ is defined by the set of ordered pairs

$$f \setminus g = \{(e, \{\langle x, T_{f \setminus g(e)}(x), I_{f \setminus g(e)}(x), F_{f \setminus g(e)}(x)\rangle : x \in X\}) : e \in E\}$$

here, $T_{f \setminus g(e)}(x)$, $I_{f \setminus g(e)}(x)$ and $F_{f \setminus g(e)}(x)$ are defined by

$$T_{f \setminus g(e)}(x) = \begin{cases} T_{f(e)}(x) - T_{g(e)}(x), & T_{f(e)}(x) > T_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

$$I_{f \setminus g(e)}(x) = \begin{cases} I_{g(e)}(x) - I_{f(e)}(x), & I_{f(e)}(x) < I_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

$$F_{f \setminus g(e)}(x) = \begin{cases} F_{g(e)}(x) - F_{f(e)}(x), & G_{f(e)}(x) < G_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

Definition 19: Let $f, g \in \mathsf{NS}$. Then 'OR' product of ns-sets f and g denoted by $f \wedge g$, is defined as follow

$$f \bigvee g = \{((e, e'), \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{g(e)}(x), F_{f(e)}(x) \wedge F_{g(e)}(x)\rangle : x \in X\}) : (e, e') \in E \times E\}.$$

Definition 20: Let $f, g \in \mathsf{NS}$. Then 'AND' product of ns-sets f and g denoted by $f \vee g$, is defined as follow

$$f \bigwedge g = \{((e, e'), \{\langle x, T_{f(e)}(x) \wedge T_{g(e)}(x), I_{f(e)}(x) \vee I_{g(e)}(x), F_{f(e)}(x) \vee F_{g(e)}(x)\rangle : x \in X\}) : (e, e') \in E \times E\}.$$

Proposition 5: Let $f, g \in \mathrm{NS}$. Then,

1. $(f \vee g)^{\tilde{c}} = f^{\tilde{c}} \wedge g^{\tilde{c}}$
2. $(f \wedge g)^{\tilde{c}} = f^{\tilde{c}} \vee g^{\tilde{c}}$

Proof: The proof is clear from Definition 19 and 20.

5. Decision Making Method

In this section we will construct a decision making method over the neutrosophic soft sets. Firstly, we will define some notions that necessary to construct algorithm of decision making method.

Definition 21: Let $X = \{x_1, x_2, ... x_m\}$ be an initial universe, $E = \{e_1, e_2, ... e_n\}$ be a parameter set and f be a neutrosophic soft set over X. Then, according to the Table of "Saaty Rating Scale" relative parameter matrix d_E is defined as follow

$$d_E = \begin{bmatrix} 1 & d_E(e_1,e_2) & \cdots & d_E(e_1,e_n) \\ d_E(e_2,e_1) & 1 & \cdots & d_E(e_2,e_n) \\ \vdots & \vdots & \vdots & \vdots \\ d_E(e_n,e_1) & d_E(e_n,e_2) & \cdots & 1 \end{bmatrix}$$

If $d_E(e_i, e_j) = d_{ij}$, we can write matrix

$$d_E = \begin{bmatrix} 1 & d_{11} & \cdots & d_{1n} \\ d_{21} & 1 & \cdots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & 1 \end{bmatrix}.$$

Here, d_{12} means that how much important e_1 by e_2. For example, if e_1 is much more important by e_2, then we can write $d_{12} = 5$ from Table 1.

Table 1. The Saaty Rating Scale

Intensity importance	Definition	Explanation
1	Equal importance	Two factor contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favourone over the other
5	Much more important	Experience and judgement strongly favourone over the other
7	Very much more important	Experience and judgement very strongly favourone over the other. Its importance is demonstrated in practice
9	Absolutely more important	The evidence favouring one over other is of the highest possiple validity
2,4,6,8	Intermediate values	When compromise is needed

Definition 22: Let f be a neutrosophic soft set and d_E be a relative parameter matrix of f. Then, score of parameter e_i, denoted by c_i and is calculated as follows

$$c_i = \sum_{j=1}^{n} d_{ij}$$

Definition 23: Normalized relative parameter matrix (nd_E for short) of relative parameter matrix d_E, denoted by $\hat{d}$, is defined as follow

$$nd_E = \begin{bmatrix} \frac{1}{c_1} & \frac{d_{12}}{c_1} & \cdots & \frac{d_{1n}}{c_1} \\ \frac{d_{21}}{c_2} & \frac{1}{c_2} & \cdots & \frac{d_{2n}}{c_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d_{n1}}{c_n} & \frac{d_{n2}}{c_n} & \cdots & \frac{1}{c_n} \end{bmatrix}$$

If $\frac{d_{ij}}{c_i} = \hat{d}_{ij}$, *we can write matrix* nd_E

$$\hat{d} = \begin{bmatrix} \hat{d}_{11} & \hat{d}_{12} & \Lambda & \hat{d}_{1n} \\ \hat{d}_{21} & \hat{d}_{22} & \Lambda & \hat{d}_{2n} \\ M & M & O & M \\ \hat{d}_{n1} & \hat{d}_{n2} & \Lambda & \hat{d}_{nn} \end{bmatrix}$$

Definition 24: Let f be a neutrosophic soft set and $\hat{d}$ be a normalized parameter matrix of f. Then, weight of parameter $e_j \in E$, denoted by $w(e_j)$ and is formulatred as follow,

$$w(e_j) = \frac{1}{|E|}\sum_{i=1}^{n} \hat{d}_{ij}$$

Now, we construct compare matrices of elements of X in neutrosophic sets $f(e)$, $\forall e \in E$

Definition 25: Let E be a parameter set and f be a neutrosophic soft set over X. Then, for all $e \in E$, compression matrices of f, denoted $X_{f(e)}$ is defined as follow

$$X_{f(e)} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mm} \end{bmatrix}$$

$$x_{ij} = \frac{\Delta_{T(e)}(x_{ij}) + \Delta_{I(e)}(x_{ij}) + \Delta_{F(e)}(x_{ij}) + 1}{2}$$

such that

$$\Delta_{T(e)}(x_{ij}) = T(e)(x_i) - T(e)(x_j)$$

$$\Delta_{I(e)}(x_{ij}) = I(e)(x_i) - I(e)(x_j)$$

$$\Delta_{F(e)}(x_{ij}) = F(e)(x_i) - F(e)(x_j)$$

Definition 26: Let $X_{f(e)}$ be compare matrix for $e \in E$. Then, weight of $x_j \in X$ related to parameter $e \in E$, denoted by $W_{f(e)}(x_j)$ is defined as follow,

$$W_{f(e)}(x_j) = \frac{1}{|X|}\sum_{i=1}^{m} x_{ij}$$

Definition 27: Let E be a parameter set, X be an initial universe and $w(e)$ and $W_{f(e)}(x_j)$ be weight of parameter e and membership degree of x_j which related to $e_j \in E$, respectively. Then, decision set, denoted D_E, is defined by the set of ordered pairs

$$D_E = \{(x_j, F(x_j)) : x_j \in X\}$$

where

$$F(x_j) = \frac{1}{|E|}\sum_{j=1}^{n} w(e_j) \times W_{f(e)}(x_j)$$

Now, we construct a neutrosophic soft set decision making method by the following algorithm;

Algorithm 1

Step 1: Input the neutrosophic soft set f,

Step 2: Construct the normalized parameter matrix,

Step 3: Compute the weight of each parameters,

Step 4: Construct the compare matrix for each parameter,

Step 5: Compute membership degree, for all $x_j \in X$

Step 6: Construct decision set D_E

Step 7: The optimal decision is to select $x_k = maxF(x_j)$.

Example 6: Let X be the set of blouses under consideration and E is the set of parameters. Each parameters is a neutrosophic word or sentence involving neutrosophic words. Consider $E = \{bright,\ cheap,\ colorful,\ cotton\}$. Suppose that, there are five blouses in the universe X given by $X = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose that,

Step 1: Let us consider the decision making problem involving the neutrosophic soft set in [2]

$$f(Bright) = \{\langle x_1,.5,.6,.3\rangle, \langle x_2,.4,.7,.2\rangle, \langle x_3,.6,.2,.3\rangle, \langle x_4,.7,.3,.2\rangle, \langle x_5,.8,.2,.3\rangle,\}$$
$$f(Cheap) = \{\langle x_1,.6,.3,.5\rangle, \langle x_2,.7,.4,.3\rangle, \langle x_3,.8,.1,.2\rangle, \langle x_4,.7,.1,.3\rangle, \langle x_5,.8,.3,.4\rangle,\}$$
$$f(Colorful) = \{\langle x_1,.7,.4,.3\rangle, \langle x_2,.6,.1,.2\rangle, \langle x_3,.7,.2,.5\rangle, \langle x_4,.5,.2,.6\rangle, \langle x_5,.7,.3,.2\rangle,\}$$
$$f(Cotton) = \{\langle x_1,.4,.3,.7\rangle, \langle x_2,.5,.4,.2\rangle, \langle x_3,.7,.4,.3\rangle, \langle x_4,.2,.4,.5\rangle, \langle x_5,.6,.4,.4\rangle,\}$$

Step 2:

$$d_E = \begin{bmatrix} 1 & 1/3 & 5 & 1/3 \\ 3 & 1 & 2 & 3 \\ 1/5 & 1/2 & 1 & 2 \\ 3 & 1/3 & 1/2 & 1 \end{bmatrix}$$

$c_1 = 6.67$, $c_2 = 9$, $c_3 = 3.7$ and $c_4 = 4.88$ and

$$\hat{d}_E = \begin{bmatrix} .15 & .05 & .75 & .05 \\ .33 & .11 & .22 & .33 \\ .05 & .14 & .27 & .54 \\ .62 & .07 & .10 & .21 \end{bmatrix}$$

Step 3: From normalized matrix, weight of parameters are obtained as $w(e_1) = .29$, $w(e_2) = .09$, $w(e_3) = .34$ and $w(e_4) = .28$.

Step 4: For each parameter, compare matrices and normalized compare matrices are constructed as follow

Let us consider parameter "bright". Then,

$$X_{f(bright)} = \begin{bmatrix} .50 & .10 & .25 & .20 & .15 \\ .45 & .50 & .20 & .15 & .10 \\ .75 & .80 & .50 & .45 & .40 \\ .80 & .85 & .55 & .50 & .45 \\ .85 & .90 & .60 & .55 & .50 \end{bmatrix}, \quad X_{f(cheap)} = \begin{bmatrix} .50 & .40 & .15 & .25 & .35 \\ .50 & .50 & .30 & .35 & .45 \\ .85 & .75 & .50 & .60 & .70 \\ .75 & .65 & .40 & .50 & .60 \\ .65 & .55 & .30 & .40 & .50 \end{bmatrix}$$

and

$$X_{f(colorful)} = \begin{bmatrix} .50 & .35 & .55 & .65 & .40 \\ .65 & .50 & .65 & .18 & .55 \\ .50 & .35 & .50 & .65 & .40 \\ .35 & .30 & .15 & .50 & .25 \\ .40 & .45 & .60 & .75 & .50 \end{bmatrix}, \quad X_{f(cotton)} = \begin{bmatrix} .50 & .25 & .35 & .50 & .15 \\ .75 & .50 & .60 & .75 & .40 \\ .65 & .40 & .50 & .65 & .30 \\ .50 & .25 & .35 & .50 & .15 \\ .85 & .60 & .70 & .85 & .50 \end{bmatrix}$$

Step 5: For all $x_j \in X$ and $e \in E$,

$W_{f(bright)}(x_1) = .67, W_{f(bright)}(x_2) = .63, W_{f(bright)}(x_3) = .42, W_{f(bright)}(x_4) = .37, W_{f(bright)}(x_5) = .32$

$W_{f(cheap)}(x_1) = .80, W_{f(cheap)}(x_2) = .57, W_{f(cheap)}(x_3) = .33, W_{f(cheap)}(x_4) = .42, W_{f(cheap)}(x_5) = .52$

$W_{f(colorful)}(x_1) = .48, W_{f(colorful)}(x_2) = .39, W_{f(colorful)}(x_3) = .49, W_{f(colorful)}(x_4) = .55, W_{f(colorful)}(x_5) = .42$
$W_{f(cotton)}(x_1) = .65, W_{f(cotton)}(x_2) = .40, W_{f(cotton)}(x_3) = .50, W_{f(cotton)}(x_4) = .65, W_{f(cotton)}(x_5) = .30$

Step 6: By using step 3 and step 5, D_E is constructed as follow

$$D_E = \{(x_1, 0.15), (x_2, 0.12), (x_3, 0.11), (x_4, 0.13), (x_5, 0.09)\}$$

Step 7: Note that, membership degree of x_1 is greater than the other. Therefore, optimal decision is x_1 for this decision making problem.

6. Group Decision Making

In this section, we constructed a group decision making method using intersection of neutrosophic soft sets and Algorithm 1.

Let $X = \{x_1, x_2, ..., x_n\}$ be an initial universe and let $d = \{d^1, d^2, ..., d^m\}$ be a decision maker set and $E = \{e_1, e_2, ..., e_k\}$ be a set of parameters. Then, this method can be described by the following steps

Algorithm 2

Step 1: Each decision-maker d^i construct own neutrosophic soft set, denoted by f_{d_i}, over U and parameter set E.

Step 2: Let for $p, r \le k$, $[d^i_{pr}]$ a relative parameter matrix of decision-maker $d^i \in D$ based on the Saaty Rating Scale. Decision-maker d^i gives his/her evaluations separately and independently according to his/her own preference based on Saaty Rating Scale. In this way, each decision-maker d^i presents a relative parameter matrix.

$$[d^i_{pr}] = \begin{bmatrix} d^i_{11} & d^i_{12} & \Lambda & d^i_{1k} \\ d^i_{21} & d^i_{22} & \Lambda & d^i_{2k} \\ M & M & O & M \\ d^i_{k1} & d^i_{k2} & M & d^i_{kk} \end{bmatrix}$$

here d^i_{pr} is equal $d_E(e_p, e_r)$ that in Definition 21.

Step 3: Arithmetic mean matrix is constructed by using the the relative parameter matrix of each decision-maker d^i. It will be denoted by $[i_{pr}]$ and will be computed as in follow

$$i_{pr} = \frac{1}{|d|}\sum_{i=1}^{m} d^i_{pr}$$

Step 4: Normalized parameter matrix, is constructed using the arithmetic mean matrix $[i_{pr}]$, it will be shown $[\hat{i}_{pr}]$ and weight of each parameter $e_i \in E$ ($w(e_i)$) is computed.

Step 5: Intersection of neutrosophic soft sets (it will be denoted by $I_{f_{d_i}}$) which are constructed by decision makers is found.

$$I_{f_d} = \bigcap_{i=1}^{m} f_{d^i}$$

Step 6: Based on the matrix I_{f_d}, for each element of $e \in E$ compare matrix, denoted by $I_{f_d(e)}$ is constructed.

Step 7: By the $I_{f_d(e)}$, weight of each element of X denoted by $W_{I_{f_d(e)}}(x_i)$, are computed.

Step 8: Decision set D_E is constructed by using values of $w(e)$ and $W_{I_{f_d}}(x)$. Namely;

$$D_E = \{(x_i, F(x_i)) : x_i \in X\}$$

and

$$F(x_i) = \frac{1}{|E|}\sum_{j=1}^{n} w(e_j) \times W_{I_{f(e)}}(x_i)$$

Step 9: From the decision set, $x_k = maxF(x_i)$ is selected as optimal decision.

Example 7: Assume that a company wants to fill a position. There are 6 candidates who fill in a form in order to apply formally for the position. There are three decision makers; one of them is from the department of human resources and the others is from the board of directors. They want to interview the candidates, but it is very difficult to make it all of them. Let $d = \{d_1, d_2, d_3\}$ be a decision makers set, $X = \{x_1, x_2, x_3, x_4, x_5\}$ be set of candidates and $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set such that parameters e_1, e_2, e_3 and e_4 stand for ”experience”,”computer knowledge”, ”higher education” and ”good health”, respectively.

Step 1: Let each decision maker construct neutrosophic soft sets over X by own interview:

$$f_{d^1} = \left\{\begin{matrix} f_{d^1}(e_1) = \{\langle x_1,.4,.2,.7\rangle, \langle x_2,.5,.6,.2\rangle, \langle x_3,.7,.3,.3\rangle, \langle x_4,.6,.5,.4\rangle, \langle x_5,.3,.5,.5\rangle\}, \\ f_{d^1}(e_2) = \{\langle x_1,.3,.5,.2\rangle, \langle x_2,.4,.4,.3\rangle, \langle x_3,.5,.7,.8\rangle, \langle x_4,.7,.1,.3\rangle, \langle x_5,.6,.3,.2\rangle\}, \\ f_{d^1}(e_3) = \{\langle x_1,.7,.4,.3\rangle, \langle x_2,.6,.1,.5\rangle, \langle x_3,.5,.2,.4\rangle, \langle x_4,.2,.2,.6\rangle, \langle x_5,.3,.3,.6\rangle\}, \\ f_{d^1}(e_4) = \{\langle x_1,.7,.3,.5\rangle, \langle x_2,.3,.5,.3\rangle, \langle x_3,.2,.4,.3\rangle, \langle x_4,.4,.2,.5\rangle, \langle x_5,.5,.2,.6\rangle\} \end{matrix}\right\}$$

$$f_{d^2}=\begin{Bmatrix} f_{d^2}(e_1)=\{\langle x_1,.5,.2,.3\rangle,\langle x_2,.3,.5,.6\rangle,\langle x_3,.4,.3,.3\rangle,\langle x_4,.2,.5,.4\rangle,\langle x_5,.5,.5,.5\rangle\}, \\ f_{d^2}(e_2)=\{\langle x_1,.5,.4,.6\rangle,\langle x_2,.7,.2,.5\rangle,\langle x_3,.6,.3,.5\rangle,\langle x_4,.7,.2,.3\rangle,\langle x_5,.6,.4,.2\rangle\}, \\ f_{d^2}(e_3)=\{\langle x_1,.6,.2,.5\rangle,\langle x_2,.4,.4,.6\rangle,\langle x_3,.2,.5,.4\rangle,\langle x_4,.3,.5,.4\rangle,\langle x_5,.3,.3,.6\rangle\}, \\ f_{d^2}(e_4)=\{\langle x_1,.3,.4,.5\rangle,\langle x_2,.4,.3,.2\rangle,\langle x_3,.4,.4,.3\rangle,\langle x_4,.4,.2,.5\rangle,\langle x_5,.2,.5,.6\rangle\} \end{Bmatrix}$$

and

$$f_{d^3}=\begin{Bmatrix} f_{d^3}(e_1)=\{\langle x_1,.4,.5,.7\rangle,\langle x_2,.5,.3,.4\rangle,\langle x_3,.7,.3,.5\rangle,\langle x_4,.4,.5,.3\rangle,\langle x_5,.7,.8,.6\rangle\}, \\ f_{d^3}(e_2)=\{\langle x_1,.6,.2,.6\rangle,\langle x_2,.4,.3,.5\rangle,\langle x_3,.5,.4,.7\rangle,\langle x_4,.3,.1,.5\rangle,\langle x_5,.4,.3,.1\rangle\}, \\ f_{d^3}(e_3)=\{\langle x_1,.4,.3,.2\rangle,\langle x_2,.6,.7,.2\rangle,\langle x_3,.3,.5,.2\rangle,\langle x_4,.6,.6,.4\rangle,\langle x_5,.6,.5,.5\rangle\}, \\ f_{d^3}(e_4)=\{\langle x_1,.5,.3,.1\rangle,\langle x_2,.2,.5,.2\rangle,\langle x_3,.5,.5,.4\rangle,\langle x_4,.5,.2,.5\rangle,\langle x_5,.5,.3,.6\rangle\} \end{Bmatrix}$$

Step 2: Relative parameter matrix of each decision maker are as in follow

$$[d^1_{pr}]=\begin{bmatrix} 1 & 3 & 1/5 & 2 \\ 1/3 & 1 & 3 & 6 \\ 5 & 1/3 & 1 & 1/5 \\ 1/2 & 1/6 & 5 & 1 \end{bmatrix} \quad [d^2_{pr}]=\begin{bmatrix} 1 & 5 & 1/7 & 2 \\ 1/5 & 1 & 1/2 & 6 \\ 7 & 2 & 1 & 1/3 \\ 1/2 & 1/6 & 3 & 1 \end{bmatrix}$$

and

$$[d^3_{pr}]=\begin{bmatrix} 1 & 3 & 1/3 & 4 \\ 1/3 & 1 & 1/3 & 1/6 \\ 3 & 3 & 1 & 1/2 \\ 1/4 & 6 & 2 & 1 \end{bmatrix}$$

Step 3: $[i_{pr}]$ can be obtained as follow,

$$[i_{pr}]=\begin{bmatrix} 1 & 3.67 & .23 & 2.67 \\ .29 & 1 & 1.28 & 4.06 \\ 5 & 1.78 & 1 & .34 \\ .42 & 4.06 & 3.33 & 1 \end{bmatrix}$$

Step 4: $[\hat{i}_{pr}]$ and weight of each parameter can be obtained as follow,

$$[\hat{i}_{pr}]=\begin{bmatrix} .13 & .49 & .03 & .35 \\ .04 & .15 & .19 & .61 \\ .62 & .22 & .12 & .04 \\ .05 & .46 & .38 & .11 \end{bmatrix}$$

and $w(e_1)=.21$, $w(e_2)=0.33$, $w(e_3)=.18$ $w(e_4)=.28$.

Step 5: Intersection of neutrosophic soft sets f_{d^1}, f_{d^2} and f_{d^3} is as follow;

$$I_{f_d}=\begin{Bmatrix} I_{f_d}(e_1)=\{\langle x_1,.4,.5,.7\rangle,\langle x_2,.3,.6,.6\rangle,\langle x_3,.4,.3,.5\rangle,\langle x_4,.2,.5,.5\rangle,\langle x_5,.3,.8,.6\rangle\}, \\ I_{f_d}(e_2)=\{\langle x_1,.3,.5,.6\rangle,\langle x_2,.4,.4,.5\rangle,\langle x_3,.5,.7,.8\rangle,\langle x_4,.3,.2,.5\rangle,\langle x_5,.4,.4,.2\rangle\}, \\ I_{f_d}(e_3)=\{\langle x_1,.6,.5,.5\rangle,\langle x_2,.4,.7,.6\rangle,\langle x_3,.2,.5,.4\rangle,\langle x_4,.2,.6,.6\rangle,\langle x_5,.3,.5,.6\rangle\}, \\ I_{f_d}(e_4)=\{\langle x_1,.3,.4,.5\rangle,\langle x_2,.2,.5,.3\rangle,\langle x_3,.2,.5,.4\rangle,\langle x_4,.4,.2,.5\rangle,\langle x_5,.2,.5,.6\rangle\} \end{Bmatrix}$$

Step 6: For each parameter, compare matrices of elements of X are obtained as in follows;

$$I_{f_d(e_1)}=\begin{bmatrix} .50 & .55 & .30 & .45 & .65 \\ .45 & .50 & .25 & .40 & .60 \\ .70 & .75 & .50 & .35 & .85 \\ .55 & .60 & .65 & .50 & .70 \\ .65 & .40 & .15 & .30 & .70 \end{bmatrix}, \quad I_{f_d(e_2)}=\begin{bmatrix} .50 & .35 & .60 & .30 & .20 \\ .65 & .50 & .75 & .35 & .35 \\ .40 & .25 & .50 & .20 & .10 \\ .70 & .65 & .80 & .50 & .40 \\ .80 & .65 & .90 & .60 & .50 \end{bmatrix}$$

and

$$I_{f_d(e_3)}=\begin{bmatrix} .50 & .75 & .65 & .80 & .70 \\ .25 & .50 & .40 & .55 & .45 \\ .35 & .60 & .50 & .65 & .55 \\ .20 & .45 & .35 & .50 & .45 \\ .30 & .55 & .45 & .55 & .50 \end{bmatrix}, \quad I_{f_d(e_4)}=\begin{bmatrix} .50 & .50 & .55 & .35 & .65 \\ .50 & .50 & .45 & .40 & .65 \\ .45 & .55 & .50 & .30 & .60 \\ .65 & .60 & .70 & .50 & .80 \\ .35 & .35 & .40 & .20 & .50 \end{bmatrix}$$

Step 7: Membership degrees of elements of X related to each parameter $e \in E$ are obtained as follows;

$W_{f_d(e_1)}(x_1)=.57$, $W_{f_d(e_1)}(x_2)=.56$, $W_{f_d(e_1)}(x_3)=.37$, $W_{f_d(e_1)}(x_4)=.40$ and $W_{f_d(e_1)}(x_5)=.66$

$W_{f_d(e_2)}(x_1)=.61$, $W_{f_d(e_2)}(x_2)=.48$, $W_{f_d(e_2)}(x_3)=.71$, $W_{f_d(e_2)}(x_4)=.39$ and $W_{f_d(e_2)}(x_5)=.31$

$W_{f_d(e_3)}(x_1)=.32$, $W_{f_d(e_3)}(x_2)=.57$, $W_{f_d(e_3)}(x_3)=.47$, $W_{f_d(e_3)}(x_4)=.61$ and $W_{f_d(e_3)}(x_5)=.53$

$W_{f_d(e_4)}(x_1)=.49$, $W_{f_d(e_4)}(x_2)=.50$, $W_{f_d(e_4)}(x_3)=.52$, $W_{f_d(e_4)}(x_4)=.35$ and $W_{f_d(e_4)}(x_5)=.64$

Step 8:

$$\begin{aligned}F(x_1) &= \frac{1}{|E|}\sum_{j=1}^{n} w(e_j)\times W_{f_d(e_j)}(x_1)\\ &= \frac{1}{4}(.21\times .57+.33\times .61+.18\times .32+.28\times .49)\\ &= .126\end{aligned}$$

similarly $F(x_2)=.130$, $F(x_3)=.136$, $F(x_4)=.105$ and $F(x_5)=.129$. Then, we get

$$D_E = \{(x_1,.126),(x_2,.130),(x_3,.136),(x_4,.105),(x_5,.129)\}$$

Step 9: Note that, membership degree of x_3 is greater than membership degrees of the others. Therefore, optimal decision is x_3 for this decision making problem.

7. Conclusion

In this paper, we firstly investigate neutrosophic soft sets given paper of Maji [10] and then we redefine notion of neutrosophic soft set and neutrosophic soft set operations. Finally, we present two applications of neutrosophic soft sets in decision making problem.

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