Florentin Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications (book), Pons Editions, Belgium, 323 pp., 2017.

CHAPTER X

X.1. Neutrosophic Multiset

Let $\mathcal{U}$ be a universe of discourse, and $M \subset \mathcal{U}$.

X.1.1. Definition

A *Neutrosophic Multiset* M is a neutrosophic set where one or more elements are repeated with the same neutrosophic components, or with different neutrosophic components.

X.1.2. Examples

$A = \{a(0.6, 0.3, 0.1), b(0.8, 0.4, 0.2), c(0.5, 0.1, 0.3)\}$ is a neutrosophic set (not a neutrosophic multiset).

But

$B = \{a(0.6, 0.3, 0.1), a(0.6, 0.3, 0.1), b(0.8, 0.4, 0.2)\}$

is a neutrosophic multiset, since the element a is repeated; we say that the element a has *neutrosophic multiplicity 2* with the same neutrosophic components.

While

$$C = \begin{Bmatrix} a(0.6, 0.3, 0.1), a(0.7, 0.1, 0.2), \\ a(0.5, 0.4, 0.3), c(0.5, 0.1, 0.3) \end{Bmatrix}$$

is also a neutrosophic multiset, since the element a is repeated (it has *neutrosophic multiplicity 3*), but with different neutrosophic components, since, for example, during the time, the neutrosophic membership of an element may change.

If the element a is repeated k times keeping the same neutrosophic components (t_a, i_a, f_a), we say that a has *multiplicity* k.

But if there is some change in the neutrosophic components of a, we say that a has the *neutrosophic multiplicity* k.

Therefore, we define in general the *Neutrosophic Multiplicity Function*:

$nm: \mathcal{U} \to \mathbb{N}$,

where $\mathbb{N} = \{1, 2, 3, \dots, \infty\}$,

and for any $a \in A$ one has (1)

$$nm(a) = \{(k_1, \langle t_1, i_1, f_1 \rangle), (k_2, \langle t_2, i_2, f_2 \rangle), \dots, (k_j, \langle t_j, i_j, f_j \rangle), \dots\}$$

which means that a is repeated k_1 times with the neutrosophic components $\langle t_1, i_1, f_1 \rangle$; a is repeated k_2 times with the neutrosophic components $\langle t_2, i_2, f_2 \rangle$, ..., a is repeated k_j times with the neutrosophic components $\langle t_j, i_j, f_j \rangle$, ..., and so on.

Of course, all $k_1, k_2, \dots, k_j, \dots \in \mathbb{N}$, and $\langle t_p, i_p, f_p \rangle \neq \langle t_r, i_r, f_r \rangle$, for $p \neq r$, with $p, r \in \mathbb{N}$.

Also, all neutrosophic components are with respect to the set A. Then, a neutrosophic multiset A can be written as:

$\big(A, nm(a)\big)$

or $\{(a, nm(a), \text{for } a \in A)\}$.

X.1.3. Examples of operations with neutrosophic multisets.

Let's have:

$A = \{5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.4,0.1,0.3\rangle}, 6_{\langle 0.2,0.7,0.0\rangle}\}$

$B = \{5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.8,0.1,0.1\rangle}, 6_{\langle 0.9,0.0,0.0\rangle}\}$

$C = \{5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\}$.

Then:

X.1.3.1. Intersection of Neutrosophic Multisets.

$A \cap B = \{5_{\langle 0.6,0.3,0.2\rangle}\}$.

X.1.3.2. Union of Neutrosophic Multisets.

$$A \cup B = \left\{\begin{matrix} 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.4,0.1,0.3\rangle}, 5_{\langle 0.8,0.1,0.1\rangle}, \\ 6_{\langle 0.2,0.7,0.0\rangle}, 6_{\langle 0.9,0.0,0.0\rangle} \end{matrix}\right\}.$$

X.1.3.3. Inclusion of Neutrosophic Multisets.

$C \subset A$, but $C \not\subset B$

X.1.3.4. Cardinality of Neutrosophic Multisets.

$Card(A) = 4$, and $Card(B) = 3$, where $Card(\cdot)$ means cardinal.

X.1.3.5. Cartesian Product of Neutrosophic Multisets.

$$B \times C = \left\{ \begin{array}{c} \left(5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right), \left(5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right), \\ \left(5_{\langle 0.8,0.1,0.1\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right), \left(5_{\langle 0.8,0.1,0.1\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right), \\ \left(6_{\langle 0.9,0.0,0.0\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right), \left(6_{\langle 0.9,0.0,0.0\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}\right) \end{array} \right\}.$$

X.1.3.6. Difference of Neutrosophic Multisets.

$$A - B = \{5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.4,0.1,0.3\rangle}, 6_{\langle 0.2,0.7,0.0\rangle}\}$$

$$A - C = \{5_{\langle 0.4,0.1,0.3\rangle}, 6_{\langle 0.2,0.7,0.0\rangle}\}$$

$$C - B = \{5_{\langle 0.6,0.3,0.2\rangle}\}$$

X.1.3.7. Sum of Neutrosophic Multisets.

$$A \uplus B = \left\{ \begin{array}{c} 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.4,0.1,0.3\rangle}, 5_{\langle 0.8,0.1,0.1\rangle}, \\ 6_{\langle 0.2,0.7,0.9\rangle}, 6_{\langle 0.9,0.0,0.0\rangle} \end{array} \right\}$$

$$B \uplus B = \left\{ \begin{array}{c} 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.6,0.3,0.2\rangle}, 5_{\langle 0.8,0.1,0.1\rangle}, 5_{\langle 0.8,0.1,0.1\rangle}, \\ 6_{\langle 0.9,0.0,0.0\rangle}, 6_{\langle 0.9,0.0,0.0\rangle} \end{array} \right\}.$$

Let's compute the neutrosophic multiplicity function, with respect to several of the previous neutrosophic multisets.

$$nm_A: A \to \mathbb{N}$$

$nm_A(5) = \{(2, \langle 0.6, 0.3, 0.2 \rangle), (1, \langle 0.4, 0.1, 0.3 \rangle)\}$

$nm_A(6) = \{(1, \langle 0.2, 0.7, 0.0 \rangle)\}$.

$nm_B: B \to \mathbb{N}$

$nm_B(5) = \{(1, \langle 0.6, 0.3, 0.2 \rangle), (1, \langle 0.8, 0.1, 0.1 \rangle)\}$

$nm_B(6) = \{(1, \langle 0.2, 0.7, 0.0 \rangle)\}$.

$nm_C: C \to \mathbb{N}$

$nm_C(5) = \{(2, \langle 0.6, 0.3, 0.2 \rangle)\}$

References

1. Eric W. Weisstein, *Multiset*, MathWorld, A Wolfram Web Resource.
http://mathworld.wolfram.com/Multiset.html

2. F. Smarandache, *Neutrosphic Multiset Applied in Physical Proceses*, Actualization of the Internet of Things, a FIAP Industrial Physics Conference, Monterey, California, Jan. 2017

X.2. Neutrosophic Multiset Applied in Physical Processes

Let *U* be a universe of discourse and a set $M \subseteq U$. The *Neutrosophic Multiset M* is defined as a neutrosophic set with the property that one or more elements are repeated either with the same neutrosophic components, or with different neutrosophic components.

For example, *Q = {a(0.6,0.3,0.2), a(0.6,0.3,0.2), a(0.5,0.4,0.1), b(0.7,0.1,0.1)}* is a neutrosophic multiset.

The Neutrosophic Multiplicity Function is defined as:

$nm: U \rightarrow N = \{1, 2, 3, \dots\}$,

and for each x∈M one has

$$nm(x) = \{(k_1, < t_1, i_1, f_1 >,), (k_2, < t_2, i_2, f_2 >), \dots, (k_j, < t_j, i_j, f_j >), \dots\}, \quad (1)$$

which means that in the set *M* the element *x* is repeated k_1 times with the neutrosophic components $<t_1,i_1,f_1>$, and k_2 times with the neutrosophic components $<t_2,i_2,f_2>)$, …, k_j times with the neutrosophic components $<t_j,i_j,f_j>$), … and so on. Of course, $<t_p,i_p,f_p>$) $\neq$ $<t_r,i_r,f_r>$ for $p \neq r$.

For example, with respect to the above neutrosophic multiset Q,

$$nm(a) = \{(2, <0.6,0.3,0.2>), (1, <0.5,0.4,0.1>)\}.$$

Neutrosophic multiset is used in time series, and in representing instances of the physical process at different times, since its neutrosophic components change in time.

X.3. Neutrosophic Complex Multiset

Let $\mathcal{U}$ be a universe of discourse, and $\mathcal{S} \subset \mathcal{U}$.

A *Neutrosophic Complex Multiset* $\mathcal{S}$ is a neutrosophic complex set, which has one or more elements that repeat either with the same complex neutrosophic components, or with different other complex neutrosophic components.

Example of Neutrosophic Complex Set.

$$B_1 = \begin{Bmatrix} a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right), \\ b\left(0.5e^{j(0.4)}, 0.2e^{j(0.3)}, 0.1e^{j(0.2)}\right) \end{Bmatrix}$$

is a neutrosophic complex set.

Examples of Neutrosophic Complex Multiset.

$$B_2 = \begin{Bmatrix} a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right), \\ a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right) \end{Bmatrix}$$

is a neutrosophic complex multiset because the element a repeats with the same neutrosophic complex components.

$$B_3 = \begin{Bmatrix} a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right), \\ a\left(0.4e^{j(0.3)}, 0.2e^{j(0.1)}, 0.7e^{j(0.4)}\right), \\ b\left(0.5e^{j(0.4)}, 0.2e^{j(0.3)}, 0.1e^{j(0.2)}\right) \end{Bmatrix}.$$

is a neutrosophic complex multiset because the element a repeats, but with different neutrosophic complex components.

$$B_4 = \left\{ \begin{array}{l} a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right), \\ a\left(0.3e^{j(0.2)}, 0.1e^{j(0.1)}, 0.8e^{j(0.5)}\right), \\ a\left(0.7e^{j(0.6)}, 0.2e^{j(0.1)}, 0.1e^{j(0.0)}\right), \\ b\left(0.7e^{j(0.2)}, 0.0e^{j(0.3)}, 0.4e^{j(0.2)}\right) \end{array} \right\}.$$

is a neutrosophic complex multiset because the element "a" repeats once with the same neutrosophic components, and afterwards with different neutrosophic components.

Similarly, we define the *Neutrosophic Complex Multiplicity Function*:

$ncm: \mathcal{U} \to N = \{1, 2, 3, \dots\}$

for $a \in \mathcal{S}$ one has

$$ncm(a): \{(k_1, < t_1 e_{j\alpha_1}, i_1 e_{j\beta_1}, f_1 e_{j\gamma_1} >), (k_2, < t_2 e_{j\alpha_2}, i_2 e_{j\beta_2}, f_2 e_{j\gamma_2} >), \dots, (k_n, < t_n e_{j\alpha_n}, i_n e_{j\beta_n}, f_n e_{j\gamma_n} >), \dots\}.$$

Whence, a neutrosophic complex multiset $\mathcal{S}$ can be written as $(\mathcal{S}, ncm(a))$ or $\{(a, ncm(a)), \text{for } a \in \mathcal{S}\}$.

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The Neutrosophic Multisets and the Neutrosophic Multiset Algebraic Structures were introduced by Florentin Smarandache in 2016.