n-Valued Refined Neutrosophic Soft Sets and its Applications in Decision Making Problems and Medical Diagnosis

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Abstract In this work we use the concept of a n-valued refined neutrosophic soft sets and its properties to solve decision making problems, Also a similarity measure between two n-valued refined neutrosophic soft sets are proposed. A medical diagnosis (MD) method is established for n-valued refined neutrosophic soft set setting using similarity measures. Lastly a numerical example is given to demonstrate the possible application of similarity measures in medical diagnosis (MD).

Keywords neutrosophic set · neutrosophic soft set · n-valued refined neutrosophic set · n-valued refined neutrosophic soft set · Similarity measures · Decision making.

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1 Introduction

Neutrosophic set was introduced in 1995 by Florentin Smarandache, who coined the words "neutrosophy" and its derivative "neutrosophic". Then he Smarandache introduced the concept of neutrosophic set in [18] and [17] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Smarandache In 2005 [20] also studied the relation between neutrosophic set and intuitionistic fuzzy sets and he showed that the neutrosophic set is a generalization of the intuitionistic fuzzy sets. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every notion or idea $\langle A \rangle$ together with its opposite or negation $\langle antiA \rangle$ and with their spectrum of neutralities $\langle neutA \rangle$ in between them (i.e. notions or ideas supporting neither $\langle A \rangle$ nor $\langle antiA \rangle$). The neutrosophic numerical components (t,i,f) are crisp numbers, intervals, or in general

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subsets of the unitary standard or nonstandard unit interval. In 2015 Smarandache [21] presented a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic. He showed generalizations of 2-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' 3-symbol valued logics or Belnap's 4-symbol valued logic to the most general n-symbol or numerical valued refined neutrosophic logic. Also he gave a generalizations for n-valued refined neutrosophic set. In 2015 Agboola [1] developed refined neutrosophic algebraic structures by studding refined neutrosophic group and he presented some of its elementary properties. Broumi et. al. in [6] defined the concept of n-valued interval neutrosophic sets and introduced the basic operations of this concept and some distances between n-valued interval neutrosophic sets are proposed . Also, they proposed an efficient approach for group multi-criteria decision making based on n-valued interval neutrosophic sets and give an application of n-valued interval neutrosophic sets in medical diagnosis problem. Smarandache in 2015 [19] gave a short history about: the neutrosophic set, neutrosophic numerical components and neutrosophic literal components, neutrosophic numbers, neutrosophic intervals, neutrosophic dual number, neutrosophic special dual number, neutrosophic special quasi dual number, neutrosophic linguistic number, neutrosophic linguistic interval-style number, neutrosophic hypercomplex numbers of dimension n, and elementary neutrosophic algebraic structures. He also gave their generalizations to refined neutrosophic set, respectively refined neutrosophic numerical and literal components, then refined neutrosophic numbers and refined neutrosophic algebraic structures, and set-style neutrosophic numbers. A short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic as presented in [21], [23], [22]. Then some authors studied the similarity measures between some of these concepts such as: Broumi and Smarandache in 2014 [8] proposed the cosine similarity measure of neutrosophic refined (multi-) sets where the cosine similarity measure of neutrosophic refined sets is the extension of improved cosine similarity measure of single valued neutrosophic. They also presented the application of medical diagnosis using this cosine similarity measure of neutrosophic refined set. Also they in 2015 [9] presented a new distance measure between neutrosophic refined sets on the basis of extended Hausdorff distance of neutrosophic set and studied some of their basic properties and using this extended to solve medical diagnosis problem. In 2016 Broumi and Deli [5] proposed the correlation measure of neutrosophic refined(multi-) sets is where The concept of this correlation measure of neutrosophic refined sets is the extension of correlation measure of neutrosophic sets and intuitionistic fuzzy multi-sets. They using this measure to solve medical diagnosis and pattern recognition problems. Since the similarity measure is an important tool in pattern recognition and fault diagnosis, Jun Ye in 2015 [11] proposed two cotangent similarity measures for single-valued neutrosophic sets (SVNSs) based on cotangent function. Then, he introduced the weighted cotangent similarity measures and studied the comparison between the cotangent similarity measures of SVNSs and existing cosine similarity measure of SVNSs. He also applied the cotangent similarity measures to the fault diagnosis of steam turbine. In 1999, Molodtsov [15] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. Maji et. al. [14] in 2003 studied soft set and gave some operations related to this theory. As a combination of neutrosophic set and soft set Maji [13] introduced neutrosophic soft set (NSS in short), established its application in decision making. In 2013 Said and Smarandache [7] defined the concept of intuitionistic neutrosophic soft set and introduced some operations on intuitionistic neutrosophic soft set and some properties of this concept have been established. Mehmet et. al. in 2015 [10] introduced the concept of neutrosophic soft expert set they also defined its basic operations, namely complement, union, intersection, AND and OR, and studied some of their properties and gave an application of this concept in a decision- making problem. Alkhazaleh [3] in 2016 introduced the concept of time-neutrosophic soft set as a generalization of neutrosophic soft set and studied some of its properties. Also, he defined its basic operations, complement, union, intersection, "AND" and "OR" and studied their properties and gave hypothetical application of this concept in decision making problems. In 2015 Karaaslan [12] In studied the concept of single-valued neutrosophic refined soft set as an extension of single-valued neutrosophic refined set. Also, he defined the set theoretical operations between two single-valued neutrosophic refined soft sets and investigated some basic properties of these operations. He also proposed two methods to calculate correlation coefficient between two single-valued neutrosophic refined soft sets, and gave a clustering analysis application of one of proposed methods. In 2015 Mukherjee and Sarkar [16] proposed a new method of measuring similarity measure and weighted similarity measure between two neutrosophic soft sets (NSSs). They also gave a comparative study between the existing similarity measures for neutrosophic soft sets. A decision making method is established for neutrosophic soft set setting using similarity measures and they gave a numerical example to demonstrate the possible application of similarity measures in pattern recognition problems. As a generalization of neutrosophic soft set Alkhazaleh in 2016 [2] introduced the concept of n-valued refined neutrosophic soft set and studied some of its properties. He also, defined its basic operations, complement, union intersection, "AND" and "OR" and studied their properties. In this study, after giving some definitions related to the n-valued refined neutrosophic soft set (n-VRNS-set), we use these concepts to solve decision making problems, Also a similarity measure between two n-valued refined neutrosophic soft sets are proposed. A medical diagnosis method (MD) is established for n-valued refined neutrosophic soft set setting using similarity measures. Lastly a numerical example is given to demonstrate the possible application of similarity measures in medical diagnosis (MD).

2 Preliminary

In this section we recall some definitions and properties regarding neutrosophic set theory, soft set theory, n-valued refined neutrosophic, neutrosophic soft set theory and n-valued refined neutrosophic soft set required in this paper.

Definition 1 [20] A neutrosophic set A on the universe of discourse X is defined as

$$A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$$

where $T; I; F : X \to]^{-}0; 1^{+}[$ and

$$^{-}0 \leqslant T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

Molodtsov defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2 [15] A pair (F,A) is called a *soft set* over U, where F is a mapping

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 3 [13] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 4 [2] Let U be an initial universe set and E be a set of parameters, $T_j = \left\{T_1, T_2, ..., T_p\right\}$ be a set types of truths, $I_k = \{I_1, I_2, ..., I_r\}$ be a set types of indeterminacies and $F_k = \{F_1, F_2, ..., F_s\}$ be a set types of falsities and $n = p + r + s$ where all subcomponents $T_j; I_k; F_l$ subsets of $[0,1]$. Consider $A \subset E$. Let $P(U)$ denotes the set of all n-Valued refined neutrosophic sets of U. The collection (f_n, A) is termed to be the n-valued refined neutrosophic soft set over U, where f_n is a mapping given by $f_n : A \to P(U)$.

Example 1 (4-valued refined neutrosophic soft set)[2]
Let $U = \{u_1, u_2\}$ be a set of universe, $E = \{e_1, e_2, e_3\}$ a set of parameters. Let the Indeterminacy I is refined (split) as $Un = Unknown$, and $C = contradiction$. T, F, Un and C are subsets of $[0,1]$. Then, we get the following case of 4-Valued refined neutrosophic soft set:

$$f_4(e_1) = \left\{ \frac{u_1}{\langle 0.5; (0.2, 0.3); 0.4 \rangle}, \frac{u_2}{\langle 0.7; (0.1, 0.5); 0.5 \rangle} \right\},$$

$$f_4(e_2) = \left\{ \frac{u_1}{\langle 0.3; (0.3, 0.4); 0.5 \rangle}, \frac{u_2}{\langle 0.3; (0.2, 0.3); 0.4 \rangle} \right\},$$

$$f_4(e_3) = \left\{ \frac{u_1}{\langle 0.6; (0.3, 0.1); 0.2 \rangle}, \frac{u_2}{\langle 0.5; (0.1, 0.2); 0.2 \rangle} \right\},$$

and we can write the 4-valued refined neutrosophic soft set (f_4, E) as consisting of the following collection of approximations:

$$\begin{aligned}(f_4, E) = \Bigg\{ &\left(e_1, \left\{ \frac{u_1}{\langle 0.5; (0.2, 0.3); 0.4 \rangle}, \frac{u_2}{\langle 0.7; (0.1, 0.5); 0.5 \rangle} \right\} \right), \\ &\left(e_2, \left\{ \frac{u_1}{\langle 0.3; (0.3, 0.4); 0.5 \rangle}, \frac{u_2}{\langle 0.3; (0.2, 0.3); 0.4 \rangle} \right\} \right), \\ &\left(e_3, \left\{ \frac{u_1}{\langle 0.6; (0.3, 0.1); 0.2 \rangle}, \frac{u_2}{\langle 0.5; (0.1, 0.2); 0.2 \rangle} \right\} \right) \Bigg\}.\end{aligned}$$

Definition 5 [2] Let (f_n,A) and (g_n,B) be two n-valued refined neutrosophic soft sets over the common universe U. (f_n,A) is said to be n-valued refined neutrosophic soft subset of (g_n,B) if $A \subset B$; and $T_f^j(e)(x) \leq T_g^j(e)(x)$ where $j \in \{1,2,...,p\}$; $I_f^k(e)(x) \leq I_g^k(e)(x)$ where $k \in \{1,2,...,r\}$ and $F_f^l(e)(x) \geq F_g^l(e)(x)$ where $l \in \{1,2,...,s\}$; $\forall e \in A; x \in U$. We denote it by $(f_n,A) \subseteq (g_n,B)$. (f_n,A) is said to be n-valued refined neutrosophic soft super set of (g_n,B) if (g_n,B) is an n-valued refined neutrosophic soft subset of (f_n,A). We denote it by $(f_n,A) \supseteq (g_n,B)$.

Definition 6 [2] Suppose $p = r$, the complement of an n-valued refined neutrosophic soft set (f_n,A) denoted by $(f_n;A)^c$ and is denoted as $(f_n,A)^c = (f_n^c, \rceil A)$; where $f_n^c : \rceil A \rightarrow P(U)$ is a mapping given by $f_n^c(x) =$ n-valued refined neutrosophic soft complement with $T_{f^c}^j(x) = F_f^j(x)$, $I_{f^c}^k(x) = I_f^k(x)$ and $F_{f^c}^j(x) = T_f^j(x)$ where $j \in \{1,2,...,p\}$ and $k \in \{1,2,...,r\}$.

Definition 7 [2] Let (f_n,A) and (g_n,B) be two n-valued refined neutrosophic soft sets over the common universe U, we say that (f_n,A) and (g_n,B) are symmetric and denoted by $|(f_n,A)| \equiv |(g_n,B)|$ iff

1. $|T_f| = |T_g|$,
2. $|I_f| = |I_g|$,
3. $|F_f| = |F_g|$.

Definition 8 [2] Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U . Then the union of (f_n,A) and (g_n,B) is denoted by '$(f_n,A) \cup (g_n,B)$' and is defined by $(f_n,A) \cup (g_n,B) = (h,C)$, where $C = A \cup B$ and the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of (h,C) are as follows:

$\forall j \in \{1,2,...,p\}$

$$T_h^j(e)(m) = T_f^j(e)(m); \quad \text{if } e \in A - B;$$

$$= T_g^j(e)(m); \quad \text{if } e \in B - A;$$

$$= max(T_f^j(e)(m); T_g^j(e)(m)); \quad \text{if } e \in A \cap B.$$

$\forall k \in \{1,2,...,r\}$

$$I_h^k(e)(m) = I_f^k(e)(m); \quad \text{if } e \in A - B;$$

$$= I_g^k(e)(m); \quad \text{if } e \in B - A;$$

$$= \frac{I_f^k(e)(m) + I_g^k(e)(m))}{2}; \quad \text{if } e \in A \cap B.$$

and $\forall l \in \{1,2,...,s\}$

$F_h^l(e)(m) = F_f^l(e)(m);$ if $e \in A - B;$

$= F_g^l(e)(m);$ if $e \in B - A;$

$= min(F_f^l(e)(m); F_g^l(e)(m));$ if $e \in A \cap B.$

Definition 9 [2] Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U . Then the intersection of (f_n,A) and (g_n,B) is denoted by '$(f_n,A) \cap (g_n,B)$' and is defined by $(f_n,A) \cap (g_n,B) = (d,C)$, where $C = A \cup B$ and the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of (d,C) are as follows: $\forall e \in C$

$\forall j \in \{1,2,...,p\}$, $T_d^j(e)(m) = min(T_f^j(e)(m); T_g^j(e)(m));.$

$\forall k \in \{1,2,...,r\}$, $I_d^k(e)(m) = \dfrac{I_f^k(e)(m) + I_g^k(e)(m))}{2}$

and $\forall l \in \{1,2,...,s\}$, $F_d^l(e)(m) = max(F_f^l(e)(m); F_g^l(e)(m)).$

Definition 10 [2] Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U. Then the 'AND' operation on them is denoted by '$(f_n,A) \bigvee (g_n,B)$' and is defined by $(f_n,A) \bigvee (g_n,B) = (q_n, A \times B)$, where the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of $(q_n, A \times B)$ are as follows: $\forall \alpha \in A, \forall \beta \in B$.

$\forall j \in \{1,2,...,p\}$, $T_q^j(\alpha,\beta)(m) = min(T_f^j(\alpha)(m); T_g^j(\beta)(m));$

$\forall k \in \{1,2,...,r\}$, $I_q^k(\alpha,\beta)(m) = \dfrac{I_f^k(\alpha)(m) + I_g^k(\beta)(m))}{2}$ and

$\forall l \in \{1,2,...,s\}$, $F_q^l(\alpha,\beta)(m) = max(F_f^l(\alpha)(m); F_g^l(\beta)(m)).$

Definition 11 [2] Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U. Then the 'OR' operation on them is denoted by '$(f_n,A) \bigwedge (g_n,B)$' and is defined by $(f_n,A) \bigwedge (g_n,B) = (q_n, A \times B)$, where the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of $(q_n, A \times B)$ are as follows: $\forall \alpha \in A, \forall \beta \in B$.

$\forall j \in \{1,2,...,p\}$, $T_q^j(\alpha,\beta)(m) = max(T_f^j(\alpha)(m); T_g^j(\beta)(m));$

$\forall k \in \{1,2,...,r\}$, $I_q^k(\alpha,\beta)(m) = \dfrac{I_f^k(\alpha)(m) + I_g^k(\beta)(m))}{2}$ and

$\forall l \in \{1,2,...,s\}$, $F_q^l(\alpha,\beta)(m) = min(F_f^l(\alpha)(m); F_g^l(\beta)(m)).$

3 An Application of n-VRNSS in Decision Making

In this section, we present an application of n-VRNSS in a decision making problem. Assume that a company wants to fill a position. There are two candidates who form the universe $U = \{u_1, u_2\}$, the hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3\}$, the parameters e_i $(i = 1,2,3)$ stand for "experience", "computer knowledge" and "good speaking" respectively. Let the truth T is refined (split) as $T_A = AbsoluteTruth$, $T_R = RelativeTruth$, indeterminacy I is refined (split) as $I_A = AbsoluteIndeterminacy$, $I_R = RelativeIndeterminacy$ and and the falsity F is refined (split) as $F_A = Absolutefalsity$, $I_R = Relativefalsity$. T_A, T_R, I_A, I_R F_A and F_R are subsets of $[0,1]$, such that $n = p+r+s$ where $n = 6, p = 2, r = 2, s = 2$. Then, after a serious discussion the committee constructs the following case of 6-Valued refined neutrosophic soft set

Example 2 [2]

$$f_5(e_1) = \left\{ \frac{u_1}{\langle(0.5,0.4);(0.2,0.3);(0.4,0.3)\rangle}, \frac{u_2}{\langle(0.7,0.4);(0.1,0.5);(0.5,0.1)\rangle} \right\},$$

$$f_5(e_2) = \left\{ \frac{u_1}{\langle(0.3,0.5);(0.3,0.4);(0.5,0.4)\rangle}, \frac{u_2}{\langle(0.3,0.3);(0.2,0.3);(0.4,0.8)\rangle} \right\},$$

$$f_5(e_3) = \left\{ \frac{u_1}{\langle(0.6,0.3);(0.3,0.1);(0.2,0.4)\rangle}, \frac{u_2}{\langle(0.5,0.7);(0.1,0.2);(0.2,0.2)\rangle} \right\},$$

and we can write the 6-valued refined neutrosophic soft set (f_6, E) as consisting of the following collection of approximations:

$$(f_6,E) = \left\{ \left(e_1, \left\{ \frac{u_1}{\langle(0.5,0.4);(0.2,0.3);(0.4,0.3)\rangle}, \frac{u_2}{\langle(0.7,0.4);(0.1,0.5);(0.5,0.1)\rangle} \right\} \right), \right.$$
$$\left(e_2, \left\{ \frac{u_1}{\langle(0.3,0.5);(0.3,0.4);(0.5,0.4)\rangle}, \frac{u_2}{\langle(0.3,0.3);(0.2,0.3);(0.4,0.8)\rangle} \right\} \right),$$
$$\left. \left(e_3, \left\{ \frac{u_1}{\langle(0.6,0.3);(0.3,0.1);(0.2,0.4)\rangle}, \frac{u_2}{\langle(0.5,0.7);(0.1,0.2);(0.2,0.2)\rangle} \right\} \right) \right\}.$$

Also we can represent the above set as shown in Table 1.

Table 1 6-valued refined neutrosophic soft set (f_6, E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T_A	0.5	0.3	0.6	0.7	0.3	0.5
T_R	0.4	0.5	0.3	0.4	0.3	0.7
I_A	0.2	0.3	0.3	0.1	0.2	0.1
I_R	0.3	0.4	0.1	0.5	0.3	0.2
F_A	0.4	0.5	0.2	0.5	0.4	0.2
F_R	0.3	0.4	0.4	0.1	0.8	0.2

The following algorithm may be followed by the company to fill the position.

1. Input the n-VRNSS (F_n, E).
2. Find the neutrosophic soft set (F,E),(Table 2) where:
   (a) $\forall e \in E$, the truth-memberships $T_{(u_i,e)} = \frac{1}{p} \sum_{j=1}^{p} T_j(u_i, e)$.
   (b) $\forall e \in E$, the indeterminacy-memberships $I_{(u_i,e)} = \frac{1}{r} \sum_{j=1}^{r} I_j(u_i, e)$.
   (c) $\forall e \in E$, the falsity-memberships $F_{(u_i,e)} = \frac{1}{s} \sum_{j=1}^{s} F_j(u_i, e_i)$.
3. Find $\alpha_j = \sum_i T(u_{ij})$.
4. Find $\beta_j = \sum_i I(u_{ij})$.
5. Find $\gamma_j = \sum_i F(u_{ij})$.
6. Find $\delta_j = \alpha_j + \beta_j - \gamma_j$.
7. Find m, for which $\delta_m = \max \delta_j$. Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the company using its option.

Table 2 Neutrosophic soft set (f,E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T	0.45	0.4	0.45	0.55	0.3	0.6
I	0.25	0.35	0.2	0.3	0.25	0.15
F	0.35	0.45	0.3	0.3	0.6	0.2

Now we use this algorithm to find the best choice for the company to fill the position. From Table 2 we have the following:

Table 3: $\delta_j = \alpha_j + \beta_j - \gamma_j$.

$\alpha_j = \sum_i u_{ij}$	$\beta_j = \sum_i u_{ij}$	$\gamma_j = \sum_i u_{ij}$	$\delta_j = \alpha_j + \beta_j - \gamma_j$.
$\alpha_1 = 1.3$	$\beta_1 = 0.8$	$\gamma_1 = 1.1$	$\delta_1 = 1$
$\alpha_2 = 1.45$	$\beta_2 = 0.7$	$\gamma_2 = 1.1$	$\delta_2 = 1.05$

Then $\max \delta_j = \delta_2$, so the committee will choose candidate 2 for the job.

4 Similarity measure for n-valued refined neutrosophic soft set(n-VRNSSs)

In this section we have proposed a new method for measuring similarity measure and weighted similarity measure for n-VRNSSs and some basic properties are also studied.

Definition 12 Let $U = \{x_1, x_2, x_3, ..., x_d\}$ be the universe of discourse and $E = \{e_1, e_2, e_3, ..., e_k\}$ be the set of parameters and let (f_n, E) and (g_n, E) be two n-valued refined neutrosophic soft sets over $U(E)$. Then the similarity measure between n-VNSSs (f_n, E) and (g_n, E) where $n = p + r + s$, is denoted by $S(f_n, g_n)$ and is defined as follows :

$$S(f_n, g_n) = \frac{1}{3dk} \sum_{i=1}^{d} \sum_{j=1}^{k} \left(3 - \frac{1}{p} \sum_{l=1}^{p} \left| T^l{}_{f_n}(x_i)(e_j) - T^l{}_{g_n}(x_i)(e_j) \right| - \frac{1}{r} \sum_{l=1}^{r} \left| I^l{}_{f_n}(x_i)(e_j) - I^l{}_{g_n}(x_i)(e_j) \right| - \frac{1}{s} \sum_{l=1}^{s} \left| F^l{}_{f_n}(x_i)(e_j) - F^l{}_{g_n}(x_i)(e_j) \right| \right) \quad (1)$$

Example 3 Let $U = \{u_1, u_2\}$ be a set of universe, $E = \{e_1, e_2, e_3\}$ a set of parameters. Let the Indeterminacy I is refined (split) as $Un = Unknown$, and $C = contradiction$. T, F, Un and C are subsets of $[0, 1]$. Let:

$$(f_4, E) = \left\{ \left(e_1, \left\{ \frac{u_1}{\langle 0.3; (0.3, 0.5); 0.2 \rangle}, \frac{u_2}{\langle 0.8; (0.3, 0.6); 0.3 \rangle} \right\} \right), \left(e_2, \left\{ \frac{u_1}{\langle 0.4; (0.4, 0.6); 0.4 \rangle}, \frac{u_2}{\langle 0.5; (0.3, 0.4); 0.3 \rangle} \right\} \right), \left(e_3, \left\{ \frac{u_1}{\langle 0.7; (0.5, 0.1); 0.3 \rangle}, \frac{u_2}{\langle 0.6; (0.3, 0.1); 0.3 \rangle} \right\} \right) \right\}.$$

and

$$(g_4, E) = \left\{ \left(e_1, \left\{ \frac{u_1}{\langle 0.5; (0.2, 0.3); 0.4 \rangle}, \frac{u_2}{\langle 0.7; (0.1, 0.5); 0.5 \rangle} \right\} \right), \left(e_2, \left\{ \frac{u_1}{\langle 0.3; (0.3, 0.4); 0.5 \rangle}, \frac{u_2}{\langle 0.3; (0.2, 0.3); 0.4 \rangle} \right\} \right), \left(e_3, \left\{ \frac{u_1}{\langle 0.6; (0.3, 0.1); 0.2 \rangle}, \frac{u_2}{\langle 0.5; (0.1, 0.2); 0.2 \rangle} \right\} \right) \right\}.$$

Then we can represent the above sets as shown in Table 4 and 5.

Table 4 4-valued refined neutrosophic soft set (f_4,E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T	0.3	0.4	0.7	0.8	0.5	0.6
Un	0.3	0.4	0.5	0.3	0.3	0.3
C	0.5	0.6	0.1	0.6	0.4	0.1
F	0.2	0.4	0.3	0.3	0.3	0.3

Table 5 4-valued refined neutrosophic soft set (g_4,E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T	0.5	0.3	0.6	0.7	0.3	0.5
Un	0.2	0.3	0.3	0.1	0.2	0.1
C	0.3	0.4	0.1	0.5	0.3	0.2
F	0.4	0.5	0.2	0.5	0.4	0.2

Now by using definition 12 we can found the similarity measure between (f_4,E) and (g_4,E) as follows:

$$S(f_4,g_4)=\frac{1}{18}\sum_{i=1}^{2}\sum_{j=1}^{3}\Bigg(3-\frac{1}{1}\sum_{l=1}^{1}\left|T^l{}_f(u_i)(e_j)-T^l{}_g(u_i)(e_j)\right|$$

$$-\frac{1}{2}\sum_{l=1}^{2}\left|I^l{}_f(u_i)(e_j)-I^l{}_g(u_i)(e_j)\right|$$

$$-\frac{1}{1}\sum_{l=1}^{1}\left|F^l{}_f(u_i)(e_j)-F^l{}_g(u_i)(e_j)\right|\Bigg)$$

$$=\frac{1}{18}\sum_{i=1}^{2}\sum_{j=1}^{3}\big(3-\left|T_f(u_i)(e_j)-T_g(u_i)(e_j)\right|$$

$$-\frac{1}{2}\sum_{l=1}^{2}\left|I^l{}_f(u_i)(e_j)-I^l{}_g(u_i)(e_j)\right|$$

$$-\left|F_f(u_i)(e_j)-F_g(u_i)(e_j)\right|\big)$$

After some calculations, the similarity measure between (f_4,E) and (g_4,E) is given

by $S(f_4,g_4)\simeq 0.87$.

Theorem 1 *If $S(f_n,g_n)$ be the similarity measure between two n-VRNSSs (f_n,E) and (g_n,E) then:*

1. $0\leq S(f_n,g_n)\leq 1$,
2. $S(f_n,g_n)=S(g_n,f_n)$,
3. $S(f_n,f_n)=1$,
4. $(f_n,E)\cap(g_n,E)=\phi\Leftrightarrow S(f_n,g_n)=0$.
5. *If* $(f_n,E)\subseteq(g_n,E)$ *and* $(g_n,E)\subseteq(z_n,E)$, *then* $S(f_n,z_n)\leq S(g_n,z_n)$.

Proof The proof is straightforward from Definition 12.

Definition 13 Let (f_n, E) and (g_n, E) be two n-VRNSSs over the same universe $(U; E)$. We call the two n-VRNSSs to be significantly similar if $S(f_n, g_n) \geq \dfrac{1}{2}$

5 Application of This Similarity Measure in Medical Diagnosis

In the following example which is given by Alkhazaleh and Salleh [4] we will try to estimate the possibility that a sick person having certain visible symptoms is suffering from dengue fever. For this we first construct a model n-VRNSS for dengue fever and the n-VRNSS of symptoms for the sick person. Next we find the similarity measure of these two sets. If they are significantly similar then we conclude that the person is possibly suffering from dengue fever. Let the Indeterminacy I is refined (split) as $Un = Unknown$, and $C = contradiction$. T, F, Un and C are subsets of $[0,1]$. Let our universal set contain only two elements “yes” and “no”, that is, $U = \{y, n\}$. Here the set of parameters E is the set of certain visible symptoms. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}\}$, where e_1 = body temperature, e_2 = cough with chest congestion, e_3 = loose motion, e_4 = chills, e_5 = headache, e_6 = low heart rate (bradycardia), e_7 = pain upon moving the eyes, e_8 = breathing trouble, e_9 = a flushing or pale pink rash comes over the face, e_{10} = low blood pressure (hypotension) and e_{11} = loss of appetite.
Our model n-VRNSS for dengue fever f_4 is given in Table 6 and this can be prepared with the help of a physician. After talking to the sick person we can construct his n-VRNSS g_4 as in Table 6. Now we find the similarity measure of these two sets (as in Example 3), here $S(f_4, g_4) \cong 0.62 < \frac{1}{2}$. Hence the two n-VRNSSs are significantly similar. Therefore, we conclude that the person is suffering from dengue fever.

Table 6: Model n-VRNSS for dengue fever and sick person

	Model n-VRNSS for dengue fever f_4		Model n-VRNSS for sick person g_4	
e_i	y	n	y	n
e_1	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.4;(0.2,0.3);0.5\rangle$	$\langle 0.8;(0.2,0.2);0.2\rangle$
e_2	$\langle 0;(0,0);1\rangle$	$\langle 1;(0,0);0\rangle$	$\langle 0.8;(0.3,0.2);0.1\rangle$	$\langle 0.3;(0.1,0.2);0.7\rangle$
e_3	$\langle 0;(0,0);1\rangle$	$\langle 1;(0,0);0\rangle$	$\langle 0.5;(0.4,0.2);0.4\rangle$	$\langle 0.4;(0.2,0.4);0.5\rangle$
e_4	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.7;(0.4,0.2);0.4\rangle$	$\langle 0.3;(0.3,0.2);0.5\rangle$
e_5	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.9;(0.3,0.4);0.2\rangle$	$\langle 0.1;(0.2,0.4);0.8\rangle$
e_6	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.6;(0.5,0.3);0.3\rangle$	$\langle 0.2;(0.3,0.2);0.6\rangle$
e_7	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.5;(0.4,0.1);0.3\rangle$	$\langle 0.3;(0.3,0.3);0.6\rangle$
e_8	$\langle 0;(0,0);1\rangle$	$\langle 1;(0,0);0\rangle$	$\langle 0.8;(0.3,0.3);0.5\rangle$	$\langle 0.2;(0.1,0.4);0.6\rangle$
e_9	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.7;(0.2,0.1);0.4\rangle$	$\langle 0.3;(0.1,0.1);0.5\rangle$
e_{10}	$\langle 1;(0,0);0\rangle$	$\langle 0;(0,0);1\rangle$	$\langle 0.6;(0.1,0.1);0.3\rangle$	$\langle 0.4;(0.2,0.3);0.5\rangle$
e_{11}	$\langle 0;(0,0);1\rangle$	$\langle 1;(0,0);0\rangle$	$\langle 0.3;(0.2,0.2);0.6\rangle$	$\langle 0.6;(0.1,0.4);0.3\rangle$

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