Neutrosophic Ideal Theory
Neutrosophic Local Function and Generated Neutrosophic Topology

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ABSTRACT

Abstract In this paper we introduce the notion of ideals on neutrosophic set which is considered as a generalization of fuzzy and fuzzy intuitionistic ideals studies in [9,11] , the important neutrosophic ideals has been given in [4]. The concept of neutrosophic local function is also introduced for a neutrosophic topological space. These concepts are discussed with a view to find new nutrosophic topology from the original one in [8]. The basic structure, especially a basis for such generated neutrosophic topologies and several relations between different neutrosophic ideals and neutrosophic topologies are also studied here. Possible application to GIS topology rules are touched upon.

KEYWORDS: Neutrosophic Set, Intuitionistic Fuzzy Ideal, Fuzzy Ideal, Neutrosophic Ideal, Neutrosophic Topology.

1-INTRODUCTION

The neutrosophic set concept was introduced by Smarandache [12, 13]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9, 10]. The fuzzy set was introduced by Zadeh [14] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama at el [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α-cut and neutrosophic ideals, which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2-TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [12, 13], and Salama at el. [4, 5, 6, 7, 8, 9, 10].

3- NEUTROSOPHIC IDEALS [4].

Definition.3.1

Let X is non-empty set and L a non–empty family of NSs. We will call L is a neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

- $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic ideal L is called a σ-neutrosophic ideal if $\{A_j\}_{j \in N} \leq L$, implies $\vee_{j \in J} A_j \in L$ (countable additivity).

The smallest and largest neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X. Also, $N.L_f$, $N.L_c$ are denoting the neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NS : B \subseteq A\}$ is an NL on X. This is called the principal NL of all NSs of denoted by NL $\langle A \rangle$.

Remark 3.1

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $O_N \in L$.

Example.3.1

Any Initiutionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$.

Example.3.2

Let $X = \{a, b, c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family $L = \{O_N, A, B, D\}$ of NSs is an NL on X.

Example.3.3

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

X	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.3.3

Let L_1 and L_2 be two NL on X. Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2 if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2.

Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in NSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.3.1

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic ideal on X,

In fact L is the smallest upper bound of the set of the L_j in the ordered set of all neutrosophic ideals on X.

Remark.3.2

The neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all neutrosophic ideals on X.

Proposition.3.3

A neutrosophic set A in neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof

(Necessity)Suppose A is a base of L. Then clearly every member of L contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.3.4

For a neutrosophic ideal L_1 with base A, is finer than a fuzzy ideal L_2 with base B iff every member of B contained in A.

Proof

Immediate consequence of Definitions

Corollary.3.1

Two neutrosophic ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.3.1

Let $\eta = \{\langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J\}$ be a non empty collection of neutrosophic subsets of X. Then there exists a neutrosophic ideal L (η) = {A $\in$ NSs: A $\subseteq \vee$ A_j} on X for some finite collection {A_j: j = 1,2, ..., n $\subseteq \eta$}.

Proof

Clear.

Remark.3.3

ii) The neutrosophic ideal L (η) defined above is said to be generated by η and η is called sub base of L(η).

Corollary.3.2

Let L_1 be an neutrosophic ideal on X and A $\in$ NSs, then there is a neutrosophic ideal L_2 which is finer than L_1

and such that A $\in L_2$ iff

A $\vee$ B $\in L_2$ for each B $\in L_1$.

Corollary.3.3

Let $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in L_1$ and $B = \langle x, \mu_B, \sigma_B, \nu_B \rangle \in L_2$, where L_1 and L_2 are neutrosophic ideals on the set X. then the neutrosophic set $A*B = \langle \mu_{A*B}(x), \sigma_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ on X where $\mu_{A*B}(x) = \vee \{\mu_A(x) \wedge \mu_B(x) : x \in X\}$, $\sigma_{A*B}(x)$ may be $= \vee \{\sigma_A(x) \wedge \sigma_B(x)\}$ or $\wedge \{\sigma_A(x) \vee \sigma_B(x)\}$ and $\nu_{A*B}(x) = \wedge \{\nu_A(x) \vee \nu_B(x) : x \in X\}$.

4. Neutrosophic local Functions

Definition.4.1. Let (X,τ) be a neutrosophic topological spaces (NTS for short) and L be neutrsophic ideal (NL, for short) on X. Let A be any NS of X. Then the neutrosophic local function $NA^*(L,\tau)$ of A is the union of all neutrosophic points(NP, for short) $C(\alpha,\beta,\gamma)$ such that if $U \in N(C(\alpha,\beta,\gamma))$ and $NA^*(L,\tau) = \vee \{C(\alpha,\beta,\gamma) \in X : A \wedge U \notin L$ for every U nbd of $C(\alpha,\beta,\gamma)\}$, $NA^*(L,\tau)$ is called a neutrosophic local function of A with respect to τ and L which it will be denoted by $NA^*(L,\tau)$, or simply $NA^*(L)$.

Example .4.1. One may easily verify that.

If L={0_N}, then N $A^*(L,\tau) = Ncl(A)$, for any neutrosophic set $A \in NSs$ on X.

If L = all NSs on X then $NA^*(L,\tau) = 0_N$, for any $A \in NSs$ on X .

Theorem.4.1. Let (X,τ) be a NTS and L_1, L_2 be two neutrosophic ideals on X. Then for any neutrosophic sets A, B of X. then the following statements are verified

i) $A \subseteq B \Rightarrow NA^*(L,\tau) \subseteq NB^*(L,\tau)$,

ii) $L_1 \subseteq L_2 \Rightarrow NA^*(L_2,\tau) \subseteq NA^*(L_1,\tau)$.

iii) $NA^* = Ncl(A^*) \subseteq Ncl(A)$.

iv) $NA^{**} \subseteq NA^*$.

v) $N(A \vee B)^* = NA^* \vee NB^*$.,

vi) $N(A \wedge B)^*(L) \leq NA^*(L) \wedge NB^*(L)$.

vii) $\ell \in L \Rightarrow N(A \vee \ell)^* = NA^*$.

viii) $NA^*(L,\tau)$ is neutrosophic closed set .

Proof.

i) Since $A \subseteq B$, let $p = C(\alpha,\beta,\gamma) \in NA^*(L_1)$ then $A \wedge U \notin L$ for every $U \in N(p)$. By hypothesis we get $B \wedge U \notin L$, then $p = C(\alpha,\beta,\gamma) \in NB^*(L_1)$.

ii) Clearly. $L_1 \subseteq L_2$ implies $NA^*(L_2,\tau) \subseteq NA^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p = C(\alpha,\beta,\gamma) \in NA^*$ but $C(\alpha,\beta,\gamma)$ may not be contained in $NA^*(L_2)$.

iii) Since $\{O_N\} \subseteq L$ for any NL on X, therefore by (ii) and Example 3.1, $NA^*(L) \subseteq NA^*(O_N) = Ncl(A)$ for any NS A on X. Suppose $p_1 = C_1(\alpha,\beta,\gamma) \in Ncl(NA^*(L_1))$. So for every $U \in N(p_1)$, $NA^* \wedge U \neq O_N$, there exists $p_2 = C_2(\alpha,\beta) \in A^*(L_1) \wedge U)$ such that for every V nbd of $p_2 \in N(p_2)$, $A \wedge U \notin L$. Since $U \wedge V \in N(p_2)$ then $A \wedge (U \cap V) \notin L$ which leads to $A \wedge U \notin L$, for every $U \in N(C(\alpha,\beta))$ therefore $p_1 = C(\alpha,\beta) \in (A^*(L))$

and so $Ncl(NA^*) \leq NA^*$ While, the other inclusion follows directly. Hence $NA^* = Ncl(NA^*)$.But the inequality $NA^* \leq Ncl(NA^*)$.

iv) The inclusion $NA^* \vee NB^* \leq N(A \vee B)^*$ follows directly by (i). To show the other implication, let $p = C(\alpha,\beta,\gamma) \in N(A \vee B)^*$ then for every $U \in N(p)$, $(A \vee B) \wedge U \notin L, i.e, (A \wedge U) \vee (B \wedge U) \notin L.$ then, we have two cases $A \wedge U \notin L$ and $B \wedge U \in L$ or the converse, this means that exist $U_1, U_2 \in N(C(\alpha,\beta,\gamma))$ such that $A \wedge U_1 \notin L$, $B \wedge U_1 \notin L$, $A \wedge U_2 \notin L$ and $B \wedge U_2 \notin L$. Then $A \wedge (U_1 \wedge U_2) \in L$ and $B \wedge (U_1 \wedge U_2) \in L$ this gives $(A \vee B) \wedge (U_1 \wedge U_2) \in L$, $U_1 \wedge U_2 \in N(C(\alpha,\beta,\gamma))$ which contradicts the hypothesis. Hence the equality holds in various cases.

vi) By (iii), we have $NA^{**} = Ncl(NA^*)^* \leq Ncl(NA^*) = NA^*$

Let (X,τ) be a GIFTS and L be GIFL on X . Let us define the neutrosophic closure operator $cl^*(A) = A \cup A^*$ for any GIFS A of X. Clearly, let $Ncl^*(A)$ is a neutrosophic operator. Let $N\tau^*(L)$ be NT generated by Ncl^* .i.e $N\tau^*(L) = \{A : Ncl^*(A^c) = A^c\}$. Now $L = \{O_N\} \Rightarrow Ncl^*(A) = A \cup NA^* = A \cup Ncl(A)$ for every neutrosophic set A. So, $N\tau^*(\{O_N\}) = \tau$. Again $L = \{all$ NSs on X$\} \Rightarrow Ncl^*(A) = A$, because $NA^* = O_N$, for every neutrosophic set A so $N\tau^*(L)$ is the neutrosophic discrete topology on X. So we can conclude by Theorem 4.1.(ii). $N\tau^*(\{O_N\}) = N\tau^*(L)$ i.e. $N\tau \subseteq N\tau^*$, for any neutrosophic ideal L_1 on X. In particular, we have for two neutrosophic ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow N\tau^*(L_1) \subseteq N\tau^*(L_2)$.

Theorem.4.2. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any neutrosophic ideal L on X, $\tau_1 \leq \tau_2$ implies $NA^*(L,\tau_2) \subseteq NA^*(L,\tau_1)$, for every A $\in L$ then $N\tau^*_1 \subseteq N\tau^*_2$

Proof. Clear.

A basis $N\beta(L,\tau)$ for $N\tau^*(L)$ can be described as follows:

$N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Then we have the following theorem

Theorem 4.3. $N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Forms a basis for the generated NT of the NT (X,τ) with neutrosophic ideal L on X.

Proof. Straight forward.

The relationship between τ and $_N\tau^*$(L) established throughout the following result which have an immediately proof .

Theorem 4.4. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any neutrosophic ideal L on X, $\tau_1 \subseteq \tau_2$ implies $N\tau^*_1 \subseteq N\tau^*_2$.

Theorem 4.5 : Let (X,τ) be a NTS and L_1, L_2 be two neutrosophic ideals on X . Then for any neutrosophic set A in X, we have

i) $NA^*(L_1 \vee L_2,\tau) = NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$ .ii) $N\tau^*(L_1 \vee L_2) = (N\tau^*(L_1))^*(L_2) \wedge N(\tau^*(L_2))^*(L_1)$

Proof Let $p = C(\alpha,\beta) \notin (L_1 \vee L_2, \tau)$, this means that there exists $U_p \in N(P)$ such that $A \wedge U_p \in (L_1 \vee L_2)$ i.e. There exists $\ell_1 \in L_1$ and $\ell_2 \in L_2$ such that $A \wedge U_p \in (\ell_1 \vee \ell_2)$ because of the heredity of L_1, and assuming $\ell_1 \wedge \ell_2 = O_N$.Thus we have $(A \wedge U_p) - \ell_1 = \ell_2$ and $(A \wedge U_p) - \ell_2 = \ell_1$ therefore $(U_p - \ell_1) \wedge A = \ell_2 \in L_2$ and $(U_p - \ell_2) \wedge A = \ell_1 \in L_1$. Hence $p = C(\alpha,\beta,\gamma) \notin NA^*(L_2, N\tau^*(L_1))$ or $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$, because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $NA^*(L_1 \vee L_2,\tau) \geq NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$. .To show the second inclusion, let us assume $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$. This implies that there exist $U_p \in N(P)$ and $\ell_2 \in L_2$ such that $(U_p - \ell_2) \wedge A \in L_1$. By the heredity of L_2, if we assume that $\ell_2 \leq A$ and define $\ell_1 = (U_p - \ell_2) \wedge A$. Then we have $A \wedge U_p \in (\ell_1 \vee \ell_2) \in L_1 \vee L_2$. Thus,

$NA^*(L_1 \vee L_2, \tau) \leq NA^*(L_1, \tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$ and similarly, we can get $A^*(L_1 \vee L_2, \tau) \leq A^*(L_2, \tau^*(L_1))$. This gives the other inclusion, which complete the proof.

Corollary 4.1. Let (X, τ) be a NTS with neutrosophic ideal L on X. Then

i) $NA^*(L,\tau) = NA^*(L,\tau^*)$ and $N\tau^*(L) = N(N\tau^*(L))^*(L)$.

ii) $N\tau^*(L_1 \vee L_2) = (N\tau^*(L_1)) \vee (N\tau^*(L_2))$

Proof. Follows by applying the previous statement.

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