Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The purpose of this paper is to introduce a new type of neutrosophic crisp set as the *- neutrosophic crisp sets as a generalization to star intuitionistic set due to Indira et al. [4], and study some of its properties. Finally we introduce and study the notion of *- neutrosophic relation and some of its properties.

Keywords: Neutrosophic Crisp Set; Star Intuitionistic Sets; Neutrosophic Relations; Neutrosophic Data.

1 Introduction
The fundamental concepts of neutrosophic set, introduced by Smarandache in [31, 32, 33], and Salama et al. in [5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 12, 22, 34] such as a neutrosophic set theory. In this paper we introduce a new type of neutrosophic crisp set as the *- neutrosophic crisp set, and study some of its properties. Finally we introduce and study the notion of *- neutrosophic relation and some of its properties. Possible applications to mathematical computer are touched upon.

2 Terminologies
We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [31, 32, 33], and Salama et al. in [5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where \[ \left[ 0, 1 \right] \] is nonstandard unit interval.

3 *- Neutrosophic Crisp Sets
We shall now consider some possible definitions for a new type of neutrosophic crisp set

Definition 3.1
Let \( X \) be a non-empty fixed set. A neutrosophic crisp set (NCS for short) \( A \) is an object having the form 
\[
A = \left\{ A_1, A_2, A_3 \right\}
\]
Then we define the *- neutrosophic set \( A^* \) as 
\[
A^* = \left\{ A_1 \cap (A_2 \cup A_3)^c, A_2 \cap (A_1 \cup A_3)^c, A_3 \cap (A_1 \cup A_2)^c \right\}
\]
where \( A_1, A_2 \) and \( A_3 \) are subsets of \( X \) such that 
\[
M = A_1 \cap (A_2 \cup A_3)^c, \quad S = A_2 \cap (A_1 \cup A_3)^c \quad \text{and} \quad R = A_3 \cap (A_1 \cup A_2)^c.
\]
A *- neutrosophic crisp set is an object having the form 
\[
A^* = \left\{ M, S, R \right\}
\]

Lemma 3.1
Let \( X \) be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) \( A \) is an object having the form 
\[
A = \left\{ A_1, A_2, A_3 \right\}
\]
Then we define the *- neutrosophic relation and some of its properties. Possible applications to mathematical computer are touched upon.
\[ R_1 = A_3 \cap (A_1 \cup A_2)^c, \quad M_2 = B_1 \cap (B_2 \cup B_3)^c, \]
\[ S_2 = B_2 \cap (B_1 \cup B_3)^c, \quad \text{and} \]
\[ R_2 = B_3 \cap (B_1 \cup B_2). \]
Then \( A \subseteq B \) implies \( A^* \subseteq B^* \).

**Proof**

Given \( A \subseteq B \). Then it is easy to prove that \( M_1 \subseteq M_2 \), \( S_1 \subseteq S_2 \), \( R_1 \supseteq R_2 \) or \( M_1 \subseteq M_2 \), \( S_1 \subseteq S_2 \), \( R_1 \supseteq R_2 \).

So \( A^* \subseteq B^* \).

**Remark 3.1**

1) All types of \( \phi_N \) and \( \phi_N \) are concedes.
2) All types of \( X_N \) and \( X_N \) are concedes.
3) \( A^* = B^* \) iff \( A^* \subseteq B^* \) and \( B^* \subseteq A^* \).

**Definition 3.8**

Let \( X \) be a non-empty set, and \( A^* = \{M, S, R\} \) be a *-neutrosophic crisp set on a NCS \( A = \{A_1, A_2, A_3\} \) where \( M = A_1 \cap (A_2 \cup A_3)^c \), \( S = A_2 \cap (A_1 \cup A_3)^c \), \( R = A_3 \cap (A_1 \cup A_2)^c \). Then the complement of the set \( A^* \) (\( A^* \), for short) may be defined as three kinds of complements:

1) Type 1:
   \[ (C_1) \quad A^* = \{M^c, S^c, R^c\}. \]

2) Type 2:
   \[ (C_2) \quad A^* = \{R, S, M\}. \]

3) Type 3:
   \[ (C_3) \quad A^* = \{R, S^c, M\}. \]

**Definition 2.3**

Let \( X \) be a non-empty fixed set, two neutrosophic crisp sets \( A, B \) are having the form \( A = \{A_1, A_2, A_3\} \), \( B = \{B_1, B_2, B_3\} \), and two *-neutrosophic crisp sets \( A^* = \{M_1, S_1, R_1\}, B^* = \{M_2, S_2, R_2\} \) where \( M_1 = A_1 \cap (A_2 \cup A_3)^c \), \( S_1 = A_2 \cap (A_1 \cup A_3)^c \), \( R_1 = A_3 \cap (A_1 \cup A_2)^c \), \( M_2 = B_1 \cap (B_2 \cup B_3)^c \), \( S_2 = B_2 \cap (B_1 \cup B_3)^c \), and \( R_2 = B_3 \cap (B_1 \cup B_2)^c \). Then

1) \( A^* \cap B^* \) may be defined as two types:
   i) Type 1: \( A^* \cap B^* = \{M_1 \cap M_2, S_2 \cup S_2, R_3 \cup R_3\} \) or
   ii) Type 2: \( A^* \cap B^* = \{M_1 \cap M_2, S_2 \cup S_2, R_3 \cap R_3\} \)

2) \( A^* \cup B^* \) may be defined as two types:
   i) Type 1: \( A^* \cup B^* = \{M_1 \cup M_1, S_2 \cup S_2, R_3 \cap R_3\} \) or
   ii) Type 2: \( A^* \cup B^* = \{M_1 \cup M_1, S_2 \cup S_2, R_3 \cap R_3\} \).

**Lemma 3.1**

Let \( A^*, B^* \) are *-neutrosophic crisp sets. Then \( A^* - B^* = A^* \cap B^* \)

It is easy to show that L.H.S is also a *-neutrosophic crisp set.

**Example 3.2**

Let \( X = \{a, b, c, d, e, f, g\} \), \( A = \{(a, b, c, d), \{e, f, g\}\} \), \( B = \{(a, b, c), \{d, e, f\}\} \), \( C = \{(a, b, c, d), \{e, f, a\}\} \), \( D = \{(a, b), \{e, f, a\}\} \) are NCS. Then \( A^* = \{(a, b, c, d), \{e, f, g\}\} \), \( B^* = \{(a, b, c), \{d, e, f\}\} \), \( C^* = \{(b), \{c, d\}, \{e\}\} \), \( D^* = \{(a, b), \{e, f, a\}\} \).

The complement may be equal as:

1) \( A^* = \{(e, f), \{a, b, c, d\}, \{a, b, c\}\} \), \( A^* = \{(f), \{a, b, c, d\}, \{a, b, c\}\} \)

2) \( C^* = \{(a, c, d), \{a, b, e, f\}\}, \{a, b, c, d\}\} \)

3) \( A^* \cup B^* = \{(a, b, c, d), \{e, f\}\}, \{a, b, c\} \), \( A^* \cup B^* = \{(a, b, c), \{d, e, f\}\}, \{a, b, c\} \)

4) \( A^* \cap B^* = \{(a, b, c), \{d, e, f\}\}, \{a, b, c\} \).

**Proposition 3.1**

Let \( \{A_j^*: j \in J\} \) be arbitrary family of *-neutrosophic crisp subsets on \( X \), then

1) \( \cap A_j^* \) may be defined two types as:
   i) Type 1: \( \cap A_j^* = \left(\left(\cap M_j \cap \cap S_j \cap \cap R_j\right)\right) \) or
   ii) Type 2: \( \cap A_j^* = \left(\left(\cap M_j \cap \cap S_j \cap \cap R_j\right)\right) \)

2) \( \cup A_j^* \) may be defined two types as:
   i) Type 1: \( \cup A_j^* = \left(\cup M_j \cup \cup S_j \cup \cup R_j\right) \) or
   ii) Type 2: \( \cup A_j^* = \left(\cup M_j \cup \cup S_j \cup \cup R_j\right) \)

**Corollary 3.2**

Let \( \{A_j^*: i \in J\} \) be NCSs in \( X \) where \( i \in J \), where \( J \) is an index set and \( \{A_j^*\} \) are corresponding *-neutrosophic crisp subsets on \( X \) then

1) \( A_i^* \subseteq B_i^* \) for each \( i \in J \), \( A_i^* \subseteq B_i^* \), \( B_i^* \subseteq A_i^* \)
2) \( B_i^* \subseteq A_i^* \) for each \( i \in J \), \( B_i^* \subseteq A_i^* \)
3) \( \left(\cap A_i^*\right)^c = \cap A_i^* \), \( \left(\cup A_i^*\right)^c = \cup A_i^* \).
d) $A_x^* \subseteq B^* \Leftrightarrow B_x^* \subseteq A_x^*$.

e) $A_x^* = A$.

f) $\phi_N^* = X_N; X_N^* = \phi_N^*$.

Now we shall define the image and preimage of *- neutrosophic crisp set.

Let $X, Y$ be two non-empty fixed sets and $f : X \to Y$ be a function and $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$ are neutrosophic crisp sets on $X$ and $Y$ respectively, $A^* = \{M_1, S_1, R_1\}$, $B^* = \{M_2, S_2, R_2\}$ be the *- neutrosophic crisp sets on $X$ and $Y$ respectively.

**Definition 3.9**

(a) If $B^*$ is a *- NCS in $Y$, then the preimage of $B^*$ under $f$, denoted by $f^{-1}(B^*)$, is a *- NCS in $X$ defined by $f^{-1}(B^*) = \{f^{-1}(M_1), f^{-1}(S_2), f^{-1}(R_2)\}$.

(b) If $A^*$ is a *- NCS in $X$, then the image of $A^*$ under $f$, denoted by $f(A^*)$, is the *- NCS in $Y$ defined by $f(A^*) = \{f(M_1), f(S_1), f(R_2)^f\}$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

**Corollary 3.2**

Let $A^* = \{A_i^*: i \in J\}$ be a family of *- NCS in $X$, and $B^* = \{B_j^*: j \in K\}$ *- NCS in $Y$, and $f : X \to Y$ a function. Then

(a) $A_1^* \subseteq A_2^* \Leftrightarrow f(A_1^*) \subseteq f(A_2^*)$.

(b) $B_1^* \subseteq B_2^* \Leftrightarrow f^{-1}(B_1^*) \subseteq f^{-1}(B_2^*)$.

(c) $f^{-1}(f(A^*)) = A^*$ and if $f$ is injective, then $A^* = f^{-1}(f(A^*))$.

(d) $f^{-1}(f(B^*)) = B^*$ and if $f$ is surjective, then $f^{-1}(f(B^*)) = B^*$.

(e) $f(\cap A_i^*) = \cap f(A_i^*)$; $f(\cap A_i^*) \subseteq \cap f(A_i^*)$ and if $f$ is injective, then $f(\cap A_i^*) = \cap f(A_i^*)$.

(f) $f^{-1}(Y_X) = X_f^{-1}$, $f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N$, $f(X_X) = Y_X$ if $f$ is surjective.

(h) If $f$ is surjective, then $(f(A^*))^f \subseteq f(A^*)^f$. If furthermore $f$ is injective, then have $(ff(A^*))^f = f(A^*)^f$.

**Proof**

Clear by definitions.

### 4 *- Neutrosophic Crisp Set Relations

Here we give the definition relation on *- neutrosophic crisp sets and study of its properties.

Let $X, Y$ and $Z$ be three ordinary nonempty sets.

**Definition 4.1**

Let $X$ be a non-empty fixed set, two neutrosophic crisp sets $A$, $B$ are having the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$, and two *- neutrosophic crisp sets $A^* = \{M_1, S_1, R_1\}$, $B^* = \{M_2, S_2, R_2\}$ where $M_1 = A_1 \cap (A_2 \cup A_3)$, $S_1 = A_2 \cap (A_1 \cup A_3)$, $R_1 = A_3 \cap (A_1 \cup A_2)$, $M_2 = B_1 \cap (B_2 \cup B_3)$, $S_2 = B_2 \cap (B_1 \cup B_3)$, and $R_2 = B_3 \cap (B_1 \cup B_2)$.

(i) The product of two *- neutrosophic crisp sets $A^*$ and $B^*$ is a *- neutrosophic crisp set $A^* \times B^*$ given by $A^* \times B^* = \{M_1 \times M_2, S_1 \times S_2, R_1 \times R_2\}$ on $X \times Y$.

(ii) We will call a *- neutrosophic crisp relation $R^* \subseteq A^* \times B^*$ on the direct product $X \times Y$.

The collection of all *- neutrosophic crisp relations on $X \times Y$ is denoted as $SNCR(X \times Y)$.

**Definition 4.2**

Let $R^*$ be a *- neutrosophic crisp relation on $X \times Y$, then the inverse of $R^*$ is denoted by $R^{*-1}$ where $R^{*-1} \subseteq A^* \times B^*$ on $X \times Y$ then $R^{*-1} \subseteq B^* \times A^*$ on $Y \times X$.

**Example 4.1**

Let $X = \{a, b, c, d, e, f\}$, $A = \{(a, b, c, d, e, f), \{e, f\}\}$, $B = \{(a, b, c, d, e, f), \{e\}\}$, are NCS. Then $A^* = \{\{a, b, c, d, e, f\}, \{e, f\}\}$, $B^* = \{\{a, b, c, d, e, f\}, \{e\}\}$, then the product of two *- neutrosophic crisp sets given by $A^* \times B^* = \{(a, a, b, (a, b), (a, c), (a, b), (c, a), (b, a), (c, b), (c, c), (e, d), (f, e)), \{(e, f)\}\}$ and $B^* \times A^* = \{(a, a, b, (a, b), (a, c), (a, d), (a, b), (b, a), (c, b), (c, c), (e, d), (f, e)), \{(e, f)\}\}$.

We can define the operations of *- neutrosophic crisp relations.

**Definition 4.3**

Let $R^*$ and $S^*$ be two *- neutrosophic crisp relations between $X$ and $Y$ for every $(x, y) \in X \times Y$ and NCSS $A$.
and $B$ in the form $A = \{A_1, A_2, A_3\}$, $A^*$ on $X$, $B = \{B_1, B_2, B_3\}$, $B^*$ on $Y$ Then we can defined the following operations
i) $R \subseteq S$ may be defined as two types
   a) Type1: $R \subseteq S \iff M_{1_1} \subseteq M_{1_1}$, $S_{1_1} \subseteq S_{1_1}$, $R_{1_1} \subseteq R_{1_1}$
   b) Type2: $R \subseteq S^* \iff M_{1_1} \subseteq M_{1_1}$, $S_{1_1} \subseteq S_{1_1}$, $R_{1_1} \subseteq R_{1_1}$

ii) $R^* \cup S^*$ may be defined as two types
   a) Type1: $R^* \cup S^* = \{M_{1_1} \cup M_{1_1}, S_{1_1} \cup S_{1_1}, R_{1_1} \cup R_{1_1}\}$
   b) Type2: $R^* \cup S^* = \{M_{1_1} \cup M_{1_1}, S_{1_1} \cup S_{1_1}, R_{1_1} \cup R_{1_1}\}$

iii) $R^* \cap S^*$ may be defined as two types
   a) Type1: $R^* \cap S^* = \{M_{1_1} \cap M_{1_1}, S_{1_1} \cap S_{1_1}, R_{1_1} \cap R_{1_1}\}$
   b) Type2: $R^* \cap S^* = \{M_{1_1} \cap M_{1_1}, S_{1_1} \cap S_{1_1}, R_{1_1} \cap R_{1_1}\}$

Theorem 4.1
Let $R^*$, $S^*$ and $Q^*$ be three $^*$- neutrosophic crisp relations between $X$ and $Y$ for every $(x, y) \in X \times Y$, then
i) $R^* \subseteq S^*$ $\implies$ $R^{-1} \subseteq S^{-1}$
ii) $(R^* \cup S^*)^{-1} \subseteq S^{-1}$
iii) $(R^* \cap S^*)^{-1} \subseteq \cap S^{-1}$
iv) $(R^*)^{-1} = R^*$.

v) $R^* \cap (S^* \cup Q^*) = (R^* \cap S^*) \cup (R^* \cap Q^*)$.
vi) $R^* \cup (S^* \cap Q^*) = (R^* \cup S^*) \cap (R^* \cup Q^*)$.

vii) If $S^* \subseteq R^*$, $Q^* \subseteq R^*$, then $S^* \subseteq Q^* \subseteq R^*$.

Proof
Clear

Definition 5.4
The $^*$- neutrosophic crisp relation $I^*$ in $SNCR^*(X \times X)$, the $^*$- neutrosophic crisp relation of identity may be defined as two types
i) Type1: $I^* = \{(A \times A), (A^* \times A^*), (\phi^*, \phi^*)\}$
ii) Type2: $I^* = \{(A \times A^*), (\phi^*, \phi^*)\}$

Now we define two composite relations of $^*$- neutrosophic crisp sets.

Definition 5.5
Let $R^*$ be a $^*$- neutrosophic crisp relation in $X \times Y$, and $S^*$ be a neutrosophic crisp relation in $Y \times Z$. Then the composition of $R^*$ and $S^*$, $R^* \circ S^*$ be a $^*$- neutrosophic crisp relation in $X \times Y \times Z$ as a definition may be defined as two types
i) Type1:
$R^* \circ S^* \leftrightarrow (R^* \circ S^*)_{(x, z)}$
$= \{(M_{1_1} \times M_{1_1}) \cap (M_{1_1} \times M_{1_1})\}$
$= \{(S_{1_1} \times S_{1_1}) \cup (S_{1_1} \times S_{1_1})\}$
$\cup (R_{1_1} \times R_{1_1}) \cap (R_{1_1} \times R_{1_1})\}$

ii) Type2:
$R^* \circ S^* \leftrightarrow (R^* \circ S^*)_{(x, z)}$
$= \{(M_{1_1} \times M_{1_1}) \cup (M_{1_1} \times M_{1_1})\}$
$\cup (S_{1_1} \times S_{1_1}) \cup (S_{1_1} \times S_{1_1})\}$
$\cup (R_{1_1} \times R_{1_1}) \cup (R_{1_1} \times R_{1_1})\}$

Theorem 4.2
Let $R^*$ be a $^*$- neutrosophic crisp relation in $X \times Y$, and $S$ be a $^*$- neutrosophic crisp relation in $Y \times Z$ then $(R^* \circ S^*)^{-1} = S^{-1} \circ R^{-1}$

Proof
Let $R^* \subseteq A^* \times B^*$ on $X \times Y$ then $R^{-1} \subseteq B \times A$.
$S^* \subseteq B^* \times D^*$ on $Y \times Z$ then $S^{-1} \subseteq D^* \times B^*$, from Definition 4.3 and similarly we can $I^*_{(R^* \circ S^*)^{-1}}(x, z) = I^*_{S^{-1}}(x, z)$ and $I^*_{R^{-1}}(x, z)$ then
$(R^* \circ S^*)^{-1} = S^{-1} \circ R^{-1}$.

References

Received: October 10, 2014. Accepted: October 29, 2014

A. A. Salama and Hewayda Elghawalby,* Neutrosophic Crisp Set & *- Neutrosophic Crisp relations