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Neutrosophic Soft Matrix And It's Application in Solving Group Decision Making Problems from Medical Science

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Abstract: The main purpose of this paper is to introduce the concept of Neutrosophic Soft Matrix(NS-Matrix). We have proposed different types of NS-Matrix along with various operations on them. A new methodology, named as NSM -Algorithm based on some of these new matrix operations, has been developed to solve neutrosophic soft set based real life group decision making problems efficiently. Finally NSM-Algorithm has been applied to solve the problems of diagnosis of a disease from the myriad of symptoms as well as to evaluate the effectiveness of different habits of human being responsible for a disease from medical science.

Keywords: Choice Matrix, Group Decision, Neutrosophic Soft Matrix(NS-Matrix), NSM -Algorithm

1 Introduction

The concept of fuzzy sets was introduced by Zadeh(1965). Since then the fuzzy sets and fuzzy logic have been applied in many real life problems in uncertain, ambiguous environment. The traditional fuzzy sets is characterized by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy sets. Consequently the concept of interval valued fuzzy sets was proposed (Turksen 1986) to capture the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on , we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is appropriate for such a situation. Intuitionistic fuzzy sets(IFS) introduced by Atanassov (1986) is appropriate for such a situation.

But the intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache (2005) introduced the concept of neutrosophic set(NS) which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

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In our regular everyday life we face situations which require procedures allowing certain flexibility in information processing capacity. Soft set theory (Molodtsov 1999; Maji et al. 2002, 2003) addressed those problems successfully. In their early work soft set was described purely as a mathematical method to model uncertainties. The researchers can pick any kind of parameters of any nature they wish in order to facilitate the decision making procedure as there is a varied way of picturing the objects. Maji et al.(2002, 2003) have done further research on soft set theory. Presence of vagueness demanded Fuzzy Soft Set (FSS) (Maji et al. 2001; Basu et al. 2012) to come into picture. But satisfactory evaluation of membership values is not always possible because of the insufficiency in the available information (besides the presence of vagueness) situation. Evaluation of non-membership values is also not always possible for the same reason and as a result there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations Intuitionistic Fuzzy Soft Set theory (IFSS)(Maji et al. 2001) may be more applicable. Now in the perlance of soft set theory there is hardly any limitation to select the nature of the criteria and as most of the parameters or criteria(which are words or sentences) are neutrosophic in nature, Maji (2012, 2013) has been motivated to combine the concept of soft set and neutrosophic set to make the new mathematical model neutrosophic soft set and has given an algorithm to solve a decision making problem. But till now there does not exist any method for solving neutrosophic soft set based group decision making problem.

Group Decision is the academic and professional field that aims to improve collective decision process by taking into account the needs and opinions of every group assisting groups or individuals within groups, as they interact and collaborate to reach a collective decision.

Cagman and Enginoglu(2010) introduced a new soft set based decision making method which selects a set of optimum elements from the alternatives. In the same year, the same authors have proposed the definition of soft matrices which are representations of soft sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer. Then Basu et al.(2012) have done further work on soft matrices.

Since there does not exist any method for solving neutrosophic soft set based group decision making problem and soft matrices have been shown to be very efficient to solve group decision making problems we are motivated to introduce the concept of neutrosophic soft matrix and their various operations to solve neutrosophic soft set based group decision making problems.

In this paper we have proposed the concept of neutrosophic soft matrix. Then we have defined different types of neutrosophic soft matrices with giving proper examples. Here we have also proposed the concept of choice matrix associated with a neutrosophic soft set. Moreover we have introduced some operations on neutrosophic soft matrices and choice matrices. Then based on some of these new matrix operations a new efficient solution procedure, named as NSM-Algorithm, has been developed to solve neutrosophic soft set based real life decision making problems which may contain more than one decision maker. The speciality of this new approach is that it may solve any neutrosophic soft set based decision making problem involving a large number of decision makers very easily and the computational procedure is also very simple. At last we have applied the NSM-Algorithm to solve some interesting problems of medical science.

2 Preliminaries

Definition: (Molodtsov 1999)

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U. Let $A \subseteq E$. A pair (F_A, E) is called a **soft set** over U, where F_A is a mapping given by, $F_A : E \to P(U)$ such that $F_A(e) = f$ if $e \notin A$.

Here F_A is called approximate function of the soft set (F_A, E). The set $F_A(e)$ is called e-approximate value set which consists of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition (Smarandache 2005)

A **neutrosophic set** A on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\},\ where\ T, I, F : X \to \left]{}^{-}0,1^{+}\right[$ and ${}^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$; T, I, F are called neutrosophic components.

"Neutrosophic" etymologically comes from "neutro-sophy" (French *neutre* < Latin *neuter*, neutral and Greek *sophia*, skill/wisdom) which means knowledge of neutral thought.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $\left]{}^{-}0,1^{+}\right[$. The non-standard finite numbers $1^{+} = 1 + d$, where 1 is the standard part and d is the non-standard part and ${}^{-}0 = 0 - d$, where 0 is its standard part and d is non-standard part. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $\left]{}^{-}0,1^{+}\right[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Any element neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between 0 and 1 or even less than 0 or greater than 1.

Thus $x(0.5, 0.2, 0.3)$ belongs to A (which means, with a probability of 50 percent x is in A, with a probability of 30 percent x is not in A and the rest is undecidable); or $y(0,0,1)$ belongs to A (which normally means y is not for sure in A); or $z(0,1,0)$ belongs to A (which means one does know absolutely nothing about z's affiliation with A); here $0.5 + 0.2 + 0.3 = 1$; thus A **is a NS and an IFS too**.

The subsets representing the appurtenance, indeterminacy and falsity may overlap, say the element

$z(0.30, 0.51, 0.28)$ and in this case $0.30 + 0.51 + 0.28 > 1$; then **B is a NS but is not an IFS; we can call it paraconsistent set** (from paraconsistent logic, which deals with paraconsistent information).

Or, another example, say the element $z(0.1, 0.3, 0.4)$ belongs to the set C, and here $0.1 + 0.3 + 0.4 < 1$; then B **is a NS but is not an IFS; we can call it intuitionistic set** (from intuitionistic logic, which deals with incomplete information).

Remarkably, in a NS one can have elements which have paraconsistent information (sum of components >1), or, incomplete information (sum of components <1), or, consistent information (in the case when the sum of components $=1$).

Definition (Maji 2012, 2013)

Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let N^U denotes the set of all neutrosophic sets of U. The collection $(\breve{F}, A)$ is termed to be the **neutrosophic soft set** over U, where $\breve{F}$ is a mapping given by $\breve{F} : A \to N^U$.

For illustration we consider an example.

Example 2.1

Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words.

Consider $E = \{$ beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive $\}$. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter beautiful, e_2 stands for the parameter wooden, e_3 stands for the parameter costly and the parameter e_4 stands for moderate. Suppose that,

$$F(beautiful) = \{<h_1,0.5,0.6,0.3>,<h_2,0.4,0.7,0.6>,<h_3,0.6,0.2,0.3>, \quad <h_4,0.7,0.3,0.2>,<h_5,0.8,0.2,0.3>\} \;,$$
$$F(wooden) = \{<h_1,0.6,0.3,0.5>,<h_2,0.7,0.4,0.3>,<h_3,0.8,0.1,0.2>, \quad <h_4,0.7,0.1,0.3>,<h_5,0.8,0.3,0.6>\},$$
$$F(costly) = \{<h_1,0.7,0.4,0.3>,<h_2,0.6,0.7,0.2>,<h_3,0.7,0.2,0.5>, \quad <h_4,0.5,0.2,0.6>,<h_5,0.7,0.3,0.4>\},$$
$$F(moderate) = \{<h_1,0.8,0.6,0.4>,<h_2,0.7,0.9,0.6>,<h_3,0.7,0.6,0.4>, \quad <h_4,0.7,0.8,0.6>,<h_5,0.9,0.5,0.7>\}.$$

The neutrosophic soft set (NSS) $(\breve{F}, E)$ is a parameterized family $\{\breve{F}(e_i); i = 1,..,10\}$ of all neutrosophic sets of U and describes a collection of approximation of an object. The mapping $\breve{F}$ here is houses(.), where dot(.) is to be filled up by a parameter $e \in E$.

Therefore, $\breve{F}(e_1)$ means houses(beautiful) whose functional-value is the neutrosophic set

$$\{<h_1,0.5,0.6,0.3>,<h_2,0.4,0.7,0.6>,<h_3,0.6,0.2,0.3>,<h_4,0.7,0.3,0.2>, \quad <h_5,0.8,0.2,0.3>\}.$$

Thus we can view the neutrosophic soft set (NSS) (F, A) as a collection of approximation as below:

$$(\breve{F}, A) = \{beautiful\ houses = \{<h_1,0.5,0.6,0.3>,<h_2,0.4,0.7,0.6>,<h_3,0.6,0.2,0.3>,$$
$$<h_4,0.7,0.3,0.2>,<h_5,0.8,0.2,0.3>\}, wooden\ houses = \{<h_1,0.6,0.3,0.5>,$$
$$<h_2,0.7,0.4,0.3>,<h_3,0.8,0.1,0.2>,<h_4,0.7,0.1,0.3>,<h_5,0.8,0.3,0.6>\},$$
$$costly\ houses = \{<h_1,0.7,0.4,0.3>,<h_2,0.6,0.7,0.2>,<h_3,0.7,0.2,0.5>,$$

$< h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >\}$, *moderate houses* $= \{< h_1, 0.8, 0.6, 0.4 >,$
$< h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >, < h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 >\}\}$

where each approximation has two parts: (i) a predicate p, and (ii) an approximate value-set v (or simply to be called value-set v).

For example, for the approximation

beautiful houses $= \{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >,$
$< h_5, 0.8, 0.2, 0.3 >\}$

we have (i) the predicate part *beautiful houses* and (ii) the approximate value-set is $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$.

Thus, a neutrosophic soft set $(\breve{F}, E)$ can be viewed as a collection of approximations like $(\breve{F}, E) = \{p_1 = v_1, p_2 = v_2, ..., p_{10} = v_{10}\}$. For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the neutrosophic soft set in the above example). In this table, the entries are c_{ij} corresponding to the house h_i and the parameter e_j , where c_{ij} **= (true-membership value of** h_i**, indeterminacy-membership value of** h_i**, falsity-membership value of** h_i **) in** $\breve{F}(e_j)$. The tabular representation of the neutrosophic soft set $(\breve{F}, A)$ is as follow:

Table-I: Tabular representation of the NSS $(\breve{F}, A)$

	beautiful	*wooden*	*costly*	*moderate*
h_1	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
h_2	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6, 0.7, 0.2)	(0.7, 0.9, 0.6)
h_3	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
h_4	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5, 0.2, 0.6)	(0.7, 0.8, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Definition: (Cagman and Enginoglu 2010)

Let (F_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) : e \in A, u \in F_A(e)\}$$

which is called a relation form of (F_A, E). Now the **characteristic function** of R_A is written by,

$$c_{R_A} : U \times E \to \{0,1\},\ c_{R_A} = \begin{cases} 1, (u,e) \in R_A \\ 0, (ij,e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, ..., u_m\}$, $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

Table-II: Tabular representation of R_A of the soft set (F_A, E)

	e_1	e_2	....	e_n
u_1	$c_{R_A}(u_1, e_1)$	$c_{R_A}(u_1, e_2)$	....	$c_{R_A}(u_1, e_n)$
u_2	$c_{R_A}(u_2, e_1)$	$c_{R_A}(u_2, e_2)$	....	$c_{R_A}(u_2, e_n)$
....	....	....	....	....
u_m	$c_{R_A}(u_m, e_1)$	$c_{R_A}(u_m, e_2)$	....	$c_{R_A}(u_m, e_n)$

If $a_{ij} = c_{R_A}(u_i, e_j)$, we can define a matrix

$$[a_{ij}]_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{pmatrix}$$

which is called a **soft matrix** of order $m \times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m \times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

3 Some New Concepts of Matrices in Neutrosophic Soft Set Theory

3.1 Definitions

Neutrosophic Soft Matrix:

Let $(\tilde{F}_A, E)$ be a neutrosophic soft set over U, where $\tilde{F}_A$ is a mapping given by, $\tilde{F}_A : E \to N^U$ such that $\tilde{F}_A(e) = f$ if $e \notin A$ where N^U is the set of all neutrosophic sets over U and f is a null neutrosophic set. Then a subset of $U \times E$ is uniquely defined by

$$\tilde{R}_A = \{(u,e) : e \in A, u \in \tilde{F}_A(e)\}$$

which is called a relation form of $(\breve{F}_A, E)$. Now the relation $\breve{R}_A$ is characterized by the truth-membership function $T_A : U \times E \to [0,1]$, indeterminacy-membership function $I_A : U \times E \to [0,1]$ and the falsity-membership function $F_A : U \times E \to [0,1]$, where $T_A(u,e)$ is the true-membership value, $I_A(u,e)$ is the indeterminacy-membership value and $F_A(u,e)$ is the falsity-membership value of the object u associated with the parameter e.

Now if the set of universe $U = \{u_1, u_2, ..., u_m\}$ and the set of parameters $E = \{e_1, e_2, ..., e_n\}$, then $\breve{R}_A$ can be presented by a table as in the following form

Table-III: Tabular representation of $\breve{R}_A$ of the NSS $(\breve{F}_A, E)$

	e_1	e_2	....	e_n
u_1	$(T_{A11}, I_{A11}, F_{A11})$	$(T_{A12}, I_{A12}, F_{A12})$	....	$(T_{A1n}, I_{A1n}, F_{A1n})$
u_2	$(T_{A21}, I_{A21}, F_{A21})$	$(T_{A22}, I_{A22}, F_{A22})$	....	$(T_{A2n}, I_{A2n}, F_{A2n})$
....	....	....	....	....
u_m	$(T_{Am1}, I_{Am1}, F_{Am1})$	$(T_{Am2}, I_{Am2}, F_{Am2})$	....	$(T_{Amn}, I_{Amn}, F_{Amn})$

where $(T_{Amn}, I_{Amn}, F_{Amn}) = (T_A(u_m, e_n), I_A(u_m, e_n), F_A(u_m, e_n))$

If $a_{ij} = (T_A(u_i, e_j), I_A(u_i, e_j), F_A(u_i, e_j))$, we can define a matrix

$$(\breve{a}_{ij})_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{pmatrix}$$

which is called an **neutrosophic soft matrix** of order $m \times n$ corresponding to the neutrosophic soft set $(\breve{F}_A, E)$ over U. A neutrosophic soft set $(\breve{F}_A, E)$ is uniquely characterized by the matrix $(\breve{a}_{ij})_{m \times n}$. Therefore we shall identify any neutrosophic soft set with its neutrosophic soft matrix and use these two concepts as interchangeable.

Example 3.1

Let U be the set of five towns, given by, $U = \{t_1, t_2, t_3, t_4, t_5\}$.

Let E be the set of parameters (each parameter is a neutrosophic word), given by,

$E = \{ \textit{highly}, \textit{immensely}, \textit{moderately}, \textit{average}, \textit{less} \} = \{e_1, e_2, e_3, e_4, e_5\}$ (say)

Let $A \subset E$, given by,

$A = \{e_1, e_2, e_3, e_5\}$ (say)

Now suppose that, $\breve{F}_A$ is a mapping, defined as populated towns(.) and given by,

$$\breve{F}_A(e_1) = \{t_1/(.2,.3,.7), t_2/(.8,.2,.1), t_3/(.4,.5,.2), t_4/(.6,.2,.3), t_5/(.7,.3,.2)\},$$
$$\breve{F}_A(e_2) = \{t_1/(0,0,1), t_2/(.9,.4,.1), t_3/(.3,.2,.6), t_4/(.4,.3,.6), t_5/(.6,.4,.3)\},$$
$$\breve{F}_A(e_3) = \{t_1/(.3,.3,.5), t_2/(.4,.2,.6), t_3/(.8,.5,.2), t_4/(.1,.4,.8), t_5/(.3,.5,.7)\},$$
$$\breve{F}_A(e_5) = \{t_1/(.9,.5,.2), t_2/(.1,.4,.8), t_3/(.5,.2,.4), t_4/(.3,.3,.5), t_5/(.2,.4,.8)\}$$

Then the Neutrosophic Soft Set

$$(\breve{F}_A, E) = \{\textit{highly populated town} = \{t_1/(.2,.3,.7), t_2/(.8,.2,.1), t_3/(.4,.5,.2), t_4/(.6,.2,.3), t_5/(.7,.3,.2)\},$$
$$\textit{immensely populated town} = \{t_1/(0,0,1), t_2/(.9,.4,.1), t_3/(.3,.2,.6), t_4/(.4,.3,.6), t_5/(.6,.4,.3)\},$$
$$\textit{moderately populated town} = \{t_1/(.3,.3,.5), t_2/(.4,.2,.6), t_3/(.8,.5,.2), t_4/(.1,.4,.8), t_5/(.3,.5,.7)\},$$
$$\textit{less populated town} = \{t_1/(.9,.5,.2), t_2/(.1,.4,.8), t_3/(.5,.2,.4), t_4/(.3,.3,.5), t_5/(.2,.4,.8)\}\}$$

Therefore the relation form of $(\breve{F}_A, E)$ is written by,

$$\breve{R}_A = \{(\{t_1/(.2,.3,.7), t_2/(.8,.2,.1), t_3/(.4,.5,.2), t_4/(.6,.2,.3), t_5/(.7,.3,.2)\}, e_1),$$
$$(\{t_1/(0,0,1), t_2/(.9,.4,.1), t_3/(.3,.2,.6), t_4/(.4,.3,.6), t_5/(.6,.4,.3)\}, e_2),$$
$$(\{t_1/(.3,.3,.5), t_2/(.4,.2,.6), t_3/(.8,.5,.2), t_4/(.1,.4,.8), t_5/(.3,.5,.7)\}, e_3),$$
$$(\{t_1/(.9,.5,.2), t_2/(.1,.4,.8), t_3/(.5,.2,.4), t_4/(.3,.3,.5), t_5/(.2,.4,.8)\}, e_5)\}$$

Hence the neutrosophic soft matrix $(\breve{a}_{ij})$ is written by,

$$(\breve{a}_{ij}) = \begin{pmatrix} (.2,.3,.7) & (0,0,1) & (.3,.3,.5) & (0,0,1) & (.9,.5,.2) \\ (.8,.2,.1) & (.9,.4,.1) & (.4,.2,.6) & (0,0,1) & (.1,.4,.8) \\ (.4,.5,.2) & (.3,.2,.6) & (.8,.5,.2) & (0,0,1) & (.5,.2,.4) \\ (.6,.2,.3) & (.4,.3,.6) & (.1,.4,.8) & (0,0,1) & (.3,.3,.5) \\ (.7,.3,.2) & (.6,.4,.3) & (.3,.5,.7) & (0,0,1) & (.2,.4,.8) \end{pmatrix}$$

Row-Neutrosophic Soft Matrix:

A neutrosophic soft matrix of order $1\times n$ i.e., with a single row is called a row-neutrosophic soft matrix. Physically, a row-neutrosophic soft matrix formally corresponds to a neutrosophic soft set whose universal set contains only one object.

Column-Neutrosophic Soft Matrix:

A neutrosophic soft matrix of order $m\times 1$ i.e., with a single column is called a **column-neutrosophic soft matrix**. Physically, a column-neutrosophic soft matrix formally corresponds to a neutrosophic soft set whose parameter set contains only one parameter.

Square Neutrosophic Soft Matrix:

A neutrosophic soft matrix of order $m\times n$ is said to be a **square neutrosophic soft matrix** if $m = n$ ie., the number of rows and the number of columns are equal. That means a square-neutrosophic soft matrix is formally equal to a neutrosophic soft set having the same number of objects and parameters.

Example 3.2

Consider the example 3.1

Here since the neutrosophic soft matrix $(\tilde{a}_{ij})$ contains five rows and five columns, so it is a square-neutrosophic soft matrix.

Complement of a neutrosophic soft matrix:

Let $(\tilde{a}_{ij})$ be an $m\times n$ neutrosophic soft matrix, where $\tilde{a}_{ij} = (T_{ij}, I_{ij}, F_{ij})\forall i, j$. Then the **complement** of $(\tilde{a}_{ij})$ is denoted by $(\tilde{a}_{ij})^o$ and is defined by,

$(\tilde{a}_{ij})^o = (\tilde{c}_{ij})$, where $(\tilde{c}_{ij})$ is also a neutrosophic soft matrix of order $m\times n$ and $\tilde{c}_{ij} = (F_{ij}, I_{ij}, T_{ij})\forall i, j$.

Example 3.3

Consider the example 3.1

Then the complement of $(\tilde{a}_{ij})$ is,

$$(\tilde{a}_{ij})^o = \begin{pmatrix} (.7,.3,.2) & (1,0,0) & (.5,.3,.3) & (1,0,0) & (.2,.5,.9) \\ (.1,.2,.8) & (.1,.4,.9) & (.6,.2,.4) & (1,0,0) & (.8,.4,.1) \\ (.2,.5,.4) & (.6,.2,.3) & (.2,.5,.8) & (1,0,0) & (.5,.3,.3) \\ (.3,.2,.6) & (.6,.3,.4) & (.8,.4,.1) & (1,0,0) & (.5,.3,.3) \\ (.2,.3,.7) & (.3,.4,.6) & (.7,.5,.3) & (1,0,0) & (.8,.4,.2) \end{pmatrix}$$

Null Neutrosophic Soft Matrix:

A neutrosophic soft matrix of order $m \times n$ is said to be a **null neutrosophic soft matrix or zero neutrosophic soft matrix** if all of its elements are $(0,0,1)$. A null neutrosophic soft matrix is denoted by, $\breve{\Phi}$. Now the neutrosophic soft set associated with a null neutrosophic soft matrix must be a null neutrosophic soft set.

Complete or Absolute Neutrosophic Soft Matrix:

A neutrosophic soft matrix of order $m \times n$ is said to be a **complete or absolute neutrosophic soft matrix** if all of its elements are $(1,0,0)$. A complete or absolute neutrosophic soft matrix is denoted by, C_A. Now the neutrosophic soft set associated with an absolute neutrosophic soft matrix must be an absolute neutrosophic soft set.

Diagonal Neutrosophic Soft Matrix:

A square neutrosophic soft matrix of order $m \times n$ is said to be a **diagonal-neutrosophic soft matrix** if all of its non-diagonal elements are $(0,0,1)$.

Choice Matrix:

It is a square matrix whose rows and columns both indicate parameters(which are neutrosophic words or sentences involving neutrosophic words). If $\breve{x}$ is a choice matrix, then its element $\breve{x}_{ij}$ is defined as follows:

$$\breve{x}_{ij} = \begin{cases} (1,0,0) \text{ } when\ i^{th}\ and\ j^{th}\ parameters\ are\ both\ choice\ parameters\ of\ the\ decision\ makers \\ (0,0,1) \text{ } otherwise, i.e.\ when\ atleast\ one\ of\ the\ i^{th}\ or\ j^{th}\ parameters\ be\ not\ under\ choice \end{cases}$$

There are different types of choice matrices according to the number of decision makers. We may realize this by the following example.

Example 3.4

Suppose that U be a set of four shopping malls, say, $U = \{s_1, s_2, s_3, s_4\}$

Let E be a set of parameters, given by,

$E = \{\ costly, excellent\ work\ culture, assured\ sale, good\ location, cheap\} = \{e_1, e_2, e_3, e_4, e_5\}$ (say)

Now let the neutrosophic soft set $(\breve{F}, A)$ describing the quality of the shopping malls, is given by,

$$\begin{aligned}(\breve{F}, E) \\ = \{\ & costly\ shopping\ malls = \{s_1/(0.9,0.1,0.3), s_2/(0.2,0.3,0.7), s_3/(0.4,0.3,0.5), s_4/(0.8,0.4,0.1)\}, \\ & shop.wt.exclnt.wrkculture. = \{s_1/(0.8,0.2,0.1), s_2/(0.3,0.4,0.5), s_3/(0.5,0.3,0.4), s_4/(0.4,0.2,0.5)\}, \\ & shop.wt.asrd.sale = \{s_1/(0.9,0.4,0.1), s_2/(0.2,0.2,0.7), s_3/(0.4,0.3,0.5), s_4/(0.8,0.5,0.1)\},\end{aligned}$$

$shop.\,wt.\,good\ location = \{s_1/(0.7,0.3,0.3), s_2/(0.9,0.6,0.1), s_3/(0.4,0.3,0.5), s_4/(0.8,0.1,0.2)\}$,
$cheap\ shop. = \{s_1/(0.1,0.4,0.8), s_2/(0.7,0.3,0.1), s_3/(0.5,0.2,0.3), s_4/(0.2,0.4,0.7)\}\}$

Suppose Mr.Ram wants to buy a shopping mall on the basis of his choice parameters excellent work culture, assured sale and cheap which form a subset A of the parameter set E.

Therefore $A = \{e_2, e_3, e_5\}$

Now **the choice matrix of Mr.Ram** is,

$$(x_{ij})_A = e_A \begin{pmatrix} & & e_A & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \end{pmatrix}$$

Now suppose Mr.Ram and Mr.Shyam together wants to buy a shopping mall according to their choice parameters. Let the choice parameter set of Mr.Shyam be, $B = \{e_1, e_2, e_3, e_4\}$ Then **the combined choice matrix of Mr.Ram and Mr.Shyam** is

$$(x_{ij})_{(A,B)} = e_A \begin{pmatrix} & & e_B & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \end{pmatrix}$$

[Here the entries $e_{ij} = (1,0,0)$ indicates that e_i is a choice parameter of Mr.Ram and e_j is a choice parameter of Mr.Shyam. Now $e_{ij} = (0,0,1)$ indicates either e_i fails to be a choice parameter of Mr.Ram or e_j fails to be a choice parameter of Mr.Shyam.]

Again the above combined choice matrix of Mr.Ram and Mr.Shyam may be also presented in its transpose form as,

$$
(\breve{x}_{ij})_{(B,A)} = e_B \begin{pmatrix} & & e_A & & \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}
$$

Now let us see the form of the combined choice matrix associated with three decision makers. Suppose that Mr.Kartik is willing to buy a shopping mall together with Mr.Ram and Mr.Shyam on the basis of his choice parameters excellent work culture, assured sale and good location which form a subset C of the parameter set E.

Therefore $C = \{e_2, e_3, e_4\}$

Then **the combined choice matrix of Mr.Ram, Mr.Shyam and Mr.Kartik** will be of three different types which are as follows,

$$
\text{(i)} \quad (\breve{x}_{ij})_{(C,A\wedge B)} = e_C \begin{pmatrix} & & e_{(A\wedge B)} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}
$$

[Since the set of common choice parameters of Mr.Ram and Mr.Shyam is, $A \wedge B = \{e_2, e_3\}$. Here the entries $\breve{e}_{ij} = (1,0,0)$ indicates that e_i is a choice parameter of Mr.Kartik and e_j is a common choice parameter of Mr.Ram and Mr.Shyam. Now $\breve{e}_{ij} = (0,0,1)$ indicates either e_i fails to be a choice parameter of Mr.Kartik or e_j fails to be a common choice parameter of Mr.Ram and Mr.Shyam.]

$$(\breve{x}_{ij})_{(A,B\wedge C)} = e_A \begin{pmatrix} & & e_{(B\wedge C)} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (1,0,0) & (0,0,1) \end{pmatrix} [\,Since\ B\wedge C = \{e_2, e_3, e_4\}]$$

(ii)

$$(\breve{x}_{ij})_{(B,C\wedge A)} = e_B \begin{pmatrix} & & e_{(C\wedge A)} & & \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix} [\,Since\ C\wedge A = \{e_2, e_3\}]$$

(iii)

Symmetric Neutrosophic Soft Matrix:

A square neutrosophic soft matrix $\breve{A}$ of order $n\times n$ is said to be a **symmetric neutrosophic soft matrix**, if its transpose be equal to it, i.e., if $\breve{A}^T = \breve{A}$. Hence the neutrosophic soft matrix $(\breve{a}_{ij})$ is symmetric, if $\breve{a}_{ij} = \breve{a}_{ji}, \forall i, j$.

Therefore if $(\breve{a}_{ij})$ be a symmetric neutrosophic soft matrix then the neutrosophic soft sets associated with $(\breve{a}_{ij})$ and $(\breve{a}_{ij})^T$ both be the same.

3.2 Operations

Transpose of a Square Neutrosophic Soft Matrix:

The transpose of a square neutrosophic soft matrix $(\breve{a}_{ij})$ of order $m\times n$ is another square neutrosophic soft matrix of order $n\times m$ obtained from $(\breve{a}_{ij})$ by interchanging its rows and columns. It is denoted by $(\breve{a}_{ij})^T$. Therefore the neutrosophic soft set associated with $(\breve{a}_{ij})^T$ becomes a new neutrosophic soft set over the same universe and over the same set of parameters.

Example 3.5

Consider the example 3.1. Here $(\breve{F}_A, E)$ be a neutrosophic soft set over the universe U and over the set of parameters E, given by,

$$(\breve{F}_A, E) = \{\textit{highly populated town} = \{t_1/(.2,.3,.7), t_2/(.8,.2,.1), t_3/(.4,.5,.2), t_4/(.6,.2,.3), t_5/(.7,.3,.2)\},$$
$$\textit{immensely populated town} = \{t_1/(0,0,1), t_2/(.9,.4,.1), t_3/(.3,.2,.6), t_4/(.4,.3,.6), t_5/(.6,.4,.3)\},$$
$$\textit{moderately populated town} = \{t_1/(.3,.3,.5), t_2/(.4,.2,.6), t_3/(.8,.5,.2), t_4/(.1,.4,.8), t_5/(.3,.5,.7)\},$$
$$\textit{less populated town} = \{t_1/(.9,.5,.2), t_2/(.1,.4,.8), t_3/(.5,.2,.4), t_4/(.3,.3,.5), t_5/(.2,.4,.8)\}\}$$

whose associated neutrosophic soft matrix is,

$$(\breve{a}_{ij}) = \begin{pmatrix} (.2,.3,.7) & (0,0,1) & (.3,.3,.5) & (0,0,1) & (.9,.5,.2) \\ (.8,.2,.1) & (.9,.4,.1) & (.4,.2,.6) & (0,0,1) & (.1,.4,.8) \\ (.4,.5,.2) & (.3,.2,.6) & (.8,.5,.2) & (0,0,1) & (.5,.2,.4) \\ (.6,.2,.3) & (.4,.3,.6) & (.1,.4,.8) & (0,0,1) & (.3,.3,.5) \\ (.7,.3,.2) & (.6,.4,.3) & (.3,.5,.7) & (0,0,1) & (.2,.4,.8) \end{pmatrix}$$

Now its transpose neutrosophic soft matrix is,

$$(\breve{a}_{ij})^T = \begin{pmatrix} (.2,.3,.7) & (.8,.2,.1) & (.4,.5,.2) & (.6,.2,.3) & (.7,.3,.2) \\ (0,0,1) & (.9,.4,.1) & (.3,.2,.6) & (.4,.3,.6) & (.6,.4,.3) \\ (.3,.3,.5) & (.4,.2,.6) & (.8,.5,.2) & (.1,.4,.8) & (.3,.5,.7) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (.9,.5,.2) & (.1,.4,.8) & (.5,.2,.4) & (.3,.3,.5) & (.2,.4,.8) \end{pmatrix}$$

Therefore the neutrosophic soft set associated with $(\breve{a}_{ij})^T$ is,

$$(\breve{G}_B, E) = \{\textit{highly populated town} = \{t_1/(.2,.3,.7), t_2/(0,0,1), t_3/(.3,.3,.5), t_4/(0,0,1), t_5/(.9,.5,.2)\},$$
$$\textit{immensely populated town} = \{t_1/(.8,.2,.1), t_2/(.9,.4,.1), t_3/(.4,.2,.6), t_4/(0,0,1), t_5/(.1,.4,.8)\},$$
$$\textit{moderately populated town} = \{t_1/(.4,.5,.2), t_2/(.3,.2,.6), t_3/(.8,.5,.2), t_4/(0,0,1), t_5/(.5,.2,.4)\},$$
$$\textit{average populated town} = \{t_1/(.6,.2,.3), t_2/(.4,.3,.6), t_3/(.1,.4,.8), t_4/(0,0,1), t_5/(.3,.3,.5)\},$$
$$\textit{less populated town} = \{t_1/(.7,.3,.2), t_2/(.6,.4,.3), t_3/(.3,.5,.7), t_4/(0,0,1), t_5/(.2,.4,.8)\}\}$$

where B = { highly, immensely, moderately, average, less } $\subseteq E$ and G_B is a mapping from B to N^U.

Addition of Neutrosophic Soft Matrices:

Two neutrosophic soft matrices $\breve{A}$ and $\breve{B}$ are said to be conformable for addition, if they be of the same order. The addition of two neutrosophic soft matrices $(\breve{a}_{ij})$ and $(\breve{b}_{ij})$ of order $m \times n$ is defined by,

$(\breve{a}_{ij}) \oplus (\breve{b}_{ij}) = (\breve{c}_{ij})$, where $(\breve{c}_{ij})$ is also an $m \times n$ neutrosophic soft matrix and

$$\breve{c}_{ij} = (max\{T_{\breve{a}_{ij}}, T_{\breve{b}_{ij}}\}, \frac{I_{\breve{a}_{ij}} + I_{\breve{b}_{ij}}}{2}, min\{F_{\breve{a}_{ij}}, F_{\breve{b}_{ij}}\}) \forall i, j.$$

Example 3.6

Consider the neutrosophic soft matrix of example 3.1,

$$(\breve{a}_{ij}) = \begin{pmatrix} (.2,.3,.7) & (0,0,1) & (.3,.3,.5) & (0,0,1) & (.9,.5,.2) \\ (.8,.2,.1) & (.9,.4,.1) & (.4,.2,.6) & (0,0,1) & (.1,.4,.8) \\ (.4,.5,.2) & (.3,.2,.6) & (.8,.5,.2) & (0,0,1) & (.5,.2,.4) \\ (.6,.2,.3) & (.4,.3,.6) & (.1,.4,.8) & (0,0,1) & (.3,.3,.5) \\ (.7,.3,.2) & (.6,.4,.3) & (.3,.5,.7) & (0,0,1) & (.2,.4,.8) \end{pmatrix}$$

Now consider another neutrosophic soft matrix $(\breve{b}_{ij})$ associated with the neutrosophic soft set $(\breve{G}_B, E)$ (also describing the pollution of the cities) over the same universe U.

Let $B = \{e_1, e_4, e_5\} \subset E$ and

$$\begin{aligned}(\breve{G}, B) = \{ & \textit{highly populated town} = \{t_1/(.3,.5,.7), t_2/(.9,.3,.1), t_3/(.4,.4,.5), t_4/(.7,.3,.2), t_5/(.6,.4,.2)\}, \\ & \textit{average populated town} = \{t_1/(.2,.4,.7), t_2/(.3,.5,.7), t_3/(.7,.2,.1), t_4/(.2,.6,.8), t_5/(.3,.4,.6)\}, \\ & \textit{less populated town} = \{t_1/(.8,.3,.1), t_2/(.2,.4,.7), t_3/(.6,.5,.4), t_4/(.3,.2,.5), t_5/(.2,.4,.6)\}\}\end{aligned}$$

and then the relation form of $(\breve{G}_B, E)$ is written by,

$$\begin{aligned}\breve{R}_B = \{ & (\{t_1/(.3,.5,.7), t_2/(.9,.3,.1), t_3/(.4,.4,.5), t_4/(.7,.3,.2), t_5/(.6,.4,.2)\}, e_1), \\ & (\{t_1/(.2,.4,.7), t_2/(.3,.5,.7), t_3/(.7,.2,.1), t_4/(.2,.6,.8), t_5/(.3,.4,.6)\}, e_2), \\ & (\{t_1/(.8,.3,.1), t_2/(.2,.4,.7), t_3/(.6,.5,.4), t_4/(.3,.2,.5), t_5/(.2,.4,.6)\}, e_5)\}\end{aligned}$$

Hence the neutrosophic soft matrix $(\breve{b}_{ij})$ is written by,

$$(\breve{b}_{ij}) = \begin{pmatrix} (.3,.5,.7) & (0,0,1) & (0,0,1) & (.2,.4,.7) & (.8,.3,.1) \\ (.9,.3,.1) & (0,0,1) & (0,0,1) & (.3,.5,.7) & (.2,.4,.7) \\ (.4,.4,.5) & (0,0,1) & (0,0,1) & (.7,.2,.1) & (.6,.5,.4) \\ (.7,.3,.2) & (0,0,1) & (0,0,1) & (.2,.6,.8) & (.3,.2,.5) \\ (.6,.4,.2) & (0,0,1) & (0,0,1) & (.3,.4,.6) & (.2,.4,.6) \end{pmatrix}$$

Therefore the sum of the neutrosophic soft matrices $(\breve{a}_{ij})$ and $(\breve{b}_{ij})$ is,

$$(\breve{a}_{ij}) \oplus (\breve{b}_{ij}) = \begin{pmatrix} (0.3,0.4,0.7) & (0,0,1) & (0.3,0.15,0.5) & (0.2,0.2,0.7) & (0.9,0.4,0.1) \\ (0.9,0.25,0.1) & (0.9,0.2,0.1) & (0.4,0.1,0.6) & (0.3,.25,0.7) & (0.2,0.4,0.7) \\ (0.4,0.45,0.2) & (0.3,0.1,0.6) & (0.8,0.25,0.1) & (0.7,0.1,0.1) & (0.6,0.35,0.4) \\ (0.7,0.25,0.2) & (0.4,0.15,0.6) & (0.1,0.2,0.8) & (0.2,0.3,0.8) & (0.3,0.25,0.5) \\ (0.7,0.35,0.2) & (0.6,0.2,0.3) & (0.3,.25,0.7) & (0.3,0.2,0.6) & (0.2,0.4,0.6) \end{pmatrix}$$

Subtraction of Neutrosophic Soft Matrices:

Two neutrosophic soft matrices $\breve{A}$ and $\breve{B}$ are said to be conformable for subtraction, if they be of the same order. For any two neutrosophic soft matrices $(\breve{a}_{ij})$ and $(\breve{b}_{ij})$ of order $m\times n$, the subtraction of $(\breve{b}_{ij})$ from $(\breve{a}_{ij})$ is defined as,

$(\breve{a}_{ij})!(\breve{b}_{ij}) = (\breve{c}_{ij})$, where $(\breve{c}_{ij})$ is also an $m\times n$ neutrosophic soft matrix and

$\breve{c}_{ij} = (min\{T_{\breve{a}_{ij}}, T_{\breve{b}^o_{ij}}\}, \frac{I_{\breve{a}_{ij}} + I_{\breve{b}_{ij}}}{2}, max\{F_{\breve{a}_{ij}}, F_{\breve{b}^o_{ij}}\}) \forall i, j$ where $(\breve{b}^o_{ij})$ is the complement of $(\breve{b}_{ij})$

Example 3.7

Consider the neutrosophic soft matrices $(\breve{a}_{ij})$ and $(\breve{b}_{ij})$ of example 3.6. Now,

$$(\breve{a}_{ij}) = \begin{pmatrix} (.2,.3,.7) & (0,0,1) & (.3,.3,.5) & (0,0,1) & (.9,.5,.2) \\ (.8,.2,.1) & (.9,.4,.1) & (.4,.2,.6) & (0,0,1) & (.1,.4,.8) \\ (.4,.5,.2) & (.3,.2,.6) & (.8,.5,.2) & (0,0,1) & (.5,.2,.4) \\ (.6,.2,.3) & (.4,.3,.6) & (.1,.4,.8) & (0,0,1) & (.3,.3,.5) \\ (.7,.3,.2) & (.6,.4,.3) & (.3,.5,.7) & (0,0,1) & (.2,.4,.8) \end{pmatrix} \text{ and}$$

$$(\breve{b}_{ij})^o = \begin{pmatrix} (.7,.5,.3) & (1,0,0) & (1,0,0) & (.7,.4,.2) & (.1,.3,.8) \\ (.1,.3,.9) & (1,0,0) & (1,0,0) & (.7,.5,.3) & (.7,.4,.2) \\ (.5,.4,.4) & (1,0,0) & (1,0,0) & (.1,.2,.7) & (.4,.5,.6) \\ (.2,.3,.7) & (1,0,0) & (1,0,0) & (.8,.6,.2) & (.5,.2,.3) \\ (.2,.4,.6) & (1,0,0) & (1,0,0) & (.6,.4,.3) & (.6,.4,.2) \end{pmatrix}$$

Therefore the subtraction of the neutrosophic soft matrix $(\tilde{b}_{ij})$ from the neutrosophic soft matrix $(\tilde{a}_{ij})$ is,

$$(\tilde{a}_{ij})!(\tilde{b}_{ij}) = \begin{pmatrix} (.2,.4,.7) & (0,0,1) & (0.3,.15,.5) & (0,0.2,1) & (.1,.4,.8) \\ (.1,.25,.9) & (0.9,.2,.1) & (0.4,.1,.6) & (0,.25,1) & (.1,.4,.8) \\ (.4,.45,.4) & (0.3,.1,.6) & (0.8,.25,.2) & (0,.1,1) & (.4,.35,.6) \\ (.2,.25,.7) & (0.4,.15,.6) & (0.1,.2,.8) & (0,0.3,1) & (.3,.25,.5) \\ (.2,.35,.6) & (0.6,.2,.3) & (0.3,.25,.7) & (0,0.2,1) & (.2,.4,.8) \end{pmatrix}$$

Product of a Neutrosophic Soft Matrix with a Choice Matrix:

Let U be the set of universe and E be the set of parameters. Suppose that $\tilde{A}$ be any neutrosophic soft matrix and $\tilde{b}$ be any choice matrix of a decision maker concerned with the same universe U and E. Now if the number of columns of the neutrosophic soft matrix $\tilde{A}$ be equal to the number of rows of the choice matrix $\tilde{b}$, then $\tilde{A}$ and $\tilde{b}$ are said to be conformable for the product ($\tilde{A}\otimes\tilde{b}$) and the product ($\tilde{A}\otimes\tilde{b}$) becomes a neutrosophic soft matrix. We may denote the product by $\tilde{A}\otimes\tilde{b}$ or simply by $\tilde{A}\tilde{b}$.

If $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$ and $\tilde{b} = (\tilde{b}_{jk})_{n\times p}$, then $\tilde{A}\tilde{b} = (\tilde{c}_{ik})$

where $\tilde{c}_{ik} = (max_{j=1}^{n} min\{T_{\tilde{a}_{ij}}, T_{\tilde{b}_{jk}}\}, average_{j=1}^{n} average\{I_{\tilde{a}_{ij}}, I_{\tilde{b}_{jk}}\}, min_{j=1}^{n} max\{F_{\tilde{a}_{ij}}, F_{\tilde{b}_{jk}}\})$

It is to be noted that, $\tilde{b}\tilde{A}$ cannot be defined here.

Example 3.8

Let U be the set of four dresses, given by, $U = \{d_1, d_2, d_3, d_4\}$. Let E be the set of parameters, given by, $E = \{$ *cheap*, *beautiful*, *comfortable*, *gorgeous* $\} = \{e_1, e_2, e_3, e_4\}$ (say). Suppose that the set of choice parameters of Mr.X be, $A = \{e_1, e_3\}$. Now let according to the choice parameters of Mr.X, we have the neutrosophic soft set $(\tilde{F}, A)$ which describes the attractiveness of the dresses and the neutrosophic soft matrix of the neutrosophic soft set $(\tilde{F}, A)$ be,

$$(\tilde{a}_{ij}) = \begin{pmatrix} (0.8,0.4,0.1) & (0.2,0.3,0.7) & (0.7,0.2,0.2) & (0.3,0.1,0.5) \\ (0.3,0.4,0.6) & (0.7,0.3,0.1) & (0.4,0.2,0.6) & (0.8,0.5,0.1) \\ (0.7,0.3,0.2) & (0.4,0.2,0.5) & (0.5,0.1,0.3) & (0.6,0.4,0.2) \\ (0.5,0.2,0.4) & (0.1,0.5,0.8) & (0.9,0.6,0.1) & (0.2,0.3,0.7) \end{pmatrix}$$

Again the choice matrix of Mr.X is,

$$(\breve{x}_{ij})_A = e_A\begin{pmatrix} & e_A & & \\ (1,0,0) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}$$

Since the number of columns of the neutrosophic soft matrix $(\breve{a}_{ij})$ is equal to the number of rows of the choice matrix $(\breve{x}_{ij})_A$, they are conformable for the product. Therefore

$$\begin{pmatrix} (0.8,0.4,0.1) & (0.2,0.3,0.7) & (0.7,0.2,0.2) & (0.3,0.1,0.5) \\ (0.3,0.4,0.6) & (0.7,0.3,0.1) & (0.4,0.2,0.6) & (0.8,0.5,0.1) \\ (0.7,0.3,0.2) & (0.4,0.2,0.5) & (0.5,0.1,0.3) & (0.6,0.4,0.2) \\ (0.5,0.2,0.4) & (0.1,0.5,0.8) & (0.9,0.6,0.1) & (0.2,0.3,0.7) \end{pmatrix} \otimes \begin{pmatrix} (1,0,0) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.2375,0.1) & (0.0,0.2375,1.0) & (0.8,0.2375,0.1) & (0.0,0.2375,1.0) \\ (0.4,0.175,0.6) & (0.0,0.175,1.0) & (0.4,0.175,0.6) & (0.0,0.175,1.0) \\ (0.7,0.2375,0.2) & (0.0,0.2375,1.0) & (0.7,0.2375,0.2) & (0.0,0.2375,1.0) \\ (0.9,0.2,0.1) & (0.0,0.2,1.0) & (0.9,0.2,0.1) & (0.0,0.2,1.0) \end{pmatrix}$$

3.3 Properties

Let $\breve{A}$ and $\breve{B}$ be two neutrosophic soft matrices of order $m \times n$. Then

(i) $\breve{A} \oplus \breve{B} = \breve{B} \oplus \breve{A}$

(ii) $(\breve{A} \oplus \breve{B}) \oplus \breve{C} = \breve{A} \oplus (\breve{B} \oplus \breve{C})$

(iii) $\breve{A} ! \breve{B} \neq \breve{B} ! \breve{A}$

(iv) $(\breve{A} ! \breve{B}) ! \breve{C} \neq \breve{A} ! (\breve{B} ! \breve{C})$

(v) $\breve{A} \oplus \breve{A}^o \neq C_A$

(vi) $\breve{A} ! \breve{A} \neq \breve{\Phi}$

Proof: The proofs of (i)-(vi) are directly obtained from the definitions of addition, subtraction and complement.

3.4 Theorems

Theorem 1: If A be a square neutrosophic soft matrix of order $n\times n$, then $(A^T)^T = A$

Proof: Let $A=(\tilde{a}_{ij})_{n\times n}$.

Then by definition, $A^T = (\tilde{b}_{ij})_{n\times n}$ *where* $b_{ij} = a_{ji} \forall i, j$. *ie.,* $A^T = (\tilde{a}_{ji})_{n\times n}$.

Therefore $(A^T)^T = (\tilde{c}_{ij})_{n\times n}$ *where* $c_{ij} = a_{ij}$ *ie.,* $(A^T)^T = (\tilde{a}_{ij})_{n\times n} = A$ (Proved)

Theorem 2: If A and B be two square neutrosophic soft matrices of order $n\times n$, then $(A\oplus B)^T = A^T \oplus B^T$

Proof: Let $A=(\tilde{a}_{ij})_{n\times n}$ and $B=(\tilde{b}_{ij})_{n\times n}$. Then $L.H.S = (A\oplus B)^T = C^T$ *where* $C=(\tilde{c}_{ij})_{n\times n}$

$$= (\tilde{c}_{ji})_{n\times n} \text{ where } c_{ji} = (max\{T_{a_{ji}}, T_{b_{ji}}\}, \frac{I_{a_{ji}} + I_{b_{ji}}}{2}, min\{F_{a_{ji}}, F_{b_{ji}}\}) \forall i, j$$

and

$$R.H.S = A^T \oplus B^T = (\tilde{a}_{ji})_{n\times n} \oplus (\tilde{b}_{ji})_{n\times n}$$

$$= (\tilde{d}_{ji})_{n\times n} \text{ where } d_{ji} = (max\{T_{a_{ji}}, T_{b_{ji}}\}, \frac{I_{a_{ji}} + I_{b_{ji}}}{2}, min\{F_{a_{ji}}, F_{b_{ji}}\}) \forall i, j = C^T = L.H.S$$

Hence $(A\oplus B)^T = A^T \oplus B^T$ (Proved)

Theorem 3: If A be a square neutrosophic soft matrix of order $n\times n$, then $(A\oplus A^T)$ is symmetric.

Proof: Let $A=(\tilde{a}_{ij})_{n\times n}$.

Then by definition, $A^T = (\tilde{a}_{ji})_{n\times n}$. Now

$$A\oplus A^T = (\tilde{a}_{ij})_{n\times n} \oplus (\tilde{a}_{ji})_{n\times n}$$

$$= (\tilde{c}_{ij})_{n\times n} \text{ where } c_{ij} = (max\{T_{a_{ij}}, T_{a_{ji}}\}, \frac{I_{a_{ij}} + I_{a_{ji}}}{2}, min\{F_{a_{ij}}, F_{a_{ji}}\}) \forall i, j.$$

Now $c_{ji} = (max\{T_{a_{ji}}, T_{a_{ij}}\}, \frac{I_{a_{ji}} + I_{a_{ij}}}{2}, min\{F_{a_{ji}}, F_{a_{ij}}\}) = c_{ij} \forall i, j$

Therefore $(c_{ij})_{n\times n}$ ie., $(A\oplus A^T)$ is symmetric. (Proved)

Theorem 4:

If A and B be two square neutrosophic soft matrices of order $n\times n$ and if A and B be symmetric, then $A\oplus B$ is symmetric.

Proof: Since A and B be symmetric,

$$A^T = A \text{ and } B^T = B$$

Therefore $A^T \oplus B^T = A\oplus B$

Thus from Theorem 2 we have,

$$(A\oplus B)^T = A^T \oplus B^T = A\oplus B$$

Hence $A\oplus B$ is symmetric. (Proved)

4 A Generalized Neutrosophic Soft Set Based Group Decision Making Problem

Let N number of decision makers want to select an object jointly from the m number of objects which have n number of features i.e., parameters(E). Suppose that each decision maker has freedom to take his decision of inclusion and evaluation of parameters associated with the selected object, i.e., each decision maker has his own choice parameters belonging to the parameter set E and has his own view of evaluation. Here it is assumed that the parameter evaluation of the objects by the decision makers must be neutrosophic and may be presented in linguistic form or neutrosophic soft set format, alternatively, in the form of neutrosophic soft matrix. Now the problem is to find out the object out of these m objects which satisfies all the choice parameters of all decision makers jointly as much as possible.

5 A New Approach to Solve Neutrosophic Soft Set Based Group Decision Making Problems

This new approach is specially based on choice matrices. These choice matrices represent the choice parameters of the decision makers and also help us to solve the neutrosophic soft set (or neutrosophic soft matrix) based decision making problems with least computational complexity.

The Stepwise Solving Procedure: To solve such type of neutrosophic soft set (or neutrosophic soft matrix) based decision making problems, we are presenting the following stepwise procedure which comprises of the newly proposed choice matrices, neutrosophic soft matrices and the operations on them.

NSM-Algorithm

Step 1: If the parameter evaluation of the objects by the decision makers are not given in neutrosophic soft matrix form, then first construct the neutrosophic soft matrices according to the given evaluations.

Step 2: Construct the combined choice matrix with respect to the choice parameters of the decision makers.

Step 3: Compute the product neutrosophic soft matrices by multiplying each given neutrosophic soft matrix with the combined choice matrix as per the rule of multiplication of neutrosophic soft matrices.

Step 4: Compute the sum of these product neutrosophic soft matrices to have the resultant neutrosophic soft matrix(R_{NS}).

Step 5: Then compute the weight of each object(O_i) by adding the true-membership values of the entries of its concerned row(i-th row) of R_{NS} and denote it as $W(O_i)$.

Step 6: The object having the highest weight becomes the optimal choice object. If more than one object have the highest weight then go to the next step.

Step 7: Now we have to consider the sum of the falsity-membership values (Θ) of the entries of the rows associated with those equal weighted objects. The object with the minimum Θ-value will be the optimal choice object. Now if the Θ-values of those objects also be the same, then go to the next step.

Step 8: Now consider the sum of the indeterminacy values (Ψ) of the entries of the rows associated with those equal Θ-valued objects. Now if the Ψ-values of those objects also be the same, any one of them may be chosen as the optimal choice object.

To illustrate the basic idea of the NSM-algorithm, now we apply it to the following neutrosophic soft set (or, neutrosophic soft matrix) based decision making problems.

Example 5.1: Let U be the set of four story books, given by, $U = \{b_1, b_2, b_3, b_4\}$. Let E be the set of parameters, given by, $E = \{\,romantic, thriller, comic, horror\,\} = \{e_1, e_2, e_3, e_4\}$ (say). Suppose that, three sisters Shayana, Dayana and Nayna together want to buy a story book among these four books for their youngest brother Ohm according to their choice parameters, $P = \{e_1, e_3\}, Q = \{e_2, e_3\}, R = \{e_1, e_4\}$ respectively. Now let according to the choice parameter evaluation of the books by Shayana, Dayana and Nayna, we have the neutrosophic soft sets $(\breve{F}_P, E)$, $(\breve{G}_Q, E)$, $(\breve{H}_R, E)$ which describe the nature of the books according to Shayana, Dayana and Nayna respectively and given by,

$$(\breve{F}_A, E) = \{\,romantic\,books = \{b_1/(0.9,0.6,0.1), b_2/(0.3,0.2,0.5), b_3/(0.7,0.4,0.1), b_4/(0.2,03,0.7)\},$$
$$comic\,books = \{b_1/(1,0,0), b_2/(0.6,0.4,0.3), b_3/(0.3,0.2,0.5), b_4/(0.2,0.3,0.7)\}\}$$

$$(\breve{G}_B, E) = \{thriller\,books = \{b_1/(0.4,0.3,0.6), b_2/(0.8,0.6,0.1), b_3/(0.5,0.1,0.2), b_4/(0.3,0.2,0.5)\},$$
$$comic\,books = \{b_1/(0.8,0.5,0.1), b_2/(0.6,0.3,0.2), b_3/(0.4,0.2,0.5), b_4/(0.2,0.3,0.8)\}\}$$

$$(\breve{H}_C, E) = \{\,romantic\,books = \{b_1/(0.9,0.6,0.1), b_2/(0.4,0.2,0.5), b_3/(0.6,0.1,0.3), b_4/(0.3,0.4,0.5)\},$$
$$horror\,books = \{b_1/(0.2,0.3,0.7), b_2/(0.3,0.1,0.5), b_3/(0.6,0.4,0.2), b_4/(0.9,0.2,0)\}\}$$

The problem is to select the story book among the four books which satisfies the choice parameters of Shayana, Dayana and Nayna as much as possible.

Now let us apply our newly proposed NSM -algorithm to solve this problem.

(1) The neutrosophic soft matrices of the neutrosophic soft sets $(\breve{F}_P, E), (\breve{G}_Q, E)$ and $(\breve{H}_R, E)$ are respectively,

$$(\breve{p}_{ij}) = \begin{pmatrix} (0.9,0.6,0.1) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0.3,0.2,0.5) & (0,0,1) & (0.6,0.4,0.3) & (0,0,1) \\ (0.7,0.4,0.1) & (0,0,1) & (0.3,0.2,0.5) & (0,0,1) \\ (0.2,0.3,0.7) & (0,0,1) & (0.2,0.3,0.7) & (0,0,1) \end{pmatrix},$$

$$(\breve{q}_{ik}) = \begin{pmatrix} (0,0,1) & (0.4,0.3,0.6) & (0.8,0.5,0.1) & (0,0,1) \\ (0,0,1) & (0.8,0.6,0.1) & (0.6,0.3,0.2) & (0,0,1) \\ (0,0,1) & (0.5,0.1,0.2) & (0.4,0.2,0.5) & (0,0,1) \\ (0,0,1) & (0.3,0.2,0.5) & (0.2,0.3,0.8) & (0,0,1) \end{pmatrix},$$

$$(\breve{r}_{il}) = \begin{pmatrix} (0.9,0.6,0.1) & (0,0,1) & (0,0,1) & (0.2,0.3,0.7) \\ (0.4,0.2,0.5) & (0,0,1) & (0,0,1) & (0.3,0.1,0.5) \\ (0.6,0.1,0.3) & (0,0,1) & (0,0,1) & (0.6,0.4,0.2) \\ (0.3,0.4,0.5) & (0,0,1) & (0,0,1) & (0.9,0.2,0) \end{pmatrix}$$

(2) The combined choice matrices of Shayana, Dayana and Nayna in different forms are,

$$e_P \begin{pmatrix} & e_{Q\wedge R} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix} \text{[Since } Q \wedge R = f, P = \{e_1, e_3\}]$$

$$e_Q \begin{pmatrix} & e_{R\wedge P} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (0,0,1) & (0,0,1) & (0,0,01) \\ (1,0,0) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix} \text{ [Since } R\wedge P=\{e_1\}, Q=\{e_2,e_3\}]$$

$$e_R \begin{pmatrix} & e_{P\wedge Q} & & \\ (0,0,1) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,0,0) & (0,0,1) \end{pmatrix} \text{ [Since } P\wedge Q=\{e_3\}, R=\{e_1,e_4\}]$$

(3) Corresponding product neutrosophic soft matrices are,

$$U_P \begin{pmatrix} & e_P & & \\ (0.9,0.6,0.1) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0.3,0.2,0.5) & (0,0,1) & (0.6,0.4,0.3) & (0,0,1) \\ (0.7,0.4,0.1) & (0,0,1) & (0.3,0.2,0.5) & (0,0,1) \\ (0.2,0.3,0.7) & (0,0,1) & (0.2,0.3,0.7) & (0,0,1) \end{pmatrix} \otimes$$

$$e_P \begin{pmatrix} & e_{Q\wedge R} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix} = \begin{pmatrix} (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \end{pmatrix}$$

$$U_Q \begin{pmatrix} & e_Q & & \\ (0,0,1) & (0.4,0.3,0.6) & (0.8,0.5,0.1) & (0,0,1) \\ (0,0,1) & (0.8,0.6,0.1) & (0.6,0.3,0.2) & (0,0,1) \\ (0,0,1) & (0.5,0.1,0.2) & (0.4,0.2,0.5) & (0,0,1) \\ (0,0,1) & (0.3,0.2,0.5) & (0.2,0.3,0.8) & (0,0,1) \end{pmatrix} \otimes$$

$$e_Q\begin{pmatrix} & e_{R\wedge P} & & \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (0,0,1) & (0,0,1) & (0,0,01) \\ (1,0,0) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}$$

$$=\begin{pmatrix} (0.8,0.1,0.1) & (0,0.1,1) & (0,0.1,1) & (0,0.1,1) \\ (0.8,0.1125,0.1) & (0,0.1125,1) & (0,0.1125,1) & (0,0.1125,1) \\ (0.5,0.375,0.2) & (0,0.375,1) & (0,0.375,1) & (0,0.375,1) \\ (0.3,0.625,0.5) & (0,0.625,1) & (0,0.625,1) & (0,0.625,1) \end{pmatrix}$$

$$U_R\begin{pmatrix} & e_R & & \\ (0.9,0.6,0.1) & (0,0,1) & (0,0,1) & (0.2,0.3,0.7) \\ (0.4,0.2,0.5) & (0,0,1) & (0,0,1) & (0.3,0.1,0.5) \\ (0.6,0.1,0.3) & (0,0,1) & (0,0,1) & (0.6,0.4,0.2) \\ (0.3,0.4,0.5) & (0,0,1) & (0,0,1) & (0.9,0.2,0) \end{pmatrix} \otimes$$

$$e_R\begin{pmatrix} & e_{P\wedge Q} & & \\ (0,0,1) & (0,0,1) & (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (1,0,0) & (0,0,1) \end{pmatrix} = \begin{pmatrix} (0,0.1125,1) & (0,0.1125,1) & (0.9,0.1125,0.1) & (0,0.1125,1) \\ (0,0.375,1) & (0,0.375,1) & (0.4,0.375,0.5) & (0,0.375,1) \\ (0,0.625,1) & (0,0.625,1) & (0.6,0.625,0.2) & (0,0.625,1) \\ (0,0.15,1) & (0,0.15,1) & (0.9,0.15,0) & (0,0.15,1) \end{pmatrix}$$

[As per the rule of multiplication of neutrosophic soft matrices.]

(4) The sum of these product neutrosophic soft matrices is,

$$\begin{pmatrix} (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \\ (0,0.75,1) & (0,0.75,1) & (0,0.75,1) & (0,0.75,1) \end{pmatrix}$$

$$\oplus\begin{pmatrix} (0.8,0.1,0.1) & (0,0.1,1) & (0,0.1,1) & (0,0.1,1) \\ (0.8,0.1125,0.1) & (0,0.1125,1) & (0,0.1125,1) & (0,0.1125,1) \\ (0.5,0.375,0.2) & (0,0.375,1) & (0,0.375,1) & (0,0.375,1) \\ (0.3,0.625,0.5) & (0,0.625,1) & (0,0.625,1) & (0,0.625,1) \end{pmatrix}$$

$$\oplus\begin{pmatrix} (0,0.1125,1) & (0,0.1125,1) & (0.9,0.1125,0.1) & (0,0.1125,1) \\ (0,0.375,1) & (0,0.375,1) & (0.4,0.375,0.5) & (0,0.375,1) \\ (0,0.625,1) & (0,0.625,1) & (0.6,0.625,0.2) & (0,0.625,1) \\ (0,0.15,1) & (0,0.15,1) & (0.9,0.15,0) & (0,0.15,1) \end{pmatrix}$$

$$=\begin{pmatrix} (0.8,0.32083,0.1) & (0,0.32083,1) & (0.9,0.32083,0.1) & (0,0.32083,1) \\ (0.8,0.4125,0.1) & (0,0.4125,1) & (0.4,0.4125,0.5) & (0,0.4125,1) \\ (0.5,0.5833,0.2) & (0,0.5833,1) & (0.6,0.5833,0.2) & (0,0.5833,1) \\ (0.3,0.5083,0.5) & (0,0.5083,1) & (0.9,0.5083,0) & (0,0.5083,1) \end{pmatrix} = R_{NS}$$

(5) Now the weights of the story books are,

- $W(b_1) = 0.8+0+0.9+0 = 1.7$
- $W(b_2) = 0.8+0+0.4+0 = 1.2$
- $W(b_3) = 0.5+0+0.6+0 = 1.1$
- $W(b_4) = 0.3+0+0.9+0 = 1.2$

(6) The story book associated with the first row of the resultant neutrosophic soft matrix(R_{NS}) has the highest weight $(W(d_1) = 1.7)$, therefore b_1 be the optimal choice book. Hence Shayana, Dayana and Nayna will buy the story book b_1 for their youngest brother according to their choice parameters.

6 Applications in Medical Science

In medical science there also exist various types of neutrosophic soft set based decision making problems and we may apply the NSM -Algorithm for solving those problems. Now we will discuss two different problems which appear very common in medical science with their solutions.

Problem 6.1 Generally in medical science a patient suffering from a disease may have multiple symptoms. Again it is also observed that there are certain symptoms which may be common to more than one diseases leading to diagnostic dilemma.

Now we consider from medical science (Hauser, Stephen, Braunwald, Fauci and Kasper 2001a, 2001b; Tierney 2003) four symptoms such as abdominal pain, fever, nausea vomiting, diarrhea which have more or less contribution in four diseases such as typhoid, peptic ulcer, food poisoning, acute viral hepatitis. Now, from medical statistics, the degree of availability of these four symptoms in these four diseases are observed as follows. The degree of belongingness of all the symptoms abdominal pain, fever, nausea vomiting and diarrhea for the diseases typhoid, peptic ulcer, food poisoning and acute viral hepatitis are $\{(0.3,0.2,0.6),(0.8,0.5,0.1),(0.1,0.4,0.7),(0.2,0.3,0.7)\},\{(0.9,0.6,0.1),(0.2,0.3,0.6),(0.1,0.7,0.8),(0.1,0.2,0.7)\},\{(0.6,0.4,0.2),(0.3,0.2,0.6),(0.6,0.4,0.3),(0.7,0.3,0.2)\}$ and $\{(0.2,0.4,0.6),(0.6,0.1,0.2),(0.5,0.3,0.4),(0.1,0.5,0.7)\}$ respectively.

Suppose a patient who is suffering from a disease, have the symptoms P (abdominal pain, fever and diarrhea). Now the problem is how a doctor detects the actual disease among these four diseases for that patient. Now we will solve this problem by applying NSM -Algorithm.

Here $U = \{typhoid, \ peptic\ ulcer, \ food\ \ poisoning, acute\ viral\ hepatitis\} = \{d_1, d_2, d_3, d_4\}$,

$E = \{abdominal\ \ pain, \ fever, nausea\ vomiting, diarrhea\} = \{e_1, e_2, e_3, e_4\}$ and $P = \{e_1, e_2, e_4\} \subset E$

(1) The neutrosophic soft matrix obtained from the given data is,

$$(\breve{a}_{ij}) = \begin{pmatrix} (0.3,0.2,0.6) & (0.8,0.5,0.1) & (0.1,0.4,0.7) & (0.2,0.3,0.7) \\ (0.9,0.6,0.1) & (0.2,0.3,0.6) & (0.1,0.7,0.8) & (0.1,0.2,0.7) \\ (0.6,0.4,0.2) & (0.3,0.2,0.6) & (0.6,0.4,0.3) & (0.7,0.3,0.2) \\ (0.2,0.4,0.6) & (0.6,0.1,0.2) & (0.5,0.3,0.4) & (0.1,0.5,0.7) \end{pmatrix}$$

(2) The choice matrix of the patient is,

$$e_P \begin{pmatrix} & e_P & & \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \end{pmatrix} \text{[Since } P = \{e_1, e_2, e_4\}]$$

(3) and (4) Corresponding product neutrosophic soft matrix is,

$$U_A\begin{pmatrix} & e_P & & \\ (0.3,0.2,0.6) & (0.8,0.5,0.1) & (0.1,0.4,0.7) & (0.2,0.3,0.7) \\ (0.9,0.6,0.1) & (0.2,0.3,0.6) & (0.1,0.7,0.8) & (0.1,0.2,0.7) \\ (0.6,0.4,0.2) & (0.3,0.2,0.6) & (0.6,0.4,0.3) & (0.7,0.3,0.2) \\ (0.2,0.4,0.6) & (0.6,0.1,0.2) & (0.5,0.3,0.4) & (0.1,0.5,0.7) \end{pmatrix} \otimes$$

$$e_P\begin{pmatrix} & e_P & & \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \end{pmatrix}$$

$$=\begin{pmatrix} (0.8,0.175,0.1) & (0.8,0.175,0.1) & (0,0.175,1) & (0.8,0.175,0.1) \\ (0.9,0.225,0.1) & (0.9,0.225,0.1) & (0,0.225,1) & (0.9,0.225,0.1) \\ (0.7,0.1625,0.2) & (0.7,0.1625,0.2) & (0,0.1625,1) & (0.7,0.1625,0.2) \\ (0.6,0.1625,0.2) & (0.6,0.1625,0.2) & (0,0.1625,1) & (0.6,0.1625,0.2) \end{pmatrix} = R_{NS}$$

[As per the rule of multiplication of neutrosophic soft matrices.]

(5) Now the weights of the diseases are,

- $W(d_1) = 0.8+0.8+0+0.8 = 2.4$
- $W(d_2) = 0.9+0.9+0+0.9 = 2.7$
- $W(d_3) = 0.7+0.7+0+0.7 = 2.1$
- $W(d_4) = 0.6+0.6+0+0.6 = 1.8$

(6) The disease associated with the second row of the resultant neutrosophic soft matrix(R_{NS}) has the highest weight $(W(d_2) = 2.7)$, therefore d_2 be the optimal choice disease. **Hence the patient is suffering from the disease peptic ulcer(d_2).**

Problem 6.2: In medical science (Carranza 2006) there are different types of diseases and various types of reasons are responsible for them. Now suppose that according to Dr.X. personal habits are responsible for dental caries (0.7,0.1,0.2), for gum disease (0.8,0.2,0.1), for oral ulcer (0.8,0.4,0.2); food habits are responsible for dental caries (0.8,0.4,0.1), for gum disease (0.7,0.3,0.3), for oral ulcer (0.4,0.2,0.5). Again let according to Dr.Y personal habits are responsible for dental caries (0.6,0.2,0.3), for gum disease (0.8,0.1,0.2), for oral ulcer (0.9,0.4,0.1); food habits are responsible for dental caries $(0.8,0.2,0.1)$, for gum disease $(0.7,0.3,0.2)$, for oral ulcer $(0.5,0.2,0.4)$ and hereditary factor is also responsible for dental caries $(0.2,0.5,0.7)$, for gum disease $(0.4,0.2,0.3)$, for oral ulcer $(0.6,0.1,0.3)$. Now the problem is to find out the disease which is mostly affected by the personal habits, food habits and hereditary factors of a human being according to both Dr.X and Dr.Y simultaneously.

Now we will solve this problem by applying NSM-Algorithm.

Here $U = \{dental\ caries, gum\ disease, oral\ ulcer\} = \{d_1, d_2, d_3\}$,

$E = \{personal\ habits, food\ habits, hereditary\ factors\} = \{e_1, e_2, e_3\}$. The choice parameter set of Dr.X is, $A = \{e_1, e_2\} \subset E$ and the choice parameter set of Dr.Y is, $A = \{e_1, e_2, e_3\} \subseteq E$

(1) The neutrosophic soft matrices according to Dr.X and Dr.Y are respectively,

$$(\breve{a}_{ij}) = \begin{pmatrix} (0.7,0.1,0.2) & (0.8,0.4,0.1) & (0,0,1) \\ (0.8,0.2,0.1) & (0.7,0.3,0.3) & (0,0,1) \\ (0.8,0.4,0.2) & (0.4,0.2,0.5) & (0,0,1) \end{pmatrix}$$

$$(\breve{b}_{ik}) = \begin{pmatrix} (0.6,0.2,0.3) & (0.8,0.2,0.1) & (0.2,0.5,0.7) \\ (0.8,0.1,0.2) & (0.7,0.3,0.2) & (0.4,0.2,0.3) \\ (0.9,0.4,0.1) & (0.5,0.2,0.4) & (0.6,0.1,0.3) \end{pmatrix}$$

(2) The combined choice matrices of Mr.X and Mr.Y in different forms are,

$$e_A \begin{pmatrix} & e_B & \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}$$

$$e_B \begin{pmatrix} & e_A & \\ (1,0,0) & (1,0,0) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) \end{pmatrix}$$

(3) Corresponding product neutrosophic soft matrices are,

$$U_A\begin{pmatrix} & e_A & \\ (0.7,0.1,0.2) & (0.8,0.4,0.1) & (0,0,1) \\ (0.8,0.2,0.1) & (0.7,0.3,0.3) & (0,0,1) \\ (0.8,0.4,0.2) & (0.4,0.2,0.5) & (0,0,1) \end{pmatrix} \otimes \quad e_A\begin{pmatrix} & e_B & \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) & (0,0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.0833,0.1) & (0.8,0.0833,0.1) & (0.8,0.0833,0.1) \\ (0.8,0.0833,0.1) & (0.8,0.0833,0.1) & (0.8,0.0833,0.1) \\ (0.8,0.1,0.2) & (0.8,0.1,0.2) & (0.8,0.1,0.2) \end{pmatrix}$$

$$U_B\begin{pmatrix} & e_B & \\ (0.6,0.2,0.3) & (0.8,0.2,0.1) & (0.2,0.5,0.7) \\ (0.8,0.1,0.2) & (0.7,0.3,0.2) & (0.4,0.2,0.3) \\ (0.9,0.4,0.1) & (0.5,0.2,0.4) & (0.6,0.1,0.3) \end{pmatrix} \otimes \quad e_E\begin{pmatrix} & e_A & \\ (1,0,0) & (1,0,0) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) \\ (1,0,0) & (1,0,0) & (0,0,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.15,0.1) & (0.8,0.15,0.1) & (0,0.15,1) \\ (0.8,0.1,0.2) & (0.8,0.1,0.2) & (0,0.1,1) \\ (0.9,0.1166,0.1) & (0.9,0.1166,0.1) & (0,0.1166,1) \end{pmatrix}$$

(4) The sum of these product neutrosophic soft matrices is,

$$\begin{pmatrix} (0.8,0.0833,0.1) & (0.8,0.0833,0.1) & (0.8,0.0833,0.1) \\ (0.8,0.0833,0.1) & (0.8,0.0833,0.1) & (0.8,0.0833,0.1) \\ (0.8,0.1,0.2) & (0.8,0.1,0.2) & (0.8,0.1,0.2) \end{pmatrix} \oplus$$

$$\begin{pmatrix} (0.8,0.15,0.1) & (0.8,0.15,0.1) & (0,0.15,1) \\ (0.8,0.1,0.2) & (0.8,0.1,0.2) & (0,0.1,1) \\ (0.9,0.1166,0.1) & (0.9,0.1166,0.1) & (0,0.1166,1) \end{pmatrix}$$

$$= \begin{pmatrix} (0.8,0.11665,0.1) & (0.8,0.11665,0.1) & (0.8,0.11665,0.1) \\ (0.8,0.09165,0.1) & (0.8,0.09165,0.1) & (0.8,0.09165,0.1) \\ (0.9,0.1083,0.1) & (0.9,0.1083,0.1) & (0.8,0.1083,0.2) \end{pmatrix} = R_{NS}$$

(5) Now the weights of the diseases are,

- $W(d_1) = 0.8 + 0.8 + 0.8 = 2.4$
- $W(d_2) = 0.8 + 0.8 + 0.8 = 2.4$
- $W(d_3) = 0.9 + 0.9 + 0.8 = 2.6$

(6) The disease associated with the third row of the resultant neutrosophic soft matrix(R_{NS}) has the highest weight $(W(d_3) = 2.6)$, therefore d_3 be the optimal choice disease. **Hence oral ulcer(d_3) is mostly affected by the personal habits, food habits and hereditary factor according to the both doctors.**

7 Conclusion

In this paper we have proposed the concept of neutrosophic soft matrix and after that different types of matrices in neutrosophic soft set theory have been defined. Then we have introduced here some new operations and properties on these matrices. Furthermore an efficient solution procedure named as NSM -Algorithm has been developed to solve neutrosophic soft set(or neutrosophic soft matrix) based group decision making problems and it has been applied in medical science to the problems of diagnosis of a disease from the myriad of symptoms as well as to evaluate the effectiveness of different habits of human being responsible for a disease.

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