

## SOFT NEUTROSOPHIC LEFT ALMOST SEMIGROUP

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In this paper we have extended neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideals, neutrosophic prime ideals, neutrosophic semiprime ideals, neutrosophic strong irreducible ideals to soft neutrosophic LA-semigroup, soft neutrosophic sub LA-semigroup, soft neutrosophic ideals, soft neutrosophic prime ideals, soft neutrosophic semiprime ideals and soft strong irreducible neutrosophic ideals respectively. We have found some new notions related to the strong or pure part of neutrosophy and we give explanation with necessary illustrative examples. We have also given rigorous theorems and propositions. The notion of soft neutrosophic homomorphism is presented at the end.

*Keywords:* Neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideal, soft LA-semigroup, soft LA-subsemigroup, soft ideal, soft neutrosophic LA-semigroup, soft sub-neutrosophic LA-semigroup, soft neutrosophic ideal.

### 1. INTRODUCTION

In 1995, Florentin Smarandache introduced the concept of neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set [1], intuitionistic fuzzy set [2] and interval valued fuzzy set [3]. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic algebraic structures in [11]. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loops, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Neutrosophic LA-semigroup is already introduced. It is basically a midway algebraic structure between neutrosophic groupoid and commutative neutrosophic semigroups. This is in fact a generalization of neutrosophic semigroup theory. In neutrosophic LA-semigroup we have two basic types of notions and they are traditional notions as well as strong or pure neutrosophic notions. It is also an extension of LA-semigroup and involves the origin of neutralities or indeterminacy factor in LA-semigroup structure. This is a rich structure because of the indeterminacy's presence in all the corresponding notions of LA-semigroup and this property makes the differences between approaches of an LA-semigroup and a neutrosophic LA-semigroup. Molodstov introduced the concept of soft set theory which is free from the problems of parameterization inadequacy.

In his paper [11], he presented the fundamental results of new theory and successfully applied it into

several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability. After getting a high attention of researchers, soft set theory is applied in many fields successfully and so as in the field of LA-semigroup theory. A soft LA-semigroup means the parameterized collection of sub LA-semigroup over an LA-semigroup. It is more general concept than the concept of LA-semigroup.

We have further generalized this idea by adding neutrosophy and extended operations of soft set theory. In this paper we introduced the basic concepts of soft neutrosophic LA-semigroup. In the proceeding section we define soft neutrosophic LA-semigroup and characterized with some of their properties. Soft neutrosophic ideal over a neutrosophic LA-semigroup and soft neutrosophic ideal of a neutrosophic LA-semigroup is given in the further sections and studied some of their related results. In the last section, the concept of soft homomorphism of a soft LA-semigroup is extended to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup.

## 2. PRELIMINARIES

### 2.1. Definition 1

Let  $(S, *)$  be an LA-semigroup and let  $\langle S \cup I \rangle = \{a + bI : a, b \in S\}$ . The neutrosophic LA-semigroup is generated by  $S$  and  $I$  under  $*$  denoted as  $N(S) = \{\langle S \cup I \rangle, *\}$ , where  $I$  is called the neutrosophic element with property  $I^2 = I$ . For an integer  $n$ ,  $n + I$  and  $nI$  are neutrosophic elements and  $0.I = 0.I^{-1}$ , the inverse of  $I$  is not defined and hence does not exist. Similarly we can define neutrosophic RA-semigroup on the same lines.

**Definition 2** Let  $N(S)$  be a neutrosophic LA-semigroup and  $N(H)$  be a proper subset of  $N(S)$ . Then  $N(H)$  is called a neutrosophic sub LA-semigroup if  $N(H)$  itself is a neutrosophic LA-semigroup under the operation of  $N(S)$ .

**Definition 3** A neutrosophic sub LA-semigroup  $N(H)$  is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of  $N(H)$  are neutrosophic elements.

**Definition 4** Let  $N(S)$  be a neutrosophic LA-semigroup and  $N(K)$  be a subset of  $N(S)$ . Then  $N(K)$  is called Left (right) neutrosophic ideal of  $N(S)$  if  $N(S)N(K) \subseteq N(K)$ ,  $\{N(K)N(S) \subseteq N(K)\}$ . If  $N(K)$  is both left and right neutrosophic ideal, then  $N(K)$  is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

**Definition 5** A neutrosophic ideal  $N(P)$  of a neutrosophic LA-semigroup  $N(S)$  with left identity  $e$  is called prime neutrosophic ideal if  $N(A)N(B) \subseteq N(P)$  implies either  $N(A) \subseteq N(P)$  or  $N(B) \subseteq N(P)$ , where  $N(A), N(B)$  are neutrosophic ideals of  $N(S)$ .

**Definition 6** A neutrosophic LA-semigroup  $N(S)$  is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

**Definition 7** A neutrosophic ideal  $N(P)$  is called semiprime neutrosophic ideal if  $N(T).N(T) \subseteq N(P)$  implies  $N(T) \subseteq N(P)$  for any neutrosophic ideal  $N(T)$  of  $N(S)$ .

**Definition 8** A neutrosophic LA-semigroup  $N(S)$  is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of  $N(S)$  is semiprime neutrosophic ideal.

**Definition 9** A neutrosophic ideal  $N(R)$  of a neutrosophic LA-semigroup  $N(S)$  is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals  $N(H), N(K)$  of  $N(S)$   $N(H) \cap N(K) \subseteq N(R)$  implies  $N(H) \subseteq N(R)$  or  $N(K) \subseteq N(R)$ .

**Definition 10** Let  $S, T$  be two LA-semigroups and  $\phi: S \rightarrow T$  be a mapping from  $S$  to  $T$ . Let  $N(S)$  and  $N(T)$  be the corresponding neutrosophic LA-semigroups of  $S$  and  $T$  respectively. Let  $\theta: N(S) \rightarrow N(T)$  be another mapping from  $N(S)$  to  $N(T)$ . Then  $\theta$  is called neutrosophic homomorphis if  $\phi$  is homomorphism from  $S$  to  $T$ .

### 2.2 Soft Sets

Throughout this subsection  $U$  refers to an initial universe,  $E$  is a set of parameters,  $P(U)$  is the power set of  $U$ , and  $A \subseteq E$ . Molodtsov [12] defined the soft set in the following manner:

**Definition 11** A pair  $(F, A)$  is called a soft set over  $U$  where  $F$  is a mapping given by

$$F : A \rightarrow P(U).$$

In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For  $a \in A$ ,  $F(a)$  may be considered as the set of  $a$ -elements of the soft set  $(F, A)$ , or as the set of  $a$ -approximate elements of the soft set.

**Definition 12** For two soft sets  $(F, A)$  and  $(H, B)$  over  $U$ ,  $(F, A)$  is called a soft subset of  $(H, B)$  if

- 1)  $A \subseteq B$  and
- 2)  $F(a) \subseteq H(a)$ , for all  $a \in A$ .

This relationship is denoted by  $(F, A) \subset (H, B)$ . Similarly  $(F, A)$  is called a soft superset of  $(H, B)$  if  $(H, B)$  is a soft subset of  $(F, A)$  which is denoted by  $(F, A) \supset (H, B)$ .

**Definition 13** Let  $(F, A)$  and  $(G, B)$  be two soft sets over a common universe  $U$  such that  $A \cap B \neq \phi$ . Then their restricted intersection is denoted by  $(F, A) \cap_R (G, B) = (H, C)$  where  $(H, C)$  is defined as  $H(c) = F(c) \cap G(c)$  for all  $c \in C = A \cap B$ .

**Definition 14** The extended intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write  $(F, A) \cap_e (G, B) = (H, C)$ .

**Definition 15** The restricted union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined as the soft set  $(H, C) = (F, A) \cup_R (G, B)$  where  $C = A \cap B$  and  $H(c) = F(c) \cup G(c)$  for all  $c \in C$ .

**Definition 16** The extended union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined

$$\text{as } H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write  $(F, A) \cup_{\varepsilon} (G, B) = (H, C)$ .

### 2.3 Soft LA-semigroup

**Definition 17** The restricted product  $(H, C)$  of two soft sets  $(F, A)$  and  $(G, B)$  over an LA-semigroup  $S$  is defined as the soft set  $(H, C) = (F, A) \odot (G, B)$ , where  $H(c) = F(c)G(c)$  for all  $c \in C = A \cap B$ .

**Definition 18** A soft set  $(F, A)$  over  $S$  is called soft LA-semigroup over  $S$  if  $(F, A) \odot (F, A) \subseteq (F, A)$ .

**Definition 19** A soft LA-semigroup  $(F, A)$  over  $S$  is said to be soft LA-semigroup with left identity  $e$  if  $F(a) \neq \phi$  is a sub LA-semigroup with left identity  $e$ , where  $e$  is the left identity of  $S$  for all  $a \in A$ .

**Definition 20** Let  $(F, A)$  and  $(G, B)$  be two soft LA-semigroups over  $S$ . Then the operation  $*$  for soft sets is

defined as  $(F, A) * (G, B) = (H, A \times B)$ , where  $H(a, b) = F(a)G(b)$  for all  $a \in A, b \in B$  and

$A \times B$  is the Cartesian product of  $A, B$ .

**Definition 21** A soft set  $(F, A)$  over an LA-semigroup  $S$  is called a soft left (right) ideal over  $S$  if  $\tilde{A}_S \odot (F, A) \subseteq (F, A), ((F, A) \odot \tilde{A}_S \subseteq (F, A))$  where  $\tilde{A}_S$  is the absolute soft LA-semigroup over  $S$ .

**Definition 22** Let  $(F, A)$  and  $(G, B)$  be two soft LA-semigroups over  $S$ . Then the Cartesian product is defined

as  $(F, A) \times (G, B) = (H, A \times B)$ , where  $H(a, b) = F(a) \times G(b)$  for all  $a \in A$  and  $b \in B$ .

**Definition 23** Let  $(G, B)$  be a soft subset of  $(F, A)$  over  $S$ . Then  $(G, B)$  is called a soft ideal of  $(F, A)$ , if  $G(b)$  is an ideal of  $F(b)$  for all  $b \in B$ .

## 3. SOFT NEUTROSOPHIC LA-SEMIGROUPS

The definition of soft neutrosophic LA-semigroup is introduced in this section and we also examine some of their properties. Throughout this section  $N(S)$  will denote a neutrosophic LA-semigroup unless stated otherwise.

**Definition 24** Let  $(F, A)$  be a soft set over  $N(S)$ . Then  $(F, A)$  over  $N(S)$  is called soft neutrosophic LA-semigroup if  $(F, A) \odot (F, A) \subseteq (F, A)$ .

**Proposition 1** A soft set  $(F, A)$  over  $N(S)$  is a soft neutrosophic LA-semigroup if and only if  $\phi \neq F(a)$  is a

neutrosophic sub LA-semigroup of  $N(S)$  for all  $a \in A$ .

**Example 1** Let  $N(S) = \{1, 2, 3, 4, 1I, 2I, 3I, 4I\}$  be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Let  $(F, A)$  be a soft set over  $N(S)$ . Then clearly  $(F, A)$  is a soft neutrosophic LA-semigroup over  $N(S)$ , where

$$F(a_1) = \{1, 1I\}, F(a_2) = \{2, 2I\},$$

$$F(a_3) = \{3, 3I\}, F(a_4) = \{4, 4I\}.$$

**Theorem 1A** soft LA-semigroup over an LA-semigroup  $S$  is contained in a soft neutrosophic LA-semigroup over  $N(S)$ .

**Proposition 2** Let  $(F, A)$  and  $(H, B)$  be two soft neutrosophic LA-semigroup over  $N(S)$ . Then

- 1) Their extended intersection  $(F, A) \cap_{\varepsilon} (H, B)$  is a soft neutrosophic LA-semigroup over  $N(S)$
- 2) Their restricted intersection  $(F, A) \cap_R (H, B)$  is also soft neutrosophic LA-semigroup over  $N(S)$ .

**Remark 1** Let  $(F, A)$  and  $(H, B)$  be two soft neutrosophic LA-semigroup over  $N(S)$ . Then

- 1) Their extended union  $(F, A) \cup_{\varepsilon} (H, B)$  is not a soft neutrosophic LA-semigroup over  $N(S)$ .
- 2) Their restricted union  $(F, A) \cup_R (H, B)$  is not a soft neutrosophic LA-semigroup over  $N(S)$ .

**Proposition 3** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic LA-semigroup over  $N(S)$ . Then  $(F, A) \wedge (H, B)$  is also soft neutrosophic LA-semigroup if it is non-empty.

**Proposition 4** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic LA-semigroup over the neutrosophic LA-semigroup  $N(S)$ . If  $A \cap B = \emptyset$  Then their extended union  $(F, A) \cup_{\varepsilon} (G, B)$  is a soft neutrosophic LA-semigroup over  $N(S)$ .

**Definition 25** A soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is said to be a soft neutrosophic LA-semigroup with left identity  $e$  if for all  $a \in A$ , the parameterized set  $F(a)$  is a neutrosophic sub LA-

semigroup with left identity  $e$  where  $e$  is the left identity of  $N(S)$ .

**Lemma 1** Let  $(F, A)$  be a soft neutrosophic LA-semigroup with left identity  $e$  over  $N(S)$ , then

$$(F, A) \odot (F, A) = (F, A).$$

**Proposition 5** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic LA-semigroups over  $N(S)$ . Then the cartesian product of  $(F, A)$  and  $(G, B)$  is also soft neutrosophic LA-semigroup over  $N(S)$ .

**Definition 26** A soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft strong neutrosophic LA-semigroup or soft pure neutrosophic LA-semigroup if each  $F(a)$  is a strong or pure neutrosophic sub LA-semigroup for all  $a \in A$ .

**Theorem 2** All soft strong neutrosophic LA-semigroups or pure neutrosophic LA-semigroups are trivially soft neutrosophic LA-semigroups but the converse is not true in general.

**Definition 28** Let  $(F, A)$  be a soft neutrosophic LA-semigroup over  $N(S)$ . Then  $(F, A)$  is called an absolute soft neutrosophic LA-semigroup if  $F(a) = N(S)$  for all  $a \in A$ . We denote it by  $\tilde{A}_{N(S)}$ .

**Definition 29** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic LA-semigroup over  $N(S)$ . Then  $(G, B)$  is soft sub neutrosophic LA-semigroup of  $(F, A)$ , if

- 1)  $B \subseteq A$ , and
- 2)  $H(b)$  is a neutrosophic sub LA-semigroup of  $F(b)$ , for all  $b \in B$ .

**Theorem 3** Every soft LA-semigroup over  $S$  is a soft sub neutrosophic LA-semigroup of a soft neutrosophic LA-semigroup over  $N(S)$ .

**Definition 30** Let  $(G, B)$  be a soft sub-neutrosophic LA-semigroup of a soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$ . Then  $(G, B)$  is said to be soft strong or pure sub-neutrosophic LA-semigroup of  $(F, A)$  if each  $G(b)$  is strong or pure neutrosophic sub LA-semigroup of  $F(b)$ , for all  $b \in B$ .

**Theorem 4** A soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  can have soft sub LA-semigroups, soft sub-neutrosophic LA-semigroups and soft strong or pure sub-neutrosophic LA-semigroups.

**Theorem 5** If  $(F, A)$  over  $N(S)$  is a soft strong or pure neutrosophic LA-semigroup, then every soft sub-neutrosophic LA-semigroup of  $(F, A)$  is a soft strong or pure sub-neutrosophic LA-semigroup.

#### 4. SOFT NEUTROSOPHIC IDEALS OVER A NEUTROSOPHIC LA-SEMIGROUP

**Definition 31** A soft set  $(F, A)$  over a neutrosophic LA-semigroup  $N(S)$  is called a soft neutrosophic left (right) ideal over  $N(S)$  if  $\tilde{A}_{N(S)} \odot (F, A) \subseteq (F, A)$ ,  $((F, A) \odot \tilde{A}_{N(S)} \subseteq (F, A))$  where  $\tilde{A}_{N(S)}$  is the absolute soft neutrosophic LA-semigroup over  $N(S)$ . A soft set  $(F, A)$  over  $N(S)$  is a soft neutrosophic ideal if it is soft neutrosophic left ideal as well as soft neutrosophic right ideal over  $N(S)$ .

**Proposition 5** Let  $(F, A)$  be a soft set over  $N(S)$ . Then  $(F, A)$  is a soft neutrosophic ideal over  $N(S)$  if and only if  $F(a) \neq \phi$  is a neutrosophic ideal of  $N(S)$ , for all  $a \in A$ .

**Proposition 6** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic ideals over  $N(S)$ . Then

- 1) Their restricted union  $(F, A) \cup_R (G, B)$  is a soft neutrosophic ideal over  $N(S)$ .

- 2) Their restricted intersection  $(F, A) \cap_R (G, B)$  is a soft neutrosophic ideal over  $N(S)$ .
- 3) Their extended union  $(F, A) \cup_\varepsilon (G, B)$  is also a soft neutrosophic ideal over  $N(S)$ .
- 4) Their extended intersection  $(F, A) \cap_\varepsilon (G, B)$  is a soft neutrosophic ideal over  $N(S)$ .

**Proposition 7** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic ideals over  $N(S)$ . Then

1. Their *OR* operation  $(F, A) \vee (G, B)$  is a soft neutrosophic ideal over  $N(S)$ .
2. Their *AND* operation  $(F, A) \wedge (G, B)$  is a soft neutrosophic ideal over  $N(S)$ .

**Proposition 8** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic ideals over  $N(S)$ , where  $N(S)$  is a neutrosophic LA-semigroup with left identity  $e$ . Then  $(F, A) * (G, B) = (H, A \times B)$  is also a soft neutrosophic ideal over  $N(S)$ .

**Proposition 9** Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic ideals over  $N(S)$  and  $N(T)$ . Then the cartesian product  $(F, A) \times (G, B)$  is a soft neutrosophic ideal over  $N(S) \times N(T)$ .

**Definition 32** A soft neutrosophic ideal  $(F, A)$  over  $N(S)$  is called soft strong or pure neutrosophic ideal over  $N(S)$  if  $F(a)$  is a strong or pure neutrosophic ideal of  $N(S)$ , for all  $a \in A$ .

**Theorem 6** All soft strong or pure neutrosophic ideals over  $N(S)$  are trivially soft neutrosophic ideals but the converse is not true.

**Proposition 8** Let  $(F, A)$  and  $(G, B)$  be two soft strong or pure neutrosophic ideals over  $N(S)$ . Then

- 1) Their restricted union  $(F, A) \cup_R (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S)$ .
- 2) Their restricted intersection  $(F, A) \cap_R (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S)$ .
- 3) Their extended union  $(F, A) \cup_\varepsilon (G, B)$  is also a soft strong or pure neutrosophic ideal over  $N(S)$ .
- 4) Their extended intersection  $(F, A) \cap_\varepsilon (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S)$ .

**Proposition 9** Let  $(F, A)$  and  $(G, B)$  be two soft strong or pure neutrosophic ideals over  $N(S)$ . Then

- 1) Their *OR* operation  $(F, A) \vee (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S)$ .
- 2) Their *AND* operation  $(F, A) \wedge (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S)$ .

**Proposition 10** Let  $(F, A)$  and  $(G, B)$  be two soft strong or pure neutrosophic ideals over  $N(S)$ , where  $N(S)$  is a neutrosophic LA-semigroup with left identity  $e$ . Then  $(F, A) * (G, B) = (H, A \times B)$  is also a soft strong or pure neutrosophic ideal over  $N(S)$ .

**Proposition 11** Let  $(F, A)$  and  $(G, B)$  be two soft strong or pure neutrosophic ideals over  $N(S)$  and  $N(T)$  respectively. Then the cartesian product  $(F, A) \times (G, B)$  is a soft strong or pure neutrosophic ideal over  $N(S) \times N(T)$ .

## 5. SOFT NEUTROSOPHIC IDEAL OF SOFT NEUTROSOPHIC LA-SEMIGROUP

**Definition 33** Let  $(F, A)$  and  $(G, B)$  be soft neutrosophic LA-semigroups over  $N(S)$ . Then  $(G, B)$  is soft neutrosophic ideal of  $(F, A)$ , if

- 1)  $B \subseteq A$ , and
- 2)  $H(b)$  is a neutrosophic ideal of  $F(b)$ , for all  $b \in B$ .

**Proposition 12** If  $(F', A')$  and  $(G', B')$  are soft neutrosophic ideals of soft neutrosophic LA-semigroup  $(F, A)$  and  $(G, B)$  over neutrosophic LA-semigroups  $N(S)$  and  $N(T)$  respectively.

Then  $(F', A') \times (G', B')$  is a soft neutrosophic ideal of soft neutrosophic LA-semigroup  $(F, A) \times (G, B)$  over  $N(S) \times N(T)$ .

**Theorem 17** Let  $(F, A)$  be a soft neutrosophic LA-semigroup over  $N(S)$  and  $\{(H_j, B_j) : j \in J\}$  be a non-empty family of soft neutrosophic sub LA-semigroups of  $(F, A)$ . Then

- 1)  $\bigcap_{j \in J} (H_j, B_j)$  is a soft neutrosophic sub LA-semigroup of  $(F, A)$ .
- 2)  $\bigwedge_{j \in J} (H_j, B_j)$  is a soft neutrosophic sub LA-semigroup of  $(F, A)$ .
- 3)  $\bigcup_{\mathcal{E}} (H_j, B_j)$  is a soft neutrosophic sub LA-semigroup of  $(F, A)$  if  $B_j \cap B_k = \emptyset$  for all  $j, k \in J$ .

**Theorem 8** Let  $(F, A)$  be a soft neutrosophic LA-semigroup over  $N(S)$  and  $\{(H_j, B_j) : j \in J\}$  be a non-empty family of soft neutrosophic ideals of  $(F, A)$ . Then

- 1)  $\bigcap_{j \in J} (H_j, B_j)$  is a soft neutrosophic ideal of  $(F, A)$ .
- 2)  $\bigwedge_{j \in J} (H_j, B_j)$  is a soft neutrosophic ideal of  $(F, A)$ .
- 3)  $\bigcup_{\mathcal{E}} (H_j, B_j)$  is a soft neutrosophic ideal of  $(F, A)$ .
- 4)  $\bigvee_{j \in J} (H_j, B_j)$  is a soft neutrosophic ideal of  $(F, A)$ .

**Proposition 13** Let  $(F, A)$  be a soft neutrosophic LA-semigroup with left identity  $e$  over  $N(S)$  and  $(G, B)$  be a soft neutrosophic right ideal of  $(F, A)$ . Then  $(G, B)$  is also soft neutrosophic left ideal of  $(F, A)$ .

**Lemma 2** Let  $(F, A)$  be a soft neutrosophic LA-semigroup with left identity  $e$  over  $N(S)$  and  $(G, B)$  be a soft neutrosophic right ideal of  $(F, A)$ . Then  $(G, B) \odot (G, B)$  is a soft neutrosophic ideal of  $(F, A)$ .

**Definition 34** A soft neutrosophic ideal  $(G, B)$  of a soft neutrosophic LA-semigroup  $(F, A)$  is called soft



strong or pure neutrosophic ideal if  $G(b)$  is a strong or pure neutrosophic ideal of  $F(b)$  for all  $b \in B$ .

**Theorem 9** Every soft strong or pure neutrosophic ideal of a soft neutrosophic LA-semigroup is trivially a soft neutrosophic ideal but the converse is not true.

**Definition 35**A soft neutrosophic ideal  $(G, B)$  of a soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft prime neutrosophic ideal if  $(H, C) \odot (J, D) \subseteq (G, B)$  implies either  $(H, C) \subseteq (G, B)$  or  $(J, D) \subseteq (G, B)$  for soft neutrosophic ideals  $(H, C)$  and  $(J, D)$  of  $(F, A)$ .

**Definition 36**A soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft fully prime neutrosophic LA-semigroup if all the soft neutrosophic ideals of  $(F, A)$  are soft prime neutrosophic ideals.

**Definition 37** A soft neutrosophic ideal  $(G, B)$  of a soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft semiprime neutrosophic ideal if  $(H, C) \odot (H, C) \subseteq (G, B)$  implies that  $(H, C) \subseteq (G, B)$  for any soft neutrosophic ideal  $(H, C)$  of  $(F, A)$ .

**Definition 38**A soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft fully semiprime neutrosophic LA-semigroup if all the soft neutrosophic ideals of  $(F, A)$  are soft semiprime neutrosophic ideals.

**Definition 39**A soft neutrosophic ideal  $(G, B)$  of a soft neutrosophic LA-semigroup  $(F, A)$  over  $N(S)$  is called soft strongly irreducible neutrosophic ideal if  $(H, C) \cap_r (J, D) \subseteq (G, B)$  implies either  $(H, C) \subseteq (G, B)$  or  $(J, D) \subseteq (G, B)$  for soft neutrosophic ideals  $(H, C)$  and  $(J, D)$  of  $(F, A)$ .

## 6. SOFT NEUTROSOPHIC HOMOMORPHISM

**Definition 40**Let  $(F, A)$  and  $(G, B)$  be two soft neutrosophic LA-semigroups over  $N(S)$  and  $N(T)$  respectively. Let  $f : N(S) \rightarrow N(T)$  and  $g : A \rightarrow B$  be two mappings. Then  $(f, g) : (F, A) \rightarrow (G, B)$  is called soft neutrosophic homomorphism, if

- 1)  $f$  is a neutrosophic homomorphism from  $N(S)$  onto  $N(T)$ .
- 2)  $g$  is a mapping from  $A$  onto  $B$ .
- 3)  $f(F(a)) = G(g(a))$  for all  $a \in A$ .

If  $f$  is a neutrosophic isomorphism from  $N(S)$  to  $N(T)$  and  $g$  is one to one mapping from  $A$  onto  $B$ . Then  $(f, g)$  is called soft neutrosophic isomorphism from  $(F, A)$  to  $(G, B)$ .

## CONCLUSION

The literature shows us that soft LA-semigroup is a general framework than LA-semigroup but in this paper we can see that there exists a more general structure which we call soft neutrosophic LA-semigroup. A soft LA-semigroup becomes soft sub-neutrosophic LA-semigroup of the corresponding soft neutrosophic LA-semigroup. Soft neutrosophic LA-semigroup points out the indeterminacy factor involved in soft LA-semigroup. Soft neutrosophic LA-semigroup can be characterized by soft neutrosophic ideals over a soft neutrosophic LA-semigroup. We can also extend soft homomorphism of soft LA-semigroup to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup. It is also mentioned here that there is still

a space to much more work in this field and explorations of further results has still to be done, this is just a beginning.

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