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Vague –Valued Possibility Neutrosophic Vague Soft Expert Set Theory and Its Applications

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Abstract: In this paper, we first propose the concept of Vague-valued possibility Neutrosophic vague soft expert sets (VPNVSEsets in short). It is a combination of vague-valued possibility neutrosophic vague sets and soft expert sets. We also define its basic operations and study some related properties. Lastly an algorithm is proposed applied to the concept of vague-valued possibility Neutrosophic vague soft expert sets in hypothetical decision making problem. Here we associate the degree of belongingness degree of indeterminacy and non-belongingness of the elements of universe set with the vague –valued possibility set.

Keywords: Soft set, Neutrosophic soft expert set, Neutrosophic Vague soft set.

1. Introduction

Most real life problems involve data with a high level of uncertainty and imprecision. Traditionally, classical mathematical theories such as fuzzy mathematics, probability theories and interval mathematics are used to deal with uncertain and fuzziness. But all these theories have their difficulties and weakness as pointed out by Molodstov [14]. This led to the introduction of the theory of soft sets by Molodstov [14] in 1999. However, in order to handle the indeterminate and inconsistent information, neutrosophic set is defined [18]. The theory of vague set was first proposed by Gau and Buehrer [12]. It is an extension of fuzzy set theory. In 2010,W. Xu J. Ma, S. Wang and G. Hao, introduced Vague soft sets and their properties as a generalization of [12]. G. Selvachandran and A.R. salleh [19], introduced Possibility vague soft expert theory and its application in decision making.

In, [18] Smarandache talked about neutrosophic set theory. It is an important new mathematical tools for handling problems involving imprecise, indeterminacy and inconsistent data. Neutrosophic vague set was defined by S. Allehezaleh [2] in 2015. The concept of neutrosophic vague soft expert set was first introduced by Ashraf Al-Qurn and N. Hassan in 2016 [16]. It is the combination of neutrosophic vague sets and soft expert sets. In2016,[19] G.Selvachandran and Abdul Razak Salleh introduced the concept of Possibility Intuitionistic Fuzzy Soft Expert and Its Application in Decision Making . In [15], Mukherjee and Sarkar introduced the concept of possibility interval valued intuitionistic fuzzy soft expert theory which is a generalization of [20]. N. Hassan and A.Al-Quran [13], introduced Possibility Neutrosophic Vague soft expert set for decision under uncertainty. For further applications we refer the papers{[4],[5],[6],[7],[8],[9],[10],[11]}.

We first introduce the concept of vague-valued possibility neutrosophic vague soft expert set. It is a combination of vague-valued possibility neutrosophic vague set and soft expert set. The concept is to improve the reasonability of decision making in reality. Next we define its basic operation as a generalization of [13]. Finally we present an application of this concept in solving a decision making problem.

2. Preliminaries

We give some basic notions in neutrosophic vague set, neutrosophic vague soft set, soft expert set and neutrosophic soft expert set.

Definition 2.1. [2] A neutrosophic vague set A_{NV}(NVS in short) on the universe of discourse X written as A_{NV} = {<x; $\hat{T}A_{NV}(x)$; $\hat{I}A_{NV}(x)$; $\hat{F}A_{NV}(x)$>; $x \in X$} whose truth-membership, indeterminacy-membership, and falsity-membership functions is defined as $\hat{T}A_{NV}(x) = [T^-, T^+]$, $\hat{I}A_{NV}(x) = [I^-, I^+]$ and $\hat{F}A_{NV}(x) = [F^-, F^+]$, where (1) $T^+ = 1 - F^-$, (2) $F^+ = 1 - T^-$ and (3) $^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.2. [2] If Ψ_{NV} is a NVS of the universe U, where $\forall u_i \in U$, $\hat{T}\Psi_{NV}(x) = [1, 1]$, $\hat{I}\Psi_{NV}(x) = [0, 0]$, $\hat{F}\Psi_{NV}(x) = [0, 0]$, then Ψ_{NV} is called a unit NVS, where $1 \leq i \leq n$. If Φ_{NV} is a NVS of the universe U, where $\forall u_i \in U$, $\hat{T}\Psi_{NV}(x) = [0, 0]$, $\hat{I}\Psi_{NV}(x) = [1, 1]$, $\hat{F}\Psi_{NV}(x) = [1, 1]$, then Φ_{NV} is called a zero NVS, where $1 \leq i \leq n$.

Definition 2.3.[2] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, (1) $\hat{T}A_{NV}(u_i) = \hat{T}B_{NV}(u_i)$, (2) $\hat{I}A_{NV}(u_i) = \hat{I}B_{NV}(u_i)$ and (3) $\hat{F}A_{NV}(u_i) = \hat{F}B_{NV}(u_i)$, then the NVS A_{NV} is equal to B_{NV}, denoted by $A_{NV} = B_{NV}$, where $1 \leq i \leq n$.

Definition 2.4. [2] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, (1) $\hat{T}A_{NV}(u_i) \leq \hat{T}B_{NV}(u_i)$, (2)$\hat{I}A_{NV}(u_i) \geq \hat{I}B_{NV}(u_i)$ and (3)$\hat{F}A_{NV}(u_i) \geq \hat{F}B_{NV}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 2.5. [2] The complement of a NVS A_{NV} is denoted by A^c and is defined by

$\widehat{T^c}A_{NV}(x) = [1 - T^+, 1 - T^-]$,

$\widehat{I^c}A_{NV}(x) = [1 - I^+, 1 - I^-]$, and

$\widehat{F^c}A_{NV}(x) = [1 - F^+, 1 - F^-]$.

Definition 2.6.[2] The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$T_{C_{NV}}(x) = [\max(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}), \max(T^+_{A_{NV_x}}, T^+_{B_{NV_x}})]$

$I_{C_{NV}}(x) = [\min(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}), \min(I^+_{A_{NV_x}}, I^+_{B_{NV_x}})]$ and

$F_{C_{NV}}(x) = [\min(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}), \min(F^+_{A_{NV_x}}, F^+_{B_{NV_x}})]$

Definition 2.7. [2] The intersection of two NVSs A_{NV} and B_{NV} is a *NVS* C_{NV}, written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$T_{H_{NV}}(x) = [\min(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}), \min(T^+_{A_{NV_x}}, T^+_{B_{NV_x}})]$

$I_{H_{NV}}(x) = [\max(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}), \max(I^+_{A_{NV_x}}, I^+_{B_{NV_x}})]$ and

$F_{H_{NV}}(x)$= [max ($F^-_{A_{NV_x}}$, $F^-_{B_{NV_x}}$), max ($F^+_{A_{NV_x}}$, $F^+_{B_{NV_x}}$)]

Definition 2.8. [17] Let$U$ be an initial universal set . Let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and let $A\subseteq E$. A collection of pairs ($\hat{F}$, E) is called a neutrosophic vague soft set {*NVSset*} over U, where $\hat{F}$ is a mapping given by $\hat{F}$: A → $NV(U)$.

Let U be a universe. E a set of parameters. X a set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A\subseteq Z$.

Definition 2.9. [3] A pair (F, A) is called a soft expert set over U, where F is a mapping given by $F : A \to P(U)$, where $P(U)$ denotes the power set of U.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O = {1 = agree, 0 = disagree} a set of opinions. Let $Z = E \times X \times O$ and $A\subseteq Z$.

Definition 2.10. [16] A pair (F, A) is called a neutrosophic soft expert set (NSES in short) over U, where F is a mapping given by $F : A \to PN(U)$, where $PN(U)$ denotes the power neutrosophic set of U.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O = {1 = agree, 0 = disagree} a set of opinions. Let $Z = E \times X \times O$ and $A\subseteq Z$.

Definition 2.11. [16] A pair (F, A) is called a neutrosophic vague soft expert set over U, where F is a mapping given by $F : A \to NV^U$, where NV^U denotes the power neutrosophic vague set of U.

Suppose $F : A \to NV^U$ is a function defined as $F(a) = F(a)(u)$, $\forall u\in U$. For each $a_i \in A$, $F(a_i) = F(a_i)(u)$, where $F(a_i)$ represents the degree of belongingness, degree of indeterminacy and non-belongingness of the elements of U in $F(a_i)$. Hence $F(a_i)$ can be written as:

$F(a_i) = \left\{\frac{u_i}{F(a_i)(u_i)}\right\}$, for i = 1, 2, 3, …

Where $F(a_i)(u_i) = <[T^-_{F(a_i)}(u_i), T^+_{F(a_i)}(u_i)], [I^-_{F(a_i)}(u_i), I^+_{F(a_i)}(u_i)], [F^-_{F(a_i)}(u_i), F^+_{F(a_i)}(u_i)]>$ and $T^+_{F(a_i)}(u_i)$ = 1- $F^-_{F(a_i)}(u_i)$, $F^+_{F(a_i)}(u_i)$ = 1- $T^-_{F(a_i)}(u_i)$ with $[T^-_{F(a_i)}(u_i), T^+_{F(a_i)}(u_i)]$, $[I^-_{F(a_i)}(u_i), I^+_{F(a_i)}(u_i)]$ and$[F^-_{F(a_i)}(u_i), F^+_{F(a_i)}(u_i)]$ representing the truth-membership function, indeterminacy-membership function and falsity-membership function of each of the elements $u_i\in U$, respectively.

Example 2.12 [16]. Suppose that a company produced new types of its products and wishes to take the opinion of some experts concerning these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products. $E = \{e_1, e_2\}$ a set of decision parameters where $e_i(i = 1, 2)$ denotes the decision "easy to use," and "quality," respectively. Let $X = \{p, q\}$ be a set of experts. Suppose that the company has distributed a questionnaire to the two experts to make decisions on the company's products, and we get the following:

$F(e_1, p, 1)$

$$= \left\{\frac{u_1}{<[0.2,0.8]; [0.1,0.3]; [0.2,0.8]>}, \frac{u_2}{<[0.1,0.7]; [0.2,0.5]; [0.3,0.9]>}, \frac{u_3}{<[0.5,0.6]; [0.3,0.7]; [0.4,0.5]>}, \frac{u_4}{<[0.8,1]; [0.1,0.2]; [0,0.2]>},\right\}$$

$F(e_1, q, 1)$

$$= \left\{\frac{u_1}{<[0.8,0.9]; [0.3,0.4]; [0.1,0.2]>}, \frac{u_2}{<[0.2,0.4]; [0.2,0.4]; [0.6,0.8]>}, \frac{u_3}{<[0,0.5] ; [0.5,0.7]; [0.5,1]>}, \frac{u_4}{<[0.6,0.7]; [0.2,0.4]; [0.3,0.4]>}\right\}$$

$F(e_2, p, 1)$

$$= \left\{\frac{u_1}{<[0.3,0.9]; [0.1,0.3]; [0.1,0.7]>}, \frac{u_2}{<[0.2,0.5]; [0.2,0.5]; [0.5,0.8]>}, \frac{u_3}{<[0.6,0.9]; [0.1,0.7]; [0.1,0.4]>}, \frac{u_4}{<[0.2,0.4]; [0.2,0.2]; [0.6,0.8]>}\right\}$$

$F(e_2, q, 1)$

$$= \left\{\frac{u_1}{<[0.4,0.6];[0.1,0.4];[0.4,0.6]>}, \frac{u_2}{<[0.1,0.3];[0.2,0.4];[0.7,0.9]>}, \frac{u_3}{<[0.1,0.5];[0.5,0.7];[0.5,0.9]>}, \frac{u_4}{<[0.2,0.7];[0.2,0.4];[0.3,0.8]>}\right\}$$

$F(e_1, p, 0)$

$$= \left\{\frac{u_1}{<[0.2,0.8];[0.7,0.9];[0.2,0.8]>}, \frac{u_2}{<[0.3,0.9];[0.5,0.8];[0.1,0.7]>}, \frac{u_3}{<[0.4,0.5];[0.3,0.7];[0.5,0.6]>}, \frac{u_4}{<[0,0.2];[0.8,0.9];[0.8,1]>}\right\}$$

$F(e_1, q, 0)$

$$= \left\{\frac{u_1}{<[0.1,0.7];[0.7,0.9];[0.3,0.9]>}, \frac{u_2}{<[0.5,0.8];[0.5,0.8];[0.2,0.5]>}, \frac{u_3}{<[0.5,1];[0.3,0.5];[0,0.5]>}, \frac{u_4}{<[0.3,0.4];[0.6,0.8];[0.6,0.7]>}\right\}$$

$F(e_2, p, 0)$

$$= \left\{\frac{u_1}{<[0.2,0.8];[0.7,0.9];[0.2,0.8]>}, \frac{u_2}{<[0.3,0.9];[0.5,0.8];[0.1,0.7]>}, \frac{u_3}{<[0.1,0.4];[0.3,0.9];[0.6,0.9]>}, \frac{u_4}{<[0.6,0.8];[0.8,0.8];[0.2,0.4]>}\right\}$$

$F(e_2, q, 0)$

$$= \left\{\frac{u_1}{<[0.4,0.6];[0.6,0.9];[0.4,0.6]>}, \frac{u_2}{<[0.7,0.9];[0.6,0.8];[0.1,0.3]>}, \frac{u_3}{<[0.5,0.9];[0.3,0.5];[0.1,0.5]>}, \frac{u_4}{<[0.3,0.8];[0.6,0.8];[0.2,0.7]>}\right\}$$

The neutrosophic vague soft expert set (F, Z) is a parameterized family $\{F(e_i), i = 1, 2, 3, \ldots\}$ of all neutrosophic vague sets of U and describes a collection of approximation of an object.

Definition 2.13. [16]. The complement of a NVSE set (F,A) is denoted by (F, A) and is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \to NV^U$ is a mapping given by $F^c(\alpha) = \tilde{c}(F(\alpha))$, $\forall \alpha \in A$.

Where $\tilde{c}$is a neutrosophic vague complement.

.**Definition2.14. [15]** The union of two NVSE sets (F, A) and (G, B) over U, denoted by$(F, A)\tilde{\cup}$ (G, B), is a neutrosophic vague soft expert set (H, C), where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$(H, C) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A - B, \\ G(\varepsilon), \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \tilde{\cup} G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$ where $\tilde{\cup}$ denotes the union of the neutrosophic vague set

Definition 2.15. [16] The intersection of two neutrosophic vague soft expert sets (F, A) and (G, B) over a universe U, is a neutrosophic vague soft expert set (H, C), denoted by $(F, A)\tilde{\cap}$ (G, B) such that $C = A \cap B$ and $\forall e \in c$

$$(H, C) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A - B, \\ G(\varepsilon), \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \tilde{\cap} G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$ where$\tilde{\cap}$ denotes the intersection of neutrosophic vague set.

Definition 2.16 [16]. Let (F,A) and (G, B) be any two NVSE sets over a soft universe (U, Z).

Then "(F,A)AND(G, B)" denoted (F,A) $\tilde{\wedge}$ (G, B) is defined by $(F,A)\tilde{\wedge}$ $(G, B) = (H, A\times B)$, where $(H, A\times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\cap$ represents the basic intersection.

Definition 2.17[16]. Let (F,A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z).

Then "(F,A) OR (G, B)" denoted $(F,A)\tilde{\vee}$ (G, B) is defined by $(F,A)\tilde{\vee}$ $(G, B) = (H,A \times B)$, where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\cup$represents the basic union.

Definition 2.18 [16]. Let U be a Universe. E a set of parameters, X a set of experts. Q = {1 = agree, 0 = disagree} a set of opinions. Let Z = E×X×Q and A⊆Z.

Let U= {u_1,u_2, …,u_n} be a universal set of elements, let E= { e_1, e_2, e_3,------, e_m} be a universal set of parameters . Let X = {x_1,x_2, …,x_i} be a set of experts and let Q = {1 = agree, 0 = disagree} be a set of opinions. Let Z = E×X×Q and A⊆Z. Then the pair (U, Z) is called a soft universe. Let F:Z→NVSs(V), and p be a fuzzy subset of Z define by p: Z→I ^{U}, where I^U is the collection of all fuzzy subsets of U. Suppose Fp: Z→NVSs(U)×I^U be a function define by Fp= {(F(Z)(u_i), P(Z)(u_i))},for all ui∈U.. Then Fp is called a possibility neutrosophic vague soft expert set (denoted by PNVSES) over the soft universe (U, Z). For each z_i∈Z, Fp(z_i) = (F(z_i)(u_i), P(z_i)(u_i)) where F(z_i) represent the degree of belongingness degree of indeterminacy and non-belongingness of the elements of U in F(z_i) and P(z_i) represents the degree of possibility of belongingness of the elements of U in F(z_i).

3. Vague-valued possibility neutrosophic vague soft expert set

In this section we introduce the definition of a vague-valued Possibility neutrosophic vague soft expert se t(VPNVSE set)}.

Let *U* be a Universe. *E* a set of parameters. *X* a set of experts and *Q* = {1 = agree, 0 = disagree} a set of opinions. Let *Z* = *E*×*X*×*Q* and *A*⊆*Z*.

Definition3.1. Let *U*={u_1,u_2, …,u_n} be a universal set of elements, let E= { e_1, e_2,e_3,------,e_m} be a universal set of parameters . Let *X* = {x_1,x_2, …,x_i} be a set of experts and let *Q* = {1 = agree, 0 = disagree} be a set of opinions. Let *Z* = *E*×*X*×*Q* and *A*⊆*Z*. Then the pair (*U*, *Z*) is called a soft universe. Let F:Z→NVS_s(V), and p be a vague- valued subset of Z define by p: Z→V(U).
Suppose F_P: *Z*→NVS_s(*U*)×V(U) be a function define by F_P= {(*F*(*Z*)(u_i), *P*(*Z*)(u_i))}, ∀u_i∈*V*. Then F_P is called a vague- valued possibility neutrosophic vague soft expert set (denoted by V*PNVSE*S) over the soft universe (*U*, *Z*). For each z_i∈*Z*, F_P(z_i) = (*F*(z_i)(u_i), *P*(z_i)(u_i)) where *F*(z_i) represent the degree of belongingness degree of indeterminacy and non-belongingness of the elements of *U* in *F*(z_i).

So *F*(z_i)(u_i) = {$[T^-_{F(z_i)}(u_i), T^+_{F(z_i)}(u_i)]$, $[I^-_{F(z_i)}(u_i), I^+_{F(z_i)}(u_i)]$, $[F^-_{F(z_i)}(u_i), F^+_{F(z_i)}(u_i)]$}

and $T^+_{F(z_i)}(u_i) = 1 - F^-_{F(z_i)}(u_i)$, $F^+_{F(z_i)}(u_i) = 1 - T^-_{F(z_i)}(u_i)$ with [$T^-_{F(z_i)}(u_i)$, $T^+_{F(z_i)}(u_i)$] , [$I^-_{F(z_i)}(u_i)$, $I^+_{F(z_i)}(u_i)$] , [$F^-_{F(z_i)}(u_i)$, $F^+_{F(z_i)}(u_i)$] representing the truth membership function indeterminacy membership function and fails membership function of each of the elements
u_i∈*U* respectively. P(z_i) represents the vague –value [t_A(x), 1-f_A(x)],indicates that the exact grade of membership of x to A(which may be unknown but it is bounded by t_A(x) and 1-f_A(x).. Hence F_P(z_i) can be written as F_P(z_i) = {$\left\{\left(\frac{u_i}{F(z_i)(u_i)}\right), P(z_i)(u_i)\right\}$ for *i* = 1, 2, 3,……, . The V*PNVSE*S(F_P,*z*) can be written simply as F_P. If *A*⊆*Z*, it is also possible to have a V*PNVSE*S(F_P, *A*). For simplicity we take the set of opinion contains of only two values namely agree and disagree.

Suppose that a company produced new types of its products & wishes to take the opinion of some experts corresponding those products. Let U = {u_1, u_2, u_3} be a set of products. *E* = {e_1, e_2} a set of decision parameters. Here, e_i (i=1,2) denote the decision "easy to use" and "equality". Let *X* = {p, q} be a set of experts. Suppose that the company has distributed questionnaire to, the two experts to make decisions on the company products. Then we have to following.

F_p: $Z \rightarrow VNVS_s(U) \times V(U)$ is a function then

$F_p(e_1, p, 1) = \{(\frac{u_1}{[0.2,0.8];[0.1,0.3];[0.2,0.8]}, [0.3,0.5], (\frac{u_2}{[0.1,0.7];[0.2,0.5];[0.3,0.9]}, [0.5,0.7], (\frac{u_3}{[0.5,0.6];[0.3,0.7];[0.4,0.5]}, [0.7,0.9]\}$

$F_p(e_1, q, 1) = \{(\frac{u_1}{[0.8,0.9];[0.3,0.4];[0.1,0.2]}, [0.4,0.6], (\frac{u_2}{[0.2,0.4];[0.2,0.4];[0.6,0.8]}, [0.6,0.8], (\frac{u_3}{[0.0,0.5];[0.5,0.7];[0.5,1]}, [0.8,1]\}$

$F_p(e_2, p, 1) = \{(\frac{u_1}{[0.3,0.9];[0.1,0.3];[0.1,0.7]}, [0.5,0.7], (\frac{u_2}{[0.2,0.5];[0.2,0.5];[0.5,0.8]}, [0.6,0.8], (\frac{u_3}{[0.6,0.9];[0.1,0.7];[0.1,0.4]}, [0.3,0.5]\}$

$F_p(e_2, q, 1) = \{(\frac{u_1}{[0.4,0.6];[0.1,0.4];[0.4,0.6]}, [0.2,0.4], (\frac{u_2}{[0.1,0.3];[0.2,0.4];[0.7,0.9]}, [0.4,0.6], (\frac{u_3}{[0.1,0.3];[0.5,0.7];[0.7,0.9]}, [0.7,0.9]\}$

$F_p(e_1, p, 0) = \{(\frac{u_1}{[0.2,0.8];[0.7,0.9];[0.2,0.8]}, [0.1,0.3], (\frac{u_2}{[0.3,0.9];[0.5,0.8];[0.1,0.7]}, [0.3,0.6], (\frac{u_3}{[0.4,0.5];[0.3,0.7];[0.5,0.6]}, [0.5,0.7]\}$

$F_p(e_1, q, 0) = \{(\frac{u_1}{[0.1,0.2];[0.6,0.7];[0.8,0.9]}, [0.8,0.9], (\frac{u_2}{[0.6,0.8];[0.6,0.8];[0.2,0.4]}, [0.6,0.8], (\frac{u_3}{[0.5,1];[0.3,0.5];[0,0.5]}, [0.3,0.7]\}$

$F_p(e_2, p, 0) = \{(\frac{u_1}{[0.1,0.7];[0.7,0.9];[0.3,0.9]}, [0.2,0.5], (\frac{u_2}{[0.5,0.8];[0.5,0.8];[0.2,0.5]}, [0.3,0.6], (\frac{u_3}{[0.1,0.4];[0.3,0.9];[0.6,0.9]}, [0.3,0.7]\}$

$F_p(e_2, q, 0) = \{(\frac{u_1}{[0.4,0.6];[0.8,0.9];[0.4,0.6]}, [0.20.5], (\frac{u_2}{[0.7,0.9];[0.6,0.8];[0.1,0.3]}, [0.4,0.6], (\frac{u_3}{[0.5,0.9];[0.3,0.5];[0.1,0.5]}, [0.5,0.7]\}$

Thus we have the VPNVSE set (F_P, Z) as follows:

$(F_P, Z) = \{(e_1, p, 1) = \{(\frac{u_1}{[0.2,0.8];[0.1,0.3];[0.2,0.8]}, [0.3,0.5], (\frac{u_2}{[0.1,0.7];[0.2,0.5];[0.3,0.9]}, [0.5,0.7], (\frac{u_3}{[0.5,0.6];[0.3,0.7];[0.4,0.5]}, [0.7,0.9]\}$

$(e_2, p, 1) = \{(\frac{u_1}{[0.3,0.9];[0.1,0.3];[0.1,0.7]}, [0.5,0.7], (\frac{u_2}{[0.2,0.5];[0.2,0.5];[0.5,0.8]}, [0.60.8], (\frac{u_3}{[0.6,0.9];[0.1,0.7];[0.1,0.4]}, [0.3,0.5]\}$

$(e_1, q, 1) = \{(\frac{u_1}{[0.8,0.9];\ [0.3,0.4];\ [0.1,0.2]}, [0.4,0.6], (\frac{u_2}{[0.2,0.4];\ [0.2,0.4];\ [0.6,0.8]}, [0.6,0.8], (\frac{u_3}{[0.0,0.5];\ [0.5,0.7];\ [0.5,1]}, [0.8,1]\}$

$(e_2, q, 1) = \{(\frac{u_1}{[0.4,0.6];\ [0.1,0.4];\ [0.4,0.6]}, [0.2,0.4], (\frac{u_2}{[0.1,0.3];\ [0.2,0.4];\ [0.7,0.9]}, [0.4,0.6], (\frac{u_3}{[0.1,0.3];\ [0.5,0.7];\ [0.7,0.9]}, [0.7,0.9]\}$

$(e_1, p, 0) = \{(\frac{u_1}{[0.2,0.8];\ [0.7,0.9];\ [0.2,0.8]}, [0.1,0.3], (\frac{u_2}{[0.3,0.9];\ [0.5,0.8];\ [0.1,0.7]}, [0.3,0.5], (\frac{u_3}{[0.4,0.5];\ [0.3,0.7];\ [0.5,0.6]}, [0.5,0.7]\}$

$(e_2, p, 0) = \{(\frac{u_1}{[0.1,0.7];\ [0.7,0.9];\ [0.3,0.9]}, [0.2,0.5], (\frac{u_2}{[0.5,0.8];\ [0.5,0.8];\ [0.2,0.5]}, [0.3,0.6], (\frac{u_3}{[0.1,0.4];\ [0.3,0.9];\ [0.6,0.9]}, [0.3,0.7]\}$

$(e_1, q, 0) = \{(\frac{u_1}{[0.1,0.2];\ [0.6,0.7];\ [0.8,0.9]}, [0.8,0.9], (\frac{u_2}{[0.6,0.8];\ [0.6,0.8];\ [0.2,0.4]}, [0.6,0.8], (\frac{u_3}{[0.5,1];\ [0.3,0.5];\ [0,0.5]}, [0.3,0.6]\}$

$(e_2, q, 0) = \{(\frac{u_1}{[0.4,0.6];\ [0.8,0.9];\ [0.4,0.6]}, [0.2,0.3], (\frac{u_2}{[0.7,0.9];\ [0.6,0.8];\ [0.1,0.3]}, [0.4,0.6], (\frac{u_3}{[0.5,0.9];\ [0.3,0.5];\ [0.1,0.5]}, [0.5,0.7]\}$

The collection (F_p, Z) is a V*PNVSE* set over the soft inverse (U, Z).

Definition 3.3: Let (F_p, A) and (G_q, B) be two V*PNVSE* sets over the soft inverse (U, Z) then (F_p, A) is a V*PNVSE* sub set of (G_q, B) if $A \subseteq B$and for all $\varepsilon \in A$ the following conditions are satisfied.

(i) p(ε) is a vague sub set of q(ε).
(ii) F(ε)is a neutrosophic vague soft set of *G*(ε).

It is denoted by $(F_p, A) \subseteq (G_q, A)$. Then (G_q, A) is called aVague-valued possibility neutrosophic soft expert superset of (F_p, A).

Definition 3.4.Let (F_p, A) and (G_q, B) be two V*PNVSE* sets over the soft inverse (U, Z) then (F_p, A) equal to (G_q, B) if for all $\varepsilon \in A$ the following holds

(i) p(ε) = q(ε).
(ii) F(ε) = *G*(ε).

In other words $(F_p, A) = (G_q, B)$ if (F_p, A) is a subset of $(G_q, B)$and (G_q, B) is a subset of (F_p, A).

4. Basic Operations On Vague-Valued Possibility Neutrosophic Soft Expert Sets.

Now we introduce some basic operations on *PNVSE* sets. These are 'complement' Union & intersection. Then we study some of the properties related to these operations.

Definition 4.1 Let (F_P, A) be a VPNVSE set over the soft universe (U, Z) then the complement of (F_P, A) denoted by $(F_P, A)^c$ is defined as

$(F_P, A)^c = (\bar{c}(F\alpha), c(P(\alpha)))\ \forall\ \alpha \in A$.

Where $\bar{c}$ a neutrosophic vague complement and c is a Vague-valued set complement.

If A be a vague set over the universe U, then

A = [{x, $t_A(x)$, 1-f_A(x)}: x∈V} in this definition$t_A(x)$ is a lower bound on the grade of membership of x to A derived from the evidence for x and f_A(x) is a lower bound on the negation of x to A derived from the evidence against x. The vague value [$t_A(x)$, 1-f_A(x)] indicates that the exact grade of membership of x to A may be unknown, but it is bounded by $t_A(x)$ & 1-f_A(x). It is to be noted that every fuzzy set α correspondence to the following vague set: α = {(x, [$\alpha(x)$, 1- α(x)]): x∈ U} thus the notion of vague sets is a generalization of fuzzy sets. The complement of the vague set A is A^c = [{x, $f_A(x)$, 1-t_A(x)}: x∈U}.

Example 4.2: Consider the VPNVSE (F_P, A) over a soft universe (U, Z) as an example 3.2. Now by definition 4.1 $(F_P, A)^c$ is given as follows:

$(F_P, z)^c$ = {(e_1, p, 1) = {($\frac{u_1}{[0.2,0.8];\,[0.7,0.9];\,[0.2,0.8]}$,[0.5, 0.7]), ($\frac{u_2}{[0.3,0.9];\,[0.5,0.8];\,[0.1,0.7]}$, [0.3, 0.5]), ($\frac{u_3}{[0.4,0.5];\,[0.3,0.7];\,[0.5,0.6]}$, [0.1, 0.3])}, (e_2, p, 1) = {($\frac{u_1}{[0.1,0.7];\,[0.7,0.9];\,[0.3,0.9]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.8];\,[0.5,0.8];\,[0.2,0.5]}$, [0.2, 0.4]), ($\frac{u_3}{[0.1,0.4];\,[0.3,0.9];\,[0.6,0.9]}$, [0.5, 0.7])}, (e_1, q, 1) = {($\frac{u_1}{[0.1,0.2];\,[0.6,0.7];\,[0.8,0.9]}$, [0.4, 0.6]), ($\frac{u_2}{[0.6,0.8];\,[0.6,0.8];\,[0.2,0.4]}$, [0.2, 0.4]), ($\frac{u_3}{[0.5,1];\,[0.3,0.5];\,[0.0,0.5]}$, [0.0, 0.2]}, (e_2, q, 1) = {($\frac{u_1}{[0.4,0.6];\,[0.6,0.9];\,[0.4,0.6]}$, [0.6, 0.8]), ($\frac{u_2}{[0.7,0.9];\,[0.6,0.8];\,[0.1,0.3]}$, [0.4, 0.6]), ($\frac{u_3}{[0.7,0.9];\,[0.3,0.5];\,[0.1,0.3]}$, [0.1, 0.3])},(e_1, p, 0) = {($\frac{u_1}{[0.2,0.8];\,[0.1,0.3];\,[0.2,0.8]}$, [0.7, 0.9]), ($\frac{u_2}{[0.1,0.7];\,[0.2,0.5];\,[0.3,0.9]}$, [0.5, 0.7]), ($\frac{u_3}{[0.5,0.6];\,[0.3,0.7];\,[0.4,0.5]}$, [0.3, 0.5])}, (e_2, p, 0) = {($\frac{u_1}{[0.3,0.9];\,[0.1,0.3];\,[0.1,0.7]}$, [0.5, 0.8]), ($\frac{u_2}{[0.2,0.5];\,[0.2,0.5];\,[0.5,0.8]}$, [0.4, 0.7]), ($\frac{u_3}{[0.6,0.9];\,[0.1,0.7];\,[0.1,0.4]}$, [0.3, 0.7])}, (e_1, q, 0) = {($\frac{u_1}{[0.8,0.9];\,[0.3,0.4];\,[0.1,0.2]}$, [0.1, 0.2]), ($\frac{u_2}{[0.2,0.4];\,[0.2,0.4];\,[0.6,0.8]}$, [0.2, 0.4]), ($\frac{u_3}{[0,0.5];\,[0.5,0.7];\,[0.5,1]}$, [0.4, 0.7]}, (e_2, q, 0) = {($\frac{u_1}{[0.4,0.6];\,[0.1,0.2];\,[0.4,0.6]}$, [0.5, 0.8]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]}$, [0.4, 0.6]), ($\frac{u_3}{[0.1,0.5];\,[0.5,0.7];\,[0.5,0.9]}$, [0.3, 0.5])}

Proposition 4.3: Let (F_P, A) be a VPNVSE set over the soft universe (U, Z) . Here,(F_P, A) = (F(e), p(e)) then $((F_P, A)^c)^c$ = (F_P, A).

Proof: Let $(F_P, A)^c$ = (G_q, B) then by definition (G_q, B) = (G(e), q(e))

G(e) = $\bar{C}$(F(e)) and q(e) = C(p(e)). Where $\bar{c}$ a neutrosophic vague complement and c is a Vague-valued set complement.

So it follows that

$(G_q, B)^c = \{\bar{C}(G(e)), C(q(e)\}$

$= \{\bar{C}(\bar{C}(F(e))), C(C(p(e)))\}$

$= (F(e), p(e)) = (F_P, A)$

$((F(e), p(e)^c)^c = (F_P, A)$

Definition 4.4: Let (F_P, A) and (G_q, B) be two V PNVSE set over a soft universe (U, Z) then the intersection of (F_P, A) and (G_q, B) denoted by $(F_P, A)\ \bar{\cap}\ (G_q, B)$ is a VPNVSE set defined as $(F_P, A)\ \bar{\cap}\ (G_q, B) = (H_r, C)$,

where C = A ∩ B and

$r(\alpha) = p(\alpha) \cap q(\alpha)\ \forall\alpha\in C$

$H(\alpha) = F(\alpha)\ \bar{\cap}\ G(\alpha)\ \forall\alpha \in C$

$$\text{And } H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A - B \\ G(\alpha) & if\ \alpha \in B - A \\ F(\alpha) \cap G(\alpha) & if\ \alpha \in A \cap B \end{cases}$$

Definition 4.5 Let (F_P, A) and (G_q, B) be two VPNVSE sets over a soft universe (U, Z). Then the union of (F_P, A) and (G_q, B) denoted by $(F_P, A)\ \bar{\cup}(G_q, B)$ is a PNVSE set defined as $(F_P, A)\ \bar{\cup}(G_q, B) = (H_r, C)$, where C = A ∪ B and

$r(\alpha) = p(\alpha) \cup q(\alpha)\ \forall\alpha \in C$

$H(\alpha) = F(\alpha)\ \bar{\cup}\ G(\alpha)\ \forall\alpha \in C$

$$\text{And } H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A - B \\ G(\alpha) & if\ \alpha \in B - A \\ F(\alpha)\ \bar{\cup}\ G(\alpha) & if\ \alpha \in A \cup B \end{cases}$$

5. Application of vague-valued possibility neutrosophic vague soft expert in a decision making problem

A company is looking to have a person to fill the vacancy for a position in their company. Out of all the candidates were short listed - The three candidates form the universe of the element U = { u_1, u_2, u_3} were short listed out of all candidates. The hiring committee consists of hiring manager, head of the department and HR director of the firm . The committee is represented by the set X = { x, y, z} (a set of experts), while the set Q = { 1 = agree, 0 = disagree} represents the set of opinions of the hiring committee members. The hiring committee consider a set of parameters E = {e_1, e_2, e_3, e_4}. The parameters e_i (i = 1, 2, 3, 4) represents the characteristic or qualities that the candidates are assessed on namely "experience", "academic qualifications", "attitude towards the professionalism "and "technical knowledge" respectively. After finishing the interview of all the candidates and going through their certificates and other supporting papers. The hire committee constitutes the VPNVSE set (F_P, z) as follows:

(F_P, z) = [{(e_1, x, 1) = {($\frac{u_1}{[0.2,0.8];\,[0.1,0.3];\,[0.2,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.7];\,[0.2,0.5];\,[0.3,0.9]}$, [0.5, 0.7]), ($\frac{u_3}{[0.5,0.6];\,[0.3,0.7];\,[0.4,0.5]}$, [0.7, 0.9])}, (e_2, x, 1) = {($\frac{u_1}{[0.3,0.9];\,[0.1,0.3];\,[0.1,0.7]}$, [0.5, 0.7]), ($\frac{u_2}{[0.2,0.5];\,[0.2,0.5];\,[0.5,0.8]}$, [0.6, 0.8]), ($\frac{u_3}{[0.6,0.9];\,[0.1,0.7];\,[0.1,0.4]}$, [0.3, 0.5])},(e_3, x, 1) = {($\frac{u_1}{[0.2,0.7];\,[0.5,0.7];\,[0.3,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.7];\,[0.4,0.5];\,[0.3,0.9]}$, [0.2, 0.5]), ($\frac{u_3}{[0.2,0.6];\,[0.3,0.5];\,[0.4,0.8]}$, [0.4, 0.6])}, ($e_4$, x, 1) = {($\frac{u_1}{[0.2,0.3];\,[0.4,0.6];\,[0.7,0.8]}$, [0.5, 0.7]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.5];\,[0.7,0.9]}$, [0.3, 0.6]), ($\frac{u_3}{[0.3,0.4];\,[0.4,0.6];\,[0.6,0.7]}$, [0.6, 0.8])},(e_1, y, 1) = {($\frac{u_1}{[0.8,0.9];\,[0.3,0.4];\,[0.1,0.2]}$, [0.4, 0.6]), ($\frac{u_2}{[0.2,0.4];\,[0.2,0.4];\,[0.6,0.8]}$, [0.6, 0.8]), ($\frac{u_3}{[0.0,0.5];\,[0.5,0.7];\,[0.5,1]}$, [0.8, 1])}, (e_2, y, 1) = {($\frac{u_1}{[0.4,0.6];\,[0.1,0.4];\,[0.4,0.6]}$, [0.2, 0.4]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]}$, [0.4, 0.6]), ($\frac{u_3}{[0.1,0.3];\,[0.5,0.7];\,[0.7,0.9]}$, [0.7, 0.9])},(e_3, y, 1) = {($\frac{u_1}{[0.2,0.5];\,[0.4,0.6];\,[0.5,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]}$, [0.6, 0.8]), ($\frac{u_3}{[0.4,0.5];\,[0.8,0.9];\,[0.5,0.6]}$, [0.5, 0.7])}, (e_4, y, 1) = {($\frac{u_1}{[0.3,0.5];\,[0.5,0.7];\,[0.5,0.7]}$, [0.5, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.9,1];\,[0.3,0.5]}$, [0.2, 0.5]), ($\frac{u_3}{[0.6,0.9];\,[0.2,0.3];\,[0.1,0.4]}$, [0.3, 0.6])},(e_1, z, 1) = {($\frac{u_1}{[0.1,0.4];\,[0.3,0.6];\,[0.6,0.9]}$, [0.4, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.2,0.5];\,[0.3,0.5]}$, [0.3, 0.5]), ($\frac{u_3}{[0.2,0.5];\,[0.4,0.7];\,[0.5,0.8]}$, [0.7, 0.9])}, (e_2, z, 1) = {($\frac{u_1}{[0.1,0.5];\,[0.4,0.6];\,[0.5,0.9]}$, [0.4, 0.5]), ($\frac{u_2}{[0.6,0.7];\,[0.3,0.5];\,[0.3,0.4]}$, [0.3, 0.5]), ($\frac{u_3}{[0.0,0.1];\,[0.2,0.4];\,[0.9,1]}$, [0.7, 0.9])},(e_3, z, 1) = {($\frac{u_1}{[0.3,0.5];\,[0.5,0.7];\,[0.5,0.7]}$, [0.1, 0.3]), ($\frac{u_2}{[0.3,0.4];\,[0.5,0.6];\,[0.6,0.7]}$, [0.3, 0.7]), ($\frac{u_3}{[0.4,0.6];\,[0.3,0.5];\,[0.4,0.6]}$, [0.7, 0.9])}, (e_4, z, 1) = {($\frac{u_1}{[0.1,0.5];\,[0.3,0.7];\,[0.5,0.9]}$, [0.4, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.1,0.2];\,[0.3,0.5]}$, [0.3, 0.7]), ($\frac{u_3}{[0.4,0.5];\,[0.7,0.8];\,[0.5,0.6]}$, [0.7, 1])}, (e_1, x, 0) = {($\frac{u_1}{[0.2,0.8];\,[0.7,0.9];\,[0.2,0.8]}$, [0.1, 0.3]), ($\frac{u_2}{[0.3,0.9];\,[0.5,0.8];\,[0.1,0.7]}$, [0.3, 0.5]), ($\frac{u_3}{[0.4,0.5];\,[0.3,0.7];\,[0.5,0.6]}$, [0.5, 0.7])}, (e_2, x, 0) = {($\frac{u_1}{[0.1,0.7];\,[0.7,0.9];\,[0.3,0.9]}$, [0.2, 0.5]), ($\frac{u_2}{[0.5,0.8];\,[0.5,0.8];\,[0.2,0.5]}$, [0.3, 0.6]), ($\frac{u_3}{[0.1,0.4];\,[0.3,0.9];\,[0.6,0.9]}$, [0.3, 0.7])},(e_3, x, 0) = {($\frac{u_1}{[0.2,0.4];\,[0.3,0.6];\,[0.6,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.8];\,[0.3,0.6];\,[0.2,0.5]}$, [0.2, 0.5]), ($\frac{u_3}{[0.4,0.7];\,[0.7,0.8];\,[0.3,0.6]}$, [0.6, 0.9])}, (e_4, x, 0) = {($\frac{u_1}{[0.3,0.5];\,[0.7,0.9];\,[0.5,0.7]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.7];\,[0.8,0.9];\,[0.3,0.5]}$, [0.4, 0.7]), ($\frac{u_3}{[0.5,1];\,[0.3,0.5];\,[0,0.5]}$, [0.3, 0.6])},(e_1, y, 0) =

{($\frac{u_1}{[0.3,0.5]; [0.5,0.8]; [0.5,0.7]}$, [0.1, 0.3]), ($\frac{u_2}{[0.1,0.3]; [0.4,0.6]; [0.7,0.9]}$, [0.3, 0.5]), ($\frac{u_3}{[0.4,0.6]; [0.7,0.9]; [0.4,0.6]}$, [0.4, 0.7])}, (e_2, y, 0) = {($\frac{u_1}{[0.1,0.2]; [0.6,0.7]; [0.8,0.9]}$, [0.8, 0.9]), ($\frac{u_2}{[0.6,0.8]; [0.6,0.8]; [0.2,0.4]}$, [0.6, 0.8]), ($\frac{u_3}{[0.5,1]; [0.3,0.5]; [0,0.5]}$, [0.3, 0.6])},(e_3, y, 0) = {($\frac{u_1}{[0.4,0.6]; [0.8,0.9]; [0.4,0.6]}$, [0.2, 0.5]), ($\frac{u_2}{[0.7,0.9]; [0.6,0.8]; [0.1,0.3]}$, [0.4, 0.6]), ($\frac{u_3}{[0.5,0.9]; [0.3,0.5]; [0.1,0.5]}$, [0.5, 0.7])}, (e_4, y, 0) = {($\frac{u_1}{[0.3,0.5]; [0.6,0.8]; [0.5,0.7]}$, [0.4, 0.6]), ($\frac{u_2}{[0.1,0.3]; [0.4,0.6]; [0.7,0.9]}$, [0.4, 0.7]), ($\frac{u_3}{[0.2,0.6]; [0.5,0.7]; [0.4,0.8]}$, [0.1, 0.4])},(e_1, z, 0) = {($\frac{u_1}{[0.2,0.5]; [0.4,0.7]; [0.5,0.8]}$, [0.4, 0.8]), ($\frac{u_2}{[0.2,0.4]; [0.3,0.5]; [0.6,0.8]}$, [0.4, 0.7]), ($\frac{u_3}{[0.2,0.7]; [0.5,0.7]; [0.3,0.8]}$, [0.1, 0.5])}, (e_2, z, 0) = {($\frac{u_1}{[0.4,0.5]; [0.5,1]; [0.5,0.6]}$, [0.5, 0.8]), ($\frac{u_2}{[0.1,0.4]; [0.3,0.5]; [0.6,0.9]}$, [0.5, 0.8]), ($\frac{u_3}{[0.3,0.5]; [0.7,0.9]; [0.5,0.7]}$, [0.2, 0.5])},(e_3, z, 0) = {($\frac{u_1}{[0.5,0.9]; [0.6,0.8]; [0.1,0.5]}$, [0.4, 0.7]), ($\frac{u_2}{[0.2,0.5]; [0.5,1]; [0.5,0.8]}$, [0.2, 0.4]), ($\frac{u_3}{[0.4,0.7]; [0.6,0.8]; [0.3,0.6]}$, [0.6, 0.8])}, (e_4, z, 0) = {($\frac{u_1}{[0.3,0.5]; [0.6,0.8]; [0.5,0.7]}$, [0.4, 0.6]), ($\frac{u_2}{[0.1,0.4]; [0.3,0.5]; [0.6,0.9]}$, [0.6, 0.8]), ($\frac{u_3}{[0.6,0.8]; [0.5,0.7]; [0.2,0.4]}$, [0.5, 0.7])}.

The collection (F_P, z) is a VPNVSE set over the soft universe (U, Z). The VPNVSE set (F_P, Z) is used together with an algorithm to solve the decision making problem. The algorithm given below is taken by the committee to determine the most suitable candidate to be hired for the position. The sets of algorithm are as follows:

Step 1: Input the VPNVSE set (F_P, Z).

Step 2: Calculate the value of $\alpha_{F(a_i)}(u_i)$ = $T^-_{F(a_i)}(u_i)$ - $F^-_{F(a_i)}(u_i)$ for interval truth-membership part [$T^-_{F(a_i)}(u_i)$, $T^+_{F(a_i)}(u_i)$] , where $T^+_{F(a_i)}(u_i)$ = 1- $F^-_{F(a_i)}(u_i)$, for each element $u_i \in U$.

Step 3: Calculate the arithmetic overage $\beta_{F(a_i)}(u_i)$ of the end points of the interval indeterminacy membership part [$I^-_{F(a_i)}(u_i)$, $I^+_{F(a_i)}(u_i)$] , for each element $u_i \in U$.

Step 4: Find the value of $\gamma_{F(a_i)}(u_i)$ = $F^-_{F(a_i)}(u_i)$ - $T^-_{F(a_i)}(u_i)$ for interval falsity-membership part [$F^-_{F(a_i)}(u_i)$, $F^+_{F(a_i)}(u_i)$] , where $F^+_{F(a_i)}(u_i)$ = 1- $T^-_{F(a_i)}(u_i)$, for each element $u_i \in U$.

Step 5: Find $\alpha_{F(ai)}$ (u_i)- $\beta_{F(ai)}$ (u_i) -$\gamma_{F(a_i)}(u_i)$for each element $u_i \in$U.

Step 6: Find the higher numerical grade from the agree-PNVSE set & disagree-PNVSE set.

Step 7: Take the arithmetic average of [t_A, 1-f_A] of the set corresponding vague set associated with the Neutrosophic vague soft set.

Step 8: Find the higher numerical grade for the average vague set value for the highest agree-VPNVSE set & disagree-VPNVSE set

Step 9: Compute the score of each element $u_i \in U$ by taking the sum of the product of the maximum numerical grade (λ_i) with the corresponding average numerical value of vague set μ_i for the agree VPNVSE set and disagree-VPNVSE set by A_i& D_i respectively.

Step 10: Find the value r_i = A_i - D_i, for each element $u_i \in U$.

Step 11: Determine the values of highest scores = max $u_i \in \cup \{r_i\}$. Then the decision is to choose element u_i as optimal or best solution if there are more than one element.

Table-1 Value of $\alpha_{F(a_i)}(u_i)$, $\beta_{F(a_i)}(u_i)$, $\gamma_{F(a_i)}(u_i)$The value of $\alpha_{F(a_i)}(u_i)$ - $\beta_{F(a_i)}(u_i)$ - $\gamma_{F(a_i)}(u_i)$& the average of the vague set corresponding to the highest numerical grade

	u_1	u_2	u_3		u_1	u_2	u_3
(e_1, x, 1)	0, 0.2, 0	-0.2,0.35, 0.2	0.1,0.5-0.1	(e_1, x, 0)	0,0.8,0	0.2,0.65,-0.2	-0.1,0.5,0.1
	-0.2,(0.4)	-0.75, (0.6)	-0.3,(0.8)		-0.8,(0.2)	-0.25(0.4)	-0.7,(0.6)
(e_2, x, 1)	0.2, 0.2, -0.2	-0.3,0.35,0.3	0.5,0.4-0.5	(e_2, x, 0)	-0.2,0.8,0.2	0.3,0.65,-0.3	-0.5,0.6,0.5
	0.2, (0.6)	-0.85,(0.35)	0.6,(0.4)		-1.2,(0.35)	-0.05,(0.45)	-1.6,(0.5)
(e_3, x, 1)	-0.5,0.5,0.5	-0.6,0.35,0.6	-0.3,0.5,0.3	(e_3, x, 0)	-0.4,0.45,0.4	0.3,0.45,-0.3	0.1,0.75,-0.1
	-1.5,(0.6)	-1.55,(0.45)	-0.8, (0.5)		-1.25,(0.4)	0.15,(0.35)	0.55,(0.75)
(e_4, x, 1)	-0.5,0.5,0.5	-0.6,0.35,0.6	-0.3,0.5,0.3	(e_4, x, 0)	-0.2,0.8,0.2	0.2,0.85,-0.2	0.5,0.4,-0.5
	-1.5,(0.6)	-1.55,(0.45)	-1.1, (0.7)		-1.20,(0.4)	-0.45,(0.55)	0.6,(.45)
(e_1, y, 1)	0.7,0.35,-0.7	-0.4,0.3,0.4	-0.5,0.6,0.5	(e_1, y, 0)	-0.2,0.8,0.2	-0.6,0.5,0.6	0,0.8,0
	1.05,(0.5)	-1.1,(0.7)	-0.6,(0.9)		-1.05,(0.2)	-1.7,(0.4)	-0.8,(0.55)
(e_2, y, 1)	0,0.25,0	-0.6,0.3,0.6	-0.6,0.6,0.6	(e_2, y, 0)	-0.7,0.65,0.7	0.4,0.7,-0.4	0.5,0.4,-0.5
	-0.25,(0.3)	-1.5,(0.5)	-1.8,(0.8)		-2.05,(0.85)	0.1, (0.7)	0.6, (0.45)
(e_3, y, 1)	-0.3,0.5,0.3	-0.6,0.3,0.6	-0.1,0.85,0.1	(e_3, y, 0)	0,0.85,0	0.6,0.7,-0.6	0.5,0.4,-0.5

	-1.1, (0.4)	-1.5, (0.7)	-1.05, (0.6)		-0.85,(0.35)	0.5,(0.5)	0.6,(0.45)
(e4, y, 1)	-0.2,0.6,0.2	0.2,0.95,-0.2	0.5,0.25,-0.5	(e4, y, 0)	-0.2,0.7,0.2	-0.6,0.5,0.6	-0.2,0.6,0.2
	-1.0,(0.6)	-0.55,(0.35)	0.75,(0.45)		-1.1,(0.5)	-1.7,(0.55)	-1.0,(0.25)
(e1, z, 1)	-0.5,0.45,0.5	0.2,0.35,-0.2	-0.3,0.55,0.3	(e1, z, 0)	-0.3,0.55,0.3	-0.4,0.35,0.4	-0.1,0.6,0.1
	-1.45,(0.55)	0.5,(0.4)	-1.1,(0.8)		-1.15,(0.6)	-1.15,(0.55)	-0.8,(0.3)
(e2, z, 1)	-0.4,0.5,0.4	0.3,0.4,-0.3	-0.9,0.3,0.9	(e2, z, 0)	-0.1,0.75,0.1	-0.5,0.4,0.5	-0.2,0.8,0.2
	-1.3,(0.45)	0.2,(0.4)	-2.1,(0.8)		-0.95,(0.65)	-1.4,(0.65)	-1.2,(0.35)
(e3, z, 1)	-0.2,0.6,0.2	-0.3,0.55,0.3	0,0.4,0	(e3, z, 0)	0.4,0.7,-0.4	-0.3,0.75,0.3	0.1,0.7,-0.1
	-1.0,(0.2)	-1.15,(0.5)	-0.4,(0.8)		0.1,(0.55)	-1.65,(0.3)	-0.5,(0.7)
(e4, z, 1)	-0.4,0.5,0.4	0.2,0.15,-0.2	-0.1,0.75,0.1	(e4, z, 0)	-0.2,0.7,0.2	-0.5,0.4,0.5	0.4,0.6,-0.4
	-1.3,(0.55)	0.25,(0.5)	-0.95,(0.85)		-1.1,(0.5)	-1.4,(0.7)	0.2,(0.6)

Table-2

	High numerical grad for agree PNVSE set (λ_i)	High numerical average value of the vague set (μ_i) corresponding to highest numerical grad	$\lambda_i \times \mu_i$		High numerical grad for disagree PNVSE set (λ_i)	High numerical average value of the vague set (μ_i) corresponding to highest numerical grad	$\lambda_i \times \mu_i$
(e1, x, 1)	u1(-0.2)	0.4	-0.08	(e1, x, 0)	u2(-0.25)	0.4	-0.1
(e2, x, 1)	u3(0.6)	0.4	0.24	(e2, x, 0)	u2(-0.05)	0.45	-0.0225
(e3, x, 1)	u3(0.8)	0.5	-0.40	(e3, x, 0)	u3(0.55)	0.75	0.4125

(e_4, x, 1)	u_3(-1.1)	0.7	-0.77	(e_4, x, 0)	u_3(0.6)	0.45	0.27
(e_1, y, 1)	u_1(1.05)	0.5	0.525	(e_1, y, 0)	u_3(-0.8)	0.55	-0.44
(e_2, y, 1)	u_1(-0.25)	0.3	-0.075	(e_2, y, 0)	u_3(0.6)	0.45	0.27
(e_3, y, 1)	u_3(-1.05)	0.6	-0.63	(e_3, y, 0)	u_3(0.6)	0.6	0.36
(e_4, y, 1)	u_3(0.75)	0.45	0.3375	(e_4, y, 0)	u_3(-1.0)	0.25	-0.25
(e_1, z, 1)	u_2(0.05)	0.4	0.02	(e_1, z, 0)	u_3(-0.8)	0.3	-0.24
(e_2, z, 1)	u_2(0.2)	0.4	0.08	(e_2, z, 0)	u_1(-0.95)	0.65	-0.6175
(e_3, z, 1)	u_3(-0.4)	0.8	-0.32	(e_3, z, 0)	u_1(0.1)	0.55	0.055
(e_4, z, 1)	u_2(0.25)	0.5	0.125	(e_4, z, 0)	u_3(0.2)	0.6	0.12

For agree

Score u_1 = -0.08+0.525+(-0.075) = 0.370

Score u_2 = 0.02+0.08+0.125 = 0.225

Score u_3 = 0.24+(-0.40)+(-0.75)+(-0.63) +0.3375+(-0.32) = -1.5225

For disagree

Score u_1 = -0.6175+0.055 = -0.5625

Score u_2 = -0.1+(-0.0225) = -0.1225,

Score u_3 = 0.4125+0.27+0.27+0.36 +(-0.25)+(-0.24)+0.12 = 0.9425

Table 3. The score $r_i = A_i - D_i$

A_i	D_i	r_i
Score u_1 = 0.37	Score u_1 = -0.5625	0.9325
Score u_2 = 0.225	Score u_2 = -0.1225	0.3475
Score u_3 = -1.5225	Score u_3 = 0.9425	-2.465

Thus S = max $_{u_i \in U} \{r_i\}$ = r_1.So, the committee is advised to hire candidate u_1 to fill the vacant position.

6. Conclusions

We give the advances of our proposal method using VPNVSE set as compared to that PVSE set as proposed by [19]. The VPNVSE set is a generalization of PVSE set. The VPNVSE set each examine the universal U in never detail with three membership functions, especially when there are many parameters involved, where PVSE set can tell us limited information about the universal U. It can only handle the incomplete information comparing both the truth-membership value and falsity-membership values with corresponding vague set. But VPNVSE set can handle problems involving imprecise, indeterminacy and incomplete data with corresponding vague set. Thus it makes more accurate and realistic than PVSE set (PNVSE set [13]). In future many applications in decision making problems can be solved with VPNSE sets- especially in medical sciences.

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