



# Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality

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**Abstract** The concept of neutrosophic can provide a generalization of fuzzy set and intuitionistic fuzzy set that make it is the best fit in representing indeterminacy and uncertainty. Single Valued Triangular Numbers (SVTrN-numbers) is a special case of neutrosophic set that can handle ill-known quantity very difficult problems. This work intended to introduce a framework with two types of ranking methods. The results indicated that each ranking method has its own advantage. In this perspective, the weighted value and ambiguity based method gives more attention to uncertainty in ranking and evaluating ISQ as well as it takes into account cut sets of SVTrN numbers that can reflect the information on Truth-membership-membership degree, false membership-membership degree and Indeterminacy-membership degree. The value index and ambiguity index method can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

**Key words:** Single Valued Triangular Neutrosophic Number (SVTrN), Single-Valued Trapezoidal Neutrosophic Number (SVTN number), Information Systems Quality (ISQ), Multi-Criteria Decision Making (MCDM).

## 1. Introduction

The neutrosophic concept became a key research topic. Neutrosophic theory involves philosophy viewpoint which addresses nature and scope of neutralities, as well as their interactions with different ideational spectra [9]. Neutrosophic includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic that it can be applied in many fields in order to solve problems related to indeterminacy [26, 23]. Neutrosophic not only considers the truth-membership and falsity-membership but also indeterminacy. Neutrosophic can provide is a generalization of classical set, fuzzy set and intuitionistic fuzzy set [22, 25, 23]. The neutrosophic set can handle many applications in information systems and decision support systems such as relational database systems, semantic web services, and financial data set detection [28]. Neutrosophic sets can represent inconsistent and incomplete information about real world problems [27, 24]. The neutrosophic set theory can be used to handle the uncertainty that related to

ambiguity in a manner analogous to human thought [22]. In the neutrosophic set, the membership function independently indicates: Truth-membership-membership degree, false membership-membership degree, and Indeterminacy-membership degree. According to [24] neutrosophic set can exemplify ambiguous and conflicting information about real world. SVTrN-number is a special case of neutrosophic set that can handle ill-known quantity very difficult problem in Multi-Criteria Decision Making (MCDM) MCDM involves a process of solving the problem and achieving goals under asset of constraints, and it can be very difficult in some cases because of incomplete and imprecise information [1]. Also, in a MCDM problem the process of ranking alternatives with neutrosophic numbers is very difficult because neutrosophic numbers are not ranked by ordinary methods as real numbers. However, it is possible with score functions, aggregation operators, distance measures, and so on. Ye [14] introduced the notations of simplified neutrosophic sets and developed a ranking method. Then, he introduced some aggregation operators. Biswas et al. [35] developed a new approach for multi-attribute group decision making problems by extending the technique for order preference by similarity to ideal solution under single-valued neutrosophic environment. In [32] introduced combination of a neutrosophic set and a soft set that can be applied to problems that contain uncertainty. In [38] a new cross entropy measure under interval neutrosophic set (INS) environment was defined and can call IN-cross entropy measure and prove its basic properties. De and Das [20] developed a ranking method for trapezoidal intuitionistic fuzzy numbers and presented the values and ambiguities of the membership degree and the non-membership degree. Pramanik et al. [37] developed a new multi attribute group decision making (MAGDM) strategy for ranking of the alternatives based on the weighted SN-cross entropy measure between each alternative and the ideal alternative. Mitchell [2] proposed a ranking method to order triangular intuitionistic fuzzy numbers by accepting a statistical viewpoint and interpreting each

IFN as ensemble of ordinary fuzzy numbers. In [33] the notion of the interval valued neutrosophic soft set (ivn-soft sets) and generalized the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft set and neutrosophic soft set. Prakash et al [21] introduced a ranking method for both trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers using the centroid concept and showed the proposed method is flexible and effective. Pramanik et al. [39] introduced new vector similarity measures of single valued and interval neutrosophic sets by hybridizing the concepts of Dice and cosine similarity measures and presented their applications in multi attribute decision making under neutrosophic environment. Peng et al [13] introduced the concept of multi-valued neutrosophic set, gave two multi-valued neutrosophic power aggregation operators. In [11, 29] the score based method can provide a simple method to rank the Single-Valued Trapezoidal Neutrosophic Number (SVTrN number). Li [4] provides ratio ranking method for TIFNs and cut sets of intuitionistic trapezoidal fuzzy numbers. The existing methods of ranking fuzzy numbers and intuitionistic fuzzy number may be extended to SVN-numbers [10]. In [34] triangular fuzzy number neutrosophic weighted arithmetic averaging operator and triangular fuzzy number neutrosophic weighted geometric averaging operator are defined to aggregate triangular fuzzy number neutrosophic sets. Li et al. [5] introduced a ranking method of triangular intuitionistic fuzzy numbers and defined the notation of cut sets of intuitionistic fuzzy numbers and their values and ambiguities of membership and non-membership functions. The main advantage of this method that it pays more attention to the impact of uncertainty and takes into account  $\theta$ -weighted value of intuitionistic fuzzy numbers by using the concepts of cut sets of intuitionistic fuzzy numbers. Biswas et al. [36] developed a ranking method based on value and ambiguity index based of single-valued trapezoidal neutrosophic numbers. According to [3] there are many ranking methods. However, there is no unique best method exists. This paper intended to introduce a framework with two types of ranking methods. This paper is organized as the follows: the first section presents the introduction for this work; the second section provides basic definitions; the third section describes the proposed framework with two ranking methods of SVTrN-numbers with the scale based approach for evaluating ISQ; the fourth section describes a case study; the fifth section gives conclusion and future work; the final section provides references.

**2. Basic Definitions**

Fuzzy theory is an important and interesting research topic in decision-making theory and science. However, fuzzy set is characterized only by its membership function between 0 and 1, but not a non-membership function [12]. To overcome the insufficient of fuzzy set, Atanassov [19] extend-

ed fuzzy set and introduced intuitionistic fuzzy set by adding an additional non-membership degree, which may express more flexible information as compared with the fuzzy set. Intuitionistic fuzzy set can be defined as the follows:

**Definition 1.** According to [18], let E be a universe. An intuitionistic fuzzy set K over E is defined by:  $K = \{ \langle x, \mu_k(x), \gamma_k(x) \rangle : x \in E \}$  where  $\mu_k: E [0, 1]$  and  $\gamma_k: E [0, 1]$  such that  $0 \leq \mu_k(x) + \gamma_k(x) \leq 1$  for any  $x \in E$ . For each  $x \in E$ , the values,  $\mu_k(x)$  and  $\gamma_k(x)$  are degree of membership function and non-membership function of x, respectively.

Smarandache [7] introduced the concept of neutrosophic set, which is differentiated by truth-membership function, indeterminacy-membership function and falsity membership function. The concept of neutrosophic set came from a philosophical point of view to express indeterminate and inconsistent information Neutrosophic set can be defined as the follows:

**Definition 2.** According to [8], let E be a universe. Neutrosophic sets A over E is defined by:  $A = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E \}$  where  $T_A(x)$ ,  $I_A(x)$ , and  $F_A(x)$  are called truth-membership function, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by  $T_A: E] - 0, 1+[$ ,  $I_A: E ] - 0, 1+[$ ,  $F_A: E ] - 0, 1+[$  Such that.  $0 \leq (T_A(x) + I_A(x) + F_A(x)) \leq 3$

2.1. Single Valued Triangular Neutrosophic Numbers  
Single valued triangular neutrosophic numbers (SVTrN-numbers) is a special case of neutrosophic set that can handle ill-known quantity very difficult problem in multi-attribute decision making and ranking. SVTrN-numbers is suitable for the expression of incomplete, indeterminate, and inconsistent information in actual applications. Specially, it has been widely applied in many areas [16]. According to [31] the SVTrN-number  $\bar{a}$  can be defined as the follows:

**Definition 3.** As [31] [10] pointed out, Let  $\bar{a} = ((a, b, c); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  where is  $\bar{a}$  SVTrN-number whose truth-membership, indeterminacy-membership and falsity-membership functions can be respectively defined by :

$$\mu_{\bar{a}}(x) = \begin{cases} \frac{(x-a)w_{\bar{a}}}{b-a}, & a \leq x < b \\ \frac{(c-x)w_{\bar{a}}}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

$$u_{\bar{a}}(x) = \begin{cases} \frac{(b-x+u_{\bar{a}}(x-a))}{b-a}, & a \leq x < b \\ \frac{(x-b+u_{\bar{a}}(c-x))}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

$$\lambda_{\bar{a}}(x) = \begin{cases} \frac{(b-x+y_{\bar{a}}(x-a))}{b-a}, & a \leq x < b \\ \frac{(x-b+y_{\bar{a}}(c-x))}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \quad (2.3)$$

If  $a \geq 0$  and at least  $c > 0$ , then  $\bar{a} = ((a, b, c); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  is called a positive SVTrN-number, denoted by  $\bar{a} > 0$ . Likewise, If  $a \leq 0$  and at least  $c < 0$ ,  $\bar{a} = ((a, b, c); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  is called a negative SVTrN-number, denoted by  $\bar{a} < 0$ .

**Definition 4.** According to [31] let  $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ ,  $\bar{e} = ((a_2, b_2, c_2); w_{\bar{e}}, u_{\bar{e}}, y_{\bar{e}})$  be two SVTrN-numbers and  $\gamma \neq 0$  be any real number, then

$$\bar{a} + \bar{e} = ((a_1 + a_2, b_1 + b_2, c_1 + c_2); \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) \quad (2.4)$$

$$\bar{a}\bar{e} = \begin{cases} ((a_1 a_2, b_1 b_2, c_1 c_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 > 0, c_2 > 0) \\ ((a_1 c_2, b_1 b_2, c_1 a_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 < 0, c_2 > 0) \\ ((c_1 c_2, b_1 b_2, a_1 a_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 < 0, c_2 < 0) \end{cases} \quad (2.5)$$

$$\gamma_{\bar{a}} = \begin{cases} ((\gamma a_1, \gamma b_1, \gamma c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}}) & (\gamma > 0) \\ ((\gamma c_1, \gamma b_1, \gamma a_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}}) & (\gamma < 0) \end{cases} \quad (2.6)$$

### 3.1.1 Concepts of Values and Ambiguities for SVTrN-Numbers

Concept of cut (or level) sets, values, ambiguities, weighted values and weighted ambiguities of SVTrN-numbers have desired properties and can reflect information on membership degrees and non-membership degrees.

**Definition 5.** As [10] [4] pointed out, let  $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  is an arbitrary SVTrN-number. Then,

- (1)  $\alpha$ -cut set of the SVTrN-number  $\bar{a}$  for truth-membership is calculated as:

$$[L_{\bar{a}}(\alpha), R_{\bar{a}}(\alpha)] = [((w_{\bar{a}} - \alpha) a + \alpha b) / w_{\bar{a}}, ((w_{\bar{a}} - \alpha) c + \alpha b) / w_{\bar{a}}]$$

If  $f(\alpha) = \alpha$ , where  $f(\alpha) \in [0, 1]$  and  $f(\alpha)$  is monotonic and non-decreasing of  $\alpha \in [0, w_{\bar{a}}]$ , the value and ambiguity of the SVTrN-number  $\bar{a}$  can be calculated as:

$$V_{\mu}(\bar{a}) = \int_0^{w_{\bar{a}}} \left[ (a+c) + \frac{(2b-a-c)\alpha}{w_{\bar{a}}} \right] \alpha d\alpha = \left[ \frac{(a+c)\alpha^2}{2} + \frac{(2b-a-c)\alpha^3}{3w_{\bar{a}}} \right] \Big|_0^{w_{\bar{a}}} = \frac{(a+4b+c)(w_{\bar{a}})^2}{6} \quad (2.7)$$

And

$$A_{\mu}(\bar{a}) = \int_0^{w_{\bar{a}}} \left[ (c-a) - \frac{(c-a)\alpha}{w_{\bar{a}}} \right] \alpha d\alpha = \left[ \frac{(c-a)\alpha^2}{2} - \frac{(c-a)\alpha^3}{3w_{\bar{a}}} \right] \Big|_0^{w_{\bar{a}}} = \frac{(c-a)(w_{\bar{a}})^2}{6} \quad (2.8)$$

- (2)  $\beta$ -cut set of the SVTrN-number  $\bar{a}$  for indeterminacy membership is calculated as;

$$[\acute{L}_{\bar{a}}(\beta), \acute{R}_{\bar{a}}(\beta)] = [((1-\beta)b + (\beta - u_{\bar{a}})a) / (1 - u_{\bar{a}}), ((1-\beta)b + (\beta - u_{\bar{a}})c) / (1 - u_{\bar{a}})]$$

If  $g(\beta) = 1 - \beta$ , where  $g(\beta) \in [0, 1]$  and  $g(\beta)$  is monotonic and non-increasing of  $\beta \in [u_{\bar{a}}, 1]$ , the value and ambiguity of the SVTrN-number  $\bar{a}$  can be calculated, respectively, as the follows:

$$V_{\nu}(\bar{a}) = \int_{u_{\bar{a}}}^1 \left[ (a+c) + \frac{(2b-a-c)(1-\beta)}{1-u_{\bar{a}}} \right] (1-\beta) d\beta = \left[ \frac{(a+c)(1-\beta)^2}{2} + \frac{(2b-a-c)(1-\beta)^3}{3(1-u_{\bar{a}})} \right] \Big|_{u_{\bar{a}}}^1 = \frac{(a+4b+c)(1-u_{\bar{a}})^2}{6} \quad (2.9)$$

And

$$A_{\nu}(\bar{a}) = \int_{u_{\bar{a}}}^1 \left[ (c-a) - \frac{(c-a)(1-\beta)}{1-u_{\bar{a}}} \right] (1-\beta) d\beta = \left[ \frac{(c-a)(1-\beta)^2}{2} - \frac{(c-a)(1-\beta)^3}{3(1-u_{\bar{a}})} \right] \Big|_{u_{\bar{a}}}^1 = \frac{(c-a)(1-u_{\bar{a}})^2}{6} \quad (2.10)$$

- (3)  $\gamma$ -cut set of the SVTrN-number  $\bar{a}$  for falsity-membership is calculated as:

$$[\acute{L}'_{\bar{a}}(\gamma), \acute{R}'_{\bar{a}}(\gamma)] = [((1-\gamma)b + (\gamma - y_{\bar{a}})a) / (1 - y_{\bar{a}}), ((1-\gamma)b + (\gamma - y_{\bar{a}})c) / (1 - y_{\bar{a}})]$$

If  $h(\gamma) = 1 - \gamma$ , where  $h(\gamma) \in [0, 1]$  and  $h(\gamma)$  is monotonic and non-increasing of  $\gamma \in [y_{\bar{a}}, 1]$ , the value and ambiguity of the SVTrN-number  $\bar{a}$ , respectively, as;

$$V_{\lambda}(\bar{a}) = \int_{y_{\bar{a}}}^1 \left[ (a+c) + \frac{(2b-a-c)(1-\gamma)}{1-y_{\bar{a}}} \right] (1-\gamma) d\gamma = \left[ \frac{(a+c)(1-\gamma)^2}{2} - \frac{(2b-a-c)(1-\gamma)^3}{3(1-y_{\bar{a}})} \right] \Big|_{y_{\bar{a}}}^1 = \frac{(a+4b+c)(1-y_{\bar{a}})^2}{6} \quad (2.11)$$

And

$$A_{\lambda}(\bar{a}) = \int_{y_{\bar{a}}}^1 \left[ (c-a) - \frac{(c-a)(1-\gamma)}{(1-y_{\bar{a}})} \right] (1-\gamma) d\gamma = \left[ \frac{(c-a)(1-\gamma)^2}{2} - \frac{(c-a)(1-\gamma)^3}{3(1-y_{\bar{a}})} \right] \Big|_{y_{\bar{a}}}^1$$

$$= \frac{(c-a) (1-y_{\bar{a}})^2}{6} \tag{2.12}$$

The function  $f(\alpha)$  gives different weights to elements at different  $\alpha$ -cut sets and these cut sets come from values of  $\mu_{\bar{a}}(x)$  which have a considerable amount of uncertainty. Therefore,  $V_{\mu}(\bar{a})$  can reflect the information on membership degrees. Also,  $g(\beta)$  can lessen the contribution of the higher  $\beta$ -cut sets come from values of  $v_{\bar{a}}(x)$  which have a considerable amount of uncertainty. Therefore,  $V_v(\bar{a})$  can reflect the information on non-membership degrees. Likewise,  $V_{\lambda}(\bar{a})$  can reflect the information on non-membership degrees.

**3.1.2 The Weighted Values and Ambiguities of the SVTrN-numbers**

The weighted values of the SVTrN-numbers can be calculated as follows:

**Definition 6.** According to [10] let  $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  be a SVTrN-number. Then, for  $\theta \in [0, 1]$ , the  $\theta$ -weighted value of the SVTrN-number  $\bar{a}$  can be defined as:

$$V_{\theta}(\bar{a}) = (a + 4b + c)/6 [\theta w_{\bar{a}}^2 + (1-\theta) (1-u_{\bar{a}})^2 + (1-\theta) (1-y_{\bar{a}})^2] \tag{2.13}$$

The  $\theta$ -weighted ambiguity of SVTrN-number  $\bar{a}$  are defined as:

$$A_{\theta}(\bar{a}) = (c-a) /6 [\theta w_{\bar{a}}^2 + (1-\theta) (1-u_{\bar{a}})^2 + (1-\theta) (1-y_{\bar{a}})^2] \tag{2.14}$$

**Definition 7.** Let  $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$  be a SVTrN-number. Based on [10]; [20] [4] the values index and ambiguities index can be generalized to the SVTrN-numbers and they can be respectively calculated for  $\lambda \in [0, 1]$  as follows:

$$V(\bar{a}, \lambda) = (a+4b+c)/6 [\lambda w_{\bar{a}}^2 + (1-\lambda)(1-u_{\bar{a}})^2 + (1-\lambda)(1-y_{\bar{a}})^2] \tag{2.15}$$

$$= V_{\mu}(\bar{a}) \lambda + V_v(\bar{a}) (1-\lambda) + V_{\lambda}(\bar{a}) (1-\lambda) \tag{2.16}$$

And

$$A(\bar{a}, \lambda) = (c-a)/6 [\lambda w_{\bar{a}}^2 + (1-\lambda)(1-u_{\bar{a}})^2 + (1-\lambda) (1-y_{\bar{a}})^2] \tag{2.17}$$

$$= A_{\mu}(\bar{a}) \lambda + A_v(\bar{a}) (1-\lambda) + A_{\lambda}(\bar{a})(1-\lambda) \tag{2.18}$$

Where  $\lambda \in [0, 1]$  and  $\lambda$  is a weight which represents the decision maker's preference information.  $\lambda \in [0, 1/2]$  shows that the decision maker prefers pessimistic or negative feeling;  $\lambda \in [1/2, 1]$  shows that the decision maker prefers optimistic or positive feeling;  $\lambda = 1/2$  shows that the decision maker is indifferent between positive feeling and negative feeling.

$$V(\bar{a}, 1/2) = V_{\mu}(\bar{a}) 1/2 + V_v(\bar{a}) (1-1/2) + V_{\lambda}(\bar{a}) (1-1/2)$$

$$= V_{\mu}(\bar{a}) 1/2 + V_v(\bar{a}) 1/2 + V_{\lambda}(\bar{a}) 1/2 = 1/2(V_{\mu}(\bar{a}) + V_v(\bar{a}) + V_{\lambda}(\bar{a})) \tag{2.19}$$

And

$$A(\bar{a}, 1/2) = A_{\mu}(\bar{a}) 1/2 + A_v(\bar{a}) (1-1/2) + A_{\lambda}(\bar{a})(1-1/2) = A_{\mu}(\bar{a}) 1/2 + A_v(\bar{a}) 1/2 + A_{\lambda}(\bar{a}) 1/2 = 1/2 (A_{\mu}(\bar{a}) + A_v(\bar{a}) + A_{\lambda}(\bar{a})) \tag{2.20}$$

**Definition 8.** Let  $\bar{a}$  and  $\bar{e}$  be two SVTrN-numbers and  $\theta \in [0, 1]$ . For weighted values and ambiguities of the SVTrN-numbers  $\bar{a}$  and  $\bar{e}$ , the ranking order of  $\bar{a}$  and  $\bar{e}$  can be defined as;

- (1) If  $V_{\theta}(\bar{a}) > V_{\theta}(\bar{e})$ , then  $\bar{a}$  is bigger than  $\bar{e}$
- (2) If  $V_{\theta}(\bar{a}) < V_{\theta}(\bar{e})$ , then  $\bar{a}$  is smaller than  $\bar{e}$
- (3) If  $V_{\theta}(\bar{a}) = V_{\theta}(\bar{e})$ , then
  - (i) If  $A_{\theta}(\bar{a}) = A_{\theta}(\bar{e})$ , then  $\bar{a}$  is equal to  $\bar{e}$
  - (ii) If  $A_{\theta}(\bar{a}) > A_{\theta}(\bar{e})$ , then  $\bar{a}$  is bigger than  $\bar{e}$
  - (iii) If  $A_{\theta}(\bar{a}) < A_{\theta}(\bar{e})$ , then  $\bar{a}$  is smaller than  $\bar{e}$

**3. The Proposed Framework with Two Ranking Methods for Evaluating Information Systems Quality**

The proposed framework aims to introduce the scale based approach with SVTrN-numbers for evaluating ISQ. The proposed framework consists of four phases as follows:

Phase 1: Using Single Valued Triangular Neutrosophic Numbers with scale based approach

The first phase aims to enable the IS evaluator to give every quality attribute one of the scale categories. The scale ranging is designed from 0 to 1 on which the value of every attribute needs to be marked. The scale is divided into categories: Low, Not low, Very low, Completely low, More or less low, Fairly low, Essentially low, Neither low nor high, High, Not high, Very high, Completely high, More or less high, Fairly high, Essentially high, having corresponding values ((4.6; 5.5; 8.6); 0.4; 0.7; 0.2), ((4.7; 6.9; 8.5); 0.7; 0.2; 0.6), ((6.2; 7.6; 8.2); 0.4; 0.1; 0.3), ((7.1; 7.7; 8.3); 0.5; 0.2; 0.4), ((5.8; 6.9; 8.5); 0.6; 0.2; 0.3), ((5.5; 6.2; 7.3); 0.8; 0.1; 0.2), ((5.3; 6.7; 9.9); 0.3; 0.5; 0.2), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((4.4; 5.9; 7.2); 0.7; 0.2; 0.3), ((6.6; 8.8; 10); 0.6; 0.2; 0.2), ((6.3; 7.5; 8.9); 0.7; 0.4; 0.6), ((5.3; 7.3; 8.7); 0.7; 0.2; 0.8), ((6.5; 6.9; 8.5); 0.6; 0.8; 0.1), ((7.5; 7.9; 8.5); 0.8; 0.5; 0.4). The user according to his/her evaluation of every quality attribute (in table 1) gives them one of the 15 defined values.

Phase 2: Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

The second phase aims to construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker as follows: Let  $Q = (q_1, q_2, \dots, q_n)$  a set of information systems.  $C = (c_1, c_2, \dots, c_m)$  be ISQ criteria, and let  $[A_{ij}] = ((a_{ij}, b_{ij}, c_{ij}); w_{\bar{a}_{ij}})$

, $u_{\tilde{a}ij}, y_{\tilde{a}ij}$ ) ( $i \in I_m$  for ISQ criteria ,  $j \in I_n$  information systems) be a SVTrN-number. Then decision matrix can be identified as the follows:

$$[A_{ij}]_{m \times n} = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{pmatrix}$$

**Phase 3: Calculate the Comprehensive Values**

At the first, Compute the normalized decision-making matrix  $R = [r_{ij}]_{m \times n}$  and compute

$U = [u_{ij}]_{m \times n}$  as the follows:

- Compute the normalized decision-making matrix

$R = [r_{ij}]_{m \times n}$  where

$$R_{ij} = ((a_{ij}/\bar{a}^+, b_{ij}/\bar{a}^+, c_{ij}/\bar{a}^+); w_{\tilde{a}ij}, u_{\tilde{a}ij}, y_{\tilde{a}ij})$$

Such that  $\bar{a}^+ = \max \{c_{ij}, i \in I_m, j \in I_n\}$

- Compute  $U = [u_{ij}]_{m \times n}$  of  $R$ . Where,  $u_{ij} = \omega_i r_{ij}$  ( $i \in I_m$  for ISQ criteria ,  $j \in I_n$  information systems),

$\omega = (\omega_1, \omega_2 \dots \omega_m)$  be the weight vector of ISQ criteria,

where  $\omega_i \in [0, 1], i \in I_m$  and  $\sum_{i=1}^m \omega_i = 1$

Then, calculate the comprehensive values  $S_j$  as:

$$S_j = \sum_{i=1}^m u_{ij} = ((\sum_{i=1}^m \omega_i r_{ij}, \sum_{i=1}^m \omega_i r_{ij}, \sum_{i=1}^m \omega_i r_{ij}); \text{Min } w_{\tilde{a}ij}, \text{Max } u_{\tilde{a}ij}, \text{Max } y_{\tilde{a}ij})$$

$$(j \in I_n) \tag{3.1}$$

**Phase 4: Evaluate and Rank ISQ**

This phase aims to introduce two evaluating and ranking methods: (1) - weighted value and ambiguity based method, (2) the value index and ambiguity index method to give more than one option for evaluating and ranking ISQ.

**(1)- Weighted value and ambiguity method**

Firstly, calculate the value of truth-membership-degree, and indeterminacy-membership, and falsity-membership degree for each comprehensive value based on “Eq. (2.7)” “Eq. (2.9)”and “Eq. (2.11)”, respectively, as the follows:

$$V_\mu(S_j) = ((a + 4b + c) (w_{sj})^2)/6 \tag{3.2}$$

$$V_\nu(S_j) = ((a + 4b + c) (1-u_{sj})^2)/6 \tag{3.3}$$

$$V_\lambda(S_j) = ((a + 4b + c) (1-y_{sj})^2)/6 \tag{3.4}$$

And, calculate the ambiguity of truth-membership-membership degree, and indeterminacy-membership, and falsity-membership degree for each comprehensive value based on “Eq. (2.8)” “Eq. (2.10)”and “Eq. (2.12)”, respectively, as the follows:

$$A_\mu(S_j) = ((c-a) (w_{sj})^2)/6 \tag{3.5}$$

$$A_\nu(S_j) = ((c-a) (1-u_{sj})^2)/6 \tag{3.6}$$

$$A_\lambda(S_j) = ((c-a) (1-y_{sj})^2)/6 \tag{3.7}$$

Secondly, calculate the weighted values ( $\theta$  - weighted value) for each alternative as the follows:

the  $\theta$  -weighted value of each comprehensive value  $S_j$  is defined as:

$$V_\theta(S_j) = V_\mu(S_j) \theta + V_\nu(S_j)(1-\theta) + V_\lambda(S_j) (1-\theta) \tag{3.8}$$

The  $\theta$  - weighted ambiguity of a comprehensive value  $S_j$  can be defined as:

$$A_\theta(S_j) = (c-a) /6 [\theta w_j^2+ (1-\theta) (1-u_{sj})^2+ (1-\theta) (1-y_{sj})^2] \tag{3.9}$$

$$= A_\mu(S_j) \theta + A_\nu(S_j) +(1-\theta) A_\lambda(S_j) (1-\theta) \tag{3.10}$$

**4. Case study**

An IS evaluation committee wants to evaluate quality of three IS centers at three universities according eight quality characteristics based ISO/IEC 25010:  $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8)$  be quality characteristics: functionality  $c_1$ , reliability  $c_2$ , usability  $c_3$ , efficiency  $c_4$ , maintainability  $c_5$ , portability  $c_6$ , security  $c_7$ , compatibility  $c_8$ . The weight vector of the eight quality characteristics is  $\omega = (.25, .25, .30, .20, .25, .20, .20, \text{ and } .15)$ .

**Phase I: Using Single Valued Triangular Neutrosophic Numbers with scale based approach**

Apply the scale based approach to enable the IS evaluator to give every quality attribute one of the following categories: Low, Not low, Very low, Completely low, More or less low, Fairly low, Essentially low, Neither low nor high, High, Not high, Very high, Completely high, More or less high, Fairly high, Essentially high, having corresponding values ((4.6; 5.5; 8.6); 0.4; 0.7; 0.2), ((4.7; 6.9; 8.5); 0.7; 0.2; 0.6), ((6.2; 7.6; 8.2); 0.4; 0.1; 0.3), ((7.1; 7.7; 8.3); 0.5; 0.2; 0.4), ((5.8; 6.9; 8.5); 0.6; 0.2; 0.3), ((5.5; 6.2; 7.3); 0.8; 0.1; 0.2), ((5.3; 6.7; 9.9); 0.3; 0.5; 0.2), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((4.4; 5.9; 7.2); 0.7; 0.2; 0.3), ((6.6; 8.8; 10); 0.6; 0.2; 0.2), ((6.3; 7.5; 8.9); 0.7; 0.4; 0.6), ((5.3; 7.3; 8.7); 0.7; 0.2; 0.8), ((6.5; 6.9; 8.5); 0.6; 0.8; 0.1), ((7.5; 7.9; 8.5); 0.8; 0.5; 0.4). The quality attributes of the three information systems can be presented based on the scale based approach as the follows:

▪ **The first information system**

The following table represents the quality attributes of the first information system based on the scale based approach.

Table (1): The quality attributes of the first information system

ISO characteristics	Linguistic values															
	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high	
C <sub>1</sub>	√															((4.6, 5.5, 8.6); 0.4, 0.7, 0.2)
C <sub>2</sub>		√														((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)
C <sub>3</sub>			√													((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
C <sub>4</sub>				√												((7.1, 7.7, 8.3); 0.5, 0.2, 0.4)
C <sub>5</sub>					√											((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)
C <sub>6</sub>						√										((5.8, 6.9, 8.5); 0.6, 0.2, 0.3)
C <sub>7</sub>			√													((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)
C <sub>8</sub>		√														((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)

▪ **The second information system**

The following table represents the quality attributes of the second information system based on the scale based approach.

Table (2): The quality attributes of the second information system

ISO characteristics	Linguistic values															
	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high	
C <sub>1</sub>													√			((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)
C <sub>2</sub>										√						((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
C <sub>3</sub>														√		((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)
C <sub>4</sub>													√			((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)
C <sub>5</sub>										√						((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
C <sub>6</sub>											√					((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)
C <sub>7</sub>												√				((6.6, 8.8, 10); 0.6, 0.2, 0.2)
C <sub>8</sub>	√															((4.7, 6.9, 8.5); 0.7, 0.2, 0.6)

▪ **The third information system**

The following table represents the quality attributes of the third information system based on the scale based approach.

Table (3): The quality attributes of the third information system

ISO characteristics	Linguistic values																
	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high		
C <sub>1</sub>															√	((7.5, 7.9, 8.5); 0.8, 0.5, 0.4)	
C <sub>2</sub>																√	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
C <sub>3</sub>																√	((6.6, 8.8, 10); 0.6, 0.2, 0.2)
C <sub>4</sub>																√	((6.3, 7.5, 8.9); 0.7, 0.4, 0.6)
C <sub>5</sub>																√	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)
C <sub>6</sub>																√	((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)
C <sub>7</sub>																√	((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)
C <sub>8</sub>																√	((6.6, 8.8, 10); 0.6, 0.2, 0.2)

**Phase 2:** Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

Let Q= (q<sub>1</sub>, q<sub>2</sub>, q<sub>3</sub>) be a set of the three IS. C= (c<sub>1</sub>, c<sub>2</sub>, c<sub>3</sub>, c<sub>4</sub>, c<sub>5</sub>, c<sub>6</sub>, c<sub>7</sub>, c<sub>8</sub>) be ISQ criteria: functionality c<sub>1</sub>, reliability c<sub>2</sub>,

usability c<sub>3</sub>, efficiency c<sub>4</sub>, maintainability c<sub>5</sub>, portability c<sub>6</sub>, security c<sub>7</sub>, compatibility c<sub>8</sub>. Let A= [A<sub>ij</sub>]<sub>8\*3</sub> = ((a<sub>ij</sub>, b<sub>ij</sub>, c<sub>ij</sub>); w<sub>a<sub>ij</sub></sub>, u<sub>a<sub>ij</sub></sub>, y<sub>a<sub>ij</sub></sub>) (i ∈ I<sub>8</sub> for ISQ criteria, j ∈ I<sub>3</sub> the three information systems) be a SVTrN-numbers. Then

((4.6, 5.5, 8.6); 0.4, 0.7, 0.2)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)	((7.5, 7.9, 8.5); 0.8, 0.5, 0.4)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)
((7.1, 7.7, 8.3); 0.5, 0.2, 0.4)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)	((6.3, 7.5, 8.9); 0.7, 0.4, 0.6)
((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)
((5.8, 6.9, 8.5); 0.6, 0.2, 0.3)	((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)	((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)	((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((4.7, 6.9, 8.5); 0.7, 0.2, 0.6)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)

**Phase 2:** Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

Let Q= (q<sub>1</sub>, q<sub>2</sub>, q<sub>3</sub>) be a set of the three IS. C= (c<sub>1</sub>, c<sub>2</sub>, c<sub>3</sub>, c<sub>4</sub>, c<sub>5</sub>, c<sub>6</sub>, c<sub>7</sub>, c<sub>8</sub>) be ISQ criteria: functionality c<sub>1</sub>, reliability c<sub>2</sub>, usability c<sub>3</sub>, efficiency c<sub>4</sub>, maintainability c<sub>5</sub>, portability c<sub>6</sub>, security c<sub>7</sub>, compatibility c<sub>8</sub>. Let A= [A<sub>ij</sub>]<sub>8\*3</sub> = ((a<sub>ij</sub>, b<sub>ij</sub>, c<sub>ij</sub>); w<sub>a<sub>ij</sub></sub>, u<sub>a<sub>ij</sub></sub>, y<sub>a<sub>ij</sub></sub>) (i ∈ I<sub>8</sub> for ISQ criteria, j ∈ I<sub>3</sub> the three information systems) be a SVTrN-numbers. Then

((4.6, 5.5, 8.6); 0.4, 0.7, 0.2)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)	((7.5, 7.9, 8.5); 0.8, 0.5, 0.4)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)
((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)
((7.1, 7.7, 8.3); 0.5, 0.2, 0.4)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)	((6.3, 7.5, 8.9); 0.7, 0.4, 0.6)
((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)	((6.2, 8.9, 9.1); 0.6, 0.3, 0.5)	((5.3, 7.3, 8.7); 0.7, 0.2, 0.8)
((5.8, 6.9, 8.5); 0.6, 0.2, 0.3)	((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)	((4.4, 5.9, 7.2); 0.7, 0.2, 0.3)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)	((6.5, 6.9, 8.5); 0.6, 0.8, 0.1)
((6.2, 7.6, 8.2); 0.4, 0.1, 0.3)	((4.7, 6.9, 8.5); 0.7, 0.2, 0.6)	((6.6, 8.8, 10); 0.6, 0.2, 0.2)

**Phase 3:** Calculate the Comprehensive Values

Before calculating the comprehensive values, Compute the normalized decision-making matrix R= [r<sub>ij</sub>]<sub>8\*3</sub> and compute U= [u<sub>ij</sub>]<sub>8\*3</sub> as the follows:

Compute the normalized decision-making matrix R= [r<sub>ij</sub>]<sub>m\*n</sub> where

R= ((a<sub>ij</sub>/ā<sup>+</sup>, b<sub>ij</sub>/ā<sup>+</sup>, c<sub>ij</sub>/ā<sup>+</sup>); w<sub>a<sub>ij</sub></sub>, u<sub>a<sub>ij</sub></sub>, y<sub>a<sub>ij</sub></sub>), such that ā<sup>+</sup>= Max {c<sub>ij</sub>; i ∈ I<sub>m</sub>, j ∈ I<sub>n</sub>}

R=

((.46, .55, .86); 0.4, 0.7, 0.2)	((.53, .73, .87); 0.7, 0.2, 0.8)	((.75, .79, .85); 0.8, 0.5, 0.4)
((.62, .76, .82); 0.4, 0.1, 0.3)	((.62, .89, .91); 0.6, 0.3, 0.5)	((.62, .89, .91); 0.6, 0.3, 0.5)
((.62, .89, .91); 0.6, 0.3, 0.5)	((.65, .69, .85); 0.6, 0.8, 0.1)	((.66, .88, 1); 0.6, 0.2, 0.2)
((.71, .77, .83); 0.5, 0.2, 0.4)	((.53, .73, .87); 0.7, 0.2, 0.8)	((.63, .75, .89); 0.7, 0.4, 0.6)
((.44, .59, .72); 0.7, 0.2, 0.3)	((.62, .89, .91); 0.6, 0.3, 0.5)	((.53, .73, .87); 0.7, 0.2, 0.8)
((.58, .69, .85); 0.6, 0.2, 0.3)	((.44, .59, .72); 0.7, 0.2, 0.3)	((.44, .59, .72); 0.7, 0.2, 0.3)
((.62, .76, .82); 0.4, 0.1, 0.3)	((.66, .88, 1); 0.6, 0.2, 0.2)	((.65, .69, .85); 0.6, 0.8, 0.1)
((.62, .76, .82); 0.4, 0.1, 0.3)	((.47, .69, .85); 0.7, 0.2, 0.6)	((.66, .88, 1); 0.6, 0.2, 0.2)

Compute U= [u<sub>ij</sub>]<sub>m\*n</sub> of R. Where, u<sub>ij</sub>= ω<sub>i</sub> r<sub>ij</sub> (i ∈ I<sub>m</sub> for ISQ criteria, j ∈ I<sub>n</sub> information systems),

ω = (.35, .25, .30, .20, .25, .20, .30, .20) be the weight vector of ISQ criteria, where ω<sub>i</sub> ∈ [0, 1], i ∈ I<sub>m</sub>, and

$$\sum_{i=1}^m \omega_i = 1$$

Calculate the comprehensive values  $S_j$  as:

$$S_j = \sum_{i=1}^m u_{ij} \quad (j \in I_n),$$

U=

((.161,.192,.301);0.4,0.7,0.2)	((.185,.255,.304);0.7,0.2,0.8)	((.262,.276,.297);0.8,0.5,0.4)
((.155,.190,.205);0.4,0.1,0.3)	((.155,.222,.227);0.6,0.3,0.5)	((.155,.222,.227);0.6,0.3,0.5)
((.186,.267,.273);0.6,0.3,0.5)	((.195,.207,.255);0.6,0.8,0.1)	((.198,.264,.300);0.6,0.2,0.2)
((.142,.154,.166);0.5,0.2,0.4)	((.106,.146,.174);0.7,0.2,0.8)	((.126,.150,.178);0.7,0.4,0.6)
((.110,.147,.180);0.7,0.2,0.3)	((.155,.222,.227);0.6,0.3,0.5)	((.132,.182,.217);0.7,0.2,0.8)
((.116,.138,.170);0.6,0.2,0.3)	((.088,.118,.144);0.7,0.2,0.3)	((.088,.118,.144);0.7,0.2,0.3)
((.186,.228,.246);0.4,0.1,0.3)	((.198,.264,.300);0.6,0.2,0.2)	((.195,.207,.255);0.6,0.8,0.1)
((.124,.152,.164);0.4,0.1,0.3)	((.094,.138,.170);0.7,0.2,0.6)	((.132,.176,.200);0.6,0.2,0.2)

Then, calculate the comprehensive values  $S_j$  as:

$$S_j = \sum_{i=1}^m u_{ij} = ((\sum_{i=1}^m \omega_i r_{ij}, \sum_{i=1}^m \omega_i r_{ij}, \sum_{i=1}^m \omega_i r_{ij}); \text{Min } w_{\tilde{a}_{ij}}, \text{Max } u_{\tilde{a}_{ij}}, \text{Max } y_{\tilde{a}_{ij}})$$

- $S_1 = ((1.18, 1.468, 1.705); .4, .7, .5)$
- $S_2 = ((1.176, 1.572, 1.801); .6, .8, .8)$
- $S_3 = ((1.288, 1.592, 1.818); .6, .8, .8)$

Phase 4: Rank ISQ

Apply the two evaluating and ranking methods: (1) - weighted value and ambiguity based method, (2) the value index and ambiguity index method

1. Weighted value and ambiguity method

Calculate the weighted value and ambiguity of truth-membership and indeterminacy membership, and falsity-membership degree for each comprehensive value

$$V_\mu(S_1) = 1.459 (.4)^2 = .233$$

$$V_\nu(S_1) = 1.459 (1-.7)^2 = .131$$

$$V_\lambda(S_1) = 1.459 (1-.5)^2 = .364$$

$$V_\mu(S_2) = 1.544 (.6)^2 = .555;$$

$$V_\nu(S_2) = 1.544 (1-.8)^2 = .061;$$

$$V_\lambda(S_2) = 1.544 (1-.8)^2 = .061$$

$$V_\mu(S_3) = 1.581 (.6)^2 = .569;$$

$$V_\nu(S_3) = 1.581 (1-.8)^2 = .063;$$

$$V_\lambda(S_3) = 1.581 (1-.8)^2 = .063$$

$$V_\theta = .233 \theta + .131(1-\theta) + .364(1-\theta)$$

$$V_\theta = .555 \theta + .061(1-\theta) + .061(1-\theta)$$

$$V_\theta = .569 \theta + .063(1-\theta) + .063(1-\theta)$$

Thirdly, graphically represents weighted values for evaluating and ranking quality of IS. The following figure represents the weighted values of the  $S_1, S_2$  and  $S_3$

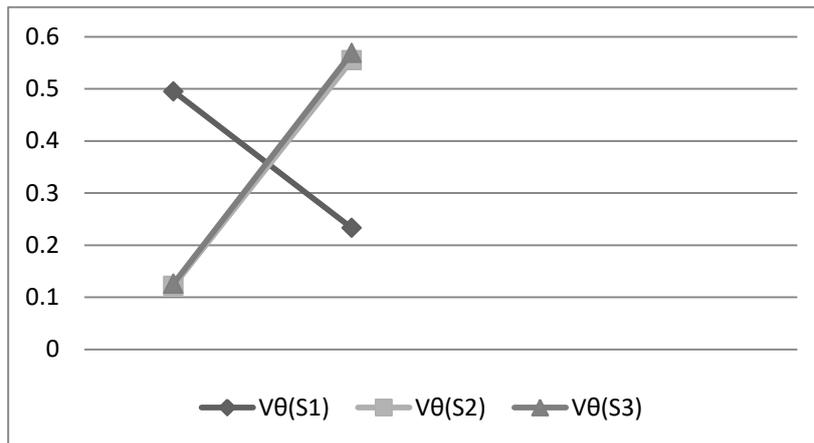


Fig. 1. The weighted values of the  $S_1, S_2$  and  $S_3$

- From figure (1) for any  $\theta \in [0, .523]$  the weighted values of the  $S_1, S_2$  and  $S_3$  can ranked as the follows:  $V_\theta(S_1) > V_\theta(S_3) > V_\theta(S_2)$ . Consequently, the quality of the first information system > the quality of the third information system > the quality of the second information system
- From figure (1), the weighted values of  $S_1$  and  $S_3$  have equal values at  $\theta = .523$ . The weighted

ambiguities of  $S_1$  and  $S_3$  can be calculated based on Eq. (3.9) as follows:

$$A_{.523}(S_1) = .0212$$

$$A_{.523}(S_3) = .0198$$

Therefore,  $S_1 > S_3$ , Consequently, the quality of the first information system is greater than the quality of the third information system

- From figure (1) for any  $\theta \in [.523, .536]$  the weighted values of the  $S_1, S_2$  and  $S_3$  can ranked as the follows:  $V_\theta(S_1) > V_\theta(S_3) > V_\theta(S_2)$ .

Consequently, the quality of the first information system > the quality of the third information system > the quality of the second information system

- From figure (1), the weighted values of  $S_1$  and  $S_2$  have equal values at  $\theta = .536$ . The weighted ambiguities of  $S_1$  and  $S_2$  can be calculated based on Eq. (4.9) as follows:

$$A_{\theta}(S_j) = (c-a) / 6 [\theta w_j^{2+} + (1-\theta) (1-u_{sj})^{2+} + (1-\theta) (1-y_{sj})^2]$$

$$A_{.536}(S_1) = .0210$$

$$A_{.536}(S_2) = .0237$$

Therefore,  $S_2 > S_1$ , Consequently, the quality of the second information system is greater than the quality of the first information system

- From figure (1) for any  $\theta \in [.536, 1]$  the weighted values of the  $S_1, S_2$  and  $S_3$  can ranked as the follows:  $V_{\theta}(S_3) > V_{\theta}(S_2) > V_{\theta}(S_1)$ . Consequently, the quality of the third information system > the quality of the second information system > the quality of the first information system

This method gives more attention to uncertainty in decision making as well as it takes into account cut sets of SVTrN numbers that can reflect the information on membership degrees and non-membership degrees. However, the calculations and graphically representation of this method become complex when alternatives increase.

**1. The value index and ambiguity index method**

Apply the value index and ambiguity index method to rank Information Systems Quality (ISQ) as the follows:

$$V_{\mu}(S_1) = 1.459 (.4)^2 = .233$$

$$V_{\nu}(S_1) = 1.459 (1-.7)^2 = .131$$

$$V_{\lambda}(S_1) = 1.459 (1-.5)^2 = .364$$

$$V_{\mu}(S_2) = 1.544 (.6)^2 = .555;$$

$$V_{\nu}(S_2) = 1.544 (1-.8)^2 = .061;$$

$$V_{\lambda}(S_2) = 1.544 (1-.8)^2 = .061$$

$$V_{\mu}(S_3) = 1.581 (.6)^2 = .569;$$

$$V_{\nu}(S_3) = 1.581 (1-.8)^2 = .063;$$

$$V_{\lambda}(S_3) = 1.581 (1-.8)^2 = .063$$

$$V(S_1, \lambda) = .233 \lambda + .131(1-\lambda) + .364(1-\lambda)$$

$$V(S_2, \lambda) = .555 \lambda + .061(1-\lambda) + .061(1-\lambda)$$

$$V(S_3, \lambda) = .569 \lambda + .063(1-\lambda) + .063(1-\lambda)$$

Table (4): Ranking results based on the Weighted Values and Ambiguities index method of SVTrN-numbers

$\lambda$	$V(S_1, \lambda)$	$V(S_2, \lambda)$	$V(S_3, \lambda)$	Ranking results
$.1 \in [0,1/2]$	.468	.165	.170	$S_1 > S_3 > S_2$
$.3 \in [0,1/2]$	.416	.251	.258	$S_1 > S_3 > S_2$

.5	.364	.338	.347	$S_1 > S_3 > S_2$
$.7 \in [1/2, 1]$	.311	.425	.436	$S_3 > S_2 > S_1$
$.8 \in [1/2, 1]$	.285	.468	.480	$S_3 > S_2 > S_1$

- (1) From table (4) values: .1 and .3 where  $\lambda \in [0, 1/2]$ , the results show when the decision maker prefers negative feeling, the ranking of quality of the three information systems is  $S_1 > S_3 > S_2$ , Consequently, the quality of the first IS > the quality of the third IS > the quality of the second IS.
- (2) From table (4) where  $\lambda = 1/2$  shows that the decision maker is indifferent between positive feeling and negative feeling, the ranking of quality of the three information systems is  $S_1 > S_3 > S_2$ , Consequently, the quality of the first IS > the quality of the third IS > the quality of the second IS.
- (3) From table (4) values: .7 and .8 where  $\lambda \in [1/2, 1]$ , the results show when the decision maker prefers positive feeling, evaluation and ranking of quality of the three information systems is  $S_3 > S_2 > S_1$ , Consequently, the quality of the third IS > the quality of the second IS > the quality of the first IS.

This method focuses on value index and ambiguity index and it can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

**5. Conclusion and Future Work**

This work intended to introduce a framework with two ranking methods of SVTrN- numbers with the scale based approach for evaluating and ranking ISQ. The proposed framework consists of four phases. The results indicated that each ranking method has its own advantage that make. In this perspective, the weighted value and ambiguity based method gives more attention to uncertainty in ranking and evaluating ISQ as well as it takes into account cut sets of SVTrN numbers that can reflect the information on membership degrees and non-membership degrees. The value index and ambiguity index can handle indeterminacy and uncertainty and it can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

For future work, SVTrN-numbers can be applied widely for more real practical applications with adapting and generalizing existing methods of ranking fuzzy numbers and intuitionistic fuzzy number to give more efficient results.

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