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Some similarity and entropy measurements of bipolar neutrosophic soft sets

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Abstract: In this paper, we proposed a different approach on bipolar neutrosophic soft sets and discussed their properties with examples which was initially introduced by Mumtaz Ali et al.[15]. Also we defined some similarity and entropy measurements between any two bipolar neutrosophic soft sets. Further, we proposed the representation of a 2-D digital image in bipolar neutrosophic soft domain. Finally, based on similarity measurements, we propose a decision making process of real-time problem in image analysis.

Keywords: Neutrosophic set, Bipolar Neutrosophic set, similarity, entropy, Digital image.

1 Introduction

In our physical world, many real life situations don't have an exact solution. For that problems, we cannot use conventional method to determine the solution. To avoid those difficulties in dealing with uncertainities, we apply the concepts of Neutrosophy. Neutrosophy is the branch of philosophy which was introduced by Florentin Smarandache [10]. Neutrosophy deals with three components truth-membership, indeterminacy-membership and falsity-membership. Apparently, in the case of uncertainty, we have different solution methods like fuzzy theory, rough theory, vague theory etc. Since Neutrosophy is the extension of fuzzy theory, it is one of the efficient method among those. By using Neutrosophy, we can analyze the origin, nature and scope of the neutralities. Neutrosophy is the base for neutrosophic sets. Neutrosophic set was introduced by Smarandache which has three components called Truth-membership, Indeterminacy-membership and Falsity-membership ranges in the non-standard interval $]^{-}0, 1^{+}[$.

But for engineering and real life problems we prefer specific solution. Since it will be difficult to apply in real life problems, Wang et al. [11] introduced the concept of single valued neutrosophic set (SVNS) which is the immediate result of neutrosophic set by taking standard interval [0,1] instead of non-standard interval $]^{-}0, 1^{+}[$. Single valued neutrosophic theory is useful in modeling uncertain imprecision. Yanhui et al. [8] proposed image segmentation through neutrosophy whereas A. A. Salama et al. [7] proposed a neutrosophic approach to grayscale images. Majundar et al. [5, 6] introduced some measures of similarity and entropy of neutrosophic sets (as well as SVNS). Aydogdu [4] proposed these similarity and entropy to Interval valued neutrosophic sets (IVNS). Also ahin and Kk [1] proposed the concepts similarity and entropy to neutrosophic soft sets.

In 2015, Deli et al. [2] introduced the concepts of bipolar neutrosophic sets (BNS) as an extension of neutrosophic sets. In 2016, Uluay et al. [3] proposed some measures of similarities of bipolar neutrosophic sets. In 2017, Mumtaz Ali et al.[15] introduced the concepts of bipolar neutrosophic soft sets which is a combined version of bipolar neutrosophic set and neutrosophic soft set. Neutrosophic set concepts are very useful in decision making problem. Abdel-Basset et al.[18, 19, 20] proposed some decision making algorithms for problems in engineering and medical fields.

In this paper, we proposed slightly different approach on bipolar neutrosophic soft sets(BNSS). Section 2 contains important preliminary definitions. In section 3, we propose different approach on bipolar neutrosophic soft set which was introduced by Ali et al.[15] and also we discuss their properties with examples. In section 4, we define entropy measurement to calculate the indeterminacy. In section 5, we defined various distances between any two BNSSs to calculate the similarity between them. In section 6, we propose the representation of 2-D digital image in bipolar neutrosophic soft domain. In section 7, we propose the decision making process of image based on similarity measurements for a real-time problem in image analysis. Finally, section 8 contains conclusion of our work.

2 Preliminaries

Definition 2.1. [12]
Let X be a universal set which contains arbitrary points x. A Neutrosophic set A is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where $T_A(x), I_A(x), F_A(x)$ referred as truth-membership function, indeterminacy-membership function and falsity-membership function respectively.
Here

$$T_A(x), I_A(x), F_A(x) : X \to]^-0, 1^+[.$$

Further it satisfies the condition

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

Example 2.2. Let $X = \{x_1, x_2, x_3\}$ be the universal set. Here, x_1, x_2, x_3 represents capacity, trustworthiness and price of a machine, respectively. Then $T_A(x), I_A(x), F_A(x)$ gives the degree of 'good service', degree of indeterminacy, degree of 'poor service' respectively. The neutrosophic set is defined by
$A = \{\langle x_1, 0.3, 0.4, 0.5\rangle, \langle x_2, 0.5, 0.2, 0.3\rangle, \langle x_3, 0.7, 0.2, 0.2\rangle\}$
where $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$

Definition 2.3. [11]
Neutrosophic set(NS) is defined over the non-standard unit interval $]^-0, 1^+[$ whereas single valued neutrosophic set is defined over standard unit interval [0,1].
It means a single valued neutrosophic set A is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where

$$T_A(x), I_A(x), F_A(x) : X \rightarrow [0, 1]$$

such that

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.4. [13, 16]
A pair (F, A) is a soft set over X if

$$F : A \rightarrow P(x)$$

That means the soft set is a parameterized family of subsets of the set X.
For any parameter $e \in A$, $F(e) \subseteq X$ is the set of e-approximation elements of the soft set (F, A).

Example 2.5. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of 2-dimensional images and let $A = \{e_1, e_2, e_3\}$ be set of parameters. where e_1=contrast, e_2=saturation and e_3=sharpness.
suppose that

$$F(e_1) = \{x_1, x_2\}$$
$$F(e_2) = \{x_1, x_3\}$$
$$F(e_3) = \{x_2, x_4\}.$$

Then, the set

$$F(A) = \{F(e_1), F(e_2), F(e_3)\}$$

is the parameterized family of subsets of X.

Definition 2.6. [14]
A neutrosophic soft set (F_A, E) over X is defined by the set

$$(F_A, E) = \left\{\langle e, F_A(e)\rangle : e \in E, F_A(e) \in NS(X)\right\}$$

where $F_A : E \longrightarrow NS(x)$ such that $F_A(e) = \varphi$ if $e \notin A$.
Also, since $F_A(e)$ is a neutrosophic set over X is defined by

$$F_A(e) = \left\{\left\langle x, u_{F_A(e)}(x), v_{F_A(e)}(x), w_{F_A(e)}(x)\right\rangle : x \in X\right\}$$

where $u_{F_A(e)}(x), v_{F_A(e)}(x), w_{F_A(e)}(x)$ represents truth-membership degree of x which holds the parameter e, indeterminacy-membership degree of x which holds the parameter e and falsity-membership degree of x which holds the parameter e.

Example 2.7. Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of houses under consideration. Let $A = \{e_1, e_2, e_3\}$ be set of parameters where e_1, e_2, e_3 represents beautiful, wooden and costly, respectively.
Then we define

$$(F_A, E) = \left\{\langle e_1, F_A(e_1)\rangle, \langle e_2, F_A(e_2)\rangle, \langle e_3, F_A(e_3)\rangle\right\}$$

Here

$$F_A(e_1) = \left\{ \langle x_1, 0.4, 0.3 \rangle , \langle x_2, 0.5, 0.6, 0.7 \rangle , \langle x_3, 0.5, 0.6, 0.7 \rangle , \langle x_4, 0.5, 0.6, 0.7 \rangle \right\}$$

$$F_A(e_1) = \left\{ \langle x_1, 0.5, 0.6, 0.3 \rangle , \langle x_2, 0.4, 0.7, 0.6 \rangle , \langle x_3, 0.6, 0.2, 0.3 \rangle , \langle x_4, 0.7, 0.2, 0.3 \rangle \right\}$$

$$F_A(e_2) = \left\{ \langle x_1, 0.6, 0.3, 0.5 \rangle , \langle x_2, 0.7, 0.4, 0.3 \rangle , \langle x_3, 0.8, 0.1, 0.2 \rangle , \langle x_4, 0.7, 0.1, 0.3 \rangle \right\}$$

$$F_A(e_3) = \left\{ \langle x_1, 0.7, 0.4, 0.3 \rangle , \langle x_2, 0.6, 0.1, 0.2 \rangle , \langle x_3, 0.7, 0.2, 0.5 \rangle , \langle x_4, 0.5, 0.2, 0.6 \rangle \right\}$$

Hence (F_A, E) is a neutrosophic soft set.

Definition 2.8. [2, 3]
Let X be the universal set which contains arbitrary points x. A bipolar neutrosophic set (BNS) A is defined by

$$A = \left\{ \langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) \rangle : x \in X \right\}$$

where

$$T^+, I^+, F^+ : E \to [0, 1] \text{ (positive membership-degrees)}$$
$$T^-, I^-, F^- : E \to [-1, 0] \text{ (negative membership-degrees)}$$

such that

$$0 \leq T^+(x) + I^+(x) + F^+(x) \leq 3 \; , \; -3 \leq T^-(x) + I^-(x) + F^-(x) \leq 0.$$

Example 2.9. Let $X = \{x_1, x_2, x_3\}$ be the universal set. A bipolar neutrosophic set (BNS) is defined by

$$\begin{aligned} A = \{ & \langle x_1, 0.3, 0.4, 0.5, -0.2, -0.4, -0.1 \rangle , \\ & \langle x_2, 0.5, 0.2, 0.3, -0.2, -0.7, -0.5 \rangle , \\ & \langle x_3, 0.7, 0.2, 0.2, -0.5, -0.4, -0.5 \rangle \} \end{aligned}$$

where $0 \leq T_A^+(x) + I_A^+(x) + F_A^+(x) \leq 3$ and $-3 \leq T_A^-(x) + I_A^-(x) + F_A^-(x) \leq 0$.
Also $T_A^+(x), I_A^+(x), F_A^+(x) \to [0, 1]$ and $T_A^-(x), I_A^-(x), F_A^-(x) \to [-1, 0]$.

3 Different approach on bipolar neutrosophic soft set

In this section, we propose a slightly different approach on bipolar neutrosophic soft sets which is the combined version of neutrosophic soft set and bipolar neutrosophic set and this was initially introduced by Mumtaz Ali et al.[15]. He defined a bipolar neutrosophic soft set associated with the whole parameter set E.

In our approach, we define a bipolar neutrosophic soft set associated with only subset of a parameter set E. Because, there is a possibility to exist different bipolar neutrosophic soft sets associated with different subsets of E.

Ali et al.[15] definition is given below.

Definition 3.1. Let U be a universe and E be a set of parameters that are describing the elements of U. A bipolar neutrosophic soft set $\mathbb{B}$ in U is defined as:

$$\mathbb{B} = \Big\{(e, \Big\{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u) : u \in U\Big\} : e \in E\Big\}$$

where $T^+, I^+, F^+ \to [0,1]$ and $T^-, I^-, F^- \to [-1,0]$. The positive membership degree $T^+(u), I^+(u), F^+(u)$, denotes the truth membership, indeterminate membership and false membership of an element corresponding to a bipolar neutrosophic soft set $\mathbb{B}$ and the negative membership degree $T^-(u), I^-(u), F^-(u)$ denotes the truth membership, indeterminate membership and false membership of an element $u \in U$ to some implicit counter-property corresponding to a bipolar neutrosophic soft set $\mathbb{B}$.

Our approach is given below.

Definition 3.2. Let X be the universe and E be the parameter set. Let A be subset of the parameter set E.

A bipolar neutrosophic soft set $\mathcal{B}$ over X is defined by

$$\mathcal{B}=(F_A, E) = \left\{\langle e, F_A(e)\rangle : e \in E, F_A(e) \in BNS(X)\right\}$$

Here

$F_A(e) = \left\{\left\langle x, u^+_{F_A(e)}(x), v^+_{F_A(e)}(x), w^+_{F_A(e)}(x), u^-_{F_A(e)}(x), v^-_{F_A(e)}(x), w^-_{F_A(e)}(x)\right\rangle : x \in X\right\}$.

where $u^+_{F_A(e)}(x), v^+_{F_A(e)}(x), w^+_{F_A(e)}(x)$ represents positive truth-membership degree , positive indeterminacy-membership degree and positive falsity-membership degree of x which holds the parametrer e, and similarly $u^-_{F_A(e)}(x), v^-_{F_A(e)}(x), w^-_{F_A(e)}(x)$ represents negative truth-membership degree , negative indeterminacy-membership degree and negative falsity-membership degree of x which holds the parameter e .

Example 3.3. Let $X = \{x_1, x_2, x_3, x_4\}$ be a universal set and let $E = \{e_1, e_2, e_3\}$ be the parameter set. Also, let $A = \{e_1, e_2\} \subseteq E$ and $B = \{e_3\} \subseteq E$ be two subsets of E.

Then we define

$$\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(e)\rangle : e \in E, F_A(e) \in BNS(X)\}$$
$$\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(e)\rangle : e \in E, G_B(e) \in BNS(X)\}$$

where,

$$F_A(e_1) = \Big\{\langle x_1, 0.5, 0.4, 0.3, -0.02, -0.4, -0.5\rangle, \langle x_2, 0.4, 0.7, 0.6, -0.3, -0.5, -0.02\rangle, \langle x_3, 0.4, 0.3, 0.5, -0.6, -0.4, -0.2\rangle, \langle x_4, 0.4, 0.6, 0.3, -0.6, -0.2, -0.3\rangle\Big\}$$

$$F_A(e_2) = \Big\{ \langle x_1, 0.6, 0.3, 0.2, -0.4, -0.5, -0.04 \rangle, \langle x_2, 0.5, 0.2, 0.3, -0.1, -0.3, -0.6 \rangle, \langle x_3, 0.3, 0.4, 0.2, -0.3, -0.4, -0.7 \rangle, \langle x_4, 0.8, 0.2, 0.01, -0.4, -0.5, -0.1 \rangle \Big\}$$

$$G_B(e_3) = \Big\{ \langle x_1, 0.6, 0.3, 0.4, -0.4, -0.5, -0.3 \rangle, \langle x_2, 0.4, 0.5, 0.1, -0.2, -0.6, -0.4 \rangle, \langle x_3, 0.2, 0.3, 0.1, -0.4, -0.4, -0.2 \rangle, \langle x_4, 0.3, 0.4, 0.4, -0.5, -0.3, -0.2 \rangle \Big\}$$

Then $\mathcal{B}_1$ and $\mathcal{B}_2$ are the parameterized family of bipolar neutrosophic soft sets over X.

3.1 Properties of Bipolar Neutrosophic soft sets

In this section, we have discussed some basic properties of Bipolar neutrosophic soft sets.

3.1.1 Subsets and Eqiuvalent sets

Let X be universal set and E be a parameter set. Let $A, B \subseteq E$. Suppose $\mathcal{B}_1$ and $\mathcal{B}_2$ be two bipolar neutrosophic soft sets. Then $\mathcal{B}_1 \subseteq \mathcal{B}_2$ if and only if $A \subseteq B$ and
$u^+_{F_A(e)}(x) \leq u^+_{G_B(e)}(x), v^+_{F_A(e)}(x) \geq v^+_{G_B(e)}(x), w^+_{F_A(e)}(x) \geq w^+_{G_B(e)}(x)$ and
$u^-_{F_A(e)}(x) \geq u^-_{G_B(e)}(x), v^-_{F_A(e)}(x) \leq v^-_{G_B(e)}(x), w^-_{F_A(e)}(x) \leq w^-_{G_B(e)}(x)$.

Also $\mathcal{B}_1$ and $\mathcal{B}_2$ are called equivalent sets only if $A = B$ and all the parameters of $\mathcal{B}_1$ and $\mathcal{B}_2$ are corresponding to each other.

Example 3.4. Suppose $\mathcal{B}_1$ and $\mathcal{B}_2$ be two bipolar neutrosophic soft sets associated with $A = \{e_2\}$ and $B = \{e_1, e_2\}$.
Let $\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(e)\rangle : e \in E\}$ and $\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(e)\rangle : e \in E\}$
Here,

$$F_A(e_2) = \Big\{ \langle x_1, 0.4, 0.3, 0.9, -0.2, -0.3, -0.4 \rangle, \ \langle x_2, 0.5, 0.6, 0.7, -0.3, -0.4, -0.6 \rangle \Big\}$$

$$G_B(e_1) = \Big\{ \langle x_1, 0.5, 0.4, 0.3, -0.6, -0.2, -0.4 \rangle, \ \langle x_2, 0.6, 0.3, 0.2, -0.5, -0.3, -0.2 \rangle \Big\}$$

$$G_B(e_2) = \Big\{ \langle x_1, 0.6, 0.4, 0.2, -0.5, -0.1, -0.1 \rangle, \ \langle x_2, 0.7, 0.6, 0.3, -0.4, -0.2, -0.3 \rangle \Big\}$$

This implies $\mathcal{B}_1 \subseteq \mathcal{B}_2$.

3.1.2 Union and Intersection

The union is defined by

$$\mathcal{B}_1 \cup \mathcal{B}_2 = (F_A \bigcup G_B) = \Big\{ \Big\langle max(u^+_{F_A(e)}(x), u^+_{G_B(e)}(x)), \frac{v^+_{F_A(e)}(x) + v^+_{G_B(e)}(x)}{2}, min(w^+_{F_A(e)}(x), w^+_{G_B(e)}(x)), min(u^-_{F_A(e)}(x), u^-_{G_B(e)}(x)), \frac{v^-_{F_A(e)}(x) + v^-_{G_B(e)}(x)}{2}, max(w^-_{F_A(e)}(x), w^-_{G_B(e)}(x)) \Big\rangle \Big\}$$

The intersection is defined by

$$\mathcal{B}_1 \cap \mathcal{B}_2 = (F_A \bigcap G_B, E) = \left\{ \left\langle min(u^+_{F_A(e)}(x), u^+_{G_B(e)}(x)), \frac{v^+_{F_A(e)}(x) + v^+_{G_B(e)}(x)}{2}, max(w^+_{F_A(e)}(x), w^+_{G_B(e)}(x)), \right.\right.$$
$$\left.\left. max(u^-_{F_A(e)}(x), u^-_{G_B(e)}(x)), \frac{v^-_{F_A(e)}(x) + v^-_{G_B(e)}(x)}{2}, min(w^-_{F_A(e)}(x), w^-_{G_B(e)}(x)) \right\rangle \right\}$$

Example 3.5. Suppose

$$\mathcal{B}_1 = (F_A, E) = \{\langle x_1, 0.4, 0.3, 0.9, -0.5, -0.2, -0.1\rangle, \langle x_2, 0.5, 0.6, 0.7, -0.3, -0.4, -0.6\rangle\}$$

$$\mathcal{B}_2 = (G_B, E) = \{\langle x_1, 0.5, 0.4, 0.3, -0.6, -0.3, -0.4\rangle, \langle x_2, 0.6, 0.3, 0.2, -0.5, -0.3, -0.2\rangle\}$$

be two bipolar neutrosophic sets. Then the union is

$$\mathcal{B}_1 \cup \mathcal{B}_2 = (F_A \bigcup G_B, E) = \{\langle x_1, 0.5, 0.35, 0.3, -0.6, -0.25, -0.1\rangle, \langle x_2, 0.6, 0.45, 0.2, -0.5, -0.35, -0.2\rangle\}$$

the intersection is

$$\mathcal{B}_1 \cap \mathcal{B}_2 = (F_A \bigcap G_B, E) = \{\langle x_1, 0.4, 0.35, 0.9, -0.3, -0.25, -0.4\rangle, \langle x_2, 0.5, 0.45, 0.7, -0.3, -0.35, -0.6\rangle\}$$

3.1.3 The complement

The complement of a BNSS is

$$\mathcal{B}^c = (F_A, E)^c = (F^c_A, \neg E) = \left\langle w^+_{F_A(e)}(x), 1 - v^+_{F_A(e)}(x), u^+_{F_A(e)}(x), w^-_{F_A(e)}(x), -1 - v^-_{F_A(e)}(x), u^-_{F_A(e)}(x) \right\rangle$$

Example 3.6. Let $\mathcal{B}$ be a bipolar neutrosophic soft set.

$$\mathcal{B} = (F_A, E) = \{\langle x_1, 0.4, 0.3, 0.9, -0.5, -0.2, -0.1\rangle, \langle x_2, 0.5, 0.6, 0.7, -0.3, -0.4, -0.6\rangle\}$$

Then the complement is defined by

$$\mathcal{B}^c = (F_A, E)^c = \{\langle x_1, 0.9, 0.7, 0.4, -0.1, -0.8, -0.5\rangle, \langle x_2, 0.7, 0.4, 0.5, -0.6, -0.6, -0.3\rangle\}$$

3.1.4 Complete BNSS and null BNSS

The complete bipolar neutrosophic soft set $comp - \mathcal{B}$ is defined by
$comp - \mathcal{B} = \{e, \rangle x_i, 1, 0, 0, 0, -1, -1\rangle : e \in E; x \in X\}$

The null bipolar neutrosophic soft set is defined by
$null - \mathcal{B} = \{e, \rangle x_i, 0, 1, 1, -1, 0, 0\rangle : e \in E; x \in X\}$

The following propositions were given by Ali et al. for bipolar neutrosophic soft set associated with the whole parameter set. These propositions are also suitable for our approach.

Proposition 3.7. *Let X be a universe and E be a parameter set. Also, $A, B, C \in E$. Let $\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(E)\rangle : e \in E, F_A(E) \in BNS(X)\}$, $\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(E)\rangle : e \in E, G_B(E) \in BNS(X)\}$, $\mathcal{B}_3 = (H_C, E) = \{\langle e, H_C(E)\rangle : e \in E, H_C(E) \in BNS(X)\}$ be three bipolar neutrosophic soft sets over X. Then,*

1. $\mathcal{B}_1 \cup \mathcal{B}_2 = \mathcal{B}_2 \cup \mathcal{B}_1$
2. $\mathcal{B}_1 \cap \mathcal{B}_2 = \mathcal{B}_2 \cap \mathcal{B}_1$
3. $\mathcal{B}_1 \cup (\mathcal{B}_2 \cup \mathcal{B}_3) = (\mathcal{B}_1 \cup \mathcal{B}_2) \cup \mathcal{B}_2$
4. $\mathcal{B}_1 \cap (\mathcal{B}_2 \cap \mathcal{B}_3) = (\mathcal{B}_1 \cap \mathcal{B}_2) \cap \mathcal{B}_2$

Proof. This proof is obvious. □

Proposition 3.8. *Let X be a universe and E be a parameter set. Also, $A, B \in E$. Let $\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(E)\rangle : e \in E, F_A(E) \in BNS(X)\}$, $\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(E)\rangle : e \in E, G_B(E) \in BNS(X)\}$ be two bipolar neutrosophic soft sets over X. Then the following De Morgan's laws are valid.*

1. $(\mathcal{B}_1 \cup \mathcal{B}_2)^c = (\mathcal{B}_1)^c \cap (\mathcal{B}_1)^c$
2. $(\mathcal{B}_1 \cap \mathcal{B}_2)^c = (\mathcal{B}_1)^c \cup (\mathcal{B}_1)^c$

Proof. Let $\mathcal{B}_1 = \left\{e, \left\langle x, u^+_{F_{A(e)}}(x), v^+_{F_{A(e)}}(x), w^+_{F_{A(e)}}(x), u^-_{F_{A(e)}}(x), v^-_{F_{A(e)}}(x), w^-_{F_{A(e)}}(x)\right\rangle : e \in E\right\}$
$\mathcal{B}_2 = \left\{e, \left\langle x, u^+_{G_{B(e)}}(x), v^+_{G_{B(e)}}(x), w^+_{G_{B(e)}}(x), u^-_{G_{B(e)}}(x), v^-_{G_{B(e)}}(x), w^-_{G_{B(e)}}(x)\right\rangle : e \in E\right\}$
Then,

$$\begin{aligned}
(\mathcal{B}_1 \cup \mathcal{B}_2)^c = &\left\{e, \left\langle x, max(u^+_{F_{A(e)}}(x), u^+_{G_{B(e)}}(x)), min(v^+_{F_{A(e)}}(x), v^+_{G_{B(e)}}(x)), min(w^+_{F_{A(e)}}(x), w^+_{G_{B(e)}}(x)),\right.\right.\\
&\left.\left. min(u^-_{F_{A(e)}}(x), u^-_{G_{B(e)}}(x)), max(v^-_{F_{A(e)}}(x), v^-_{G_{B(e)}}(x)), max(w^-_{F_{A(e)}}(x), w^-_{G_{B(e)}}(x))\right\rangle : e \in E\right\}^c\\
= &\left\{e, \left\langle x, min(w^+_{F_{A(e)}}(x), w^+_{G_{B(e)}}(x)), 1 - min(v^+_{F_{A(e)}}(x), v^+_{G_{B(e)}}(x)), max(u^+_{F_{A(e)}}(x), u^+_{G_{B(e)}}(x)),\right.\right.\\
&\left.\left. max(w^-_{F_{A(e)}}(x), w^-_{G_{B(e)}}(x)), -1 - max(v^-_{F_{A(e)}}(x), v^-_{G_{B(e)}}(x)), min(u^-_{F_{A(e)}}(x), u^-_{G_{B(e)}}(x))\right\rangle : e \in E\right\}\\
= &\left\{e, \left\langle x, min(w^+_{F_{A(e)}}(x), w^+_{G_{B(e)}}(x)), max(1 - v^+_{F_{A(e)}}(x), 1 - v^+_{G_{B(e)}}(x)), max(u^+_{F_{A(e)}}(x), u^+_{G_{B(e)}}(x)),\right.\right.\\
&\left.\left. max(w^-_{F_{A(e)}}(x), w^-_{G_{B(e)}}(x)), min(-1 - v^-_{F_{A(e)}}(x), -1 - v^-_{G_{B(e)}}(x)), min(u^-_{F_{A(e)}}(x), u^-_{G_{B(e)}}(x))\right\rangle : e \in E\right\}\\
= &\left\{e, \left\langle x, w^+_{F_{A(e)}}(x), 1 - v^+_{F_{A(e)}}(x), u^+_{F_{A(e)}}(x), w^-_{F_{A(e)}}(x), -1 - v^-_{F_{A(e)}}(x), u^-_{F_{A(e)}}(x)\right\rangle : e \in E\right\}\\
&\cap \left\{e, \left\langle x, w^+_{G_{B(e)}}(x), 1 - v^+_{G_{B(e)}}(x), u^+_{G_{B(e)}}(x), w^-_{G_{B(e)}}(x), -1 - v^-_{G_{B(e)}}(x), u^-_{G_{B(e)}}(x)\right\rangle : e \in E\right\}\\
&= (\mathcal{B}_1)^c \cap (\mathcal{B}_2)^c
\end{aligned}$$

$$(\mathcal{B}_1 \cap \mathcal{B}_2)^c = \Big\{e, \Big\langle x, min(u^+_{F_A(e)}(x), u^+_{G_B(e)}(x)), max(v^+_{F_A(e)}(x), v^+_{G_B(e)}(x)), max(w^+_{F_A(e)}(x), w^+_{G_B(e)}(x)),$$
$$max(u^-_{F_A(e)}(x), u^-_{G_B(e)}(x)), min(v^-_{F_A(e)}(x), v^-_{G_B(e)}(x)), min(w^-_{F_A(e)}(x), w^-_{G_B(e)}(x)) \Big\rangle : e \in E\Big\}^c$$
$$= \Big\{e, \Big\langle x, max(w^+_{F_A(e)}(x), w^+_{G_B(e)}(x)), 1 - max(v^+_{F_A(e)}(x), v^+_{G_B(e)}(x)), min(u^+_{F_A(e)}(x), u^+_{G_B(e)}(x)),$$
$$min(w^-_{F_A(e)}(x), w^-_{G_B(e)}(x)), -1 - min(v^-_{F_A(e)}(x), v^-_{G_B(e)}(x)), max(u^-_{F_A(e)}(x), u^-_{G_B(e)}(x)) \Big\rangle : e \in E\Big\}$$
$$= \Big\{e, \Big\langle x, max(w^+_{F_A(e)}(x), w^+_{G_B(e)}(x)), min(1 - v^+_{F_A(e)}(x), 1 - v^+_{G_B(e)}(x)), min(u^+_{F_A(e)}(x), u^+_{G_B(e)}(x)),$$
$$min(w^-_{F_A(e)}(x), w^-_{G_B(e)}(x)), max(-1 - v^-_{F_A(e)}(x), -1 - v^-_{G_B(e)}(x)), max(u^-_{F_A(e)}(x), u^-_{G_B(e)}(x)) \Big\rangle : e \in E\Big\}$$
$$= \Big\{e, \Big\langle x, w^+_{F_A(e)}(x), 1 - v^+_{F_A(e)}(x), u^+_{F_A(e)}(x), w^-_{F_A(e)}(x), -1 - v^-_{F_A(e)}(x), u^-_{F_A(e)}(x) \Big\rangle : e \in E\Big\}$$
$$\cup \Big\{e, \Big\langle x, w^+_{G_B(e)}(x), 1 - v^+_{G_B(e)}(x), u^+_{G_B(e)}(x), w^-_{G_B(e)}(x), -1 - v^-_{G_B(e)}(x), u^-_{G_B(e)}(x) \Big\rangle : e \in E\Big\}$$
$$= (\mathcal{B}_1)^c \cup (\mathcal{B}_2)^c$$

□

Proposition 3.9. *Let X be a universe and E be a parameter set. Also, $A, B, C \in E$. Let $\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(E)\rangle : e \in E, F_A(E) \in BNS(X)\}$, $\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(E)\rangle : e \in E, G_B(E) \in BNS(X)\}$, $\mathcal{B}_3 = (H_C, E) = \{\langle e, H_C(E)\rangle : e \in E, H_C(E) \in BNS(X)\}$ be three bipolar neutrosophic soft sets over X. Then,*

1. $\mathcal{B}_1 \cap (\mathcal{B}_2 \cup \mathcal{B}_3) = (\mathcal{B}_1 \cap \mathcal{B}_2) \cup (\mathcal{B}_1 \cap \mathcal{B}_3)$
2. $\mathcal{B}_1 \cup (\mathcal{B}_2 \cap \mathcal{B}_3) = (\mathcal{B}_1 \cup \mathcal{B}_2) \cap (\mathcal{B}_1 \cup \mathcal{B}_3)$

Proof. This proof is obvious.

□

4 Entropy measure of bipolar neutrosophic soft sets

Generally Entropy measures are used to calculate indeterminacy of sets. In this section, we define entropy measurement for bipolar neutrosophic soft sets.

Definition 4.1. Let $X = \{x_1, x_2, \ldots, x_m\}$ be a universe of discourse set and $E = \{e_1, e_2, \ldots, e_n\}$ be subset of a parameter set A. Let $\mathcal{B}_1 = (F_A, E)$ and $\mathcal{B}_2 = (G_A, E)$ be two bipolar neutrosophic soft sets. The mapping $\mathcal{E} : BNSS(X) \to \mathbb{R}^+ \cup \{0\}$ is called an entropy on bipolar neutrosophic soft sets if $\mathcal{E}$ satisfies the following conditions.

1. $\mathcal{E}(B) = 0$ if and only if $B \in IFSS(X)$ (Intiutionistic fuzzy soft set)

2. $\mathcal{E}(B)$ is maximum if and only if $u^{+}_{F_A(e)}(x) = v^{+}_{F_A(e)}(x) = w^{+}_{F_A(e)}(x)$ and $u^{-}_{F_A(e)}(x) = v^{-}_{F_A(e)}(x) = w^{-}_{F_A(e)}(x)$ for all $e \in E$ and $x \in X$

3. $\mathcal{E}(B) = \mathcal{E}(B^c)$ for all $B \in BNSS(X)$

4. $\mathcal{E}(B_1) \leq \mathcal{E}(B_2)$ if $B_2 \subseteq B_1$.

Definition 4.2. Let $\mathcal{B}$ be a bipolar neutrosophic soft set. Then, entropy of $\mathcal{B}$ is denoted by $\mathcal{E}(\mathcal{B})$ and defined as follows:

$$\mathcal{E}(\mathcal{B}) = 1 - \frac{1}{2mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\Bigg[\Big(u^{+}_{\mathcal{B}(e_j)}(x_i) + w^{+}_{\mathcal{B}(e_j)}(x_i)\Big)\cdot\Big|v^{+}_{\mathcal{B}(e_j)}(x_i) - v^{+}_{\mathcal{B}^c(e_j)}(x_i)\Big| - \Big(u^{-}_{\mathcal{B}(e_j)}(x_i) + w^{-}_{\mathcal{B}(e_j)}(x_i)\Big)\cdot\Big|v^{-}_{\mathcal{B}(e_j)}(x_i) - v^{-}_{\mathcal{B}^c(e_j)}(x_i)\Big|\Bigg]$$

Example 4.3. Let $X = \{x_1, x_2, x_3, x_4\}$ be a universal set and let $E = \{e_1, e_2, e_3\}$ be the parameter set. Let $A = \{e_1, e_2\}$ be a subset of E.

1. Define $\mathcal{B}_1 = (F_A, E) = \{\langle e_1, F_A(e_1)\rangle, \langle e_2, F_A(e_2)\rangle\}$
   where,

$$F_A(e_1) = \Big\{\langle x_1, 0.6, 0, 0.4, -0.3, 0, -0.7\rangle, \langle x_2, 0.3, 0, 0.7, -0.2, 0, -0.8\rangle, \langle x_3, 0.4, 0, 0.6, -0.6, 0, -0.4\rangle, \langle x_4, 0.1, 0, 0.9, -0.5, 0, -0.5\rangle\Big\}$$

$$F_A(e_2) = \Big\{\langle x_1, 0.5, 0, 0.5, -0.4, 0, -0.6\rangle, \langle x_2, 0.2, 0, 0.8, -0.1, 0, -0.9\rangle, \langle x_3, 0.3, 0, 0.7, -0.7, 0, -0.3\rangle, \langle x_4, 0.8, 0, 0.2, -0.4, 0, -0.6\rangle\Big\}$$

   Since all the indeterminacy degrees are zero, $\mathcal{B}_1$ becomes intituitionistic fuzzy soft set(IFSS).
   By Definition 4.2, $\mathcal{E}(\mathcal{B}_1) = 0$

2. Define $\mathcal{B}_2 = (F_A, E) = \{\langle e_1, F_A(e_1)\rangle, \langle e_2, F_A(e_2)\rangle\}$ where,

$$F_A(e_1) = \Big\{\langle x_1, 0.5, 0.5, 0.5, -0.9, -0.9, -0.9\rangle, \langle x_2, 0.3, 0.3, 0.3, -0.8, -0.8, -0.8\rangle, \langle x_3, 0.4, 0.4, 0.4, -0.5, -0.5, -0.5\rangle, \langle x_4, 0.5, 0.5, 0.5, -0.5, -0.5, -0.5\rangle\Big\}$$

$$F_A(e_2) = \Big\{ \langle x_1, 0.4, 0.4, 0.4, -0.4, -0.4, -0.4 \rangle, \langle x_2, 0.5, 0.5, 0.5, -0.1, -0.1, -0.1 \rangle, \langle x_3, 0.3, 0.3, 0.3, -0.5, -0.5, -0.5 \rangle, \langle x_4, 0.8, 0.8, 0.8, -0.2, -0.2, -0.2 \rangle \Big\}$$

Since truth-membership, indeterminacy and falsity-membership degrees are equal,
By Definition 4.2, $\mathcal{E}(\mathcal{B}_1) = 1$ (i.e maximum).

3. Define $\mathcal{B}_3 = (F_A, E) = \{\langle e_1, F_A(e_1)\rangle, \langle e_2, F_A(e_2)\rangle\}$ where,

$$F_A(e_1) = \Big\{ \langle x_1, 0.5, 0.4, 0.7, -0.2, -0.5, -0.7 \rangle, \langle x_2, 0.4, 0.7, 0.3, -0.6, -0.2, -0.1 \rangle, \langle x_3, 0.4, 0.6, 0.2, -0.5, -0.3, -0.7 \rangle, \langle x_4, 0.6, 0.3, 0.2, -0.7, -0.5, -0.3 \rangle \Big\}$$

$$F_A(e_2) = \Big\{ \langle x_1, 0.6, 0.3, 0.7, -0.4, -0.2, -0.4 \rangle, \langle x_2, 0.4, 0.7, 0.3, -0.7, -0.3, -0.4 \rangle, \langle x_3, 0.3, 0.5, 0.1, -0.5, -0.7, -0.3 \rangle, \langle x_4, 0.8, 0.3, 0.1, -0.5, -0.2, -0.4 \rangle \Big\}$$

Then,
$(\mathcal{B}_3)^c = (F_A^c, \neg E) = \{\langle e_1, F_A^c(e_1)\rangle, \langle e_2, F_A^c(e_2)\rangle\}$

where,

$$F_A^c(e_1) = \Big\{ \langle x_1, 0.7, 0.6, 0.5, -0.7, -0.5, -0.2 \rangle, \langle x_2, 0.3, 0.3, 0.4, -0.1, -0.8, -0.6 \rangle, \langle x_3, 0.2, 0.4, 0.4, -0.7, -0.7, -0.5 \rangle, \langle x_4, 0.2, 0.7, 0.6, -0.3, -0.5, -0.7 \rangle \Big\}$$

$$F_A^c(e_2) = \Big\{ \langle x_1, 0.7, 0.7, 0.6, -0.4, -0.8, -0.4 \rangle, \langle x_2, 0.3, 0.3, 0.4, -0.4, -0.7, -0.7 \rangle, \langle x_3, 0.1, 0.5, 0.7, -0.3, -0.3, -0.5 \rangle, \langle x_4, 0.1, 0.7, 0.8, -0.4, -0.8, -0.5 \rangle \Big\}$$

Since the sum of indeterminacy and its complement is one and complement of truth-membership becomes falsify-membership and vice versa,
By Definition 4.2, $\mathcal{E}(B) = \mathcal{E}(B^c)$ for any BNSS.

4. Let $\mathcal{B}_1 = (F_A, E) = \{\langle e, F_A(e)\rangle : e \in E\}$ and $\mathcal{B}_2 = (G_B, E) = \{\langle e, G_B(e)\rangle : e \in E\}$
Here,

$$F_A(e_2) = \Big\{ \langle x_1, 0.4, 0.3, 0.9, -0.2, -0.3, -0.4 \rangle, \ \langle x_2, 0.5, 0.6, 0.7, -0.3, -0.4, -0.6 \rangle \Big\}$$

$$G_B(e_1) = \left\{ \langle x_1, 0.5, 0.4, 0.3, -0.6, -0.2, -0.4 \rangle \, , \, \langle x_2, 0.6, 0.3, 0.2, -0.5, -0.3, -0.2 \rangle \right\}$$
$$G_B(e_2) = \left\{ \langle x_1, 0.6, 0.4, 0.2, -0.5, -0.1, -0.1 \rangle \, , \, \langle x_2, 0.7, 0.6, 0.3, -0.4, -0.2, -0.3 \rangle \right\}$$

Here $\mathcal{B}_1 \subseteq \mathcal{B}_2$.
By Definition 4.2,

$$\mathcal{E}(\mathcal{B}_1) = 0.705$$
$$\mathcal{E}(\mathcal{B}_2) = 0.6725$$

Hence

$$\mathcal{E}(\mathcal{B}_2) \leq \mathcal{E}(\mathcal{B}_1) \text{ if } \mathcal{B}_1 \subseteq \mathcal{B}_2$$

5 Distance between bipolar neutrosophic soft sets

In this section, we will define some distance measures of bipolar neutrosophic soft sets. Let X be a universe, E be a parameter set and let A, B be two subsets of E.
Let $\mathcal{B}_1 = (F_A, E)$ and $\mathcal{B}_2 = (G_B, E)$ be two bipolar neutrosophic soft sets.

Here
$$F_A(e) = \left\{ \left\langle x, u^+_{F_A(e)}(x), v^+_{F_A(e)}(x), w^+_{F_A(e)}(x), u^-_{F_A(e)}(x), v^-_{F_A(e)}(x), w^-_{F_A(e)}(x) \right\rangle : x \in X \right\}$$
$$G_B(e) = \left\{ \left\langle x, u^+_{G_B(e)}(x), v^+_{G_B(e)}(x), w^+_{G_B(e)}(x), u^-_{G_B(e)}(x), v^-_{G_B(e)}(x), w^-_{G_B(e)}(x) \right\rangle : x \in X \right\}$$

Definition 5.1. Consider the two Bipolar neutrosophic soft sets $\mathcal{B}_1 = (F_A, E)$ and $\mathcal{B}_2 = (G_B, E)$ defined above. Let d be a mapping defined as $d : BNSS(x) \times BNSS(x) \to \mathbb{R}^+ \cup \{0\}$ and it satisfies the following conditions.
$i)$ $d(\mathcal{B}_1, \mathcal{B}_2) \geq 0$
$ii)$ $d(\mathcal{B}_1, \mathcal{B}_2) = d(\mathcal{B}_2, \mathcal{B}_1)$
$iii)$ $d(\mathcal{B}_1, \mathcal{B}_2) = 0 iff \mathcal{B}_1 = \mathcal{B}_2$
$iv)$ $d(\mathcal{B}_1, \mathcal{B}_2) + d(\mathcal{B}_2, \mathcal{B}_3) \geq d(\mathcal{B}_1, \mathcal{B}_3)$ $(for\ any\ \mathcal{B}_3)$

Then, $d(\mathcal{B}_1, \mathcal{B}_2)$ is called a distance measure between two bipolar neutrosopihic soft sets $\mathcal{B}_1$ and $\mathcal{B}_2$.

Definition 5.2. A real function $\mathcal{S} : BNSS(X) \times BNSS(X) \to [0, 1]$ is called a similarity measure between two bipolar neutrosophic soft sets $\mathcal{B}_1 = [a_{ij}]_{m\times n}\ and\ \mathcal{B}_2 = [b_{ij}]_{m\times n}$ if $\mathcal{S}$ satisfies the following conditions.
$i)\mathcal{S}(\mathcal{B}_1, \mathcal{B}_2) \in [0, 1]$
$ii)\mathcal{S}(\mathcal{B}_1, \mathcal{B}_2) = \mathcal{S}(\mathcal{B}_2, \mathcal{B}_1)$
$iii)\mathcal{S}(\mathcal{B}_1, \mathcal{B}_2) = 1 iff [a_{ij}]_{m\times n} = [b_{ij}]_{m\times n}$
$iv)\mathcal{S}(\mathcal{B}_1, \mathcal{B}_3) \leq \mathcal{S}(\mathcal{B}_1, \mathcal{B}_2) + \mathcal{S}(\mathcal{B}_2, \mathcal{B}_3)\ if\ \mathcal{B}_1 \subseteq \mathcal{B}_2 \subseteq \mathcal{B}_3\ (for\ any\ \mathcal{B}_3)$

5.1 Hamming distance between two bipolar neutrosophic soft sets

$$d^{H}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{|\Delta_{ij}u(x)|+|\nabla_{ij}u(x)|+|\Delta_{ij}v(x)|+|\nabla_{ij}v(x)|+|\Delta_{ij}w(x)|+|\nabla_{ij}w(x)|}{6}.$$

where

$$\Delta_{ij}u(x)=u^{+}_{\mathcal{B}_1(e_j)}(x_i)-u^{+}_{\mathcal{B}_2(e_j)}(x_i)$$
$$\nabla_{ij}u(x)=u^{-}_{\mathcal{B}_1(e_j)}(x_i)-u^{-}_{\mathcal{B}_2(e_j)}(x_i)$$

Proof. i) Since $|\Delta_{ij}u(x)|, |\nabla_{ij}u(x)|, |\Delta_{ij}v(x)|, |\nabla_{ij}v(x)|, |\Delta_{ij}w(x)|, |\nabla_{ij}w(x)|$ are all positive, $d^{H}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)\geq 0$
ii) Since $\left|u^{+}_{\mathcal{B}_1(e_j)}(x_i)-u^{+}_{\mathcal{B}_2(e_j)}(x_i)\right|=\left|u^{+}_{\mathcal{B}_2(e_j)}(x_i)-u^{+}_{\mathcal{B}_1(e_j)}(x_i)\right|$,
$|\Delta_{ij}u(X)|$ is same for both $d^{H}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)$ and $d^{H}_{BNSS}(\mathcal{B}_2,\mathcal{B}_1)$.
Also this is true for all membership degrees.
Hence $d^{H}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)=d^{H}_{BNSS}(\mathcal{B}_2,\mathcal{B}_1)$
iii) Since $\Delta_{ij}u(X)=u^{+}_{\mathcal{B}_1(e_j)}(x_i)-u^{+}_{\mathcal{B}_2(e_j)}(x_i)$ and $\nabla_{ij}u(X)=u^{-}_{\mathcal{B}_1(e_j)}(x_i)-u^{-}_{\mathcal{B}_2(e_j)}(x_i)=0$ are both zero for $\mathcal{B}_1=\mathcal{B}_2$,
$d^{H}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)=0$ if $\mathcal{B}_1=\mathcal{B}_2$.
iv) Let

$$d^{H}_{BNSS}(B_1,B_2)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{|\Delta_{ij}u_1(x)|+|\nabla_{ij}u_1(x)|+|\Delta_{ij}v_1(x)|+|\nabla_{ij}v_1(x)|+|\Delta_{ij}w_1(x)|+|\nabla_{ij}w_1(x)|}{6}$$
$$d^{H}_{BNSS}(B_2,B_3)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{|\Delta_{ij}u_2(x)|+|\nabla_{ij}u_2(x)|+|\Delta_{ij}v_2(x)|+|\nabla_{ij}v_2(x)|+|\Delta_{ij}w_2(x)|+|\nabla_{ij}w_2(x)|}{6}$$

$$d^{H}_{BNSS}(B_1,B_2)+d^{H}_{BNSS}(B_2,B_3)$$
$$=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left|u^{+}_{\mathcal{B}_1(e_j)}(x_i)-u^{+}_{\mathcal{B}_2(e_j)}(x_i)\right|+\left|u^{+}_{\mathcal{B}_2(e_j)}(x_i)-u^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|u^{-}_{\mathcal{B}_1(e_j)}(x_i)-u^{-}_{\mathcal{B}_2(e_j)}(x_i)\right|+}{6}$$
$$\frac{\left|u^{-}_{\mathcal{B}_2(e_j)}(x_i)-u^{-}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|v^{+}_{\mathcal{B}_1(e_j)}(x_i)-v^{+}_{\mathcal{B}_2(e_j)}(x_i)\right|+\left|v^{+}_{\mathcal{B}_2(e_j)}(x_i)-v^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|+}{}$$
$$\frac{\left|v^{-}_{\mathcal{B}_1(e_j)}(x_i)-v^{-}_{\mathcal{B}_2(e_j)}(x_i)\right|+\left|v^{-}_{\mathcal{B}_2(e_j)}(x_i)-v^{-}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|w^{+}_{\mathcal{B}_1(e_j)}(x_i)-w^{+}_{\mathcal{B}_2(e_j)}(x_i)\right|+}{}$$
$$\frac{\left|w^{+}_{\mathcal{B}_2(e_j)}(x_i)-w^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|w^{-}_{\mathcal{B}_1(e_j)}(x_i)-w^{-}_{\mathcal{B}_2(e_j)}(x_i)\right|+\left|w^{-}_{\mathcal{B}_2(e_j)}(x_i)-w^{-}_{\mathcal{B}_3(e_j)}(x_i)\right|}{}$$
$$\geq\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left|u^{+}_{\mathcal{B}_1(e_j)}(x_i)-u^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|v^{+}_{\mathcal{B}_1(e_j)}(x_i)-v^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|+\left|w^{+}_{\mathcal{B}_1(e_j)}(x_i)-w^{+}_{\mathcal{B}_3(e_j)}(x_i)\right|}{6}$$

This implies

$$d^H_{BNSS}(B_1, B_2) + d^H_{BNSS}(B_2, B_3) \geq d^H_{BNSS}(B_1, B_3)$$

□

5.2 Normalized Hamming distance

$$d^{nH}_{BNSS}(B_1, B_2) = \frac{d^H_{BNSS}(B_1, B_2)}{mn}$$

Proof. Since $d^H_{BNSS}(B_1, B_2)$ satisfies definition 5.1, for any positive m, n

$$d^{nH}_{BNSS}(B_1, B_2) = \frac{d^H_{BNSS}(B_1, B_2)}{mn}$$

also satisfies definition 5.1 □

5.3 Euclidean distance between two BNSS

$$d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2) = \left[\sum_{j=1}^{n}\sum_{i=1}^{m} \frac{(\Delta_{ij}u(x))^2 + (\nabla_{ij}u(x))^2 + (\Delta_{ij}v(x))^2 + (\nabla_{ij}v(x))^2 + (\Delta_{ij}w(x))^2 + (\nabla_{ij}w(x))^2}{6}\right]^{\frac{1}{2}}$$

where

$$\Delta_{ij}u(x) = u^+_{\mathcal{B}_1(e_j)}(x_i) - u^+_{\mathcal{B}_2(e_j)}(x_i)$$
$$\nabla_{ij}u(x) = u^-_{\mathcal{B}_1(e_j)}(x_i) - u^-_{\mathcal{B}_2(e_j)}(x_i)$$

Proof. i) Since $(\Delta_{ij}u(x))^2, (\nabla_{ij}u(x))^2, (\Delta_{ij}v(x))^2, (\nabla_{ij}v(x))^2, (\Delta_{ij}w(x))^2, (\nabla_{ij}w(x))^2$ are all positive, $d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2) \geq 0$

ii) Since $(u^+_{\mathcal{B}_1(e_j)}(x_i) - u^+_{\mathcal{B}_2(e_j)}(x_i))^2 = (u^+_{\mathcal{B}_2(e_j)}(x_i) - u^+_{\mathcal{B}_1(e_j)}(x_i))^2$, $(\Delta_{ij}u(X))^2$ is same for both $d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2)$ and $d^E_{BNSS}(\mathcal{B}_2, \mathcal{B}_1)$.
Also this is true for all membership degrees.
Hence $d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2) = d^E_{BNSS}(\mathcal{B}_2, \mathcal{B}_1)$

iii) Since $\Delta_{ij}u(X) = u^+_{\mathcal{B}_1(e_j)}(x_i) - u^+_{\mathcal{B}_2(e_j)}(x_i)$ and $\nabla_{ij}u(X) = u^-_{\mathcal{B}_1(e_j)}(x_i) - u^-_{\mathcal{B}_2(e_j)}(x_i) = 0$ are both zero for $\mathcal{B}_1 = \mathcal{B}_2$,
$d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2) = 0$ if $\mathcal{B}_1 = \mathcal{B}_2$.

iv) Let

$$d^E_{BNSS}(\mathcal{B}_1, \mathcal{B}_2) = \left[\sum_{j=1}^{n}\sum_{i=1}^{m} \frac{(\Delta_{ij}u_1(x))^2 + (\nabla_{ij}u_1(x))^2 + (\Delta_{ij}v_1(x))^2 + (\nabla_{ij}v_1(x))^2 + (\Delta_{ij}w_1(x))^2 + (\nabla_{ij}w_1(x))^2}{6}\right]^{\frac{1}{2}}$$

$$d^E_{BNSS}(\mathcal{B}_2, \mathcal{B}_3) = \left[\sum_{j=1}^{n}\sum_{i=1}^{m} \frac{(\Delta_{ij}u_2(x))^2 + (\nabla_{ij}u_2(x))^2 + (\Delta_{ij}v_2(x))^2 + (\nabla_{ij}v_2(x))^2 + (\Delta_{ij}w_2(x))^2 + (\nabla_{ij}w_2(x))^2}{6}\right]^{\frac{1}{2}}$$

By the definition of Euclidean norm, we take
$d^{E}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2) = \|\mathcal{B}_1 - \mathcal{B}_2\|_2$
$d^{E}_{BNSS}(\mathcal{B}_2,\mathcal{B}_3) = \|\mathcal{B}_2 - \mathcal{B}_3\|_2$
Then, $\|\mathcal{B}_1 - \mathcal{B}_3\|_2 = \|\mathcal{B}_1 - \mathcal{B}_2 + \mathcal{B}_2 - \mathcal{B}_3\|_2$
By Triangle inequality,
$\|\mathcal{B}_1 - \mathcal{B}_3\|_2 \leq \|\mathcal{B}_1 - \mathcal{B}_2\|_2 + \|\mathcal{B}_2 - \mathcal{B}_3\|_2$
Hence $d^{E}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2) + d^{E}_{BNSS}(\mathcal{B}_2,\mathcal{B}_3) \geq d^{E}_{BNSS}(\mathcal{B}_1,\mathcal{B}_3)$

□

5.4 Normalized Euclidean distance

$$d^{nE}_{BNSS}(B_1,B_2) = \frac{d^{E}_{BNSS}(B_1,B_2)}{\sqrt{mn}}$$

Proof. Since, $d^{E}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)$ satisfies Definition 5.1,

$$d^{nE}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2) = \frac{d^{E}_{BNSS}(\mathcal{B}_1,\mathcal{B}_2)}{\sqrt{mn}}$$

also satisfies Definition 5.1 for all m, n. □

Note 5.3. From the above measurements, we conclude the following conditions.

$i)$ $0 \leq d^{H}_{BNSS}(B_1,B_2) \leq mn$ [Obviously true]
$ii)$ $0 \leq d^{nH}_{BNSS}(B_1,B_2) \leq 1$ [from *i)*]
$iii)$ $0 \leq d^{E}_{BNSS}(B_1,B_2) \leq \sqrt{mn}$ [Obvious from *i)*]
$iv)$ $0 \leq d^{nE}_{BNSS}(B_1,B_2) \leq 1$ [from *iii)*]

Based on these distance measures, we can calculate the similarity between two BNSSs using the following measures.
$i)$ $\mathcal{S}^{H}_{BNSS}(B_1,B_2) = \dfrac{1}{1 + d^{H}_{BNSS}(B_1,B_2)}$
$ii)$ $\mathcal{S}^{E}_{BNSS}(B_1,B_2) = \dfrac{1}{1 + d^{E}_{BNSS}(B_1,B_2)}$
$iii)$ $\mathcal{S}^{n}H_{BNSS}(B_1,B_2) = \dfrac{1}{1 + d^{n}H_{BNSS}(B_1,B_2)}$
$iv)$ $\mathcal{S}^{n}E_{BNSS}(B_1,B_2) = \dfrac{1}{1 + d^{n}E_{BNSS}(B_1,B_2)}$

6 Representation of image in bipolar neutrosophic soft Domain

In this section, we convert 2-dimensional digital image into bipolar neutrosophic set. A digital image contains many pixels. According to pixel intensity values, we classified digital image as foreground image and background image.

we define bipolar neutrosophic soft set as parameterization of family of subsets which contains positive mebership degrees and negative membership degrees. Here we assign positive membership degrees to foreground image and negative membership degree to background image.

For example, Let us consider a 2-dimensional digital image as $X = \{x_1, x_2, x_3, y_1, y_2, y_3\}$. Here x_1, x_2, x_3 represents foreground pixels and y_1, y_2, y_3 represents background pixels. Let $A = \{e_1, e_2, e_3\}$ be set of parameters, where e_1, e_2, e_3 denotes contrast, brightness and sharpness of given image respectively.

Define $\mathcal{B} = (F_A, E) = \langle e, F_A(e)\rangle : e \in E, F_A(e) \in BNS(X)$

Here

$$F_A(e_1) = \Bigg\{ \Big\langle x_1, u^+_{F_A(e_1)}(x_1), v^+_{F_A(e_1)}(x_1), w^+_{F_A(e_1)}(x_1), u^-_{F_A(e_1)}(x_1), v^-_{F_A(e_1)}(x_1), w^-_{F_A(e_1)}(x_1) \Big\rangle, \\ \Big\langle x_2, u^+_{F_A(e_1)}(x_2), v^+_{F_A(e_1)}(x_2), w^+_{F_A(e_1)}(x_2), u^-_{F_A(e_1)}(x_2), v^-_{F_A(e_1)}(x_2), w^-_{F_A(e_1)}(x_2) \Big\rangle, \\ \Big\langle x_3, u^+_{F_A(e_1)}(x_3), v^+_{F_A(e_1)}(x_3), w^+_{F_A(e_1)}(x_3), u^-_{F_A(e_1)}(x_3), v^-_{F_A(e_1)}(x_3), w^-_{F_A(e_1)}(x_3) \Big\rangle \Bigg\}$$

$$F_A(e_2) = \Bigg\{ \Big\langle x_1, u^+_{F_A(e_2)}(x_1), v^+_{F_A(e_2)}(x_1), w^+_{F_A(e_2)}(x_1), u^-_{F_A(e_2)}(x_1), v^-_{F_A(e_2)}(x_1), w^-_{F_A(e_2)}(x_1) \Big\rangle, \\ \Big\langle x_2, u^+_{F_A(e_2)}(x_2), v^+_{F_A(e_2)}(x_2), w^+_{F_A(e_2)}(x_2), u^-_{F_A(e_2)}(x_2), v^-_{F_A(e_2)}(x_2), w^-_{F_A(e_2)}(x_2) \Big\rangle, \\ \Big\langle x_3, u^+_{F_A(e_2)}(x_3), v^+_{F_A(e_2)}(x_3), w^+_{F_A(e_2)}(x_3), u^-_{F_A(e_2)}(x_3), v^-_{F_A(e_2)}(x_3), w^-_{F_A(e_2)}(x_3) \Big\rangle \Bigg\}$$

$$F_A(e_3) = \Bigg\{ \Big\langle x_1, u^+_{F_A(e_3)}(x_1), v^+_{F_A(e_3)}(x_1), w^+_{F_A(e_3)}(x_1), u^-_{F_A(e_3)}(x_1), v^-_{F_A(e_3)}(x_1), w^-_{F_A(e_3)}(x_1) \Big\rangle, \\ \Big\langle x_2, u^+_{F_A(e_3)}(x_2), v^+_{F_A(e_3)}(x_2), w^+_{F_A(e_3)}(x_2), u^-_{F_A(e_3)}(x_2), v^-_{F_A(e_3)}(x_2), w^-_{F_A(e_3)}(x_2) \Big\rangle, \\ \Big\langle x_3, u^+_{F_A(e_3)}(x_3), v^+_{F_A(e_3)}(x_3), w^+_{F_A(e_3)}(x_3), u^-_{F_A(e_3)}(x_3), v^-_{F_A(e_3)}(x_3), w^-_{F_A(e_3)}(x_3) \Big\rangle \Bigg\}$$

where $u^+_{F_A(e)}(x), v^+_{F_A(e)}(x), w^+_{F_A(e)}(x)$ represents positive truth-membership degree , positive indeterminacy-membership degree and positive falsity-membership degree of a pixel x which holds the parametrer e, and similarly $u^-_{F_A(e)}(x), v^-_{F_A(e)}(x), w^-_{F_A(e)}(x)$ represents negative truth-membership degree , negative indeterminacy-membership degree and negative falsity-membership degree of a pixel x which holds the parameter e .

Remark 6.1. We assume the pixels are already classified as foreground and background pixels based on their intensity values. This assumption leads us to the following conditions.

For absolute foreground pixels,
$u^+(x) = [0, 1] \qquad u^-(x) = 0$
$v^+(x) = [0, 1] \qquad v^-(x) = -1$
$w^+(x) = [0, 1] \qquad w^-(x) = -1$
For absolute background pixels,

$u^+(x) = 0 \qquad u^-(x) = [-1, 0]$
$v^+(x) = 1 \qquad v^-(x) = [-1, 0]$
$w^+(x) = 1 \qquad w^-(x) = [-1, 0]$

6.1 Pixels in BNSS domain

Digital images are just array of pixels; each and every pixel has particular intensity values. Initially, Yanhui et al.,[8, 17] proposed the technique to transform image into neutrosophic domain. In this subsection, we extend this technique to bipolar neutrosophic domain.

We allocate membership values for each pixel according to their attributes. For foreground pixels $u^+(i, j), v^+(i, j), w^+(i, j)$ named as positive truth-membership, positive indeterminacy, positive falsity-membership respectively and for background pixels $u^-(i, j), v^-(i, j), w^-(i, j)$ named as negative truth-membership, negative indeterminacy, negative falsity-membership respectively.

An arbitrary pixel can be represented as follows:
$P_{BNS}(i, j) = \{u^+(i, j), v^+(i, j), w^+(i, j), u^-(i, j), v^-(i, j), w^-(i, j)\}$.
Here

$$u^+(i, j) = \frac{\bar{g}(i, j) - \bar{g}_{min}}{\bar{g}_{max} - \bar{g}_{min}} \qquad v^+(i, j) = \frac{\delta(i, j) - \delta_{min}}{\delta_{max} - \delta_{min}}$$
$$w^+(i, j) = 1 - u^+(i, j) = \frac{\bar{g}_{max} - \bar{g}(i, j)}{\bar{g}_{max} - \bar{g}_{min}}$$
$$u^-(i, j) = \frac{\hat{g}_{min} - \hat{g}(i, j)}{\hat{g}_{max} - \hat{g}_{min}} \qquad v^-(i, j) = \frac{\delta_{min} - \delta(i, j)}{\delta_{max} - \delta_{min}}$$
$$w^-(i, j) = -1 - u^-(i, j) = \frac{\hat{g}(i, j) - \hat{g}_{max}}{\hat{g}_{max} - \hat{g}_{min}}$$

where $\bar{g}(i, j)$ represents mean intensity of foreground pixel in some neighbourhoods W and $\hat{g}(i, j)$ represents the mean intensity of background pixel in some neighbourhoods W^*.
Here

$$\bar{g}(i, j) = \frac{1}{W \times W} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} g(m, n)$$
$$\hat{g}(i, j) = \frac{1}{W^* \times W^*} \sum_{m=i-w^*/2}^{i+w^*/2} \sum_{n=j-w^*/2}^{j+w^*/2} g(m, n)$$
$$\delta(i, j) = |g(i, j) - \bar{g}(i, j)|$$
$$\delta(i, j) = |g(i, j) - \hat{g}(i, j)|$$
$$\delta_{max} = max\delta(i, j) \qquad \delta_{min} = min\delta(i, j)$$

Example 6.2. Let $X = \{f_1, f_2, b_1, b_2\}$ be pixel set of a 2-D image. Also let $E = \{e_1, e_2, e_3\}$ be the subset of the parameter set A with parameters e_1, e_2, e_3 as contrast, brightness and sharpness, respectively.

Now we define
$(F_A, E) = \{\langle e, F_A(e)\rangle : e \in E, F_A(e) \in BNS(X)\}$.
Here

$$F(e_1) = \Big\{ \langle f_1, 0.5, 0.4, 0.3, 0, -1, -1\rangle, \langle f_2, 0.4, 0.7, 0.6, 0, -1, -1\rangle, \langle f_3, 0.4, 0.3, 0.5, 0, -1, -1\rangle, \langle b_1, 0, 1, 1, -0.6, -0.2, -0.3\rangle, \langle b_2, 0, 1, 1, -0.7, -0.1, -0.3\rangle, \langle b_3, 0, 1, 1, -0.4, -0.2, -0.3\rangle \Big\}$$

$$F(e_2) = \Big\{ \langle f_1, 0.6, 0.3, 0.2, 0, -1, -1\rangle, \langle f_2, 0.5, 0.2, 0.3, 0, -1, -1\rangle, \langle f_3, 0.3, 0.4, 0.2, 0, -1, -1\rangle, \langle b_1, 0, 1, 1, -0.4, -0.5, -0.1\rangle, \langle b_2, 0, 1, 1, -0.6, -0.2, -0.3\rangle, \langle b_3, 0, 1, 1, -0.4, -0.5, -0.1\rangle \Big\}$$

$$F(e_3) = \Big\{ \langle f_1, 0.6, 0.3, 0.4, 0, -1, -1\rangle, \langle f_2, 0.4, 0.5, 0.1, 0, -1, -1\rangle, \langle f_3, 0.2, 0.3, 0.1, 0, -1, -1\rangle \langle b_1, 0, 1, 1, -0.5, -0.3, -0.2\rangle, \langle b_2, 0, 1, 1, -0.5, -0.4, -0.2\rangle, \langle b_3, 0, 1, 1, -0.7, -0.9, -0.1\rangle \Big\}$$

Then (F_A, E) is a bipolar neutrosophic soft set which is the parameterized family of soft subsets of X.

7 Decision making process based on similarity measurements

Since neutrosophic set theory deals with uncertainities, it is useful for decision making problems. Due to lack of parametrization tools in neutrosophic sets alone, we have some difficulties while making decisions. There fore, neutrosophic set along with parameters are more favorable for decision making problems.

In this evaluation criteria, we have two types of membership degrees as positive and negative membership degrees. So we consider positive membership degrees for foreground pixels and negative membership degrees for background pixels. This means, we expect maximum positive truth-membership value and minimum negative truth-membership value for foreground pixels while maximum negative truth-membership value and minimum positive truth-membership value for background pixels.

So we define ideal neutrosophic values for our criteria in the following way.

$$[f_{ij}] = \Big\{ e_j, \Big\langle max(u^+_{F(e_j)}(x_i)), min(v^+_{F(e_j)}(x_i)), min(w^+_{F(e_j)}(x_i)), max(u^-_{F(e_j)}(x_i)), min(v^-_{F(e_j)}(x_i)), min(w^-_{F(e_j)}(x_i)) \Big\rangle : e_j \in E; x_i \in X \Big\}$$

$$[b_{ij}] = \Big\{ e_j, \Big\langle min(u^+_{F(e_j)}(x_i)), max(v^+_{F(e_j)}(x_i)), max(w^+_{F(e_j)}(x_i)), min(u^-_{F(e_j)}(x_i)), max(v^-_{F(e_j)}(x_i)), max(w^-_{F(e_j)}(x_i)) \Big\rangle : e_j \in E; x_i \in X \Big\}$$

So our aim is to select the most relevant foreground and background set of pixels by their brightness, contrast level and sharpness level from the image samples of a particular image. The different types of lena images and their corresponding neutrosophic values are given below.

(a) Blur image (b) Noisy image (c) Low resolution

Figure 1: Different types of Lena images

$\mathcal{B}_1$	Brightness(e_1)	Contrast(e_2)	Sharpness(e_3)
f_1	(0.5,0.4,0.3,-0.2,-0.3,-0.9)	(0.8,0.2,0.4,-0.3,-0.4,-0.8)	(0.4,0.7,0.6,-0.2,-0.3,-0.9)
f_2	(0.2,0.3,0.7,-0.1,-0.4,-0.3)	(0.6,0.3,0.3,-0.6,-0.3,-0.5)	(0.5,0.6,0.3,-0.4,-0.6,-0.8)
b_1	(0.7,0.2,0.4,-0.5,-0.6,-0.9)	(0.5,0.6,0.2,-0.7,-0.3,-0.2)	(0.2,0.1,0.3,-0.7,-0.5,-0.5)
b_1	(0.4,0.6,0.8,-0.7,-0.3,-0.3)	(0.6,0.6,0.8,-0.7,-0.2,-0.2)	(0.3,0.4,0.3,-0.9,-0.1,-0.2)

Table 1:Neutrosophic values of (a) Blur image.

$\mathcal{B}_2$	Brightness(e_1)	Contrast(e_2)	Sharpness(e_3)
f_1	(0.6,0.5,0.4,-0.1,-0.2,-0.8)	(0.7,0.1,0.3,-0.4,-0.5,-0.9)	(0.3,0.6,0.5,-0.3,-0.4,-0.9)
f_2	(0.8,0.3,0.5,-0.4,-0.5,-0.8)	(0.4,0.5,0.1,-0.8,-0.4,-0.3)	(0.4,0.3,0.5,-0.5,-0.3,-0.5)
b_1	(0.5,0,0.2,-0.7,-0.4,-0.7)	(0.3,0.4,0.4,-0.8,-0.4,-0.3)	(0.4,0.3,0.5,-0.5,-0.3,-0.3)
b_2	(0.2,0.4,0.6,-0.9,-0.1,-0.1)	(0.4,0.4,0.8,-0.5,-0.2,-0.2)	(0.2,0.2,0.3,-0.5,-0.1,-0.4)

Table 2:Neutrosophic values of (b) Noisy image.

$\mathcal{B}_3$	Brightness(e_1)	Contrast(e_2)	Sharpness(e_3)
f_1	(0.4,0.5,0.7,-0.9,-0.8,-0.2)	(0.3,0.8,0.7,-0.6,-0.5,-0.1)	(0.7,0.4,0.5,-0.7,-0.6,-0.1)
f_2	(0.2,0.7,0.5,-0.6,-0.5,-0.2)	(0.6,0.5,0.9,-0.2,-0.6,-0.7)	(0.6,0.7,0.5,-0.5,-0.7,-0.5)
b_1	(0.5,0.4,0.8,-0.3,-0.6,-0.3)	(0.7,0.4,0.6,-0.2,-0.6,-0.7)	(0.6,0.7,0.5,-0.5,-0.7,-0.7)
b_2	(0.8,0.6,0.4,-0.1,-0.9,-0.9)	(0.6,0.6,0.2,-0.5,-0.8,-0.8)	(0.8,0.8,0.7,-0.5,-0.9,-0.6)

Table 3:Neutrosophic values of (c) Low resolution image.

Following table shows that the neutrosophic values of absolute foreground and background pixels.

$model - \mathcal{B}$	Brightness(e_1)	Contrast(e_2)	Sharpness(e_3)
f	(1,0,0,0,-1,-1)	(1,0,0,0,-1,-1)	(1,0,0,0,-1,-1)
b	(0,1,1,-1,0,0)	(0,1,1,-1,0,0)	(0,1,1,-1,0,0)

By our criteria, we define ideal neutrosobhic values as follows.

$\mathcal{B}$	Brightness(e_1)	Contrast(e_2)	Sharpness(e_3)
f_1	(0.6,0.4,0.3,-0.1,-0.8,-0.9)	(0.8,0.1,0.3,-0.6,-0.5,-0.9)	(0.7,0.4,0.5,-0.2,-0.6,-0.9)
f_2	(0.8,0.3,0.5,-0.1,-0.5,-0.9)	(0.6,0.5,0.9,-0.2,-0.3,-0.3)	(0.6,0.3,0.3,-0.4,-0.7,-0.8)
b_1	(0.5,0.4,0.8,-0.7,-0.4,-0.3)	(0.3,0.6,0.6,-0.8,-0.3,-0.2)	(0.2,0.7,0.5,-0.7,-0.3,-0.3)
b_2	(0.2,0.6,0.8,-0.9,-0.1,-0.1)	(0.4,0.6,0.8,-0.7,-0.2,-0.2)	(0.2,0.8,0.7,-0.9,-0.1,-0.2)

Now we compute the Hamming distance between our ideal bipolar neutrosophic soft set and the bipolar neutrosophic set of each images to find the similarity.

$$d^H_{BNSS}(\mathcal{B},\mathcal{B}_1) = 1.9$$
$$d^H_{BNSS}(\mathcal{B},\mathcal{B}_2) = 1.7667$$
$$d^H_{BNSS}(\mathcal{B},\mathcal{B}_3) = 3.6$$

Then the similarity values are,

$$\mathcal{S}^H_{BNSS}(\mathcal{B},\mathcal{B}_1) = \frac{1}{1+d^H_{BNSS}(\mathcal{B},\mathcal{B}_1)} = 0.3448$$
$$\mathcal{S}^H_{BNSS}(\mathcal{B},\mathcal{B}_2) = \frac{1}{1+d^H_{BNSS}(\mathcal{B},\mathcal{B}_2)} = 0.3614$$
$$\mathcal{S}^H_{BNSS}(\mathcal{B},\mathcal{B}_3) = \frac{1}{1+d^H_{BNSS}(\mathcal{B},\mathcal{B}_3)} = 0.2174$$

Based on these similarity scores, we choose $\mathcal{B}_2$ as the reliable bipolar neutrosophic soft set. This means among these three types of image samples, second image is more favorable to our criteria.

8 Conclusion and Future work

In this paper, we proposed a different approach on bipolar neutrosophic soft sets and discussed their properties which was initially introduced by Ali et al. Further we defined some distance measures between any two bipolar neutrosophic soft sets to check similarity between them. And also we defined entropy measure to calculate indeterminacy. In section 6, we gave the representation of 2-D image in bipolar neutrosophic domain. Finally, the proposed similarity measurements have been applied to decision making problem in image analysis. Our future work will include more decision making methods based upon different similarity measurements.

References

[1] Ridvan Sahin and Ahmed Kucuk. On similarity and entropy of Neutrosophic soft sets, Journal of Intelligent and Fuzzy Systems, 27(2014), 5, 2417-2430.

[2] Irfan Deli, Mumtaz Ali and Florentin Smarandache. Bipolar Neutosophic sets and Their applications based on multi-criteria decision making problems, Proceedings of Int. Conf. on Advanced Mechatronic systems, 2015, 249-254.

[3] Vakkas Ulucay, Irfan Deli and Mehmet Sahin. Similarity measures of Bipolar neutrosophic sets and their applications to multiple criteria decision making Neural Computing and Applications, 29(2018), 3, 739-748.

[4] Ali Aydogdu. On entropy and similarity measures of Interval valued Neutrosopic sets, Neutrosophic sets and systems, 9(2015), 47-49.

[5] Pinaki Majumdar and Syamal Kumar Samanta. On similarity and entropy of Neutrosophic sets, Journal of Intelligent and Fuzzy Systems, 26(2014), 3, 1245-1252.

[6] Ali Aydogdu. On similarity and entropy of Single valued neutrosophic sets, Gen. Math. Notes, 29(2015), 1, 67-74.

[7] A. A. Salama, Florentin Smarandache and Hewayda ElGhawalby. Neutrosophic approach to Grayscale image domain, Neutrosophic sets and systems, 21(2018), 13-19.

[8] Yanhui, Guo and H.D. Cheng New neutrosophic approach to image segmentation, Pattern Recognition, 42(2009), 5, 587-595.

[9] H. D. Cheng and Yanhui Guo. A new Neutrosophic approach to image thresholding, New Mathematics and Natural Computation, 4(2008), 3, 291-308.

[10] Florentin Smarandache. Neutrosophy: A new branch of philosophy, Multiple valued logic: An international journal, 8(2002), 297-384.

[11] Haibin wang, Florentin Smarandache, Yanqing Zhang and Rajshekar Sunderaraman. Single Valued Neutrosophic sets, Proceedings in Technical serise and applied Mathematics, 2012.

[12] Florentin Smarandache. Neutrosophic set: A generalization of the intituitionistic fuzzy set, International Journal of Pure and Applied Mathematics, 24(2005), 3, 287-297.

[13] P. K. Maji, R. Biswas and A. R. Roy. Soft set theory, Computers and Mathematics with applications, 45(2003), 4-5, 555-562.

[14] Irfan Deli, Selim Eraslan and Naim Cagman. Neutrosophic soft sets and their decision making based on similarity measure, Neural Computing and applications, 29(2018), 1, 187-203.

[15] Mumtaz Ali, Le Hoang Son, Irafan Deli and Nguyen Dang Tien. Bipolar neutrosophic soft sets and applications in decision making, Journal of Intelligent and Fuzzy Systems, 33(2017), 6, 4077-4087.

[16] D. Molodtsov Soft set theory - First results, Computers and Mathematics with applications, 37(1999), 4-5, 19-31.

[17] F. A. Kruse, A. B. Lefkoff, J. W. Boardman, K. B. Heidebrecht, A. T. Shapiro, P. J. Barloon, and A. F. H. Goetz. The spectral image processing system (SIPS) interactive visualization and analysis of imaging spectrometer data, AIP Conference Proceedings, 283(1993), 1, 192-201.

[18] Mohamed Abdel-Basset, Gunasekaran Manogaran, Abduallah Gamal and Florentin Smarandache. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria Design Automation for Embedded Systems, 22(2018), 3, 257-278.

[19] Mohamed Abdel-Basset, Gunasekaran Manogaran, Abduallah Gamal and Florentin Smarandache. A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection Journal of Medical Systems, 43(2019), 2, 19-41.

[20] Mohamed Abdel-Basset, M. Saleh, Abduallah Gamal and Florentin Smarandache An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number Applied Soft Computing, 77(2019), 1, 438-452.

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